Machine Design II -ME4001D

Monsoon Sem-Aug to Dec-2023

Ilango M

August 14, 2023

Table of Contents

Course syllabus

Module-1: Design of clutches, brakes, belts and chain drives: 13

Friction clutches and brakes; uniform pressure and uniform wear assumptions; design of disc and cone types of clutches and brakes; design of external contracting and internal expanding elements; band type clutches and brakes; belt and chain drives of common types; design of flat and V-belt drives; selection of roller chains.

Module-2: Design of gears and Journal bearings: 14

spur, helical, bevel and worm gears; tooth loads; gear materials; design stresses; basic tooth stresses; stress concentration; service factor; velocity factor; bending strength of gear teeth; Buckingham's equation for dynamic load; surface strength and durability; heat dissipation; design for strength and wear.
Lubrication and journal bearing design: types of lubrication and lubricants; viscosity; journal bearing with perfect lubrication; hydrodynamic theory of lubrication; design considerations; heat balance; journal bearing design.

Module-3: Rolling contact bearings and Design for manufacturing: 12

Bearing types, bearing life, static and dynamic capacity, selection of bearings with axial and radial loads, selection of tapered roller bearings, lubrication, seals, shaft, housing and mounting details.
General design recommendations for rolled sections, forgings, screw machine products, turned parts, machined round holes, parts produced on milling machine, welded parts and castings; modification of design for manufacturing easiness for typical products.

Marking scheme

- Assignments & tutorials - 20
- Mid semester exam- 30
- End semester exam- 50

What is a clutch?

- Mechanical device used in vehicles with manual transmissions to engage and disengage power transmission from the engine to the transmission.
- It allows the driver to control the power transfer from the engine to the wheels, enabling them to change gears and maintain control over the vehicle's speed.

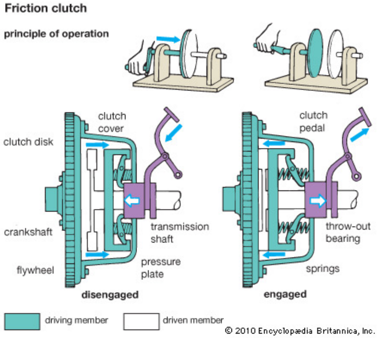


Figure: Image source: Encyclopedia Britanica

Clutch
- Positive contact
- Friction
- Electromagnetic
- Fluid coupling

Why a clutch is needed?

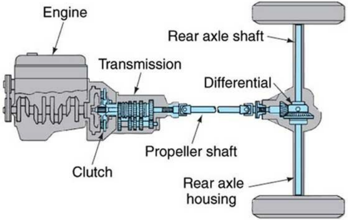


Figure: Image source: Google images

- **Smooth Gear Changes:** The clutch enables smooth gear changes in vehicles with manual transmissions.
- **Control over Power Transfer:** Using the clutch, the driver controls the power transfer from the engine to the wheels.
- **Stopping and Starting:** Clutch is vital for bringing the vehicle to a complete stop and starting from a standstill without stalling the engine.
- **Engine Protection:** Clutch acts as a safety feature, protecting the engine and other drivetrain components from sudden shocks and excessive stress. When gears are shifted without disengaging the clutch, it can lead to a phenomenon called "gear grinding."

Friction based clutches

Requirements:

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

Accordingly, the **material** choosen for clutch design should have

- High and uniform coefficient of friction.
- Should not be affected by moisture and oil.
- Should have the ability to withstand high temperatures caused by slippage.
- Should have high heat conductivity.
- Should have high resistance to wear and scoring.

Torque transmitting capacity

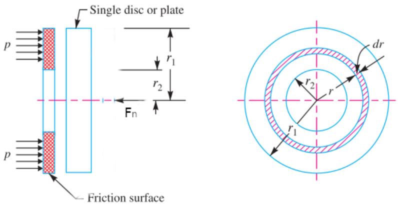


Figure: Image source: Machine design: R S Khurmi

Let T - Torque transmitted by clutch,
p - Normal pressure
r_1 and r_2 - External and internal radii
r - Mean radius of the friction face, and
μ - Coefficient of friction.

Forces acting on the shaded area Elemental normal force, $f_n = p * 2\pi r dr$,
Elemental tangential friction force, $f_t = \mu f_n \implies \mu p * 2\pi r dr$
Elemental friction torque,
$\vec{r} \times \vec{f_t} \implies dT = r * \mu p * 2\pi r dr$

$$dT = \mu p 2\pi r^2 dr \tag{1}$$

Back to recap?15

Torque transmitting capacity

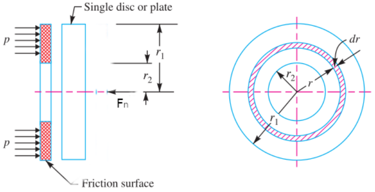


Figure: Image source: Machine design: R S Khurmi

$$dT = \mu p 2\pi r^2 dr$$

Just look at the individual terms !

