Machine design II - Gears

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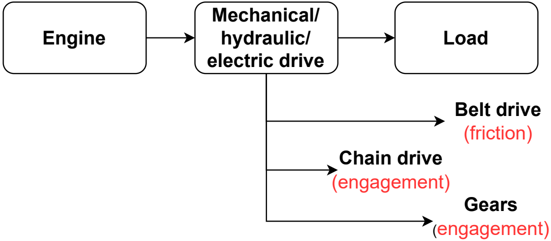
Mechanical drives

Mechanical drives are intended to transmit power over a certain distance

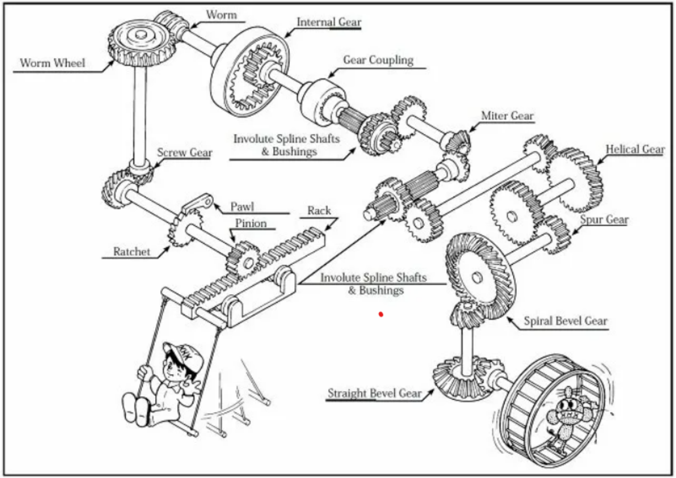


Choice of mechanical drives depends on

- Distance between the input and output shafts
- Velocity ratio- if there is slip velocity ratio will not be constant
- Shifting mechanism to obtain different speeds
- Installation and maintenance cost

Gears

Gear drives are toothed wheels which transmit power by means of sucessive engagement of teeth.



Advantages of gear drive

- Velocity ratio remains constant
- Center distance between the gears in small resulting in a compact construction
- A provision can be made in the gearbox for gear shifting, thus changing the velocity ration over a wide range.

Classification of gears

Gears are broadly classified into four groups- spur, helical, bevel and worm gears.

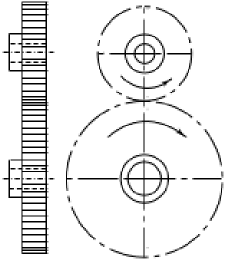


Fig. 17.1 *Spur Gears*

The teeth with involute profile are cut parallel to the axis of the shaft. Can be used only to transmit power between parallel shafts. Spur gear impose radial loads on the shaft.

The teeth of helical gears are cut at an angle with the axis of the shaft. Helical gears also have involute teeth profile in a plane perpendicular to the teeh element.

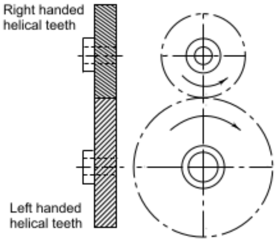


Fig. 17.2 *Helical Gears*

Helix angle in both the pinion and the gear is same; however, the hand of helix is opposite.

In helical gears, a right hand pinion meshes with an left hand gear and vice versa. Helical gear impose radial and axial thrust loads.

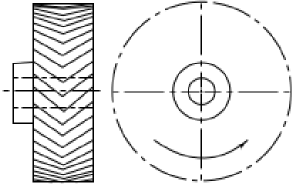


Fig. 17.3 *Herringbone Gear*

Special type of helical gears with opposing helix hand on the same gear reffered as Herringbone gears. This construction results in equal and opposite axial loads resulting in no axial load on the shaft.

Bevel gears have shape of a truncated cone. Used for transmitting power btw shafts which are $\perp$ to each other. 90^o is not a rigid condition. Tooth can be straight or spiral. They impose radial and axial loads.

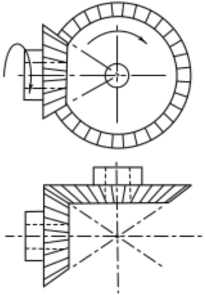


Fig. 17.4 *Bevel Gears*

Worm gear and choice of gears

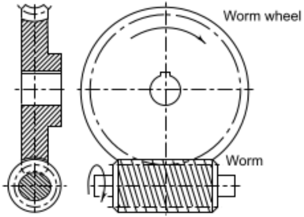


Fig. 17.5 *Warm Gears*

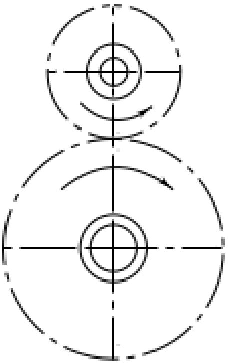
Used for shafts that won't intersect. The worm is in the form of a threaded screw meshing with a matching wheel. Worm wheel- imposes high radial load on the shaft. Worm imposes high axial load. This system is preffered for high speed reduction ratio.

