Introduction to Chain/Belt drives

Transmission of power b/n shafts

- Gears
- Belts } Shafts separated by considerable distance
- Chains

Belts → Flat belts, V belts, Toothed (or timing) belts

- Relatively quiet except for timing belts (figure)
- Slippage leads to inexact velocity ratios ∴ do not use in watches
- Slippage is sometimes used as an advantage to disengage the drive. Pulleys are moved close to eachother instead of using a clutch.
- Also provides damping in belts which reduces the chance for shock and vibration

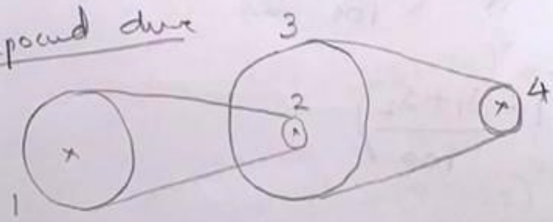
Velocity ratio of a belt drive

Length of belt that passes over driver in a second = length of belt that passes over the driven

$$d_1 N_1 = d_2 N_2 \Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

with the thickness of belt considered, $\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$

Compound drive



$V_1 = V_2$

$d_1 N_1 = d_2 N_2 \Rightarrow d_1 N_1 / d_2 = N_2$

also $d_3 N_3 = d_4 N_4$.

$N_3 = \frac{d_4 N_4}{d_3}$

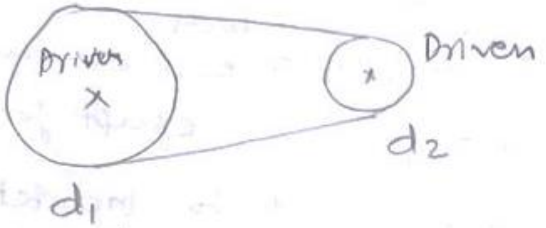
$\frac{d_1 N_1}{d_2} = \frac{d_4 N_4}{d_3} \Rightarrow \frac{N_4}{N_1} = \frac{d_1 \cdot d_3}{d_2 \, d_4}$

⇒

$$\therefore \frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

$$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

Belt slippage



$S_1 \rightarrow$ Slip of belt over driver

$N_1 \rightarrow$ ~~rpm~~/speed of driver in rpm

$$\therefore V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \cdot \frac{S_1}{100}$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

let $S_2 \rightarrow$ Slip of ~~belt~~ belt over driven

$$\frac{\pi d_2 N_2}{60} = V - \left(\frac{V \cdot S_2}{100}\right) = V\left(1 - \frac{S_2}{100}\right)$$

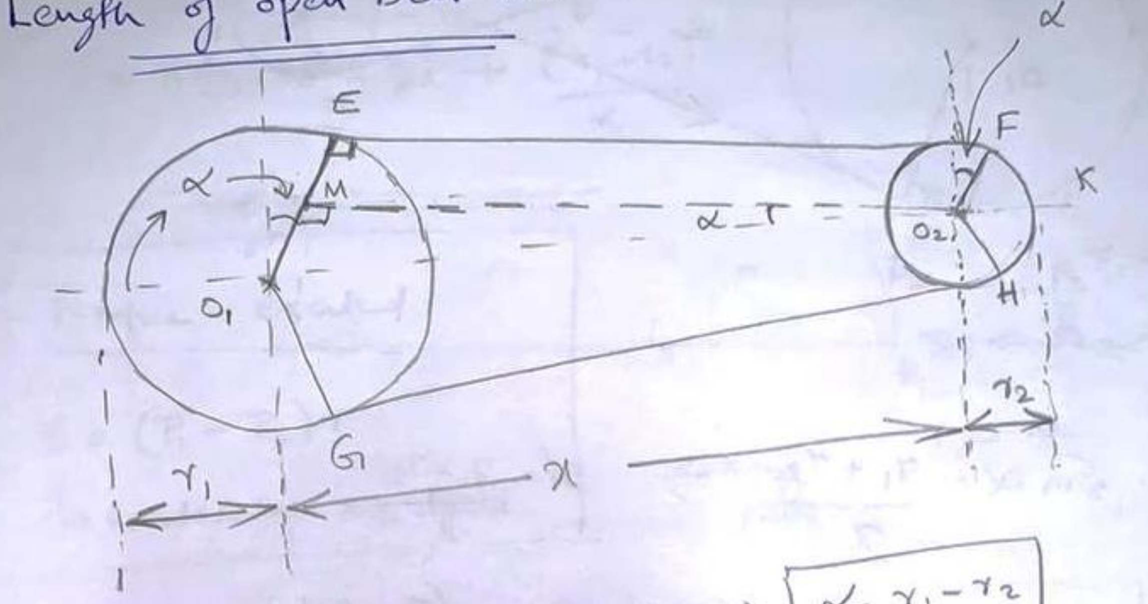
$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \left(\frac{S_1 + S_2}{100}\right)\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \left(\frac{S_1 + S_2}{100}\right)\right) \quad \text{if 't' is considered}$$

