Machine Design II -ME4001D

Monsoon Sem-Aug to Dec-2023

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Table of Contents

Course syllabus

Module-1: Design of clutches, brakes, belts and chain drives: 13

Friction clutches and brakes; uniform pressure and uniform wear assumptions; design of disc and cone types of clutches and brakes; design of external contracting and internal expanding elements; band type clutches and brakes; belt and chain drives of common types; design of flat and V-belt drives; selection of roller chains.

Module-2: Design of gears and Journal bearings: 14

Spur, helical, bevel and worm gears; tooth loads; gear materials; design stresses; basic tooth stresses; stress concentration; service factor; velocity factor; bending strength of gear teeth; Buckingham's equation for dynamic load; surface strength and durability; heat dissipation; design for strength and wear.
Lubrication and journal bearing design: types of lubrication and lubricants; viscosity; journal bearing with perfect lubrication; hydrodynamic theory of lubrication; design considerations; heat balance; journal bearing design.

Module-3: Rolling contact bearings and Design for manufacturing: 12

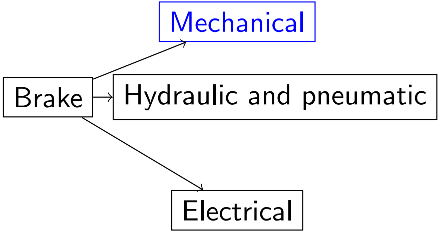
Bearing types, bearing life, static and dynamic capacity, selection of bearings with axial and radial loads, selection of tapered roller bearings, lubrication, seals, shaft, housing and mounting details.
General design recommendations for rolled sections, forgings, screw machine products, turned parts, machined round holes, parts produced on milling machine, welded parts and castings; modification of design for manufacturing easiness for typical products.

Marking scheme

- Assignments & tutorials - 20
- Mid semester exam- 30
- End semester exam- 50

What is a brake?

- Brakes are mechanical devices that help vehicles and machines slow down or stop. They do this by turning the energy of movement into heat. Brakes are essential for keeping things safe and under control when driving or using machines.
- First step in the brake design is to determine the braking torque capacity for the given application.



Brake capacity depends on

- Pressure between braking surfaces
- Contacting area of braking surface
- Coefficient of friction
- Ability to dissipate heat that is eqivalent to energy being absorbed

Mechanical brake: conversion of K.E and P.E to heat:

A rigid body in motion generally possesses both translational and rotational energies. While attempting to reduce the movement of such bodies using a brake, these energies are converted to heat.

Example:

A vehicle of mass $1200kg$ is moving down the hill at a slope of $1:5$ at $72km/h$. It is to be stopped in a distance of $50m$. If the diameter of the tyre is $600mm$, determine the average braking torque to be applied to stop the vehicle, neglecting all the frictional energy except for the brake. If the friction energy is momentarily stored in a $20kg$ cast iron brake drum, What is average temperature rise of the drum? The specific heat for cast iron may be taken as $520J/kgC$.

- The car on a down slope has a potential energy $m*g*h*s$
- $v_1 = 72*10^3/(60*60)m/s$- initial velocity, $v_2 = 0m/s$
- You can get minimum friction force required from here! Work done by friction= Friction force* stoping distance= $(P.E + \Delta K.E)$
- Equate the total energy with heat energy $m*C*\Delta T$ to get temperature rise.

Block brake with short shoe:

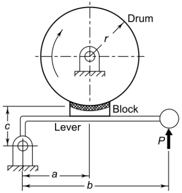


Fig. 12.2 *Block Brake*

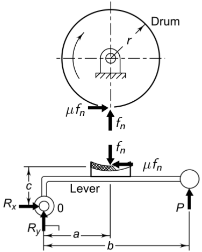


Figure: Design of machine elements: V B

As the shoe length is short we can assume uniform pressure distribution in wheel-shoe contact area

Taking moment about the pivot at O,

$pb - f_n * a + \mu f_n c = 0 \implies p = \left(\frac{a - \mu c}{b}\right) * f_n$