Elemental torque acting on the shaded area depends on

- μ - Coefficient of friction- constant
- r - radial distance- changing from inner and outer circles- known for a given clutch dimension
- p - Normal pressure acting on the plate !

Two theories are used to quantify p to obtain the torque capacity

1. Uniform pressure theory
2. Uniform wear theory

(1) Uniform pressure: for new clutches

$p = \frac{Normalforce}{Area}$

$p = \frac{F_n}{\pi(r_1^2 - r_2^2)}$

So what will be the elemental torque?

$dT = \mu \frac{F_n}{\pi(r_1^2 - r_2^2)} * 2\pi r^2 dr$

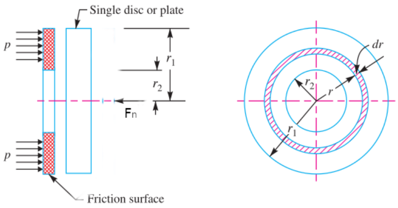


Figure: Image source: Machine design: R S Khurmi

The total frictional torque acting will be obtained by definite integral of dT between the limits r_2 and r_1

$$T = 2\mu \frac{F_n}{(r_1^2 - r_2^2)} \int_{r_1}^{r_2} r^2 dr$$
$$\implies \frac{2\mu F_n}{3} \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)} \qquad (2)$$

If there are N friction plates are in contact then

$$T = \frac{2N\mu F_n}{3} \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)} \qquad (3)$$

*Single plate clutch, usually both sides of the disc are effective, $N = 2$ at max.

Back to recap15

(2) Uniform wear: for wornout clutches

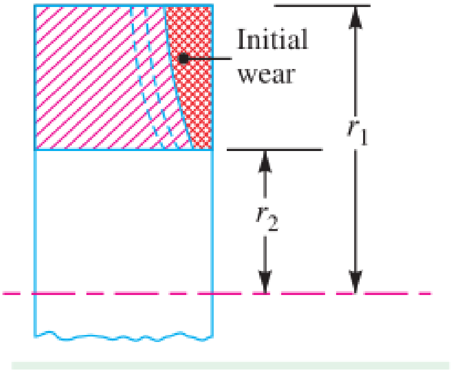


Figure: Image source: Machine design: R S Khurmi

- The axial wear is proportional to the frictional work.
- Wear is uniformly distributed over the entire surface area of the friction disk

$Frictionalwork \propto Normal\ pressure * axialvelocity$

$wear \propto p * 2\pi r\omega$

$wear = constant = p2\pi r\omega$ (Uniform wear assumption)

$\implies pr = constant$

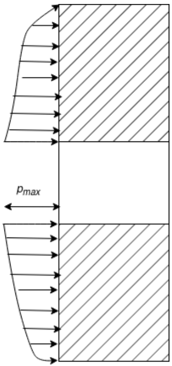
r changes from r_2- low value to r_1 high value, therefore pressure will be maximum at the inner radius r_2

(2) Uniform wear: for wornout clutches

$\implies pr = constant$

pressure is inversely proportional to radial distance

pressure will be maximum at the inner radius r_2



From equation (1), $dT = \mu pr 2\pi r dr$,
substituting in terms of maximum pressure
$dT = \mu p_{max} r_2 2\pi r dr$

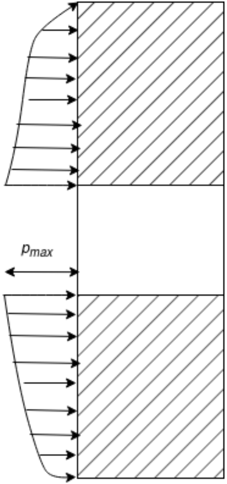
The total frictional torque will be obtained by definite integral of dT between the limits r_2 and r_1

$$\begin{aligned} T &= \mu p_{max} r_2 2\pi \int_{r_2}^{r_1} r dr \\ &= \mu p_{max} r_2 2\pi \left(\frac{r_1^2 - r_2^2}{2}\right) \end{aligned} \tag{4}$$

Wait, what is the value of p_{max}?

