

Nathan Joshua B. Sucgang  
July 11, 2024

1. Suppose that  $X_1, X_2, X_3$  are independent with common probability mass function:  $P\{X_i = 0\} = .2, P\{X_i = 1\} = .3, P\{X_i = 3\} = .5, i = 1, 2, 3$ 
  - (a) Plot the probability mass function of the average of a sample size of 2.  $X_2 = \frac{X_1 + X_2}{2}$
  - (b) Determine  $E[X_2]$  and  $Var(X_2)$
  - (c) Plot the probability mass function of the average of a sample size of 3.  $X_3 = \frac{X_1 + X_2 + X_3}{3}$
  - (d) Determine  $E[X_3]$  and  $Var(X_3)$

2. If 10 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 10 to 60) is between 30 and 40 inclusive.

Solution:

$$E[X_i] = \sum_{x_i=1}^6 \frac{1}{6}x = \frac{7}{2}$$

$$Var(X_i) = \sum_{x_i=1}^6 \frac{1}{6}\left(x - \frac{7}{2}\right)^2 = \frac{35}{12}$$

$$\text{Let } X = \sum_{x_i=1}^6 X_i$$

$$E[X_i] = \sum_{x_i=1}^{10} E(X_i) = 10\left(\frac{7}{2}\right) = 35$$

$$Var(X_i) = \sum_{x_i=1}^{10} Var[X_i] = 10\left(\frac{35}{12}\right) = \frac{350}{12}$$

$$P(30 \leq X \leq 40) = P(29.5 < X < 39.5)$$

$$= P\left(\frac{29.5 - 35}{\sqrt{350/12}} < \frac{X - 35}{\sqrt{350/12}} < \frac{39.5 - 35}{\sqrt{350/12}}\right)$$

$$= P(-1.02 \leq Z \leq 1.02)$$

$$= 2(0.8461) - 1$$

$$= 0.6922$$

3. A highway department has enough salt to handle a total of 80 inches of snowfall. Suppose the daily amount of snow has a mean of 1.5 inches and a standard deviation of .3 inch.
  - (a) Approximate the probability that the salt on hand will suffice for the next 50 days.

Solution:

- (b) What assumption did you make in solving part (a)?

Solution:

It is assumed that  $X_i$  are independent and normal distributed random variables

- (c) Do you think this assumption is justified? Explain briefly.

Solution:

This assumption is not justified because given snowfall in last few days is dependent on the atmospheric conditions, which would also affect today's rainfall.

4. The lifetime (in hours) of a type of electric bulb has expected value 500 and standard deviation 80. Approximate the probability that the sample mean of n such bulbs is greater than 525 when

- (a)  $n = 4$ ;
  - (b)  $n = 16$ ;
  - (c)  $n = 36$ ;
  - (d)  $n = 64$ .
5. Fifty-two percent of the residents of a certain city are in favor of teaching evolution in high school. Find or approximate the probability that at least 50 percent of a random sample of size  $n$  is in favor of teaching evolution, when
- (a)  $n = 10$ ;
  - (b)  $n = 100$ ;
  - (c)  $n = 1000$ ;
  - (d)  $n = 10,000$ .
6. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation  $\sigma = .1$  mg. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.
- (a) Determine a 95 percent confidence interval estimate of the true weight.
  - (b) Determine a 99 percent confidence interval estimate of the true weight.
7. The following are scores on IQ tests of a random sample of 18 students at a large eastern university. 130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142
- (a) Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university
  - (b) Construct a 95 percent lower confidence interval estimate.
  - (c) Construct a 95 percent upper confidence interval estimate.
8. The capacities (in ampere-hours) of 10 batteries were recorded as follows: 140, 136, 150, 144, 148, 152, 138, 141, 143, 151
- (a) Estimate the population variance  $\sigma^2$ .
  - (b) Compute a 99 percent two-sided confidence interval for  $\sigma^2$ .
  - (c) Compute a value  $v$  that enables us to state, with 90 percent confidence, that  $\sigma^2$  is less than  $v$ .
9. Independent random samples are taken from the output of two machines on a production line. The weight of each item is of interest. From the first machine, a sample of size 36 is taken, with sample mean weight of 120 grams and a sample variance of 4. From the second machine, a sample of size 64 is taken, with a sample mean weight of 130 grams and a sample variance of 5. It is assumed that the weights of items from the first machine are normally distributed with mean  $\mu_1$  and variance  $\sigma^2$  and that the weights of items from the second machine are normally distributed with mean  $\mu_2$  and variance  $\sigma^2$  (that is, the variances are assumed to be equal). Find a 99 percent confidence interval for  $\mu_1 - \mu_2$ , the difference in population means.
10. To estimate  $p$ , the proportion of all newborn babies that are male, the gender of 10,000 newborn babies was noted. If 5106 of them were male, determine
- (a) a 90 percent and
  - (b) a 99 percent confidence interval estimate of  $p$ .