Length of open belt drive



$\Delta O_1 O_2 M$, $\quad \sin\alpha = \dfrac{r_1 - r_2}{x} \Rightarrow \boxed{\alpha = \dfrac{r_1 - r_2}{x}}$

Also $\quad MO_2 = \sqrt{x^2 - (r_1 - r_2)^2}$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2} = x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$$

$$= x - \frac{x}{2}\left(\frac{r_1 - r_2}{x}\right)^2$$

$$= x - \frac{(r_1 - r_2)^2}{2x} = EF = GH$$

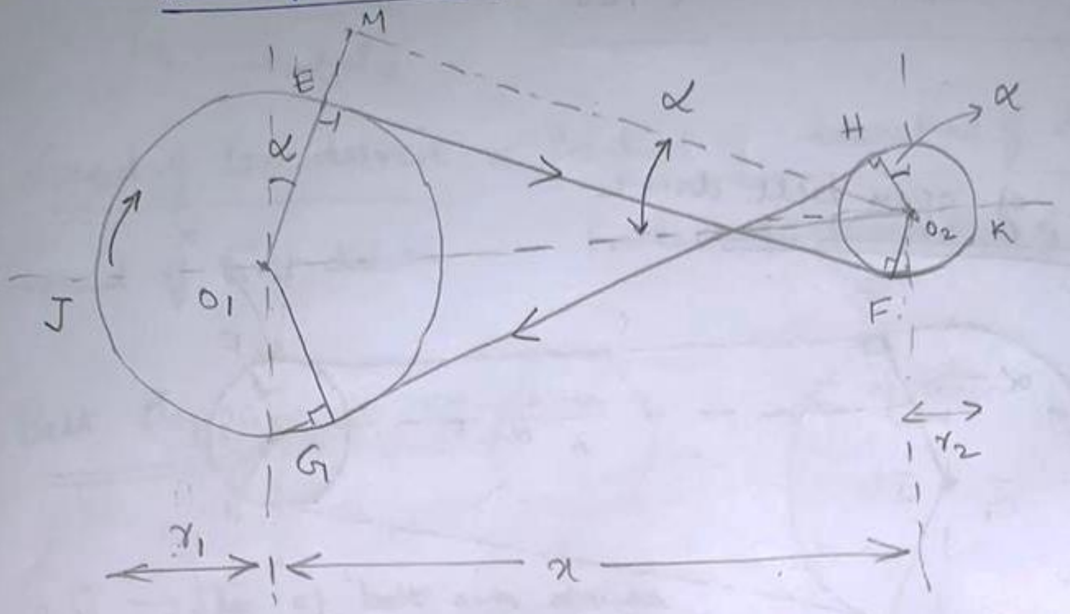
$\therefore$ Length of belt $= r_1(\pi + 2\alpha) + r_2(\pi - 2\alpha) + EF + GH$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2\frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$$

Length of cross belt



$$\sin\alpha = \frac{r_1 + r_2}{x} \Rightarrow \alpha = \frac{r_1 + r_2}{x}$$

$\therefore$ Length of belt $= (\pi + 2\alpha) r_1 + (\pi + 2\alpha) r_2 + 2 \cdot EF$

$$EF = \sqrt{x^2 - (r_1 + r_2)^2} = x\sqrt{1 + \left(\frac{r_1+r_2}{x}\right)^2} = x\left(1 - \left(\frac{r_1+r_2}{x}\right)^2\right)^{1/2}$$

$$= x\left(1 - \frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2\right)$$

$$= x - \frac{(r_1+r_2)^2}{2x}$$

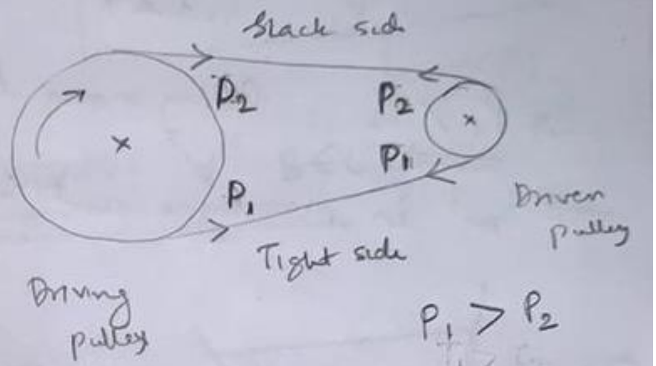
$\therefore$ Length of belt $= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1+r_2)^2}{x}$

But $\alpha = \frac{r_1 + r_2}{x}$

Length

$$= \pi(r_1+r_2) + \frac{2(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$



$P_1 > P_2$

Torque exerted

$$\tau = (P_1 - P_2)\, r$$

↳ exerted by the driver.