(2) Uniform wear: for wornout clutches

To find p_{max},



Use normal force expression,
$f_n = \int_{r_2}^{r_1} p 2\pi r dr$

But, from uniform wear theory,
$pr = constant, \implies p_{max} r_2 = constant.$
$f_n = 2\pi p_{max} r_2 \int_{r_2}^{r_1} dr$

$$p_{max} = \frac{f_n}{2\pi r_2 (r_1 - r_2)} \tag{5}$$

Substituting p_{max} from equation (7) to torque equation (4),

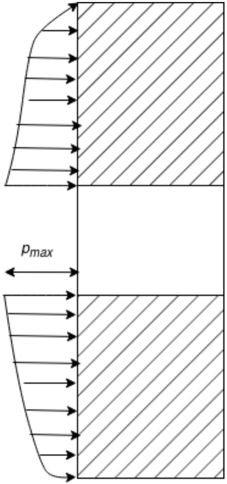
$$T = \mu f_n (r_1 + r_2) \tag{6}$$

$r_1 + r_2$ is reffered to as friction radius.

Back to recap?15

(2) Uniform wear: for wornout clutches

To find p_{max},



Use normal force expression,
$f_n = \int_{r_2}^{r_1} p 2\pi r dr$

But, from uniform wear theory,
$pr = constant, \implies p_{max} r_2 = constant.$
$f_n = 2\pi p_{max} r_2 \int_{r_2}^{r_1} dr$

$$p_{max} = \frac{f_n}{2\pi r_2 (r_1 - r_2)} \tag{7}$$

Substituting p_{max} to torque equation (4),

$$T = \mu f_n \frac{(r_1 + r_2)}{2} \tag{8}$$

$\frac{(r_1+r_2)}{2}$ is reffered to as friction radius.

Recap of single plate clutch

- Two plates- driving and driven disks are pressed against each other to transmit torque/ power
- Force/ torque trasnsmission is based on friction between surfaces (Find the friction force)
- The friction force cause rotation of the driven member about its axis of rotation- (Find the elemental torque)
- Elemental torque dT depends on pressure p and radius r refer: (1)
- We are interested in finding the torque carrying capacity of the clutch!
- Two theories are used Uniform pressure theory (3) and Uniform wear theory (8)

Uniform pressure

$$p = \frac{f_n}{\pi(r_1^2 - r_2^2)}$$

$$T = \frac{2N\mu f_n}{3} \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$$

$$T = \frac{2N\mu f_n}{3} R_{up}$$

Uniform wear

$$p_{max} r_2 = constant$$

$$T = \frac{N\mu f_n}{2}(r_1 + r_2)$$

$$T = \frac{N\mu f_n}{2}(R_{uw})$$

Problems on single plate clutch:

[1] An automotive plate clutch consists of two pairs of contacting surfaces with an asbestos friction lining. The torque transmitting capacity of the clutch is $550 Nm$. The coefficient of friction is 0.25 and the permissible intensity of pressure is $0.5 N/mm^2$. Due to space limitations, the outer diameter of the friction disk is fixed as $250 mm$. Using uniform wear theory, calculate

1. the inner diameter of the friction disk; and
2. the spring force required to keep the clutch in an engaged position.

Problems on single plate clutch:

[2] An automotive plate clutch consists of two pairs of contacting surfaces with an asbestos friction lining. The maximum engine torque is 250 Nm. The coefficient of friction is 0.35. The inner and outer diameters of friction lining are 175 and 250 mm respectively. The clamping force is provided by 9 springs, each compressed by 5 mm to give a force of 800 N, when the clutch is new.

1. What is the Factor Of Safety (FOS) with respect to (wrt) slippage when the clutch is brand new?
2. What is the FOS wrt slippage after initial wear has occured?
3. How much wear of friction lining will take place before the clutch will slip?

Problems on single plate clutch:

[3] A single plate clutch consists of one pair of contacting surfaces. Because of space limitations, the outer diameter of the friction disk is fixed as D. The permissible intensity of pressure is p_{max} and the coefficient of friction is, μ. Assuming uniform wear theory, plot the variation of the torque transmitting capacity against the ratio of diameters (d/D). Show that the torque transmitting capacity of the clutch is maximum, when (d/D) is equal to 0.577.

Multi-plate/ Multi-disk clutches

- Single plate clutch have limitations in torque carrying capacity
- Number of frictional surfaces are limited to 2
- Multiplate clutch consists of multiple driving set B and driven disks set A.

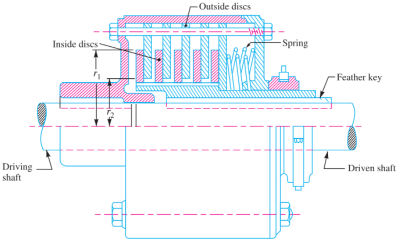


Figure: Image source: Machine design: R S Khurmi

Driving plate is connected to the input shaft
Driven plate is connected to the splined output shaft and can move axially in absence of axial force.

