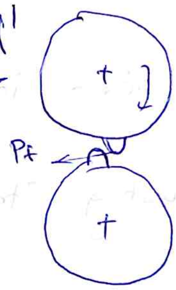
Summarizing :

- We started with force analysis of gears, where for a given Power, ~~actin~~ we can get the tangential force P_t acting on a gear

$$\underset{\text{in (kW)}}{\text{Power}} = \frac{2\pi n M_t}{60 \times 10^3} \quad , \quad M_t = \frac{P_t \times d'}{2}$$



$$\Rightarrow \boxed{P_t = \frac{\text{Power} \times 60 \times 10^6}{\pi n d'}} \quad - (1)$$

- Later in effective load on gear tooth we have seen two factors

1) The first factor is <u>Service factor</u> C_s, eventhough the gears are designed for rated torque at times there can be peak torques resulting in a higher effective torque

$$\boxed{P_{eff} = C_s P_t} \quad - (2) \quad , \text{ where } C_s = \frac{\text{Maximum torque}}{\text{rated torque}}$$

$$C_s > 1$$

2) The second factor is Velocity factor C_v – in order to account for the dynamic load there are two methods

(i) Approximate method based on Velocity factor C_v
↳ for initial gear design

(ii) Precise Calculation by using Buckingham dynamic load estimation

For the next section on estimating module based on beam strength of gear, we will start with the approximate method.

Where $P_{eff} = \frac{C_s}{C_v} P_t$, $C_v < 1$. Relation developed by Barth.

$- (3)$

(1)

On the material side, from Lewis ~~of~~ Cantilever approximation of gear tooth we have

$$S_b = m b \cancel{\sigma_b}^{S_{ut}} Y$$

where S_b is the bending strength of gear tooth

- Also we have seen in permissible bending stress that a gear undergoes fluctuating stresses. So, $\frac{S_{ut}}{3}$ should be used

$$\Rightarrow \quad S_b = m b \frac{S_{ut}}{3} Y \quad - (4)$$

Usually all designs will have a factor of safety "fos"

The bending strength of the gear tooth should be higher than P_{eff} to avoid failure from bending.

$$S_b = P_{eff} * fos$$

Equating these two we are intended in finding the desired module for the gears. From (1), (3) and (4)

$$m b \frac{S_{ut}}{3} Y = \frac{C_s}{C_v} \frac{Power \times 60 \times 10^6}{\pi n d'} \times fos$$

module $m = \frac{d'}{Z} \Rightarrow \boxed{d' = m Z}$

Rewriting

$$\boxed{m^2 = \frac{60 \times 10^6}{\pi} \left[\frac{Power \times C_s \times fos}{C_v \, n \, Z \, b \, Y \left(\frac{S_{ut}}{3}\right)} \right]} \quad - (5)$$

This is ok, if they give you width of the gear tooth,
But mostly the width of the gear is not known in initial design.

Thus in equation (5) both L.H.S and R.H.S have unknowns
to have a single variable, the initial design starts with an
assumption on ratio of $\frac{b}{m}$, this results in

$$m^3 = \left[\frac{60\times10^6}{\pi}\left[\frac{\text{Power}\times fos\times C_s}{C_v\times n\times\left(\frac{b}{m}\right)\times\left(\frac{S_{ut}}{3}\right)Y}\right]\right]$$

$$\boxed{m = \left[\frac{60\times10^6}{\pi}\left[\frac{\text{Power}\times fos\times C_s}{C_v\times n\times\underbrace{\left(\frac{b}{m}\right)}_{\text{assumption}}\times\left(\frac{S_{ut}}{3}\right)Y}\right]\right]^{1/3}} \quad -(6)$$

Should be used for initial design estimation of module of gear.

as we have used

$$P_{eff} = \frac{C_s}{C_v} P_t$$

Problem:

Design a pair of spur gears with 20° full depth involute teeth based on the Lewis equation. The velocity factor is to be used to account for dynamic load. The pinion gear is connected to 10 kW, 1440 rpm motor. The starting torque of the motor is 150% of the rated torque. The speed reduction is 4:1. The pinion as well as the gear is made of plain
(3) carbon steel $S_{ut} = 600\ N/mm^2$. The factor of safety can be taken as 1.5. Design the gears, specify their dimensions.

