Beam strength of gear tooth: → The analysis of bending stresses in gear tooth was done by Wifred Lewis 1892. Even today, Lewis equation is considered as the basic equation in gear design.

In Lewis analysis, the gear tooth is treated as a Cantilever beam

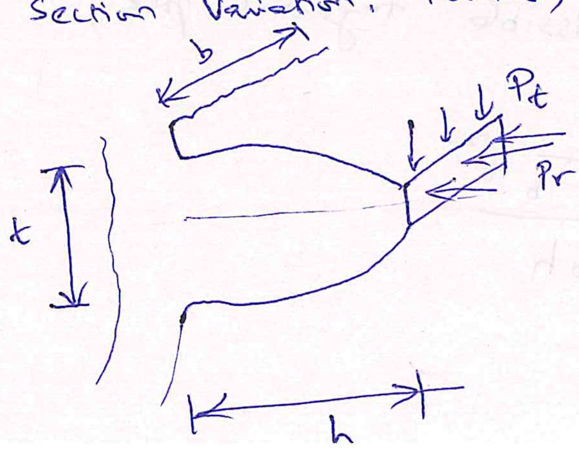
From force analysis of gears we saw that there are two components of forces P_t → tangential and P_r → radial

- Lewis considered the effect of tangential component on the gear tooth
- The tangential component ~~F~~ (P_t) is assumed to be uniformly distributed
- At any time only one gear pair teeth is in contact – most of the time more than one gear pair will be in contact.

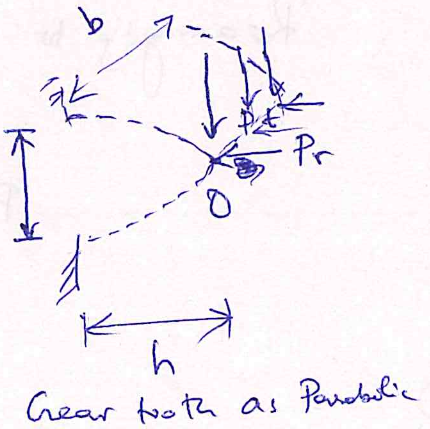
  Normal Contact Ratio of spur gears is 1.6.

If you look at the gear tooth, the cross section of the tooth varies from the fixed end to the free end.

Lewis considered an parabolic variation in cross-section for his analysis. Eventhough the shape of the gear tooth is not a parabola, this parabolic assumption simplifies the expression for cross section variation. Further, it gives a conservative estimate.



$\approx$



Gear tooth as Parabolic beam

①

Conservative estimate: The tooth is larger than the parabola at every section

From beam bending theory we have

$$\frac{M_b}{I} = \frac{\sigma_b}{y}$$

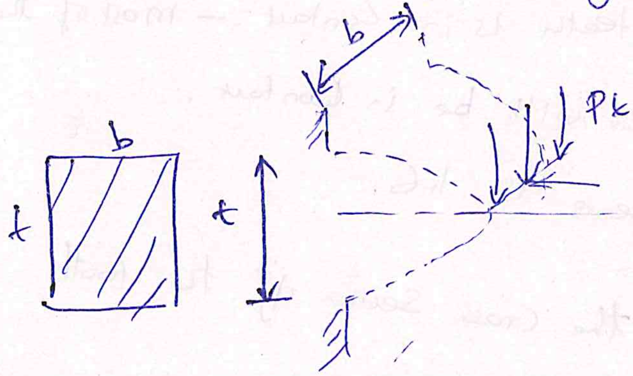
where $M_b \rightarrow$ is the bending moment

$I \rightarrow$ area moment of inertia

$\sigma_b \rightarrow$ bending stress

y - distance measured from the neutral axis

Although the cross section varies, Lewis considered the maximum cross-section for finding I, $I = \frac{bt^3}{12}$



, bending moment

$$M_b = P_t \times h$$

, for maximum bending $y = \frac{t}{2}$

$$\Rightarrow \sigma_b = \frac{M_b\, y}{I} = \frac{t}{2} \times \frac{12}{bt^3} \times P_t \times h$$

$$\Rightarrow \sigma_b = \frac{6 P_t h}{bt^2}$$

Rearranging to get permissible tangential force,

$$P_t = \frac{b\, \sigma_b\, t^2}{6h}$$

in $\boxed{P_t = \frac{b \sigma_b t^2}{6h}}$ — (1), the Variable t, b and h depends upon the size of the tooth, module and its profile.