Where p is the applied force and f_n is the normal force applie the drum

Just rewriting the equation in terms of f_n as, $f_n = \left(\frac{b}{a - \mu c}\right) p$

1. If $(a - \mu c)$ becomes zero then f_n goes unbounded for an zero force applied (p) - self locking
2. If $(a - \mu c) > 0$ then the friction force aids in braking ref to as self enegrised brake
3. If $(a - \mu c) < 0$ this results in uncontrolled braking and grabbing

Block brake with short shoe:

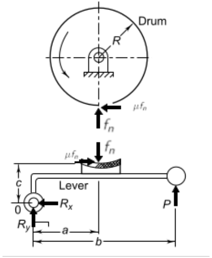


Figure: Design of machine elements: V B Bhandari

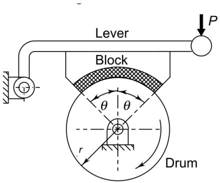
To find the reaction forces use force equilibrium for the lever body,
$R_x = \mu f_n$ and $R_y = f_n$

If the direction of rotation of the wheel is changed then
$pb - f_n * a - \mu f_n c = 0 \implies p = \left(\frac{a+\mu c}{b}\right) * f_n$
Where p is the applied force and f_n is the normal force applied
the drum
Just rewriting the equation in terms of f_n as, $f_n = \left(\frac{b}{a+\mu c}\right) p$

Therefore the braking effort depends on

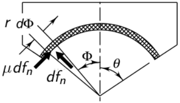
1. Direction of rotation of the drum- fwd direction one magnitude and reverse one magnitude.
2. Geometric parameters a, b, r and c- location of pivot O
3. Coefficient of friction μ

Block brake with long shoe:

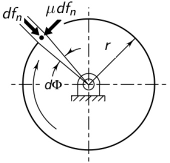


If angle of contact θ between the block shoe and the brake drum is greater than 45° then the assumption of uniform pressure distribution is not valid. In long shoe brakes the normal and friction forces are concentrated at the mid point of the shoe.

Considering an element of the friction lining, located at an angle ϕ and subtending an angle $d\phi$



(a) Forces on lining



(b) Forces on drum

Area of elemental friction lining= $r d\phi w$, area=(arc length *width)

where w is the width of the friction lining- parallel to axis of the drum.

Elemental normal force,

$df_n = p * r d\phi w$- pressure times area, elemental frictional force acting in that area is

$df_t = \mu df_n \implies \mu p * r d\phi w$

Elemental torque

$\delta M_t = r * df_t \implies$

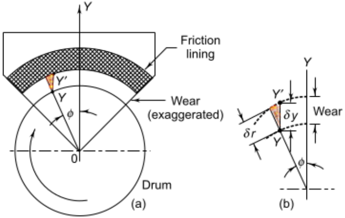
$r * \mu p r d\phi w \implies \mu r^2 w p d\phi$

$\boxed{\delta M_t = \mu r^2 w p d\phi}$- note: μ, r, w are known variables, only unknown is the pressure variation w.r.t ϕ

Block brake with long shoe:

- $\boxed{M_t = \mu r^2 w \int p d\phi}$ depends on the pressure variation, p, if we are able to bring in a relationship between p and ϕ then δM_t can be integrated - braking torque applied on the drum!.
- Brake drum is made up of a harder material than the friction lining, thus during braking wear occurs in friction lining.

- After wear the lining will retain the cylindrical shape of the wheel drum.



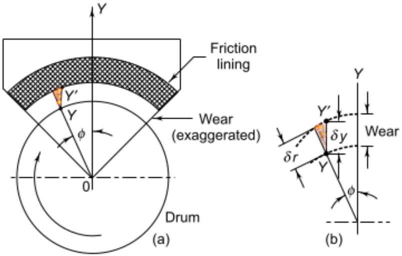
Wear on Friction Lining

Consider the shaded triangle, using similar traingles:
$\cos(\phi) = \frac{\delta r}{\delta y}$

The radial wear of the lining happens because of friction, thus the amount of radial wear δr should be proportional to the work done by the friction force. $\delta r \alpha p * \omega$- Work done is normal pressure $*$ velocity.

