Rankine Cycle - Numericals

Dr. Vinod Kumar Sharma
Department of Mechanical Engineering
NIT Calicut

Example - 1

Ques. In a reheat cycle, steam at 550°C expands in an h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a I.p. turbine to 40°C. If the maximum moisture content at the turbine exhaust is limited to 14.67%, find

(a) the reheat pressure,

(b) the pressure of steam at the inlet to the h.p. turbine,

(c) the net specific work output

(d) the cycle efficiency.

Assume all ideal processes.

Solution:

From S.T. at 40°C, 14.67% moisture

$\therefore x = 0.8533$

$p_3 = 0.0738$ bar

$h_f = 167.6$ kJ/kg;

$h_{fg} = 2406.7$ kJ/kg

$\therefore h_4 = 167.6 + 0.85322 \times 2406.7$

$h_4 = 2221.2$ kJ/kg

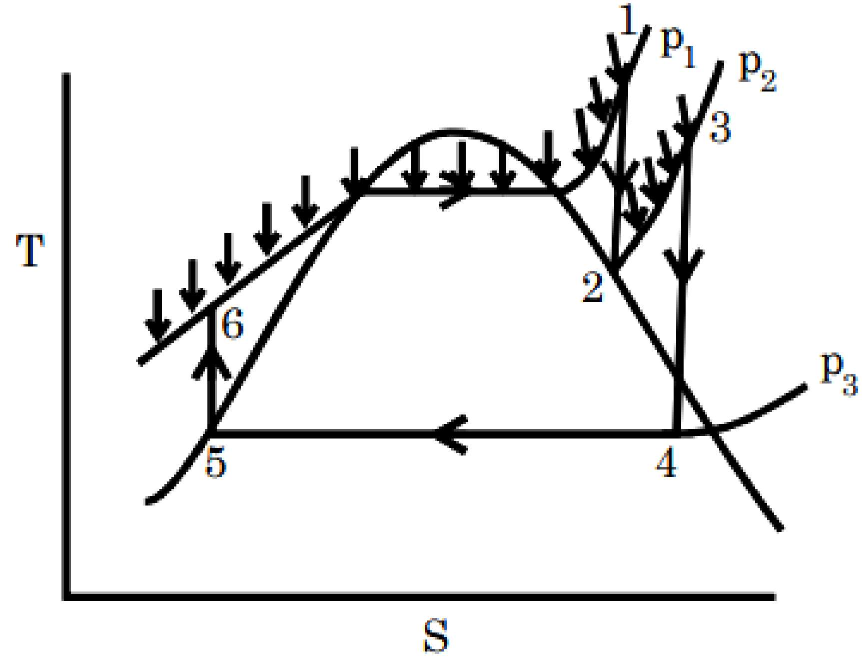
$s_f = 0.573$ kJ/kg K

$s_{fg} = 7.685$ kJ/kg K

$s_4 = 0.573 + 0.8533 \times 7.685$

$s_4 = 7.1306$ kJ/kg

$\therefore$ at 400°C and $s_4 = 7.1306$



- From Steam Table

(a) reheat pressure

Pr = 20 bar,

h_3 = 3247.6 kJ/kg

At 20 bar saturation h_2 = 2797.2 kJ/kg

$\therefore$ s_2 = 6.3366 kJ/kg K

