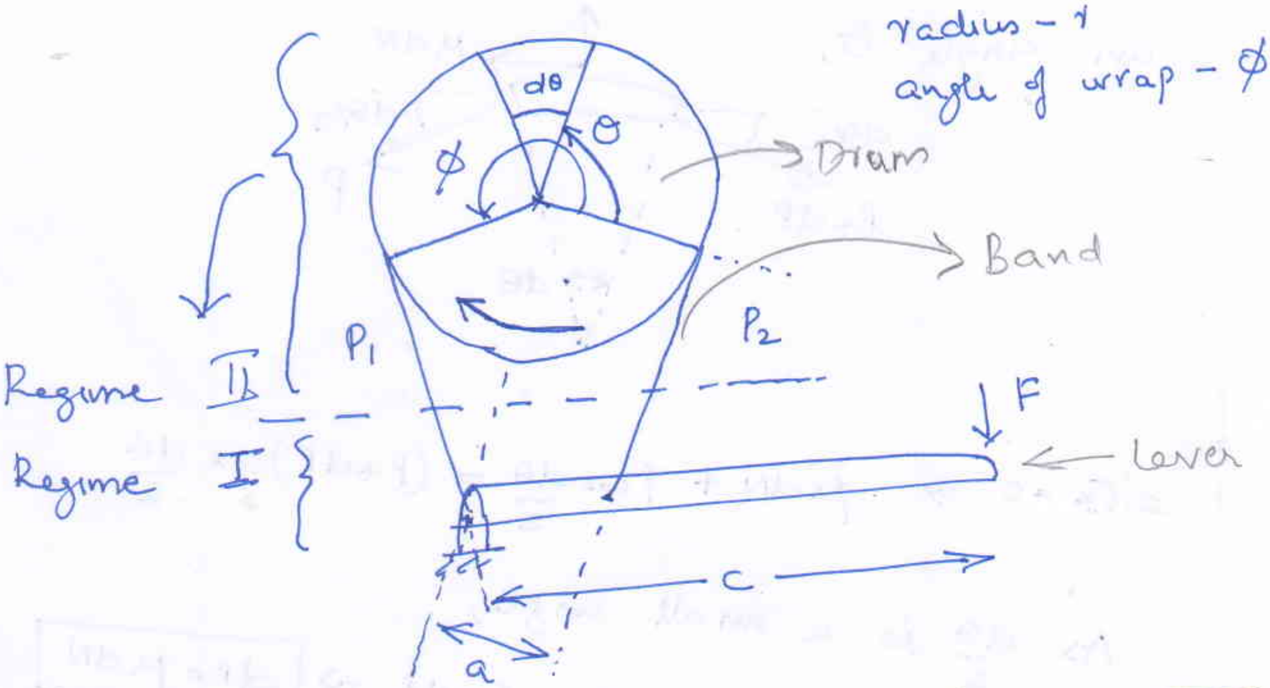
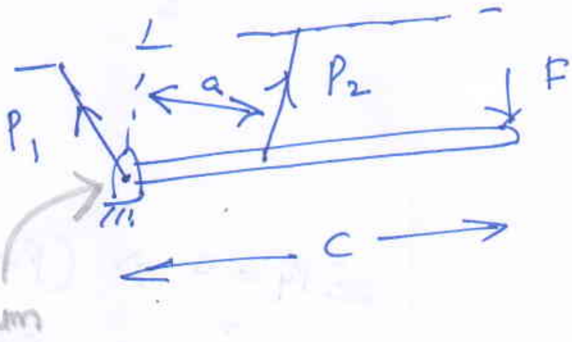
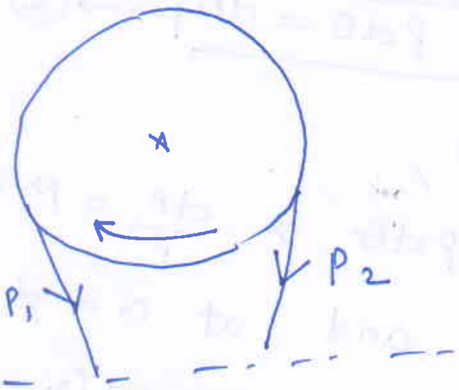
Simple Band brakes