Also if the cylindrical shape is retained then the vertical measure of wear from y to y', δy will be same at the center and distal end of the brake lining.
$\delta y = Constant = C_1$
$\implies \frac{\delta r}{\cos(\phi)} = C1$

Block brake with long shoe:



Wear on Friction Lining

Just look at the relations we are having, $\frac{\delta r}{\cos(\phi)} = C1$, $\delta r \alpha p * \omega$, if we assume that the brakes are operated at constant angular velocity ω and rewriting the expression that we are having

$p = C1\cos(\phi)$

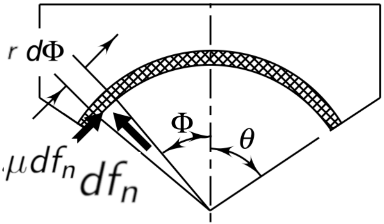
Thus the pressure p will be high at the mid point of the shoe and will reduce as ϕ increases. If the maximum pressure is assumed to be p_{max} then $p = p_{max}\cos(\phi)$

Net braking torque by integrating from $-\theta$ to θ

$$M_t = \mu p_{max} r^2 w \int_{-\theta}^{\theta} \cos(\phi) d\phi \qquad (1)$$

But most of the cases we know or in need of the normal force acting on the contact area- Just look at the previous short shoe topic f_n will be known or given.

Block brake with long shoe:



(a) Forces on lining

The elemental normal force can be summed up to f_n. But this df_n acts $\perp$ to the friction surface acting at an angle ϕ w.r.t the vertical axis.

df_n will have two components, $df_n \cos(\phi)$ as veritcal component and $df_n \sin(\phi)$ as horizontal component. Thus total vertical force f_n is,

$$f_n = \int df_n \cos(\phi) \implies \int_{-\theta}^{\theta} prd\phi w \cos(\phi)$$

writing $p = p_{max} \cos(\phi)$,

$$\begin{aligned} f_n &= p_{max} rw \int_{-\theta}^{\theta} \cos^2(\phi) d\phi \\ &= p_{max} rw \int_{-\theta}^{\theta} \left(\frac{1 + \cos(2\phi)}{2}\right) d\phi \end{aligned} \tag{2}$$

Block brake with long shoe:

$$f_n = p_{max} r w \left(\theta + \frac{\sin(2\theta)}{2} \right) \tag{3}$$

Further we can rewrite the braking torque M_t in terms of normal force as

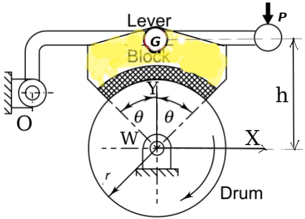
$$M_t = \left[\frac{4\mu \sin(\theta)}{2\theta + \sin(2\theta)} \right] f_n r \implies \mu' f_n r$$

Again here, look back at the M_t expression for the short shoe brake. There will be only difference with the coefficient of friction, μ' here is reffered to as the equivalent coefficient of friction for long shoe brake.

Other relationship- Force, moment equilibrium derived for short shoe brake holds here as well!

Pivoted block brake with long shoe:

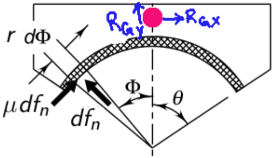
Pivot at G-



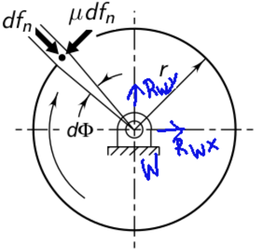
While rigidly fixing the block to the lever the friction force will try to unseat the friction lining.
Pivot at **G** adds an extra DOF allowing rotation of the block about the pivot. This extra DOF helps in attaining lesser wear and better life of the friction lining.

- All the expressions derived for block brake with long shoe holds here. We will derive it once again for continuity.
- The difference is from the reaction forces at the pivot at **G**, R_{GX} and R_{Gy}
- An geometric variable h is added- the idea is to find h such that the moment at the pivot G, M_{tG} due to the braking force is zero.
- Let us look at the individual FBD of the bodies in this brake.

