1. Big omega notation: prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

$g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants c and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n^2} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \cong 1$

example $c = 1/2$

$1 + 2/n + 4/n^2 \geq \frac{1}{2}$ $(1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1$ $(n \geq 1, n_0 = 1)$

$1 + 2/n + 4/n^2 \geq 1/2$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

2. Big theta notation. determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

$c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$

$h(n) = 4n^2 + 3n$

$h(n) \leq c_2 n^2$

$4n^2 + 3n \leq c_2 n^2 \Rightarrow 4n^2 + 3n \leq cn^2$

lets $c_2 = 5$

divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $O(n^2)$

$$1 + \frac{n}{n\log n} \le 2$$

(simplify)

$$1 + \frac{1}{\log n} \le c_2 \qquad c_2 = 2$$

$$1 + \frac{1}{\log n} \le 2 \qquad (c_2 = 2, n_0 = 2)$$

Then $h(n)$ is $O(n\log n)$

Lower bound.

$h(n) \ge c_1(n\log n)$

$h(n) = n\log n + n$

$n\log n + n \ge c_1 n\log n$

divide bothsides by $n\log n$

$$1 + \frac{n}{n\log n} \ge c_1$$

(simplify)

$$1 + \frac{1}{\log n} \ge c_1 \qquad c_1 = 1$$

$$\frac{1}{\log n} \ge 0 \text{ for all } n \ge 1$$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

3. Solve the following recurrence relations & find the order of growth of solutions $T(n) = 4T(n/2) + n_2 \,/\, T(1) = 1$.

Let $f(n) = n_3 - 2n_2 + n$ and $g(n) - n_2$ show whether $f(n) = \Omega(g(n))$ is true or false.

$f(n) \ge c_1 g(n)$

substituting $f(n)$ & $g(n)$ into this inequality we get,

$n^3 - 2n^2 + n \ge c(-n_2)$

A and c and n_0 holds $n \ge n_0$

$n^3 - 2n^2 + n \ge -cn_2$

$n^3 - 2n^2 + n + cn^2 \ge 0$

$n^3 + (c-2)n^2 + n \ge 0 \quad (n^3 \ge 0)$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \ge 0$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n_2)$

$\therefore$ The statement $f(n) = \Omega(g(n))$ is true,

4. Determine whether $h(n) = n\log n + n$ is $\Theta(\log n)$ prove a vigorous proof for you conclusion.

$c_1 n\log n \le h \le c_2 n\log n$

upper bound

$h(n) \le c_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \le c_2 n\log n$

divide both sides by $n\log n$.

$$1 + \frac{n}{n\log n} \le 2$$

$$1 + \frac{1}{\log n} \le c_2$$

$$1 + \frac{1}{\log n} \le 2$$

Then $h(n)$ is $O(n\log n)$

lower bound

$h(n) \ge c_1 n\log n$

$h(n) = n\log n + n$

$n\log n + n \ge c_1 n\log n$

divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \ge c_1$$

$$1 + \frac{1}{\log n} \ge c_1$$

$$\frac{1}{\log n} \ge 0$$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

5. solve the following reccurance relations and find the order of growth of solutions.

$T(n) = 4T(n/2) + n2 \; T(1) = 1$

Sol: $T(n) = 4T(n/2) + n^2, \; T(1) = 1$

$T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4, b = 2, f(n) = n^2$

applying master theorem,

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - \epsilon})$

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, Then $T(n) = f(n)$

$T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = aT(n/b) + f(n)$.

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, then $T(n) = f(n)$

calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$ (comparing $f(n)$ with $n^{\log_b a}$):

$f(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

Order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$

is $\Theta(n^2 \log n)$