→ FBD of Regime I



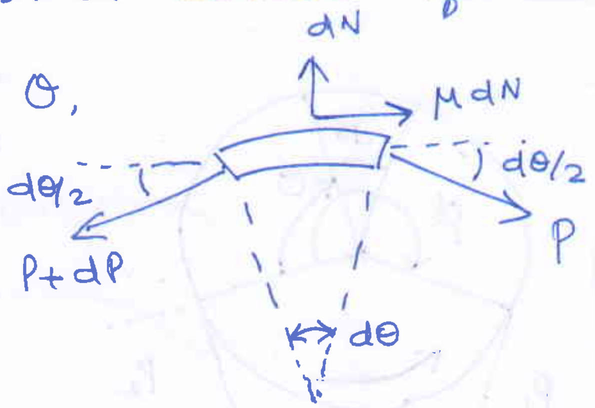
→ FBD of Regime II



→ From the FBD of Regime I, taking moments about the fulcrum of lever,

$$Fc = P_2 a \Rightarrow \boxed{F = \frac{P_2 a}{c}}$$

→ FBD of a small element of the band at ^(which is) an angle θ,



$\Sigma F_x = 0 \Rightarrow \mu dN + P\cos\frac{d\theta}{2} = (P+dP)\cos\frac{d\theta}{2}$

As $\frac{d\theta}{2}$ is a small angle,

$\mu\, dN + P = P + dP \Rightarrow \boxed{dP = \mu\, dN} \rightarrow (1)$

$\Sigma F_y = 0 \Rightarrow (P+dP)\sin\frac{d\theta}{2} + P\sin\frac{d\theta}{2} = dN$

$(P+dP)\frac{d\theta}{2} + P\frac{d\theta}{2} = dN \quad \left[\because \sin\frac{d\theta}{2} = \frac{d\theta}{2}\right]$

$2P\frac{d\theta}{2} = dN \quad \left[dP\cdot\frac{d\theta}{2} \rightarrow 0\right]$

$\boxed{P\,d\theta = dN} \rightarrow (2)$

From (1) and (2),

$dP = \mu \cdot P\,d\theta \Rightarrow \frac{dP}{P} = \mu\, d\theta.$

At $\theta = 0$, $P = P_2$ and at $\theta = \phi$, $P = P_1$

$\Rightarrow \int_{P_2}^{P_1} \frac{dP}{P} = \int_0^{\phi} \mu\, d\theta \Rightarrow [\ln P]_{P_1}^{P_2} = \mu[\theta]_0^{\phi}$

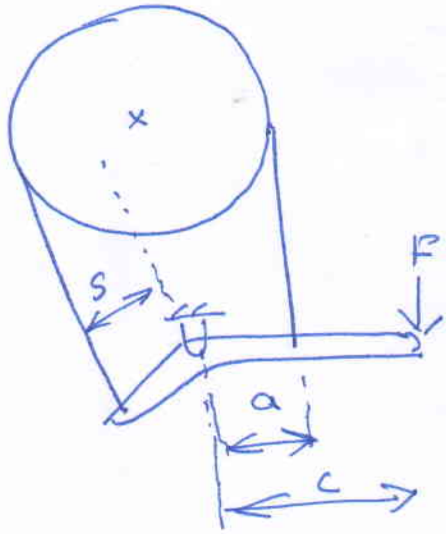
$\ln P_1 - \ln P_2 = \mu\phi \Rightarrow \ln\frac{P_1}{P_2} = \mu\phi$

$\Rightarrow \boxed{\frac{P_1}{P_2} = e^{\mu\phi}}$

The maximum normal pressure occurs when

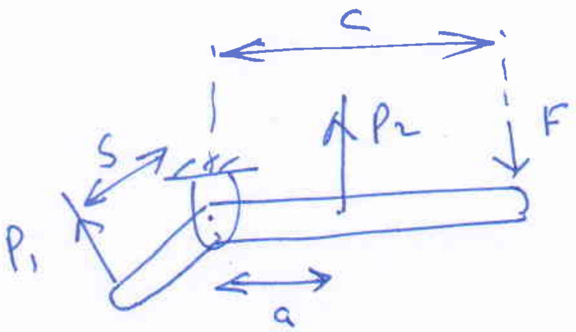
$\theta = \phi$ i.e. $P\big|_{\theta=\phi} = P_{max} = P_1$

Differential band brakes



- Neither side of the band passes through the fulcrum.

FBD of the lever



$$P_1 s + Fc = P_2 a.$$

For self locking, $P_1 s \geq P_2 a$

$$\boxed{\frac{s}{a} \geq \frac{P_2}{P_1}}$$