Lewis with his experiments on gears by changing the gear dimensions found that there exist an relationship involving the module of gears

Multiplying the numerator & denominator of (1) by the module of mating gear

$$P_t = mb\sigma_b \left[\frac{t^2}{6hm}\right]$$

Lewis defines this grouped Variables as a factor called Lewis form factor represented by Y

$$\boxed{Y = \frac{t^2}{6hm}}$$

$$\boxed{P_t = mb\sigma_b Y} \quad \text{— (2)}$$

This gives the relationship between the tangential force and the bending stress.

→ As the stress increases, force increase and vice versa.

But the gear is made up of some material, the stress that can develop in a material is limited beyond that maximum stress there is material failure! Corresponding force at maximum stress is referred as beam strength.

No one wants to use the gear to test till the material's maximum strength, but it can be used as a alarming red signal!

So to avoid confusion, this maximum value is represented as S_b

$S_b \rightarrow$ beam strength in N

σ_b – Permissible bending Stress N/mm^2

$$\boxed{S_b = m\, b\, \sigma_b\, Y} \quad - \; (3)$$

– in order to avoid failure of gear tooth due to bending, the beam strength should be higher than the tangential force acting on the gear tooth.

$$\boxed{S_b > P_t}$$

Note: There is difference between (2) and (3).

(2) is based on the stress developed

(3) is based on the material strength.

Lewis form factor is experimentally measured and will be provided in the data book.

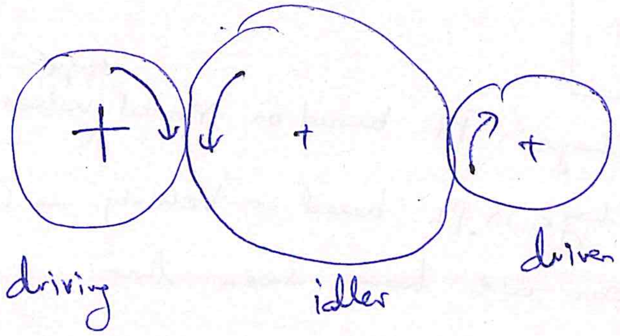
In the design of gears, two toothed wheels are there – a pinion – gear of smaller dimension and a gear.

The module, m and width, b will be same for mating gears, so when same material is used for both the pinion and gear then the pinion is the weakest.

If the materials are different then the value $\boxed{\sigma_b Y}$ determines the weaker between the pinion and gear.

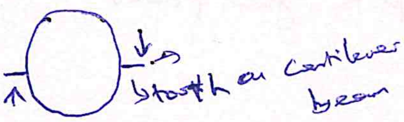
Permissible bending stress:
- Previously we have set $\sigma_b \rightarrow$ (Sut) as the material's maximum tensile stress to find beam strength, but in application the gear tooth undergoes a time varying fluctuating load.

- Considering a gear train of this form

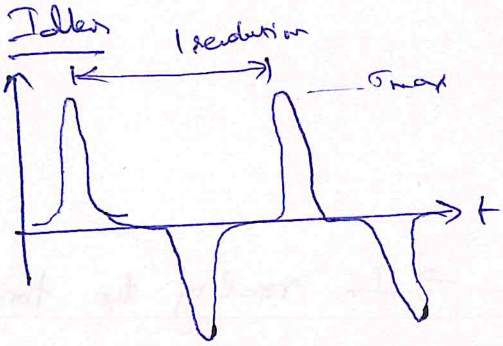


The stresses developed in driving and driven will be one directional on idler it will be bidirectional

Driving & driven



Idler



$\sigma_{mean} = \sigma_m = \dfrac{\sigma_{max}}{2}$ over one cycle.

