Essential Mathematical Formulas

Algebra

Basic Algebraic Identities

$$
\begin{aligned}
(a+b)^2 &= a^2 + 2ab + b^2 \\
(a-b)^2 &= a^2 - 2ab + b^2 \\
a^2 - b^2 &= (a+b)(a-b) \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
(a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\
a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\
a^3 - b^3 &= (a-b)(a^2 + ab + b^2)
\end{aligned}
$$

Quadratic Equations

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
\alpha + \beta &= -\frac{b}{a} \\
\alpha\beta &= \frac{c}{a}
\end{aligned}
$$

Geometry

Triangle

$$
\begin{aligned}
a^2 + b^2 &= c^2 \quad \text{(Pythagorean Theorem)} \\
\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\
\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2} \quad \text{(Heron's Formula)}
\end{aligned}
$$

Circle

$$
\begin{aligned}
C &= 2\pi r \quad \text{(Circumference)} \\
A &= \pi r^2 \quad \text{(Area)}
\end{aligned}
$$

Rectangle

$$
\begin{aligned}
P &= 2(l + w) \quad \text{(Perimeter)} \\
A &= lw \quad \text{(Area)}
\end{aligned}
$$

Parallelogram

$$
A = \text{base} \times \text{height}
$$

Trapezoid

$$A = \frac{1}{2} \times (\text{base}_1 + \text{base}_2) \times \text{height}$$

Trigonometry

Basic Trigonometric Identities

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
1 + \tan^2\theta &= \sec^2\theta \\
1 + \cot^2\theta &= \csc^2\theta
\end{aligned}$$

Angle Sum and Difference Formulas

$$\begin{aligned}
\sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\
\cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\
\tan(a \pm b) &= \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}
\end{aligned}$$

Double Angle Formulas

$$\begin{aligned}
\sin 2\theta &= 2 \sin\theta \cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \\
\tan 2\theta &= \frac{2\tan\theta}{1 - \tan^2\theta}
\end{aligned}$$

Calculus

Derivatives

$$\begin{aligned}
\frac{d}{dx} x^n &= n x^{n-1} \quad \text{(Power Rule)} \\
\frac{d}{dx}[f(x) + g(x)] &= f'(x) + g'(x) \quad \text{(Sum Rule)} \\
\frac{d}{dx}[f(x)g(x)] &= f(x)g'(x) + f'(x)g(x) \quad \text{(Product Rule)} \\
\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] &= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \quad \text{(Quotient Rule)} \\
\frac{d}{dx} f(g(x)) &= f'(g(x))g'(x) \quad \text{(Chain Rule)}
\end{aligned}$$

Integrals

$$\begin{aligned}
\int x^n \, dx &= \frac{x^{n+1}}{n+1} + C \\
\int_a^b f(x) \, dx &= F(b) - F(a) \quad \text{(Fundamental Theorem of Calculus)}
\end{aligned}$$

Probability and Statistics

Basic Probability

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Descriptive Statistics

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{(Mean)}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \quad \text{(Standard Deviation)}$$

Sequences and Series

Arithmetic Sequence

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

Geometric Sequence

$$a_n = a_1 r^{n-1}$$

$$S_n = a_1 \frac{1-r^n}{1-r} \quad \text{(for } r \neq 1\text{)}$$

Additional Useful Formulas

Distance Formula (between two points)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of a Line (slope-intercept form)

$$y = mx + b$$

Advanced Calculus and Analysis

Series and Sequences

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad \text{(Taylor Series)}$$
$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n \quad \text{(Maclaurin Series)}$$
$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad \text{for } |r| < 1 \quad \text{(Geometric Series)}$$
$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \quad \text{(Binomial Theorem)}$$
$$e^{ix} = \cos x + i \sin x \quad \text{(Euler's Formula)}$$

Multivariable Calculus

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right) \quad \text{(Gradient)}$$
$$\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \quad \text{(Divergence)}$$
$$\nabla \times \mathbf{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \quad \text{(Curl)}$$
$$\nabla^2 f = \frac{\partial^2 f}{\partial x_1^2} + \frac{\partial^2 f}{\partial x_2^2} + \cdots + \frac{\partial^2 f}{\partial x_n^2} \quad \text{(Laplacian)}$$

Advanced Integration

$$\int u\,dv = uv - \int v\,du \quad \text{(Integration by Parts)}$$
$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C \quad \text{(Trigonometric Integrals)}$$
$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C \quad \text{(Trigonometric Integrals)}$$
$$\int f(g(x))g'(x)\,dx = \int f(u)\,du \quad \text{(Integration by Substitution)}$$

Differential Equations

First-Order Linear Differential Equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$
$$y = e^{-\int P(x)\,dx} \left(\int Q(x) e^{\int P(x)\,dx}\,dx + C \right)$$

Second-Order Linear Homogeneous Differential Equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$
$$ar^2 + br + c = 0 \quad \text{(Characteristic equation)}$$

Non-Homogeneous Differential Equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$
$$\text{General solution} : y = y_c + y_p$$

Linear Algebra

Matrix Multiplication

$$(AB)_{ij} = \sum_k A_{ik}B_{kj}$$

Determinant of a 2x2 Matrix

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Determinant of a 3x3 Matrix

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Eigenvalues and Eigenvectors

$$\text{For a matrix } A, \text{ if } Av = \lambda v, \text{ then } \lambda \text{ is an eigenvalue and } v \text{ is an eigenvector.}$$
$$\text{Characteristic equation} : \det(A - \lambda I) = 0$$

Cramer's Rule

$$\text{For a system of linear equations } Ax = b :$$
$$x_i = \frac{\det(A_i)}{\det(A)}, \quad \text{where } A_i \text{ is the matrix } A \text{ with the } i\text{-th column replaced by the vector } b.$$

Probability and Statistics

Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Variance

$$\text{Var}(X) = E[(X - \mu)^2] = E[X^2] - (E[X])^2$$

Standard Deviation

$$\sigma = \sqrt{\text{Var}(X)}$$

Covariance

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$$

Correlation Coefficient

$$\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

Complex Analysis

Cauchy-Riemann Equations

$$\text{For } f(z) = u(x, y) + iv(x, y) : \\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Residue Theorem

$$\int_\gamma f(z)\,dz = 2\pi i \sum \text{Res}(f, a_i), \quad \text{where Res}(f, a_i) \text{ is the residue of } f \text{ at the isolated singularity } a_i.$$

Vector Calculus

Green's Theorem

$$\oint_C (L\,dx + M\,dy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dA$$

Stokes' Theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

Divergence Theorem

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{F})\,dV$$