1. Solve the following recurrence relation.

a) $x(n) = x(n-1)+5$ for $n>1$ with $x(1)=0$

1. Write down the first two terms to identify the pattern.

$x(1)=0$
$x(2)=x(1)+5=5$
$x(3)=x(2)+5=10$
$x(4)=x(3)+5=15$

2. Identity the pattern (or) the general term
→ The first term $x(1)=0$
The common difference $d=5$
The general formula for the n^{th} term of an AP is
$x(n)=x(1)+(n-1)d$
Substuting the given values
$x(n) = 0+(n-1)\,5=5(n-1)$
The solution is
$x(n)=5(n-1)$

b) $x(n)=3x(n-1).\ n>1$ with $x(1)=4$
1. write down the first two terms to identify the pattern
$x(1)=4$
$x(2)=3x(1)=3\cdot4=12$
$x(3)=3x(2)=36$
$x(4)=3x(3)=108$

2. Identity the general term.
→ The first term $x(1)=4$
→ The common ratio $r=3$

The general formula for the n^{th} term of a GP is
$x(n)=x(1)\cdot r^{n-1}$

Substuting the given values

$x(n) = 4 \cdot 3^{n-1}$

The solution is $x(n) = 4 \cdot 3^{n-1}$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2k$)

for $n = 2^k$, we can write recurrance in terms of k.

1) substitute $n = 2^k$ in the recurrance.

$x(2^k) = x(2^{k-1}) + 2^k$

2) write down the first few terms to identity the pattern

$x(1) = 1$

$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$

$x(4) = x(2^2) = x(4) + 8 = 7 + 8 = 15$

$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 7$

3) Identity the general term by finding the pattern

we observe that:

$x(2^k) = x(2^{k-1}) + 2^k$

we sum the series:

$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$

since $x(1) = 1!$

$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$

The geometric series with the term $a = 2$ and the last term 2^k except for the additional $+1$ term.

The sum of a geometric series s. with ratio $r = 2$

is given by

$$S = a \frac{r^n - 1}{r - 1}$$

Here $a = 2$, $r = 2$ and $n = k$:

$$S = 2 \frac{2^k - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1}$$

Adding the +1 term

$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$

$x(2^k) = 2^{k+1} = 2^{k+1} - 1$

d. $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve for $n = 3^k$)

for $n = 3^k$, we can write the recurrence in terms of k

* substitute $n = 3^k$ in the recurrence

$$x(3^k) = x(3^{k-1}) + 1$$

* write down the first few terms to identify the Pattern

$$x(1) = 1$$
$$x(3) = x(3^1) = x(1) + 1 = 1 + 1 = 2$$
$$x(9) = x(3^2) = (3) + 1 = 2 + 1 = 3$$
$$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4$$

* Identify the general term:

we observe that:

$$x(3^k) = x(3^{k-1}) + 1$$

summing up the series

$$x(3^k) = 1 + 1 + 1 + \cdots + 1$$

$x(3^k) = k + 1$. the solution is $x(3^k) = k + 1$

2. Evaluate the following recurrences complexity

(i) $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$

The recurrence relation can be solved using Iteration method.

i) substitute $n = 2^k$ in the recurrance

ii) iteration the recurrance

for $k = 0$: $T(2^0) = T(1) = T(1)$

$k = 1$: $T(2^1) = T(1) + 1$

$k = 2$: $T(2^2) = T(8) = T(n) + 1 = (T(1) + 1) + 1 = T(1) + 2$

iii) Generalize the Pattern

$$T(2^k) = T(1) + k$$

since $n = 2^k$, $k = \log_2 n$

* Assume $T(1)$ is a constant C.

$T(n) = C + \log_2 n$

The solution is

$T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + T(n/3) + (n \cdot$ where c is constant and n is input size)

The recurrance can be solved using the master's theorem. for divide and conquer recurrance of the form.