Pivoted block brake with long shoe:



(a) Forces on lining

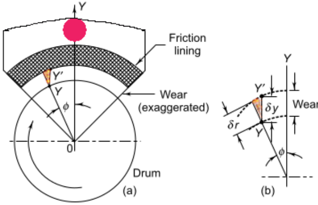


(b) Forces on drum

(1) Area of elemental friction lining$= rd\phi w$, area=(arc length *width) where w is the width of the friction lining- parallel to axis of the drum.
(2) Elemental normal force, $df_n = p * rd\phi w$- pressure times area,
(3) Elemental frictional force acting in that area is $df_t = \mu df_n \implies \mu p * rd\phi w$
(4) Elemental torque
$\delta M_t = r * df_t \implies r * \mu prd\phi w \implies \mu r^2 wpd\phi$
$\boxed{\delta M_t = \mu r^2 wpd\phi}$- note: μ, r, w are known variables, only unknown is the pressure variation w.r.t ϕ

Pivoted block brake with long shoe:

The same wear theory we discussed for long shoe without pivot holds here as well!- w.r.t wheel, there is no change.- Brake drum is made up of a harder material than the friction lining, thus during braking wear occurs in friction lining.



Wear on Friction Lining

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Also if the cylindrical shape is retained then the vertical measure of wear from y to y', δy will be same at the center and distal end of the brake lining.

$\delta y = Constant = C_1$

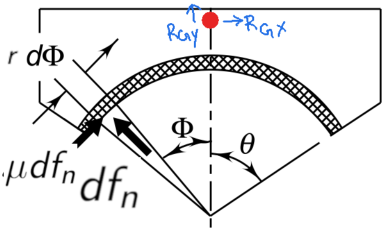
$\implies \frac{\delta r}{\cos(\phi)} = C1$. Just look at the relations we are having, $\frac{\delta r}{\cos(\phi)} = C1$, $\delta r \alpha p * \omega$, if we assume that the brakes are operated at constant angular velocity ω and rewriting the expression that we are having, $p = C1\cos(\phi)$

Pivoted block brake with long shoe:

Thus the pressure p will be high at the mid point of the shoe and will reduce as ϕ increases. If the maximum pressure is assumed to be p_{max} then $p = p_{max}\cos(\phi)$
Net braking torque by integrating from $-\theta$ to θ

$$\begin{aligned} M_t &= \mu p_{max} r^2 w \int_{-\theta}^{\theta} \cos(\phi) d\phi \\ &= 2\mu p_{max} r^2 w \sin(\theta) \end{aligned} \tag{4}$$

But most of the cases we know or in need of the normal force acting on the contact area.



(a) Forces on lining

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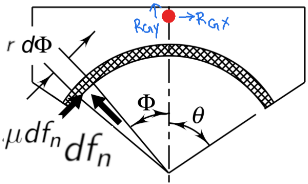
Pivoted block brake with long shoe:



(a) Forces on lining

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Thus total vertical force f_n is,

$$f_n = \int df_n \cos(\phi) \implies \int_{-\theta}^{\theta} prd\phi w \cos(\phi)$$

writing $p = p_{max}\cos(\phi)$,

$$\begin{aligned} f_n &= p_{max} rw \int_{-\theta}^{\theta} \cos^2(\phi) d\phi \\ &= p_{max} rw \int_{-\theta}^{\theta} (\frac{1+\cos(2\phi)}{2}) d\phi \end{aligned} \tag{5}$$