(b) Pressure of steam at the inlet to the h.p. turbine

At 550°C and 6.3366 kJ/kg K

From Steam Table

Pr = 200 bar

$\therefore h_1$ = 3393.5 kJ/kg

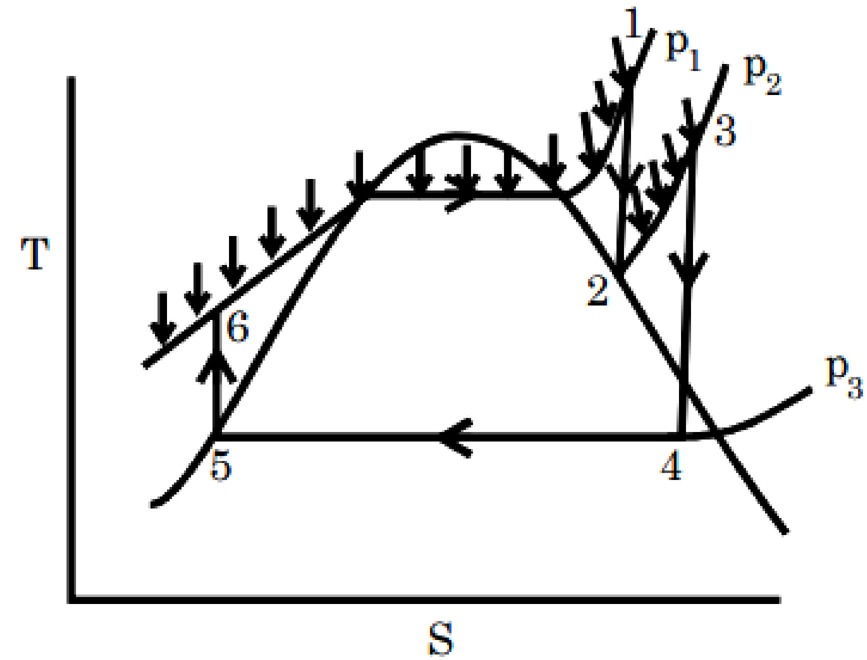
$\therefore h_5$ = 167.6 kJ/kg

W_P = 0.001 × (20000 – 7.38) kJ/kg

W_P = 20 kJ/kg

$\therefore h_6 = h_5 + W$ = 187.6 kJ/kg

$\therefore W_T = (h_1 - h_2) + (h_3 - h_4)$ = 1622.7 kJ/kg



(c) net work output

$W_{net} = W_T - W_P =$ **1602.7 kJ/kg**

Heat input $Q_1 = (h_1 - h_6) + (h_3 - h_2)$

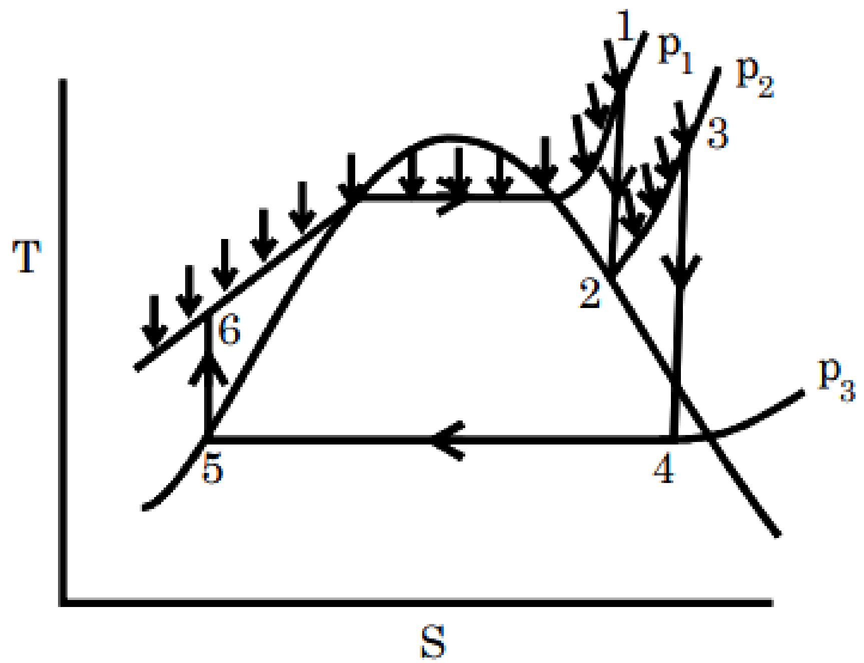
$Q_1 = (3393.5 - 187.6) + (3247.6 - 2797.2)$ kJ/kg

$\mathbf{Q_1 = 3656.3}$ **kJ/kg**

(d) Cycle efficiency

$$\eta = \frac{1602.7}{3656.3} \times 100\%$$

$\boldsymbol{\eta}$ **= 43.83 %**



Example - 2

The net power output of an ideal regenerative-reheat steam cycle is 80MW. Steam enters the high pressure (H.P.) turbine at 80 bar, 500°C. After expansion to 7 bar, some of the steam then goes to an open heater and the balance is reheated to 400°C, after which it expands to 0.07 bar. Compute

