

1. Given an array of $[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8]$ integers find the maximum and minimum product that can be obtained by multiplying two integers from the array.

Sol: array is $[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -4]$
we need to consider the largest and smallest products that can be formed by selecting two numbers from the array

1. Sort the array.

Sorted array $[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

2. Identify possible candidates for maximum product

3. Identify possible candidates for minimum product

calculating maximum product

* The two largest positive numbers are 10 and 11 $= 10 \times 11 = 110$

* The two smallest negative numbers are -9 and -8 $= -9 \times -8 = 72$

The maximum product is 110

calculating minimum products

The largest positive and negative numbers is 11 and -9

$$11 \times -9 = -99$$

The smallest negative numbers are $-9 \times -8 = 72$, -99 is smaller than 72 so maximum product = 110, and minimum product = -99

2. Demonstrate the Primary search method to search for the key = 23 from the array $= \{2, 5, 8, 12, 16, 23, 38, 56, 72, 91\}$

Sol: Given key = 23 & array $= \{2, 5, 8, 12, 16, 23, 38, 56, 72, 91\}$

* Initialise pointers

$$\text{low} = 0 \text{ and } \text{high} = 9$$

$$\text{calculate mid} = \left(\frac{\text{low} + \text{high}}{2} \right) = \left(\frac{0 + 9}{2} \right) = 4$$

compare $\text{arr}(\text{mid})$ with key:

$$\text{arr}(4) = 16$$

since $16 < 23$ update $\text{low} = \text{mid} + 1 = 5$

$$\text{calculate mid} = \left(\frac{\text{low} + \text{high}}{2} \right) = \left(\frac{5 + 9}{2} \right) = 7$$

compare $\text{arr}(\text{mid})$ with key:

$$\text{arr}(7) = 56$$

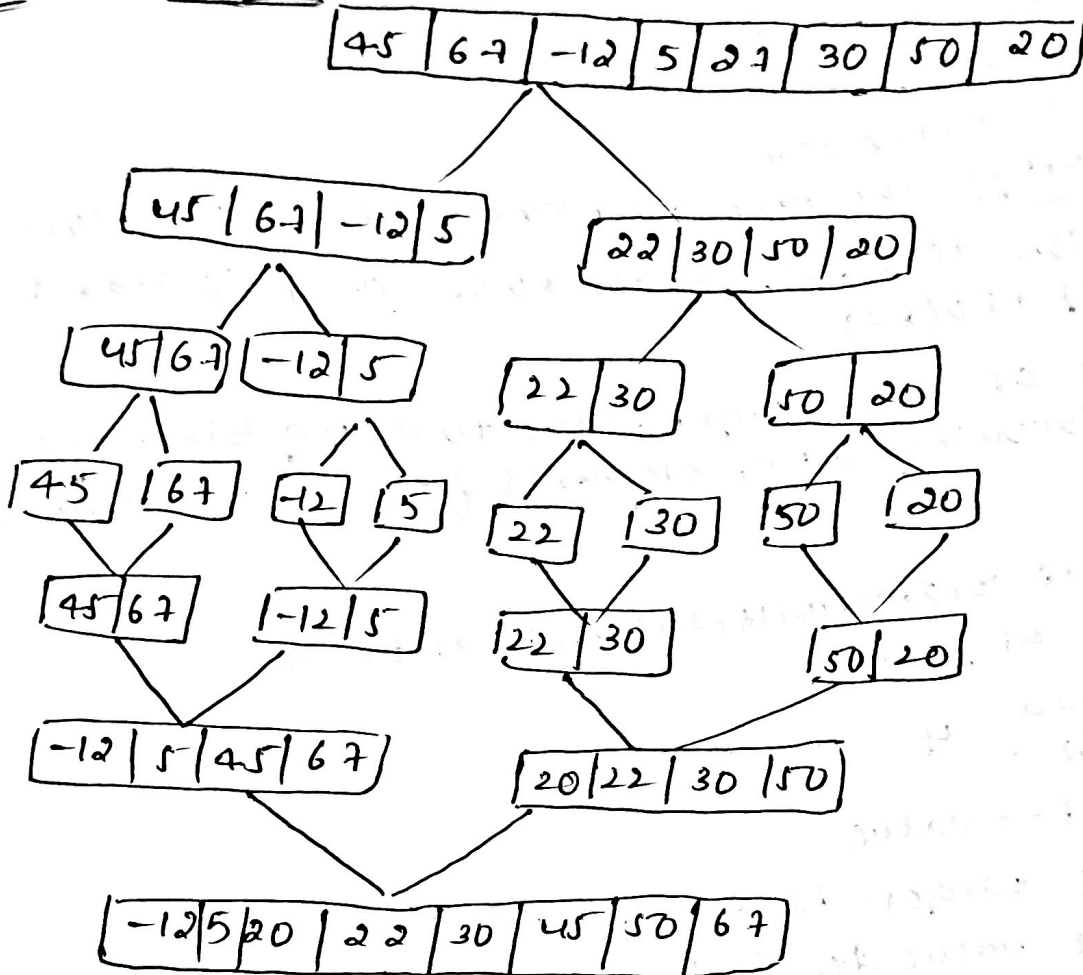
since $56 > 23$ update $\text{high} = \text{mid} - 1 = 6$

$$\text{mid} = \left(\frac{5 + 6}{2} \right) = 5 \Rightarrow \text{arr}[\text{mid}] = \text{arr}(5) = 23; 23 = 23.$$

The key is found at index 5. \therefore The key = 23 is found at index 5.

- 3) Apply merge sort and other list of 8 elements (Data: (45, 67, -12, 5, 23, 30, 50, 20)). Set up a recurrence relation for the no. of key comparisons made by merge sort.

Sol: Merge sort:



\therefore The sorted list = (-12, 5, 20, 23, 30, 45, 50, 67)

- 4) Find recurrence relation for comparisons

$$T(n) = 2 + (n/2) + O(n)$$

if $n=1$, $T(1) = 0$, base

\rightarrow At each level of recursion we make at most $n-1$ comparisons to merge two halves of size $n/2$ so it becomes

$$T(n) = 2 + (n/2) + (n-1)$$

Solving recurrence relation we get

$$T(n) = n \log_2(n) - (n+1)$$

$$\therefore T(n) = O(n \log n)$$

\therefore The recurrence relation is $T(n) = 2T(n/2) + O(n)$ or more precisely $T(n) = n \log_2(n) - n + 1$

4. Find the no. of times to perform swapping for solution sort also estimate the time complexity for others of notations sets (12, 7, 15, -2, 18, 6, 13, 4)

Sol: The selection sort algorithm always makes exactly $n-1$ swaps in the worst case, where n is the no. of elements in the list.

$S = \{12, 7, 15, -2, 18, 6, 13, 4\}$

No. of elements, $n = 8$

No. of swaps $= n-1 = 8-1 = 7$

Time complexity: The time complexity of selection sort in Big O notation is $O(n^2)$. So, the no. of swap is 7 and time complexity is $O(n^2)$

5. Find the index of the target value to using binary search team the following list of elements {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

given: list {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} and $target = 10$

low = 0, and high = 9

$$mid = \frac{0+9}{2} = \frac{0+9}{2} = 4$$

mid = 10, mid == value

since == 10 the target is found at index 4

∴ The target value 10 is found at index 4.