$$f_n = p_{max} rw \left(\theta + \frac{\sin(2\theta)}{2}\right) \tag{6}$$

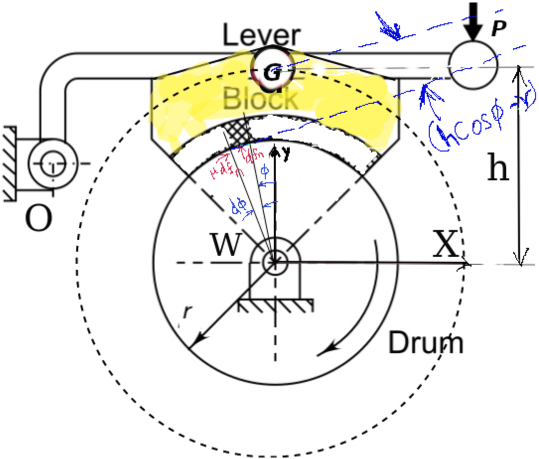
Further we can rewrite the braking torque M_t in terms of normal force as

$$M_t = \left[\frac{4\mu\sin(\theta)}{2\theta + \sin(2\theta)}\right] f_n r \implies \mu' f_n r$$

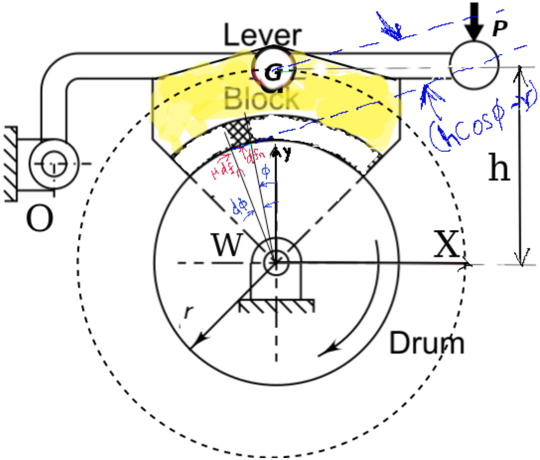
Till this there is no difference between rigidly fixed and pivoted shoe relations

Pivoted block brake with long shoe:

- The purpose of introducing an pivot with extra DOF is to tackle the friction force responsible for unseating of the friction lining.
- This useating force is tackled by choosing the geometric position of the pivot "posistion of G" w.r.t the wheel drum
- Find h such that moment due to the friction force about G is zero
- The distance from G to a tangent to the elemental area will be $(h\cos(\phi) - r)$



Pivoted block brake with long shoe:



The normal force f_n, acts symmetric about G hence it won't create any moment. The friction force μdf_n creates a moment about G with an moment arm $(h\cos(\phi) - r)$. Using distance between two parallel lines.

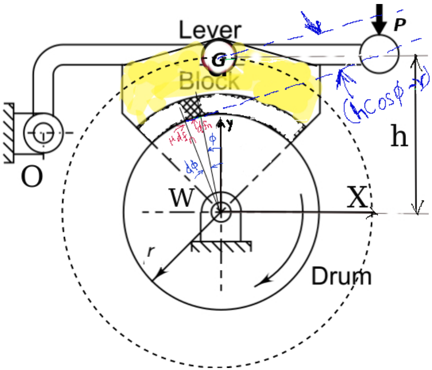
Elemental moment at G due to the elemental friction force μdf_n acting at an angle ϕ w.r.t Y axis,

$$\begin{aligned} dM_{tG} &= \mu df_n (h\cos(\phi) - r) \\ &= \mu p_{max} r d\phi w \cos(\phi)(h\cos(\phi) - r) \end{aligned} \tag{7}$$

Total moment acting at G due to friction force,

$$M_{tG} = \mu p_{max} r w \int_{-\theta}^{\theta} \cos(\phi)(h\cos(\phi) - r) d\phi$$

Pivoted block brake with long shoe:



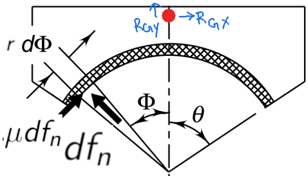
We want $M_{tG} = 0$ grouping constants and variables in the moment expression

$$
\begin{aligned}
M_{tG} &= \mu p_{max} r w \int_{-\theta}^{\theta} (h\cos^2(\phi) - r\cos(\phi)) d\phi = 0 \\
\implies & \int_{-\theta}^{\theta} (h\cos^2(\phi) - r\cos(\phi)) d\phi = 0
\end{aligned} \tag{9}
$$