(a) The reheat pressure,

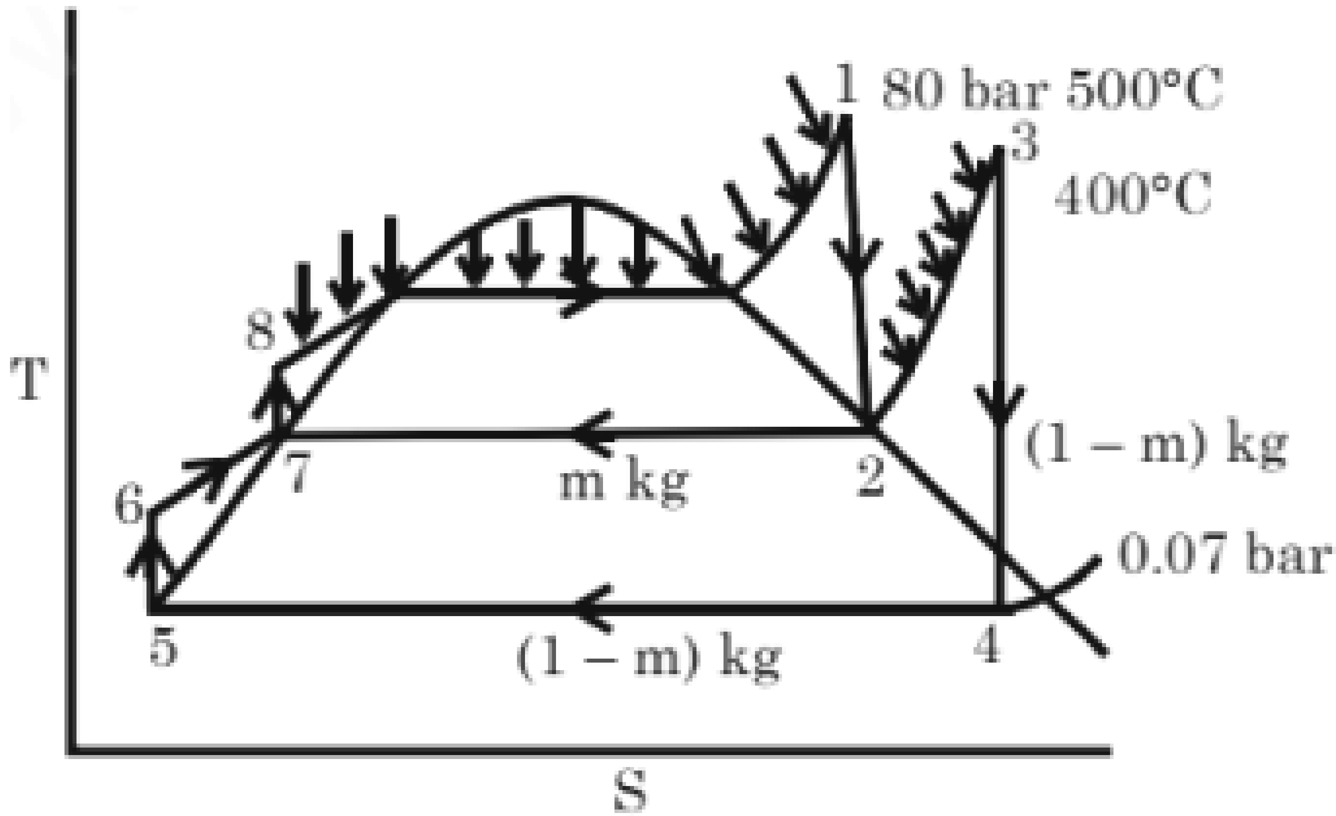
(b) What is the steam flow rate to the h.p. turbine,

(c) Calculate the cycle efficiency

(d) If there is a 10°C rise in the temperature of the cooling water, what is the rate of flow of cooling water in the condenser,

(e) If the velocity of steam flowing from the turbine to the condenser is limited to a maximum of 130 m/s, find the diameter of the connecting pipe.

