Machine design II

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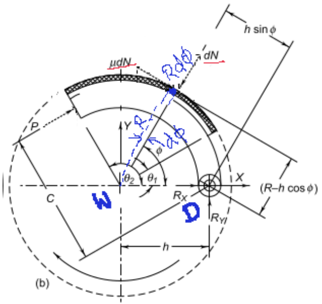
Internally expanding brake- Braking torque on wheel



Forces acting on Shoe

(4) Elemental friction force
$df = \mu dN \implies \frac{\mu p_{max} RW}{\sin(\phi_{max})} \sin(\phi) d\phi$
(5) Elemental braking torque acting on the wheel $dM_t = R * df_t$
$dM_t = \mu \frac{p_{max} R^2 W}{\sin \phi_{max}} \sin \phi d\phi$
The friction lining starts from θ_1 and ends at θ_2

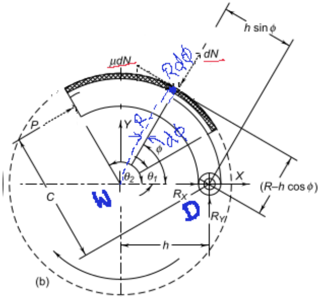
Total braking torque M_t about the wheel center W can be obtained by integrating from θ_1 to θ_2,

$$M_t = \int_{\theta_1}^{\theta_2} \frac{\mu p_{max} Rw}{\sin(\phi_{max})} \sin(\phi) d\phi$$
$$= \frac{\mu p_{max} Rw}{\sin(\phi_{max})} [-\cos(\phi)]_{\theta_1}^{\theta_2} \implies$$
$\frac{\mu p_{max} Rw}{\sin(\phi_{max})} [\cos(\theta_1) - \cos(\theta_2)]$. While considering the brake shoe, it is subjected to five forces,

- normal force, dN
- friction force, df
- Brake actuation force, P
- Reaction forces, R_{DX} and R_{DY} acting at D

Except the reaction forces, the other forces creates a turning moment of the brake shoe against the rotating wheel drum.

Internally expanding brake- Moment acting on the shoe about D



Forces acting on Shoe

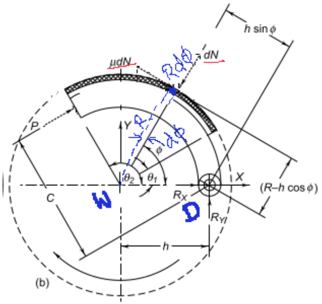
Equating the moments due the forces acting on the shoe about the pivot at D will provide necessary relationship between the applied force P and the moments due to contact forces between the shoe and the drum.

To moment due to the applied force is simple and it is $C * P$- refer the figure. Moments due to the normal and friction forces are bit tricky as they act throughout the contact surface. However, sticking to the elemental area and forces acting on it, expressions can be simplified.

In text books the moment arms are directly provided as $h\sin(\phi)$ and $(R - h\cos(\phi))$- refer figure. An interested reader would ponder upon the reason for such expressions.

We will look at how to find atleast one of them, say how the moment arm for dN is $h\sin(\phi)$

Internally expanding brake- finding the moment arm for dN



Forces acting on Shoe

Two find the moment arm distance we need to construct equation of parallel line, which is parallel to the line of action of the force about D. Find two points on the line to get its equation, line WA - W-$(0,0)$and A-$(R\cos(\phi), R\sin(\phi))$.

Equation of line WA is

$$\frac{y-0}{R\sin(\phi)-0} = \frac{x-0}{R\cos(\phi)-0} \implies y = \tan(\phi)x$$

Constructing line DB - D-$(h,0)$and B-$(h + h\cos(\phi), h\sin(\phi))$.

Equation of line DB is

$$\frac{y-0}{h\sin(\phi)-0} = \frac{x-h}{h+h\cos(\phi)-h}$$

$$\implies y = \tan(\phi)x - h\tan(\phi)$$

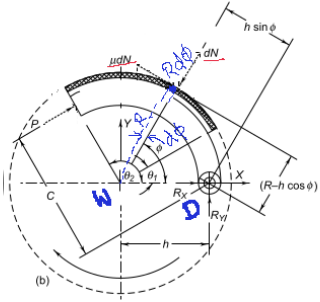
Distance betweent two parallel lines of the form $ax + by = c1$ and $ax + by = c2$ is $d = \frac{|-c1-c2|}{\sqrt{a^2+b^2}}$