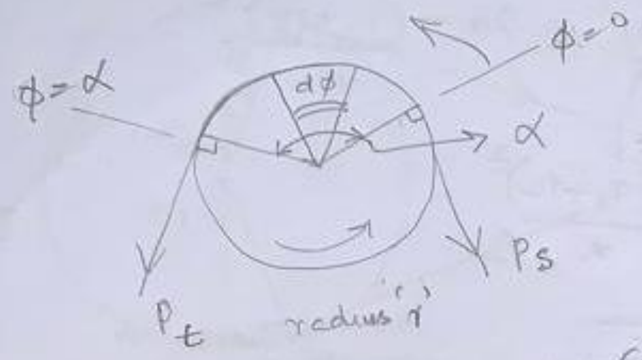
~~Power transmitted~~

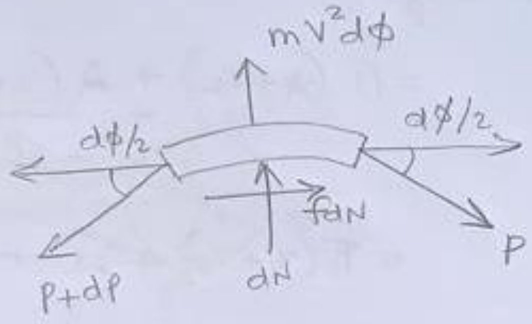
$$\cancel{Q = (P_1 - P_2)\, v}$$

Power transmitted

$$Q = (P_1 - P_2)\, v$$

ANALYSIS OF BELT TENSIONS : FLAT BELTS





→ $\underline{\underline{P_t > P_s}}$ Pulley moves ACW (This pulley is the driven pulley)

→ 'm' is mass per unit length

$P_t \rightarrow$ TIGHT

$P_s \rightarrow$ SLACK

Mass of element $= m r \, d\phi$

Centrifugal force $= (m r \, d\phi)\dfrac{v^2}{r} = m v^2 d\phi$

→ Considering the horizontal equilibrium,

$$(P+dP)\cos\frac{d\phi}{2} = f\,dN + P\cos d\phi/2$$

$$dP\cos\frac{d\phi}{2} = f\,dN \Rightarrow dP = f\,dN \text{ (or) } \boxed{dN = dP/f} \rightarrow (1)$$

→ Consider vertical equilibrium,

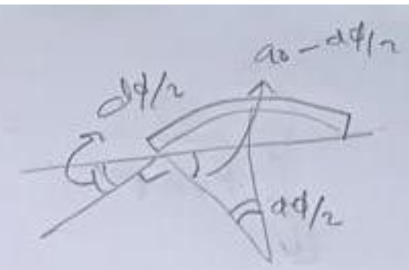
$$mv^2 d\phi + dN = (P+dP)\sin\frac{d\phi}{2} + P\sin\frac{d\phi}{2}$$

$$mv^2 d\phi + dN = P\,d\phi$$

$$\boxed{dN = (P - mv^2)\,d\phi} \longrightarrow (2)$$

Using (1), $\dfrac{dP}{f} = (P - mv^2)\,d\phi \Rightarrow \dfrac{dP}{P - mv^2} = f\,d\phi$