Solution:



From S.T of 80 bar 500°C

h_1 = 3398.3 kJ/kg

s_1 = 6.724 kJ/kg – K

s_2 = 6.725 at 6.6 bar

so Reheat pr. 6.6

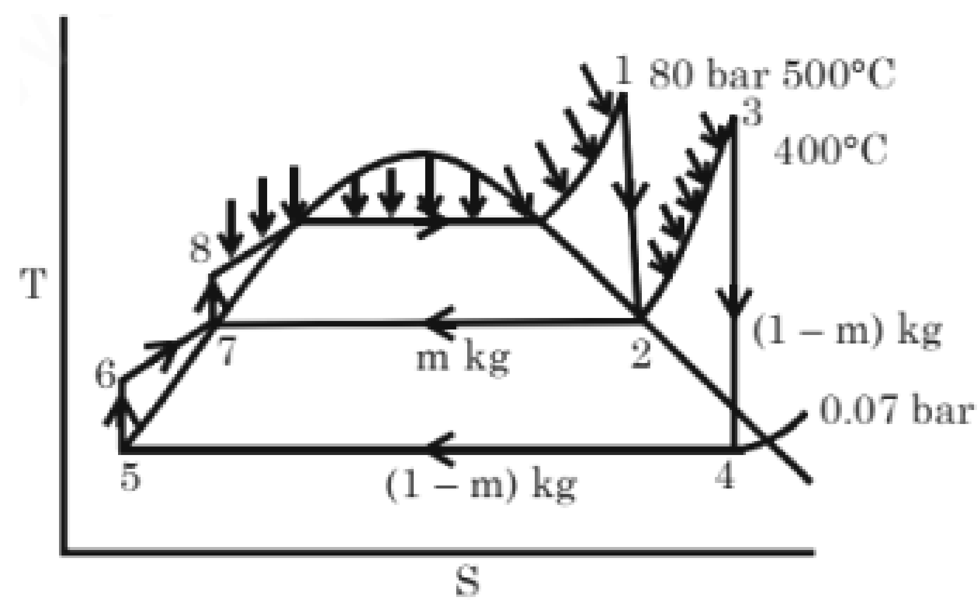
∴ h_2 = 2759.5 kJ/kg

h_3 = 3270.3 + 0.6(3268.7 – 3270.3)

h_3= 3269.3 kJ/kg

s_3 = 7.708 + 0.6 (7.635 – 7.708)

s_3 = 7.6643 kJ/kg K



At 0.07 bar

h_f = 163.4,

h_{fg} = 2409.1

h_f = 0.559,

s_{fg} = 7.71

∴ If quality is x then

7.6642 = 0.559 + (x × 7.717)