$T(n) = aT(n/b) + f(n)$

where $a=2$, $b=3$ and $f(n) = cn$.

lets determine the value of $\log_b^a$:

$\log_b a = \log_3 2$

using the propeoties of logarithms

$\log_3 2 = \dfrac{\log 2}{\log 3}$

Now we compare $f(n) = cn$ with $n^{\log_3 2}$.

$f(n) = O(n)$

$n = n^1$

since $\log_3 2$ we are in the third case of the master's theorem.

$f(n) = O(n^c)$ with $c > \log_b a$

The solution is:

$T(n) = O(f(n)) = O(cn) = O(n)$

3. consider the following reccurance algorithm?

min [A[0 ... (n-2)]

if $n=1$ return A[0]

else temp = min(1A[0 ... n-2]]

if temp <= A(n-1) return temp

else

return A[n-1]

what does this algorithm compute?

The given algorithm min(A(0...(n-1)) computes the minimum value in the array 'A'. form index '0' for '(n-1)'. it does it by reccurance finding the minimum value in the sub array A(0...(n-2)) and then compu-ting it. with the last element A[n-1] to determine the maximum value.

b) set up a reccurance relation for the algorithm basic operation count and solve it.

To determine the recurrance relation for the algorithm basic operation count, let's analize the step involved in the algorithm. the basic operation are the comparisons and function calls.

Recurrence relation setup.

1) Base case when n=1, the algorithm performs a single operation to return A[v].

2) Recursive case. when n>1, the algorithm makes a recursive call to min (A[0...(n-2)]; perform a comparison b/w temp and A(n-1)

let t(n) represent the no. of basic operation the algorithm. performs for an array of size n.

* Base case

$T(1) = 1$

* Recursive case

$T(n) = T(n-1)+1$

Here T(n-1) accounts for the operations performed by the recursive call to min (A [0...(n-2)]: and the +1 accounts for the comparison b/w temp and [n-1]

To solve this recurrence relation we can we iteration method.

$T(n) = T(n-1) + 1$

$= T(n-3)+1+1)+1 \ldots$

$1+(n-1)$

$= n$

The solution is:

$T(n) = n$

This means the algorithm performs n basic operations for an input array of size n.

4. Analyze the order of growth.

(i) $f(n) = 2n^2 + 5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation.

To analyze the order of growth and use the Ω notation, we need to compare the given function $f(n)$ and $g(n)$.

given functions:

$f(n) = 2n^2 + 5$

$g(n) = 7n$

Order of growth using $\Omega(g(n))$ notation: The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large n, P(n) grows at least as fas as g(n)

$f(n) \geq c \cdot g(n)$

Lets analyze $f(n) = 2n^2 + 5$ with respect to $s(n) = 7n$.

* Identity donimant terms.

→ The donimant terms in $f(n)$ is $2n^2$ since it grows faster then the constant terms as n increase.

* establish the inequality.

→ we want to find constant c and no such that:

$2n^2 + 5 \geq c = 7n$ for all $n \geq n_0$

* simpity the inequality:

* ignore the lower order term 5 for larger

$2n^2 \geq 7cn$

* Divide both sides by n.

$2n \geq 7c$

* solve for n

$n \geq 7/2$

* choose constants

let $c = 1$

$n \geq \frac{7 \cdot 1}{2} = 3.5$

$\therefore$ for $n \geq n$, the inequality holds:

$2n^2 + 5 \geq 7n$. for all $n \geq n$.

we have shown that there will $c = 1$ and $n_0 = n$ such that for all $n \geq n_0$.

$2n^2 + 5 \geq 7n$

Thus, we can conclude that:

$f(n) = 2n^2 + 5 = \Omega(7n)$

in Ω notation the donimant term $2n^2$ in $f(n)$ clearly grows faster than $7n$. hence.

$f(n) = \Omega(n^2)$

however, for the specific comparison asked $f(n) = \Omega(7n)$ is also correct.

showing that $f(n)$ grows at least as fast as $7n$.