SCHEMAS

Definition: s = s_1…s_i with s_i = 0,1 or *

- Schema s resent a subset of D
- Ex: o(s) = number of defined bit and s=01*1* => s contains 2^(l-o(s)) chromosomes

s	0	1	*	1	*

x∈ s	0	1	0	1	0
			1		1

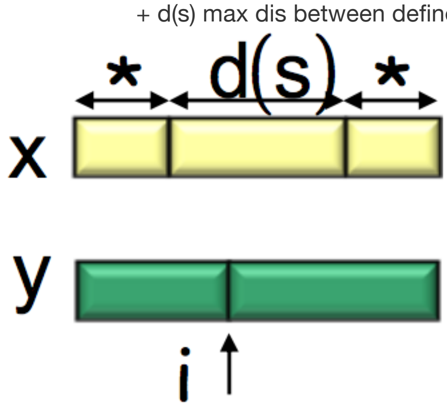
- A chromosomes belong to 2^i schema

SCHEMA THEOREM

- provide estimate of number of schemas during evolution
- Def : Population **P**, size **n**, at time **t**, schema **s**. m(s,t) = number of chromosomes of s in P at time t
- Analysis schema change after GA
- **Selection**: u^(s,t) average value f(x) over s. So good schema => ratio is high ~1

$$= \left[\frac{\hat{u}(s,t)}{\bar{f}(t)}\right] m(s,t)$$

- **Crossover:** prob p_c and (1-p_c) just copied
  + d(s) max dis between defined binary (between 2 *)



+ x is schema. If i outside d(s) = **schema reserve** (at least 1) else possible both offspring are not in s and **schema is destroyed**

+ probability that chromosomes does not produce a chromosomes of s **at most p_c*d(s)/(l-1) (l is length of representation of s)**

+ remain chromosomes after crossover

$$E(m_2(s,t)) \geq \left(1 - p_c \frac{d(s)}{l-1}\right) m_1(s,t$$

- **Mutation:**
- o(s) = number of define bit. p_m mutation prob
- x mutated belongs to s => survival prob s_m(s) = (1-p_m)^o(s)

$$E(m(s,t+1)) \geq m_2(s,t) \,.\, [1 - p_m]^{o(s)}$$

FINALLLY,

$$\underbrace{E(m(s,t+1))}_{t+1} \geq \underbrace{m(s,t)}_{t} \cdot \frac{\hat{u}(s,t)}{\bar{f}(t)} \cdot \underbrace{\left[1 - p_c \frac{d(s)}{l-1}\right]}_{\leq 1} \cdot \underbrace{[1 - p_m]^{o(s)}}_{\leq 1}$$

- Evolution will i**ncrease** the size of **good schema.** (average f(x) of s >>> average f(x) of Population)

The N^3 Argument

Theorem: Under reasonable assumptions, random population of size N **sample N^3 schemas** (100 chromo => 10^6 schemas)

Demonstration:

- let s be a schema of k-order (k defined bits), 2^l chromosomes, 2^(l-k) belong to s => random N chromosomes, N.2^-k belongs to s
- k is **highest order** of schema which is represented in population of size N, schema of k-order has **least /theta copies (number of chromosomes /theta <= combination(L,k))**. => N/{\theta} = 2^k (number of ways to assign value to k defined bits)
- now need to show that number of schema of order k > N^3 (hypothesis) <=> 2^k combination(L,k) >= (2^k \theta)^3