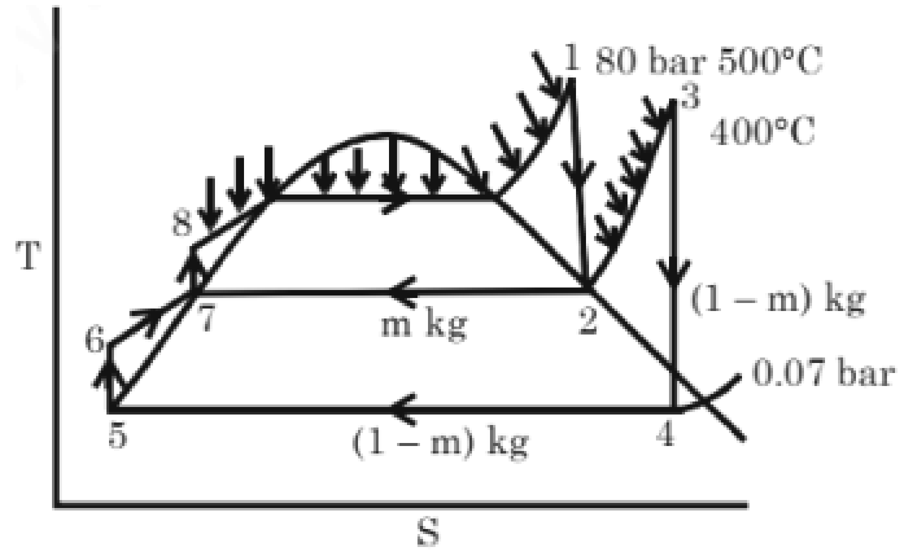
⇒ x = 0.9207

∴ h_4 = 163.4 + 0.9207 × 2409.1

h_4 = 2381.5 kJ/kg

h_5 = 163.4 kJ/kg ≈ h_6,

h_7 = 686.8 kJ/kg ≈h_8



$\therefore$ From Heat balance of heater

$$m \times h_2 + (1 - m)\, h_6 = h_7$$

$\therefore$ m = 0.2016 kg/kg of steam at H.P

$\therefore$ (1 – m) = 0.7984

$W_T = h_1 - h_2 + (1 - m)\,(h_3 - h_4)$

W_T= 1347.6kJ/kg

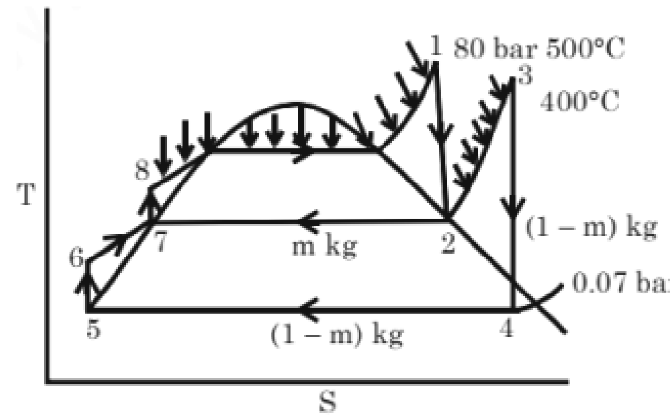
$Q = (h_1 - h_8) + (1 - m)\,(h_3 - h_2)$

Q = 3118.5 kJ/kg at H.P

(a) Reheat pr. 6.6 bar

(b) Steam flow rate at H.P = $\dfrac{80 \times 10^3}{1347.6}$

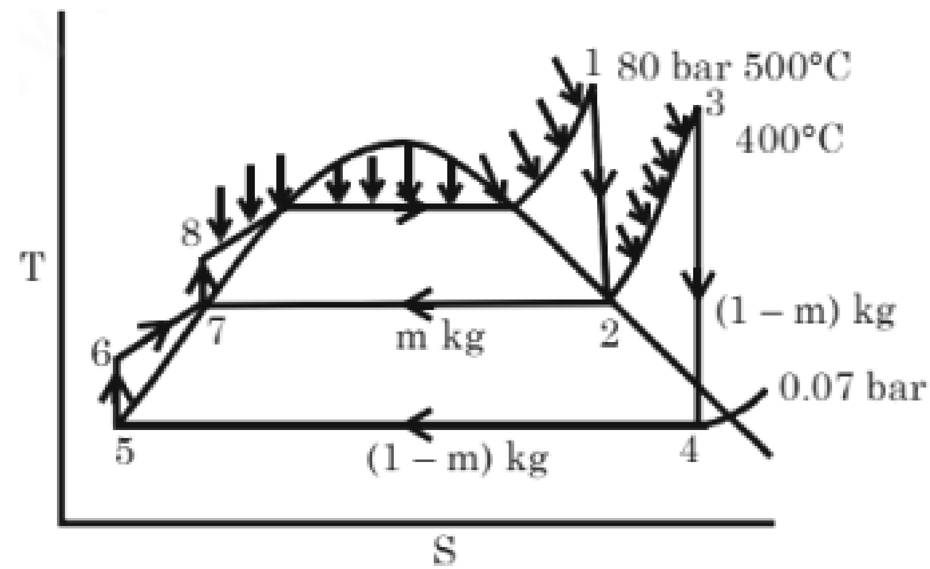
= 59.36 kg/s



(c) Cycle efficiency (η) = $\frac{W}{Q}$

$$\eta = \frac{1347.6}{3118.5} \times 100\%$$

η = 43.21%



Steam leaves the boiler in a steam turbine plant at 2 MPa, 300°C and is expanded to 3.5 kPa before entering the condenser. Compare the following four cycles:

(1) A superheated Rankine cycle.

(2) A reheat cycle, with steam reheated to 300°C at the pressure when it becomes saturated vapor.

(3) A regenerative cycle, with an open feedwater heater operating at the pressure where steam becomes saturated vapor.

(4) A regenerative cycle, with a closed feedwater heater operating at the pressure where steam becomes saturated vapor.