Utilizing trignometric identities $\cos^2(\phi) = \frac{(1+\cos(2\phi))}{2}$, the relationship between the pivot location h and shoe geometries r and θ is obtained as

$$
h = \frac{4r\sin(\theta)}{2\theta + \sin(\theta)} \tag{10}
$$

Pivoted block brake with long shoe:



(a) Forces on lining

Due to extra DOF from the pivot, there will unknown reaction forces R_{GX} and R_{GY} at the pivot G. These reaction forces can be obtained by taking force equilibrium of the friction lining. Considering $\sum F_Y = 0$,

$$R_{GY} = \int_{-\theta}^{\theta} df_n \cos(\phi) + \mu df_n \sin(\phi)$$
$$= p_{max} rw \int_{-\theta}^{\theta} (\cos^2(\phi) + \mu \cos(\phi) \sin(\phi) d\phi \tag{11}$$

Using trignometric identities $\sin(a+b)$ and $\sin(a-b)$ it can be prooved that, $\int_{-\theta}^{\theta} \cos(\phi)\sin(\phi)d\phi = 0$ as it results in an a single Sine term-odd function. Simplifying, $R_{GY} = \frac{p_{max} rw}{2}(2\theta + \sin(2\theta))$. R_{GX} can also be obtained by resolving the normal and frictional force as

$$R_{GX} = \int df_n \sin(\phi) + \mu df_n \cos(\phi) \tag{12}$$

$R_{GX} = \frac{\mu p_{max} rw}{2}(2\theta + \sin(2\theta))$

Simple band brake:

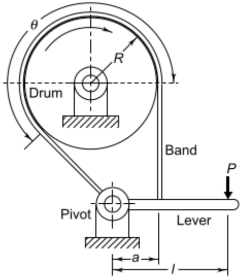


Fig. 12.20 *Simple Band Brake*

1. Flexible strip lined with friction materials
2. When one end of the steel band passes through the fulcrum of the actuating lever- simple band brake

Working of a band brake is similar to a pulley-belt operation.
Ratio of band tensions is given by $\frac{P_1}{P_2} = e^{\mu\theta}$

- P_1 Tension in tight side in N
- P_2 Tension in loose side in N
- μ Coefficient of friction
- θ Angle of wrap in *rad*

Braking torque acting on the drum,
$M_t = (P_1 - P_2) * R$

Simple band brake:

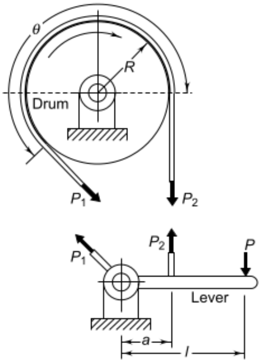


Fig. 12.21 *Free-body Diagram of Forces*

Taking moment about the pivot,
$P_2 a - P * l = 0$

Consider an elemental area of the band subtended by an angle $d\phi$

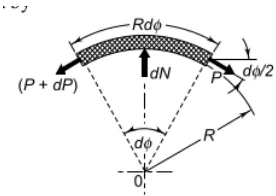


Fig. 12.22

(1) Elemental area $Rd\phi w$- arc length *width of the band. P and $P + dP$ are the band tensions in the loose and tight side respectively.
(2) Elemental normal force
$dN = pressure * area \implies p * Rd\phi w$. Note: p is pressure while P is force

Simple band brake:

Considering equilibrium of vertical forces on the element

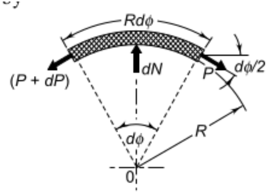


Fig. 12.22

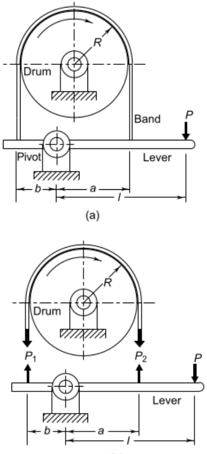
- $dN - P\sin(\frac{d\phi}{2}) - (P + dP)\sin(\frac{d\phi}{2}) = 0$, for smaller angles of $d\phi$, $\sin(\frac{d\phi}{2}) = \frac{d\phi}{2}$
- Therefore $dN = Pd\phi$, previously using pressure-area relationship we have $dN = p * Rd\phi w$