Wear strength of a gear tooth:

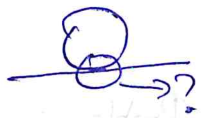
Wear failure occurs when ~~th~~ contact stress between two meeting teeth exceeds the surface endurance strength

To reduce wear on gears, the gear tooth surfaces in contact should have proper surface hardness properties.

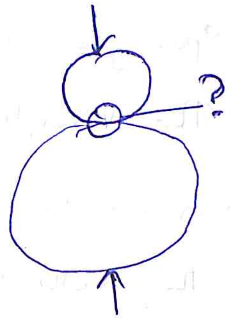
The analysis of wear strength of a gear was done by Earle Buckingham 1920's. Buckingham's equation for wear strength is based on Hertz's theory on contact.

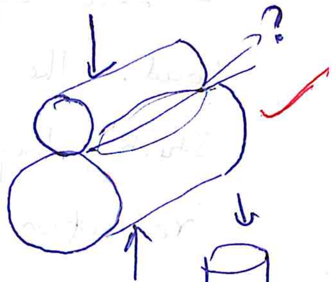
First we will look into useful/ relevant (gears) results of Hertz theory of contact stress. Hertz ~~has~~ proposed theories on contact between two elastic bodies — giving relationship between deformation in the contact area and the contact stress developed. The theory is proposed for contact between two elastic bodies with non-adhesi contact. There are five types of commonly used solutions

1) Contact between a sphere & elastic half space



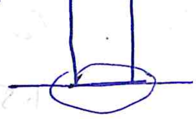
2) Contact between two spheres



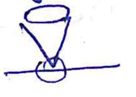


3) Contact between two cylinders with parallel axes

4) Contact between cylinder flat face with elastic half space



5) Contact between conical identer and elastic half space



Among these five types, Earle Buckingham considered the contact stress between two cylinders with parallel axes.

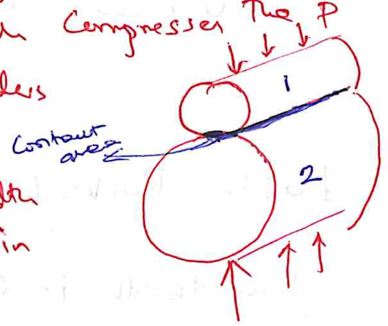
Why two cylinders with parallel axes? — Probably because, contact type in line contact in gears
type for gears

The relationship between contact stress, σ_c and the force compressing the cylinders together, P is

$$\boxed{\sigma_c = \frac{2P}{\pi b l}} \quad -\ (7)$$

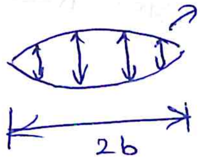
from:
Hertz theory

Please note: P is not the load as we defined in gears it is the force which compresses the two cylinders



Also:
b - is not the width of the gear as in other topic

- For time being you can ignore that we are using these for gears - if it confuses you.
- Between the two cylinders there is a contact area, Hertz found that the area of contact depends on the force P applied and elasticity of the contacting bodies.



$$\boxed{b = \left[\frac{2P(1-\mu^2)\left[\frac{1}{E_1}+\frac{1}{E_2}\right]}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]^{1/2}} \quad -\ (8)$$

E_1 & E_2 Young's modulus of the cylinder 1 & 2

$\mu \rightarrow$ Poisson's ratio

$d_1, d_2 \rightarrow$ diameters of the cylinder

(7) & (8) are from Hertz contact stress theory

Equations (7) & (8) can be clubbed together and rearranged as

$$\sigma_c^2 = 0.35 \left(\frac{P}{l}\right) \left[\frac{\frac{1}{r_1} + \frac{1}{r_2}}{\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}\right] \quad -(9)$$

Assumptions:

Of course, Hertz used assumption in his analysis such as the cylinders are made up of isotropic material, Elastic limit is not exceeded, dimensions of r_1 & r_2 are large compared to the dimension of contact area defined using b.