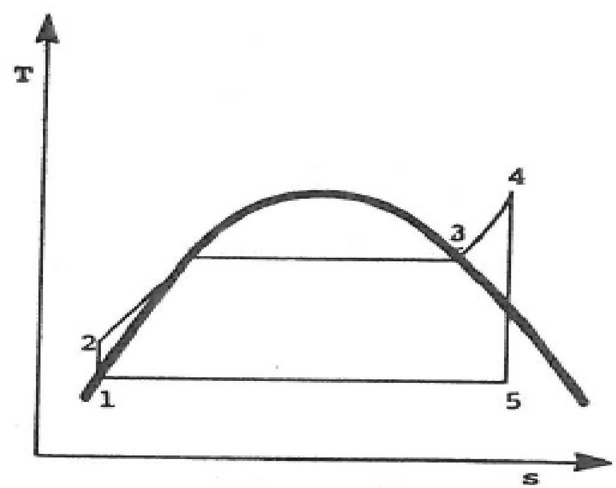


Figure 38. Rankine cycle

SOLUTION

(1) Referring to Figure 38, the steam tables show that

$$h_4 = 3{,}025 \text{ kJ/kg}$$
$$s_4 = 6.768 \text{ kJ/kg} - \text{K}$$

At $P = 3.5$ kPa,

$$s_g = 8.521 \text{ kJ/kg} - \text{K}$$
$$s_f = 0.391 \text{ kJ/kg} - \text{K}$$

Since $s_5 = s_4$, steam at 5 is a mixture of liquid and vapor. The quality is found as

$$\begin{aligned} x_5 &= \frac{s_5 - s_f}{s_{fg}} \\ &= \frac{6.768 - 0.391}{8.130} \\ &= 0.785 \end{aligned}$$

Therefore,

$$\begin{aligned} h_5 &= h_f + x_5 h_{fg} \\ &= 112 + 0.785(2{,}438) \\ &= 2{,}023 \text{ kJ/kg} \end{aligned}$$

hence

$$\begin{aligned} w_{45} &= h_4 - h_5 \\ &= 3{,}025 - 2{,}023 \\ &= 1{,}002 \text{ kJ/kg} \end{aligned}$$

Now

$$\begin{aligned} w_{12} &= h_1 - h_2 \\ &= v_f(p_1 - p_2) \\ &= 0.0010(0.0035 - 2) \times 10^3 \text{ kJ/kg} \\ &= -2 \text{ kJ/kg} \end{aligned}$$

Therefore, the net work output is

$$w = w_{45} + w_{12} = 1{,}000 \text{ kJ/kg}$$

Heat input is

$$q_{42} = h_4 - h_2$$

But

$$h_2 = h_1 - w_{12} = 112 + 2 = 114 \text{ kJ/kg}$$

therefore,

$$q_{42} = 3{,}025 - 114 = 2{,}911 \text{ kJ/kg}$$

Thus,

$$\eta = \frac{w}{q_{42}} = \frac{1{,}000}{2{,}911} = 0.344$$

Also

$$\text{Specific Steam Consumption} = \frac{3,600}{w} = \frac{3,600}{1,000} = 3.6 \text{ kg/kWh}$$

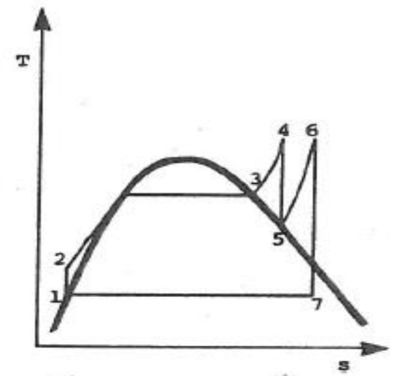


Figure 39. Reheat cycle

(2) Refer to Figure 39, and note that since

$$s_5 = s_{sat} = s_4 = 6.768 \text{ kJ/kg} - \text{K}$$

the pressure at reheat point 5 can be found using the steam tables. Interpolating between 0.55 MPa and 0.6 MPa gives

$$P_5 = 0.588 \text{ MPa.}$$

Then

$$\begin{aligned} h_5 &= 2,753 + \frac{0.588 - 0.55}{0.60 - 0.55}(2,757 - 2,753) \\ &= 2,753 + \frac{0.038}{0.05} \times 4 \\ &= 2,756 \text{ kJ/kg} \end{aligned}$$