- Equating pressure, p and the force applied P, $p = \frac{P}{Rw}$.
- The band tension varies from a higher value of P_1 on the tight side to a lower value of P_2 on the loose side.
- Therefore $p_{max} = \frac{P_1}{Rw}$

Note:Action of P_1 and P_2 will depend on the wheel drum rotation direction.

Differential band brake:

Neither end of the band passes through the pivot. Such geometry is designed for self-locking.



- Taking moment equilibrium about the pivot, $Pl + P_1 b - P_2 a = 0$, further $\frac{P_1}{P_2} = e^{\mu\theta}$
- Rewriting in useful form, $P = \frac{P_2(a - be^{\mu\theta})}{l}$
- For self locking $P = 0$ or $-ve$ this $\implies \; a <= be^{\mu\theta}$
- In general operating conditions, self-locking is undesirable as the brake is out of operator's control.
- However, advantageous where back-stop mechanism is needed.

Problems on differential band brake:

[1] The differential band brake shown in Figure 3, has a drum diameter of 600mm and the angle of contact is 240^o. The band brake is 5mm thick and 100mm wide. The coefficient of friction between the band and the drum is 0.3. If the band is subjected to a stress of 50MPa, find: (1) Least braking force required, (2) Torque applied to the brake drum.

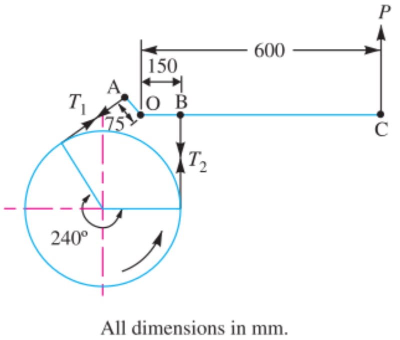


Figure: 3 Differential band brake

Disk brake:

Similar to a plate clutch- operates on friction between plates- two major differences-

- The driven plate is fixed.
- Friction lining contacts only a smaller portion of the disk.
- Area of the friction disk is small- Uniform pressure theory can be used!
- Friction radius $R_f = \frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)$

R_o = outer radius of pad (mm)
R_i = inner radius of pad (mm)
θ = angular dimension of pad (radians)

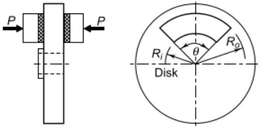


Fig. 12.27 *Disk Brake with Annular Pad*

- Braking torque capacity, $M_t = \mu P R_f$
- Area of contact, $A = \pi(R_o^2 - R_i^2)\frac{\theta}{2\pi}$
- In terms of pressure, $p = P/A$

Sometimes, disk brake with circular pads are being used -Empirical relations are used for R_f

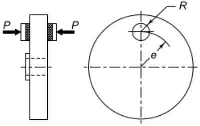


Fig. 12.28 *Disk Brake with Circular Pad*

The friction radius (R_f) of circular pad is given by,

$$R_f = \delta e \quad (12.33)$$

The values of (δ) are given in Table 12.1.

Table 12.1 *Values of δ for circular-pad caliper disk brakes*

R/e	δ
0.0	1.0000
0.1	0.9833
0.2	0.9693
0.3	0.9572
0.4	0.9467
0.5	0.9375

Problems on disk brake:

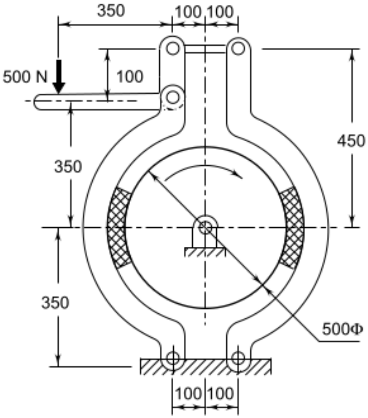
[2] Following data is given for a caliper type annular disk brake, for the front wheel of a motor cycle: Torque capacity 1500Nm, Outer radius of the pad is 150mm, inner radius of the pad is 100mm, average pressure on the pad is 2MPa, coefficient of friction is 0.35 and the number of pads is 2. Calculate the angular dimensions of the pad.