Multi-plate/ Multi-disk clutches

- Equations derived for torque carrying capacity of a single plate clutch has to be modified
- Accounting the number of contacting pairs

Uniform pressure

$$p = \frac{f_n}{\pi(r_1^2 - r_2^2)}$$

$$T = \frac{2n\mu f_n}{3}\frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}$$

$$T = \frac{2n\mu f_n}{3}R_{up}$$

Uniform wear

$$p_{max} r_2 = constant$$

$$T = \frac{n\mu f_n}{2}(r_1 + r_2)$$

$$T = \frac{n\mu f_n}{2}(R_{uw})$$

where n is the number of pairs of contacting surfaces

In the design of multi-plate clutches, very often it is required to determine the number of disks rather than the number of contacting surfaces.

Multi-plate/ Multi-disk clutches

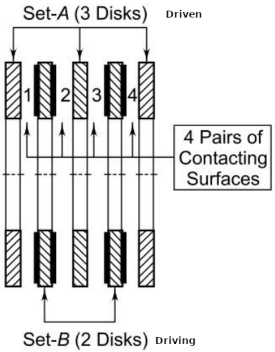


Figure: Image source: Design of machine elements: V B Bhandari

Number of disks = number of contacting pairs+1 $\implies n+1$

$n_{disks} = n_A + n_B$

- It should be noted that the outer disks have contact on one side only.
- With increase in number of contacting surfaces the torque transmitting capacity of the cluth is also increased
- Because of smaller size with high torque capacity, large amount of heat will be generated- mostly immersed in oil- wet clutch
- Presence of oil reduces the coefficient of friction!

[4] An oil immersed multi-disk clutch with moulded asbestos on one side and steel disks on the other, is used in an application. The torque transmitted by the clutch is $75 Nm$. The coefficient of friction is 0.1. The permissible intensity of pressure is $500 kPa$. The outer diameter is kept as $100 mm$ due to space limitation. Assuming uniform wear theory, calculate the inside diameter of the disks, the required number of disks and the clamping force required.

Cone clutch:

The driving and driven members are conical surfaces!- more surface area for contact, two additional parameters- α-cone angle and b-width of friction surfaces

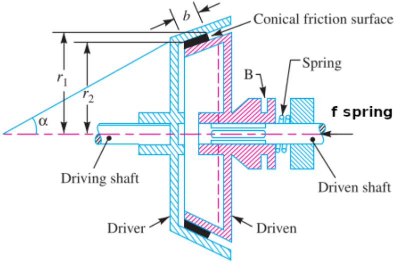
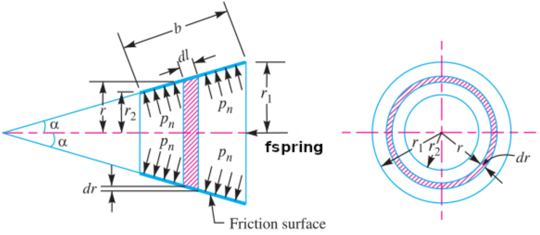


Figure: Image source: Machine Design: R S Khurmi



p_n- pressure acting normal to the contact surface

We know, elemental Friction force$= \mu f_n$ but what is f_n?, try using the normal pressure

$f_n = p_n *$ surface area of contact

$\Longrightarrow \; p_n * 2\pi r dl$. Note here!, in plate type clutches this was $p2\pi rdr$

Cone clutch:

$\sin(\alpha) = \frac{opp}{hyp} \implies \frac{dr}{dl}$, therefore $dl = \frac{dr}{\sin(\alpha)}$

Modifying $f_n = p_n * 2\pi r \frac{dr}{\sin(\alpha)}$- still this p_n is appearing,

$$f_t = \mu p_n * 2\pi r \frac{dr}{\sin(\alpha)}$$

Elemental torque dT due to friction is

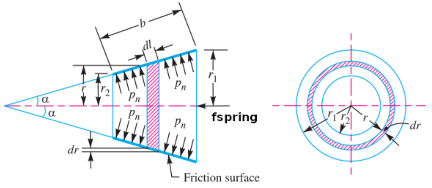
$$dT = r * \mu p_n * 2\pi r \frac{dr}{\sin(\alpha)}$$

Total torque by integrating from r_2 to r_1

$$T = \frac{\mu 2\pi}{\sin(\alpha)} \int_{r_2}^{r_1} p_n r^2 dr \tag{9}$$

While solving problems we noted that they give spring force f_{spring} than normal pressure p_n!