Applying here

$$d = \frac{h\tan(\phi)}{\sqrt{1+\tan^2(\phi)}} \implies \frac{h\tan(\phi)}{\sqrt{\frac{\cos^2(\phi)+\sin^2(\phi)}{\cos^2(\phi)}}}$$

$$d = \frac{h\tan(\phi)}{\frac{1}{\cos(\phi)}} = h\sin(\phi)$$

Internally expanding brake- moment due to the normal contact force



Forces acting on Shoe

Having found the moment arm of the force dN about D as $h\sin(\phi)$, the moment caused by this elemental normal force about D,

$dM_{DN} = h\sin(phi) * dN \implies \frac{p_{max}RWh}{\sin(\phi_{max})}\sin^2(\phi)d\phi$

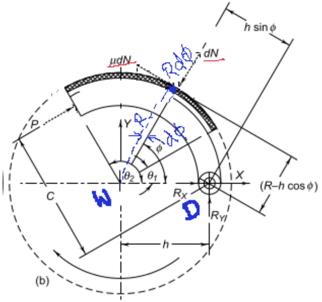
To find the net torque due to the normal forces acting on the contact area, we have to integrate from θ_1 to θ_2,

$M_{DN} = \frac{p_{max}Rwh}{\sin(\phi_{max})}\int_{\theta_1}^{\theta_2}\sin^2(\phi)d\phi$ Using $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ trigonometric identity $\sin^2(\phi)$ can be written as $\frac{1-\cos(2\phi)}{2}$

Final expression for M_{DN} after applying limits is

$M_{DN} = \frac{p_{max}Rwh}{4\sin(\phi_{max})}[2\theta_2 - \sin(2\theta_2) - 2\theta_1 + \sin(2\theta_1)]$

Internally expanding brake-moment due to frictional force



Forces acting on Shoe

The moment arm of the force df about D is $(R - h\cos(\phi))$, "can be found similar to $h\sin(\phi)$", the moment caused by this elemental friction force about D,

$$dM_{Df} = (R - h\cos(\phi)) * df \implies \frac{\mu p_{max} RWh}{\sin(\phi_{max})}(R - h\cos(\phi))\sin(\phi)d\phi$$

To find the net torque due to the friction forces acting on the contact area, we have to integrate from θ_1 to θ_2,

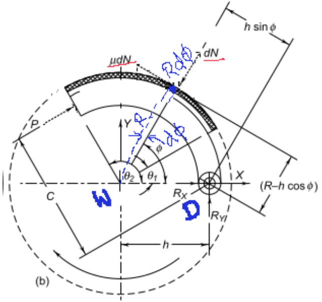
$$M_{Df} = \frac{\mu p_{max} Rwh}{\sin(\phi_{max})} \int_{\theta_1}^{\theta_2} (R\sin(\phi) - h\cos(\phi)\sin(\phi))\, d\phi$$

Using $\sin(a+b)$ and $\sin(a-b)$ trigonometric identities the trignometric terms inside integration can be simplified

M_{Df} after applying limits is

$$M_{Df} = \frac{\mu p_{max} Rw[4R(\cos(\theta_1 - \cos(\theta_2)) - h(\cos(2\theta_1) - \cos(2\theta_2)))]}{4\sin(\phi_{max})}$$

Internally expanding brake-net moment about D



Forces acting on Shoe

The net moment acting about point D will be- refer Figure

$$C * P - M_{DN} + M_{Df} = 0$$

Rewriting

$$P = \frac{M_{DN} - M_{Df}}{C}$$

Here the moment M_{Df} which is due friction reduces the actuation froce that need to be applied. The brake shoe at this mounting configuration reffered to as leading shoe.

Remember: we started with the wheel rotating clockwise.

For anti clockwise rotation of the wheel

$$P = \frac{M_{DN} + M_{Df}}{C}$$

Such a mounting configuration of the brake shoe referred to as trailing shoe.

[1] An internal-expanding brake with four identical shoes is shown in the Figure. Each hinge pin supports a pair of shoes. The actuating mechanism is designed in such a way that it produces the same force P on each of the four shoes. The face width of the friction lining is $50mm$ and the maximum intensity of normal pressure is limited to $1N/mm^2$. The coefficient of friction is 0.3. Calculate

1. The actuating force P.
2. The torque absorbing capacity of the brake.