Problems on disk brake:

[3] Following data is given for a caliper type circular pad disk brake: Torque capacity 1500 Nm, number of caliper brakes on the wheel is 3, number of pads on each calliper brake is 2, coefficient of friction is 0.35, average pressure on the pad is 2 MPa. The ratio of pad radius to the distance of the pad center from axis of disk is 0.2. Calculate the pad radius.

Problems on block brake:

[4] For a double block brake shown in the Figure, the brake drum rotates in a clockwise direction and the actuating force is 500N. The coefficient of friction between the bocks and the drum is 0.35. Calculate the braking torque capacity of the brake.



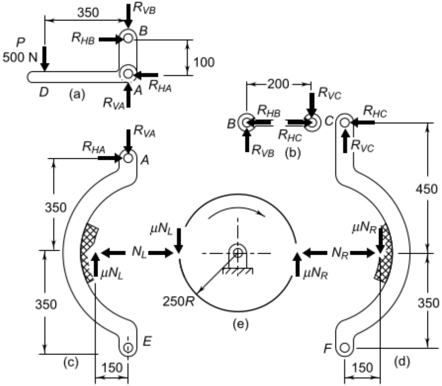


Fig. 12.7 *Free-body Diagram of Forces*

Internally expanding brake:

So far, we have seen brake shoes placed outside the wheel drum and pressed against the wheel drum producing frictional torque resisting wheel rotation. In automotive applications, the brake shoes are placed inside the wheel drum, reffered as an internally expanding brake- due to space limitations.

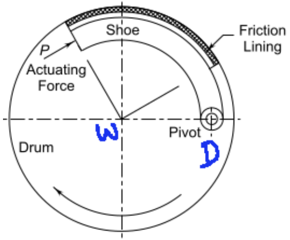
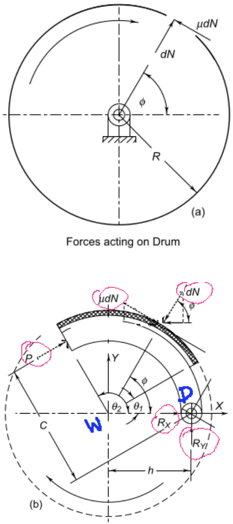


Fig. 12.16 *Internal Expanding Brake*

FBD of the wheel drum and the shoe



Internally expanding brake:

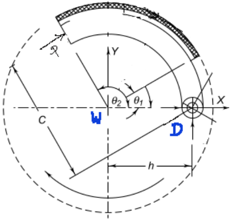
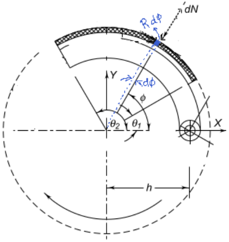


Figure: Geometry of brake shoe



Consider an elemental area on the friction lining at an angle ϕ and subtending and angle $d\phi$.

(1) Elemental area$= Rd\phi w$, where w is the width of the lining.

(2) Elemental normal force is

$dN = p * area \implies p * Rd\phi w$

(3) Assumption: Normal pressure , p is proportional to the vertical distance of the element from the pivot at D,

$p = C_1 \sin(\phi)$

If $\theta_2 > 90$, p_{max} happens at $\theta_2 = 90deg$, but if by geometry, $\theta_2 < 90$, p_{max} happens at θ_{max}

$p = \frac{p_{max}}{\sin(\phi_{max})} \sin(\phi)$

References

Bhandari, V. B (2010). *Design of Machine Elements*. Mc Graw Hill Education.

Khurmi, R. S., and J. K. Gupta (2005). *A textbook of machine design*. S. Chand publishing.