Earle Buckingham while applying Hertzian contact theory to a pair of gear teeth in contact need to provide values for r_1, r_2, P.

In the context of gears, the force P is the force acting along the pitch line. => $P = P_N$, but in force analysis of gears we have seen that, ~~from~~ only power involved in the transmission will be given to you, from there you need to find P_t and then $\boxed{P_N \text{ as } P_t / \cos\alpha.}$ -(10)

Ok, but what about r_1 & r_2 for gears?

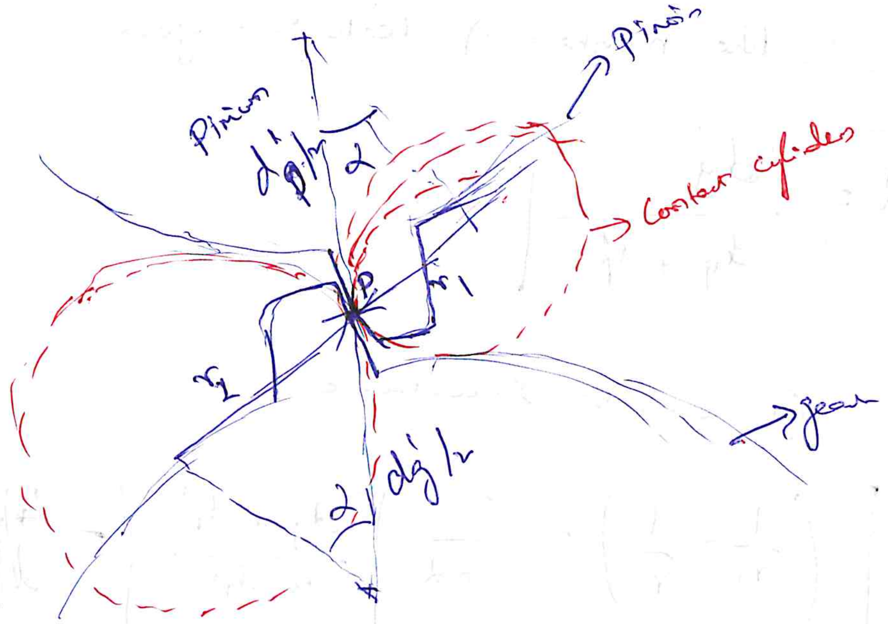
r_1 & r_2 are the radii of curvature at the pitch point.

Why so? — For this you have to overlap two figures to visualize

(6) - The contact occurs at the pitch point → when one pair of teeth carries the entire load

- When two pairs are in contact, the load "P" is imposed on the gear tooth near the pitch line area.

Refer fig: 17.42 b
and
17.43
in Bhandari



From figure you can see $\dfrac{r_1}{\frac{d_p'}{2}} = \sin\alpha \Rightarrow r_1 = \dfrac{d_p' \sin\alpha}{2}$

also $r_2 = \dfrac{d_g' \sin\alpha}{2}$, now $\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)$ in Contact Stress definition

we can rewrite interms of gear dimensions.

$$\boxed{\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{2}{\sin\alpha}\left(\frac{1}{d_p'} + \frac{1}{d_g'}\right)} \quad (11)$$

What we are trying to do in this wear strength topic ?, Similar to bending strength of gear "Lewis equation", we are trying to get the relationship between the force acting on gears P_t & contact stress developed. In Bending strength we used Cantilever beam assumption to relate P_t with σ_b, here Contact Stress to relate P_t with σ

In (11) we are having two variables d_p', d_g', lets us Combine into one by

(7) defining a factor Q, as

Ratio factor

$$\boxed{Q = \frac{2 d_g'}{d_g' + d_p'} = \frac{2 Z_g}{Z_g + Z_p}}$$

Where d' is the ~~diameter~~ pitch circle diameters of gears

$Z \rightarrow$ is the number of teeth in a gear

$$\boxed{Q = \frac{2d_g'}{d_g' + d_p'}} \quad -(12)$$

Using (12), (11) can be rewritten as

$$\boxed{\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{2}{\sin\alpha}\left[\frac{d_g' + d_p'}{d_g' d_p'}\right] = \frac{4}{Q d_p' \sin\alpha}} \quad -(13)$$

