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1. Suppose that X_1, X_2, X_3 are independent with common probability mass function: $P\{X_i = 0\} = .2, P\{X_i = 1\} = .3, P\{X_i = 3\} = .5, i = 1, 2, 3$

   (a) Plot the probability mass function of the average of a sample size of 2. $X_2 = \frac{X_1+X_2}{2}$

   (b) Determine $E[X_2]$ and $Var(X_2)$

   (c) Plot the probability mass function of the average of a sample size of 3. $X_3 = \frac{X_1+X_2+X_3}{3}$

   (d) Determine $E[X_3]$ and $Var(X_3)$

2. If 10 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 10 to 60) is between 30 and 40 inclusive.

   Solution:

   $$E[X_i] = \sum_{x_i=1}^{6} \frac{1}{6}x = \frac{7}{2}$$

   $$Var(X_i) = \sum_{x_i=1}^{6} \frac{1}{6}(x - \frac{7}{2})^2 = \frac{35}{12}$$

   Let $X = \sum_{x_i=1}^{6} X_i$

3. A highway department has enough salt to handle a total of 80 inches of snowfall. Suppose the daily amount of snow has a mean of 1.5 inches and a standard deviation of .3 inch.

   (a) Approximate the probability that the salt on hand will suffice for the next 50 days.

   (b) What assumption did you make in solving part (a)?

   (c) Do you think this assumption is justified? Explain briefly.

4. The lifetime (in hours) of a type of electric bulb has expected value 500 and standard deviation 80. Approximate the probability that the sample mean of n such bulbs is greater than 525 when

   (a) n = 4;

   (b) n = 16;

   (c) n = 36;

   (d) n = 64.

5. Fifty-two percent of the residents of a certain city are in favor of teaching evolution in high school. Find or approximate the probability that at least 50 percent of a random sample of size n is in favor of teaching evolution, when

   (a) n = 10;

   (b) n = 100;

   (c) n = 1000;

   (d) n = 10,000.

6. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = .1$ mg. Suppose that the results of five successive weighings of the same object are as follows: $3.142, 3.163, 3.155, 3.150, 3.141$.

   (a) Determine a 95 percent confidence interval estimate of the true weight.

   (b) Determine a 99 percent confidence interval estimate of the true weight.

7. The following are scores on IQ tests of a random sample of 18 students at a large eastern university. $130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142$

   (a) Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university

   (b) Construct a 95 percent lower confidence interval estimate.

   (c) Construct a 95 percent upper confidence interval estimate.

8. The capacities (in ampere-hours) of 10 batteries were recorded as follows: $140, 136, 150, 144, 148, 152, 138, 141, 143, 151$

   (a) Estimate the population variance σ^2.

   (b) Compute a 99 percent two-sided confidence interval for σ^2.

   (c) Compute a value v that enables us to state, with 90 percent confidence, that σ^2 is less than v.

9. Independent random samples are taken from the output of twomachines on a production line. The weight of each item is of interest. From the first machine, a sample of size 36 is taken, with sample mean weight of 120 grams and a sample variance of 4. From the second machine, a sample of size 64 is taken, with a sample mean weight of 130 grams and a sample variance of 5. It is assumed that the weights of items from the first machine are normally distributed with mean μ_1 and variance σ^2 and that the weights of items from the second machine are normally distributed with mean μ_2 and variance σ^2 (that is, the variances are assumed to be equal). Find a 99 percent confidence interval for $\mu_1 - \mu_2$, the difference in population means.

10. To estimate p, the proportion of all newborn babies that are male, the gender of 10,000 newborn babies was noted. If 5106 of them were male, determine

    (a) a 90 percent and

    (b) a 99 percent confidence interval estimate of p.