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1. Suppose that X_1, X_2, X_3 are independent with common probability mass function: $P\{X_i = 0\} = .2, P\{X_i = 1\} = .3, P\{X_i = 3\} = .5, i = 1, 2, 3$
 - (a) Plot the probability mass function of the average of a sample size of 2. $X_2 = \frac{X_1 + X_2}{2}$
 - (b) Determine $E[X_2]$ and $Var(X_2)$
 - (c) Plot the probability mass function of the average of a sample size of 3. $X_3 = \frac{X_1 + X_2 + X_3}{3}$
 - (d) Determine $E[X_3]$ and $Var(X_3)$
2. If 10 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 10 to 60) is between 30 and 40 inclusive.
3. A highway department has enough salt to handle a total of 80 inches of snowfall. Suppose the daily amount of snow has a mean of 1.5 inches and a standard deviation of .3 inch.
 - (a) Approximate the probability that the salt on hand will suffice for the next 50 days.
 - (b) What assumption did you make in solving part (a)?
 - (c) Do you think this assumption is justified? Explain briefly.
4. The lifetime (in hours) of a type of electric bulb has expected value 500 and standard deviation 80. Approximate the probability that the sample mean of n such bulbs is greater than 525 when
 - (a) $n = 4$;
 - (b) $n = 16$;
 - (c) $n = 36$;
 - (d) $n = 64$.
5. Fifty-two percent of the residents of a certain city are in favor of teaching evolution in high school. Find or approximate the probability that at least 50 percent of a random sample of size n is in favor of teaching evolution, when
 - (a) $n = 10$;
 - (b) $n = 100$;
 - (c) $n = 1000$;
 - (d) $n = 10,000$.
6. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = .1$ mg. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.
 - (a) Determine a 95 percent confidence interval estimate of the true weight.
 - (b) Determine a 99 percent confidence interval estimate of the true weight.
7. The following are scores on IQ tests of a random sample of 18 students at a large eastern university. 130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142
 - (a) Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university
 - (b) Construct a 95 percent lower confidence interval estimate.
 - (c) Construct a 95 percent upper confidence interval estimate.
8. The capacities (in ampere-hours) of 10 batteries were recorded as follows: 140, 136, 150, 144, 148, 152, 138, 141, 143, 151

- (a) Estimate the population variance σ^2 .
 - (b) Compute a 99 percent two-sided confidence interval for σ^2 .
 - (c) Compute a value v that enables us to state, with 90 percent confidence, that σ^2 is less than v .
9. Independent random samples are taken from the output of two machines on a production line. The weight of each item is of interest. From the first machine, a sample of size 36 is taken, with sample mean weight of 120 grams and a sample variance of 4. From the second machine, a sample of size 64 is taken, with a sample mean weight of 130 grams and a sample variance of 5. It is assumed that the weights of items from the first machine are normally distributed with mean μ_1 and variance σ^2 and that the weights of items from the second machine are normally distributed with mean μ_2 and variance σ^2 (that is, the variances are assumed to be equal). Find a 99 percent confidence interval for $\mu_1 - \mu_2$, the difference in population means.
10. To estimate p , the proportion of all newborn babies that are male, the gender of 10,000 newborn babies was noted. If 5106 of them were male, determine
- (a) a 90 percent and
 - (b) a 99 percent confidence interval estimate of p .