Utilizing (13) & (10) in (9)

$$\sigma_c^2 = \frac{1.4\, P_t}{b Q d_p' \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$$

rewritting in terms of P_t

$$\cancel{P_t = d_p b Q \sigma_c^2 \frac{\sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}}$$

$$P_t = b Q d_p' \left(\frac{\sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right) \sigma_c^2}{1.4}\right)$$

grouping the bracketed terms and represent it as K

8

$$\boxed{P_t = b Q d_p' \underset{\sim}{K}} \quad (14), \quad \text{where} \quad \boxed{K = \frac{\sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right) \sigma_c^2}{1.4}} \quad (15)$$

K has the stress component

K is the load-stress factor

Now compare this with Lewis's equation $P_t = m b \sigma_b Y$

This gives the relationship between the tangential force & contact stress developed and vice versa

Equation (14) gives the relationship between the tangential force acting on the gear tooth 'P_t' with the contact stress developed 'σ_c'.

Similar to "ultimate tensile strength", a material will have surface endurance strength. The tangential force corresponding to the surface endurance strength is called wear strength.

Wear strength is the maximum value of the tangential force that the tooth can transmit without wear failure.

To have difference between P_t & wear strength a variable S_w is used, same as (14) but with change in variable and maximum contact stress, that the material can withstand.

$$\boxed{S_w = b\,Q\,d_p'\,k} \quad -(18)$$

$\sigma_{c\,ultimate}$ should be used

Equation (18) is known as Buckingham's equation for wear.

- To have simplified expression for k, assuming same material properties

$$k = \frac{\sigma_c^2 \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}$$

$E_1 = 206$ Gpa
E_2
$\alpha = 20°$

Also the contact stress can be expressed in terms of Brinell hardness number (BHN)

$\sigma_c = 0.27$ (BHN) kgf/mm²

$$\boxed{\sigma_c = 0.27 \times 9.81 \times \text{BHN} \ \text{N/mm}^2}$$

(9)

Therefore $k = 0.156 \left(\frac{\text{BHN}}{100}\right)^2$

Applicable only for $E_1, E_2 = 206$ GPa, $\alpha = 20°$; if these values are changed then K has to be calculated

Estimation of module based on wear strength:

- As a designer of gear we want $S_w > P_{eff}$

- Considering factor of Safety $\boxed{S_w = P_{eff} \times fos}$ - (17)

Remember P_{eff} → read Effective load on gears topic, for initial design
$P_{eff} = \frac{C_s}{C_v} P_t$

- P_t → we need to calculate from the power

$$\text{Power (in kW)} = \frac{2\pi \overset{\text{in rpm}}{n} M_t}{60 \times 10^6} \quad M_t \rightarrow P_t \times \frac{d'}{2}$$

rewriting $P_t = \frac{\text{Power} \times 60 \times 10^6}{\pi n \times d'}$, in terms of module $m = \frac{d'}{z}$

$$P_t = \frac{(\text{Power in kW}) \times 60 \times 10^6}{\pi n \times m z}$$

$$\boxed{P_{eff} = \frac{C_s}{C_v} P_t \Rightarrow \frac{C_s \, \text{Power} \times 60 \times 10^6}{C_v \, \pi n m z}} \quad (18)$$

S_w on the other hand

From (16)

$S_w = b Q d_p' K$

in (16) we have pinion k so in (18) we need to

Consider pinion $\boxed{z_p \text{ and } n_p}$

as in problems only ratio will be given

$$\boxed{S_w = m \left(\frac{b}{m}\right) Q \, m z_p K} \quad -(19)$$

Equating (18) & (19) as in (17)

We get

$$\boxed{m = \left[\frac{60 \times 10^6}{\pi} \left[\frac{\overset{\text{in kW}}{\text{Power}} \times C_s \times fos}{Z_p^2 \, n_p \, C_v \left(\frac{b}{m}\right) Qk} \right] \right]^{1/3}} \quad \text{— (20)}$$