As 6 and 5 are on the same isobar, by interpolation

$$
\begin{aligned}
h_6 &= 3,065 + \frac{0.588 - 0.5}{0.60 - 0.5}(3,062 - 3,065) \\
&= 3,065 + \frac{0.088}{0.1}(-3) \\
&= 3,062.4 \text{ kJ/kg} \\
s_6 &= 7.460 + 0.88(7.373 - 7.460) \\
&= 7.460 + 0.88(-0.087) \\
&= 7.384 \text{ kJ/kg} - \text{K}
\end{aligned}
$$

At $P = 3.5$ kPa,

$$
\begin{aligned}
s_g &= 8.521 \text{ kJ/kg} - \text{K} \\
s_f &= 0.391 \text{ kJ/kg} - \text{K}
\end{aligned}
$$

Since $s_7 = s_6$, the quality at 7 is found as

$$
x_7 = \frac{7.384 - 0.391}{8.130} = 0.86.
$$

Then

$$h_7 = 112 + 0.86(2{,}438)$$
$$= 112 + 2{,}095 = 2{,}207 \text{ kJ/kg}$$

The net work output is given by

$$w = w_{45} + w_{67} + w_{12}$$
$$= (3{,}025 - 2{,}765) + (3{,}062.4 - 2{,}207) - 2$$
$$= 1{,}122.4$$

The heat input is

$$q = q_{42} + q_{65}$$
$$= 2{,}911 + (h_6 - h_5)$$
$$= 2{,}911 + (3{,}062.4 - 2{,}756)$$
$$= 3{,}217.4$$

Therefore,

$$\eta = \frac{1{,}122.4}{3{,}217.4} = 0.349$$

and

$$\text{s.s.c.} = \frac{3{,}600}{w} = \frac{3{,}600}{1{,}122.4} = 3.2 \text{ kg/kWh.}$$

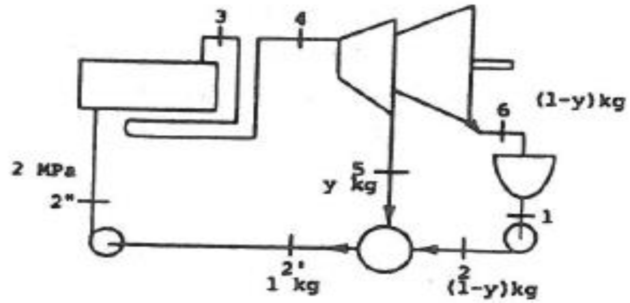


Figure 40. (a) Equipment schematic for regenerative cycle

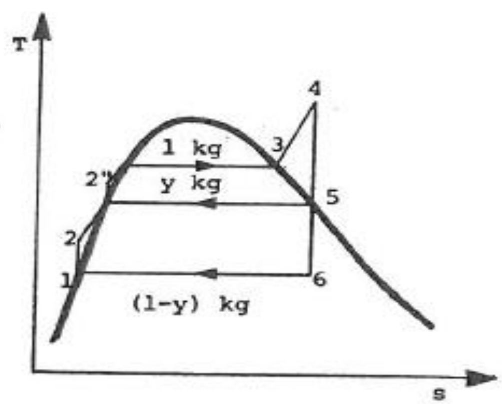


Figure 40. (b) Regenerative cycle

(3) Refer to Figures 40 (a) and 40 (b). The work is as in (b)

$$w_{45} = 269 \text{ kJ/kg}$$

Next determine the amount of steam bled off at 5. Consider an energy balance for the open feedwater heater with

$$h_{2'} = yh_s - (1-y)h_2$$

which gives

$$y = \frac{h_{2'} - h_2}{h_5 - h_2}$$

To find the value for $h_{2'}$, enter the steam tables. At 5 the pressure is known (P = 0.588 MPa) and the state of the steam is given as saturated vapor. Therefore, by interpolating between the values of 0.5 MPa and 0.6 MPa, obtain

$$\begin{aligned} h_{2'} &= 656 + \frac{0.588 - 0.55}{0.60 - 0.55}(670 - 656) \\ &= 656 + \frac{0.038}{0.05} \times 14 \\ &= 666.6 \text{ kJ/kg} \end{aligned}$$

Then

$$\begin{aligned} y &= \frac{666.6 - 114}{2,756 - 114} \\ &= \frac{552.6}{2,642} \\ &= 0.209 \end{aligned}$$