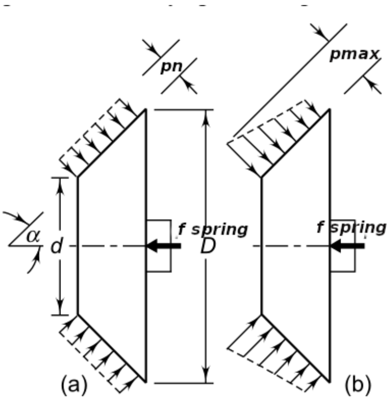
In plate clutch f_n and f_{spring} are same but here there are different



p_n is acting $\perp$ to the contact surface, w.r.t ground reference it has a horizontal component $p_n \sin(\alpha)$ and vertical component $p_n \cos(\alpha)$ Resolving and equating the forces in horizontal direction

$$f_{spring} = \int_{r_2}^{r_1} p_n \sin(\alpha) 2\pi r dl \implies \int_{r_2}^{r_1} p_n 2\pi r dr$$

Pressure p_n values for cone clutch



Uniform pressure theory:

$f_{spring} = \int_{r_2}^{r_1} p_n 2\pi r dr \implies p_n\pi(r_1^2 - r_2^2)$

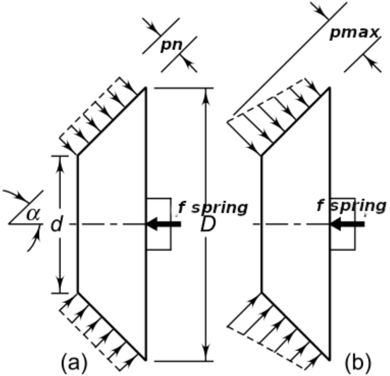
Torque transmitting capacity: refer equation (9), $T = \frac{\mu 2\pi p_n}{\sin(\alpha)} \int_{r_2}^{r_1} r^2 dr \implies \frac{\mu 2\pi p_n}{\sin(\alpha)} \frac{r_1^3 - r_2^3}{3}$

Writing in terms of spring forces,

$$T = \frac{2\mu f_{spring}}{3\sin(\alpha)} \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \tag{10}$$

Thus if $\alpha = \pi/2$ cone will become plate cluch geometry, also as α reduces the torque carrying capacity increases!

Pressure p_n values for cone clutch



Uniform wear theory:

$p_n r = constant \implies p_{max} * r_2 = constant$

$f_{spring} = \int_{r_2}^{r_1} p_{max} 2\pi r_2 dr \implies p_{max} r_2 2\pi (r_1 - r_2)$

Torque transmitting capacity: refer equation (9) $T = \frac{\mu 2\pi p_{max} r_2}{\sin(\alpha)} \int_{r_2}^{r_1} rdr \implies \frac{\mu 2\pi p_{max} r_2}{\sin(\alpha)} \frac{r_1^2 - r_2^2}{2}$

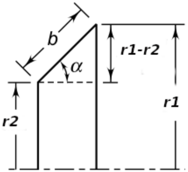
Writing in terms of spring forces,

$$T = \frac{\mu f_{spring}}{2\sin(\alpha)}(r_1 + r_2) \qquad (11)$$

So there is a change only in the denominator!, while reading just look back at the plate type clutch equations.

Friction lining width and info of cone clutch

Rember the term "b"- friction lining width!, you can get it from geometry.



$\sin(\alpha) = \frac{r_1 - r_2}{b}$ Advantages

- For same geometric dimensions, cone clutch transmits about $1/\sin(\alpha)$ times the torque capacity of a single plate clutch (Single pair contact surface)
- To avoid self engagement, $\alpha > \tan^{-1}(\mu)$

Problems on cone clutch:

[5] A cone clutch is used to connect an electric motor running at $1440 rpm$ with a machine which is stationary. The machine is equivalent to a rotor of $150 kg$ mass and radius of gyration as $250 mm$. The machine has to be brought to the full speed of $1440 rpm$ from stationary condition in $40 s$. The semi cone angle is $12.5\deg$. The mean radius of the clutch is twice the face width. The coefficient of friction is 0.2 and the normal intensity of pressure should not exceed $0.1 N/mm^2$. Assuming uniform wear criterion, calculate:

1. inner and outer diameters
2. facewidth of the friction lining
3. force required to engage the clutch
4. heat generated during each engagement

References

- Bhandari, V. B (2010). *Design of Machine Elements*. Mc Graw Hill Education.
- Khurmi, R. S., and J. K. Gupta (2005). *A textbook of machine design*. S. Chand publishing.