Stress amplitude $\sigma_a = \dfrac{\sigma_{max}}{2}$

$\sigma_{mean} = 0$, $\sigma_a = \sigma_{max}$ Stress amplitude

→ For components under fluctuating load, endurance limit is the criterion for design

- There are several factors influencing the σ_{max} peak for gears such as surface finish, size of gear tooth, reliability, stress concentration, direction of rotation

- It is difficult to quantify these influences. Earle Buckingham suggested that the endurance limit stress of a gear tooth is 1/3rd of the material's tensile strength

$$\boxed{S_e = \frac{1}{3} S_{ut}}$$

Effective load on gears: Change in P_t based on rated torque values $\rightarrow C_s$

Change in P_t based on velocity $\rightarrow C_v$

From the force analysis of gear we have seen that

$$\text{Power} = \frac{2\pi n M_t}{60}$$

n - rpm

$$M_t = \frac{P_t \times d'}{2}$$

P_t → tangential force; d' → pitch circle dia of the gear

for a given power,

$$\frac{2\pi n M_t}{60} = \text{Power}, \quad \frac{P_t \times d'}{2} = \frac{\text{Power} \times 60}{2\pi n}$$

P_t tangential force → depends on the rated power and rated speed.

→ In reality the torque developed by the power source varies based on the load. Ex: While riding a bicycle uphill you need to apply lot of force / torque.

In design we have to consider the maximum torque value that a gear can take. This is quantified using service factor, C_s.

C_s defined as $$C_s = \frac{\text{Maximum torque}}{\text{Rated torque}} = \frac{P_{t\,max}}{P_{t\,rated}}$$

Rated torque → is the torque that the system is designed to operate for long time / continuously over its operational life.

Maximum torque is the highest torque that an System can handle without any failure. Typically higher than the rated torque.

$C_s > 1$. Values of Service factor will be given in the data book.

now $P_{t_{max}} = C_s . P_t$

Change in P_t based on Velocity :

When gears rotate at appreciable speed, there is considerable dynamic forces, $P_t \rightarrow$ we have calculated based on static conditions, there is lot of contact $\rightarrow$ impact forces, elasticity of bodies, errors in manufacturing, inertia resulting in dynamic loads.

There are two methods for accounting dynamic loads

1) Approximate estimation using Velocity factor $\rightarrow$ used in initial design

2) Using Buckingham's equation $\rightarrow$ used in final stages of design

1) Barth developed empirical relations for Velocity factor C_v

Ordinary to Commercially cut gears	Accurately hobbed & generated gears	Precision gears (machining parts)
$C_v = \frac{3}{3+V}$	$C_v = \frac{6}{6+V}$	$C_v = \frac{5.6}{5.6+\sqrt{V}}$
$V < 10\,m/s$	$V < 20\,m/s$	$V > 20\,m/s$

where V is the pitch line velocity defined as $V = \frac{\pi d' n}{60 \times 10^3}$ d' in mm, n rpm, in m/s

$C_v < 1$ as $V \uparrow$, $C_s \downarrow$

Now the effective ~~tanngential~~ tangential force is $\boxed{P_{eff} = \frac{C_s}{C_V} P_t}$ → Only for initial stages of gear design.

2) In the final stages of gear design, the dynamic load is calculated by equations derived by Earle Buckingham $\boxed{1950s}$

$$P_{eff} = \underbrace{C_S P_t}_{\text{static including service factor}} + P_d \rightarrow \text{dynamic load}$$

$$P_d = \frac{21V(Ceb + P_t)}{21V + \sqrt{Ceb + P_t}}$$

P_d – incremental dynamic load

C → deformation factor in N/mm²

e – Sum of errors between two meshing teeth. (mm)

b – face width of the tooth

P_t → in N

$$C = \frac{k}{\left[\frac{1}{E_p} + \frac{1}{E_g}\right]}$$

k → constant depending on the form of the tooth → vary depending on the gear profile

E_p & E_g are the Young's modulus in N/mm²

~~The Buckingham dynamic load P_d is for~~

(8)

Review: 1) we Started with beam approximation of gear tooth

We got $P_t = \frac{b\,\sigma_b t^2}{6h}$

Lewis gave $P_t = \frac{m b\,\sigma_b \cancel{t^2}}{} \underbrace{\left[\frac{t^2}{6hm}\right]}_{Y}$ Lewis form factor

2) We saw $\boxed{S_b = m b\,\sigma_b Y}$ defining the bending strength

3) Change in P_t due to fluctuating load

$$\boxed{S_e = \frac{1}{3} S_{ut}}$$

4) Change in P_t due to 1) Service factor

2) Velocity factor

- Barth's formula for initial design
- Buckingham's formula for final design

Estimation of module based on beam Strength :

→ for a given Power, factor of Safety, ultimate tensile strength, number of teeth, find the module for initial design.