Hence,

$$\begin{aligned} w_{56} &= (1-y)(h_5-h_6) \\ &= 0.791(2{,}756-2{,}023) \\ &= 580 \text{ kJ/kg} \end{aligned}$$

also

$$\begin{aligned} w_{2'2''} &= v_f(P_{2'}-P_{2''}) \\ &= 0.0011(0.588-2)\times 10^3 \\ &= -1.1\times 1.412 \\ &= -1.55 \text{ kJ/kg} \end{aligned}$$

Therefore,

$$\begin{aligned} w &= w_{45}+w_{56}+w_{12}+w_{2'2''} \\ &= 269+580-0.791\times 2-1.55 \\ &= 845.87 \text{ kJ/kg} \end{aligned}$$

The heat input is

$$\begin{aligned} q_{42''} &= 3{,}025-(666.6+1.55) \\ &= 2{,}356.8 \text{ kJ/kg} \end{aligned}$$

The efficiency of this cycle is

$$\eta = \frac{w}{q_{42''}} = \frac{845.87}{2{,}356.8} = 0.3595$$

and

$$\text{s.s.c.} = \frac{3{,}600}{w} = \frac{3{,}600}{845.9} = 4.25 \text{ kg/k Wh..}$$

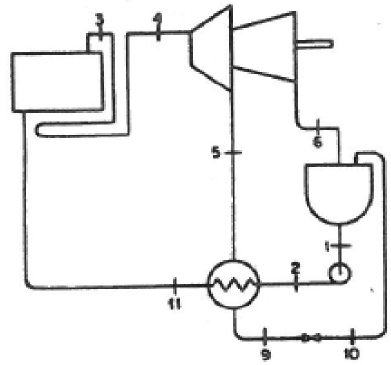


Figure 41. (a) Equipment diagram including closed heater

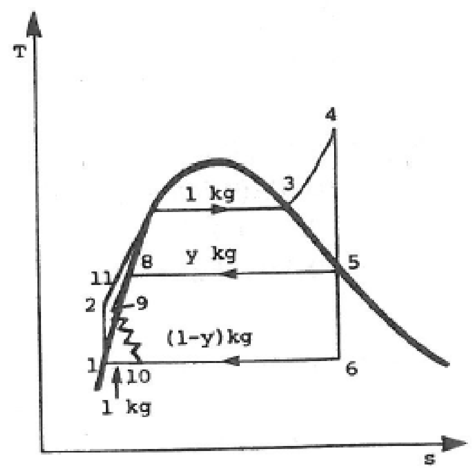


Figure 41. (b) A regenerative cycle with closed heater

(4) Refer to Figures 41 (a) and 41 (b). The work is as in part (b).

$$w_{45} = 269 \text{ kJ/kg}$$

Heat balance for the heater as a closed system gives

$$h_{21} = yh_5 - (1-y)h_2$$

giving

$$y = \frac{h_{11} - h_2}{h_5 - h_9}$$

Now in finding the enthalpies in the feed line, it is usual to make the following assumptions:

i. Neglect the feed pump term.

ii. Assume the enthalpy of the compressed liquid to be the same as that of the saturated liquid at the same temperature.

iii. Assume the states of the condensate extracted from the turbine, before and after throttling, to be the same as that of the saturated liquid at the lower pressure of the throttled liquid.

Using these assumptions

$$h_2 = h_1$$
$$h_{11} = h_8$$
$$h_9 = h_{10} = h_1$$

whence

$$y = \frac{h_8 - h_1}{h_5 - h_1}$$
$$= \frac{666.6 - 112}{2,756 - 112} = 0.209 \text{ kJ/kg}$$

Also,

$$w_{56} = 580 \text{ kJ/kg.}$$

Therefore,

$$\begin{aligned} w &= w_{45} + w_{56} + w_{12} \\ &= 269 + 580 - 2 = 847 \text{ kJ/kg} \end{aligned}$$

Heat input $q_{411} = 2{,}358.4$ kJ/kg.

Then

$$\eta = \frac{w}{q_{411}} = \frac{847}{2,358.4} = 0.360$$

and

$$\text{s.s.c.} = \frac{3,600}{w} = \frac{3,600}{847} = 4.25 \text{ kg/k Wh.}$$