$$\int_{P_s}^{P_t} \frac{dP}{(P-mv^2)} = f\int_0^{\alpha} d\phi$$

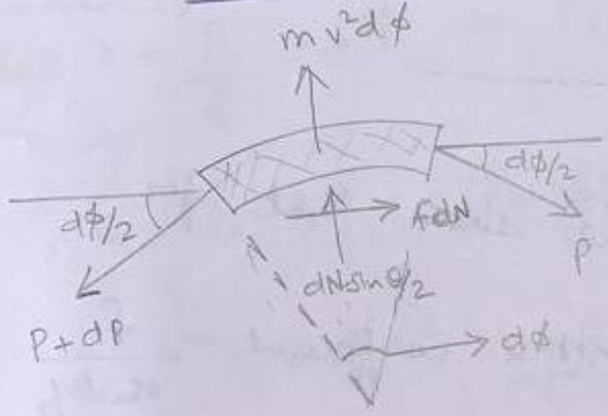


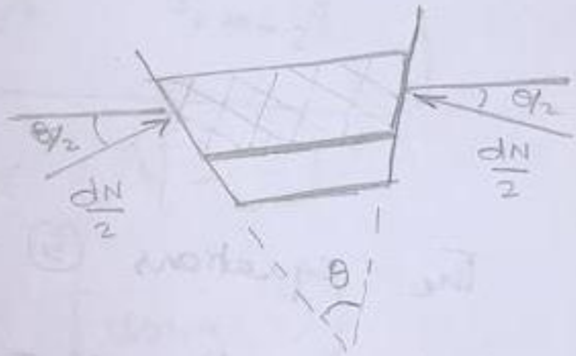
$$\left[\ln (P-mv^2)\right]_{P_s}^{P_t} = f\alpha$$

$$\ln (P_t - mv^2) - \ln (P_s - mv^2) = f\alpha$$

$$\ln \frac{(P_t - mv^2)}{(P_s - mv^2)} = f\alpha \Rightarrow \boxed{\frac{P_t - mv^2}{P_s - mv^2} = e^{f\alpha}} \to (3)$$

ANALYSIS OF BELT TENSIONS: 'V' BELTS

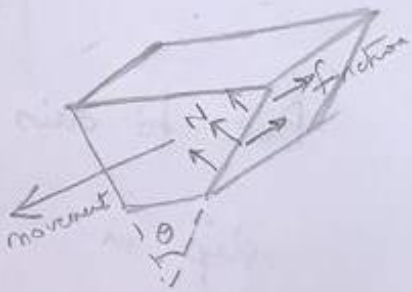




→ Horizontal equilibrium,

$$(P+dP)\cos\frac{d\phi}{2} = f\,dN + P\cos\frac{d\phi}{2}$$

$$dP = f\,dN \Rightarrow \boxed{dN = dP/f} \to (4)$$



→ Vertical eqbm

$$mv^2 d\phi + dN \sin\frac{\theta}{2} = (P+dP)\sin\frac{d\phi}{2} + P\sin\frac{d\phi}{2}$$

$$mv^2 d\phi + dN \sin\frac{\theta}{2} = P\,d\phi$$

$$mv^2 d\phi + \frac{dP}{f}\sin\frac{\theta}{2} = P\,d\phi \Rightarrow (P - mv^2)\,d\phi = \frac{dP}{f}\sin\frac{\theta}{2}$$

$$\frac{dP}{P - mv^2} = \frac{f\,d\phi}{\sin\theta/2}$$

$$\int_{P_s}^{P_t} \frac{dP}{P - mv^2} = \frac{f}{\sin\frac{\theta}{2}} \int_0^{\alpha} d\phi$$

$$\left[\ln\left(P - mv^2\right)\right]_{P_s}^{P_t} = \frac{f}{\sin\theta/2}(\alpha)$$

$$\ln\left(\frac{P_t - mv^2}{P_s - mv^2}\right) = \frac{f\alpha}{\sin\theta/2}$$

$$\boxed{\frac{P_t - mv^2}{P_s - mv^2} = e^{\left(\frac{f}{\sin\theta/2}\right)\alpha}} \longrightarrow 5$$

- The equations 5 and 3 show that for 'V' belts, the effective friction coefficient $= \frac{f}{\sin\theta/2}$

- Due to this increased frictional force, the slip in V-belt is much less compared to flat belts.

- The effect of belt speed is to reduce the net tension in the belt. Higher the belt speed, lower the effective tension in the belt.

Condition for maximum power to be transmitted

Power transmitted,

$$Q = (P_t - P_s)\,v$$

Let $\underline{(P_t - mv^2) = P_h} \longrightarrow (6)$ and $\underline{(P_s - mv^2) = P_l} \longrightarrow (7)$

Therefore, $Q = (P_h - P_l)\,v$.

But from (3), $k = \frac{P_h}{P_l} = e^{f\alpha} \Rightarrow P_l = P_h/k$.

$$\therefore\ Q = (P_h - P_l)\,v = (P_h - P_h/k)\,v = P_h\left[\frac{k-1}{k}\right] v$$

$$Q = (P_t - mv^2)\left[\frac{k-1}{k}\right] v \quad [\text{Using } (6)]$$

To find belt velocity at which transmitted power is maximum,

$$\frac{dQ}{dv} = \frac{(k-1)}{k}\left[(P_t - mv^2) + v(-2mv)\right]$$

$$\frac{dQ}{dv} = \frac{k-1}{k}\left[P_t - 3mv^2\right]$$

for maximum power to be transmitted,

$$\frac{dQ}{dv} = 0 \Rightarrow \boxed{v_{opt} = \sqrt{\frac{P_t}{3m}}}$$