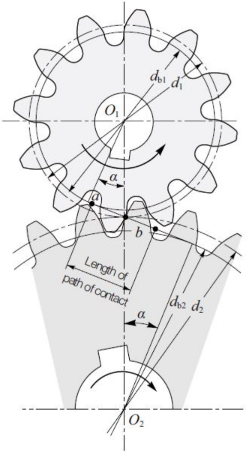
Selection of type of gears- 1^{st} step in gear design for any application

- Layout of shafts
- Speed reduction-single stage- Spur gear and Helical normally- 6 : 1 and it can go upto 10 : 1, bevel gears normally 1 : 1 and can go upto 3 : 1. For high speed reduction worm gears are preffered normally 60 : 1 and can go upto 100 : 1
- Power to be transmitted

Spur gears generate lot of noise in high speed applications- due to sudden contact. Helical gears are preffered for high speed applications as there is continuous and smooth contact.

Law of gearing





'The common normal to the tooth profile at the point of contact should always pass through a fixed point, called a pitch point, in order to obtain constant velocity ratio'.

Intuitively -Teeth on the wheels reduces slipping (by locking) between the wheels! Ok- what should be the shape, number of teeth to obtain zero slip?

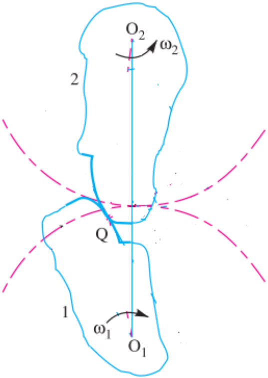


Fig. 28.7. Law of gearing.

Consider the portions of the two teeth, one on the wheel 1 and the other on the wheel 2

O_1 and O_2 are the centers of the wheel, the two wheels contact each other at the point Q

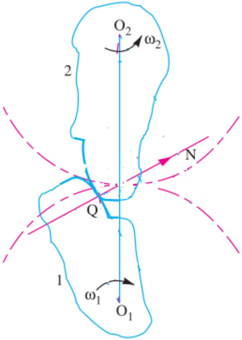


Fig. 28.7. Law of gearing.

Law of gearing

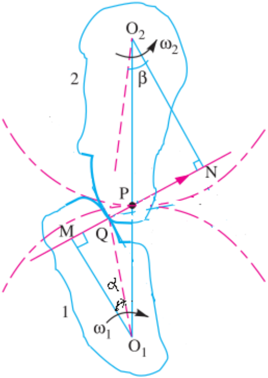
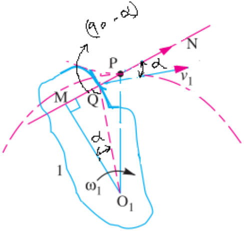


Fig. 28.7. Law of gearing.

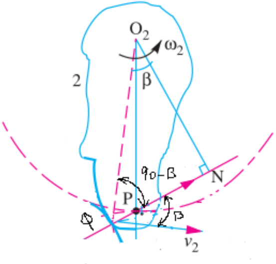
The line connecting the centers of the wheels intersect with the common normal @ P. Draw perpendicular lines O_1M and O_2N from the centers to the the normal.

Name the angle made between the lines O_1M and O_1Q as α and the angle made between the lines O_2N and O_2P as β.-So far things are defined geometrically



Have a close look at the wheel 1. The contact point Q will have a velocity of $v_1 = \omega_1 * O_1Q$, the angle between MN and the velocity at Q is α.

Similarly, closer look at the wheel 2. The contact point Q will have a velocity of $v_2 = \omega_2 * O_2 Q$, the angle between MN and the velocity at Q is β.



Now velocity along the common normal to the wheels at the point of contact will be obtained by projecting them on the normal- dot product.

In order to have zero slip the contact point velocity along the common normal should be equal resulting in $v_1 \cos(\alpha) = v_2 \cos(\beta)$

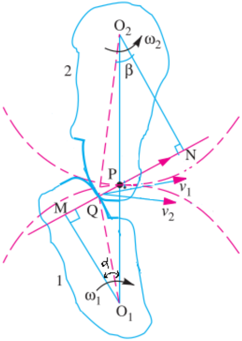


Fig. 28.7. Law of gearing.

Law of gearing

$v_1 \cos(\alpha) = v_2 \cos(\beta)$
$\omega_1 O_1 Q \cos(\alpha) = \omega_2 O_2 Q \cos(\beta)$

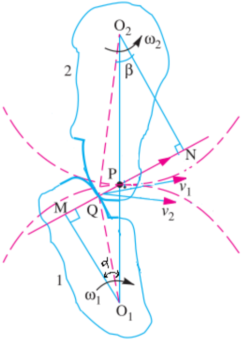


Fig. 28.7. Law of gearing.

$\cos(\alpha) = \frac{O_1 M}{O_1 Q}$ and $\cos(\beta) = \frac{O_2 N}{O_2 Q}$.
This results in $\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$

Now use similar triangles $O_1 MP$ and $O_2 NP$

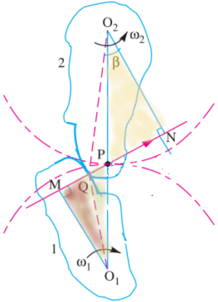


Fig. 28.7. Law of gearing.

$\frac{O_1 M}{O_2 N} = \frac{O_1 P}{O_2 P}$
Rewriting we can get $\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P}$
angular velocity ratio of the wheels is proportional to the distace of P from the centers.

Law of gearing

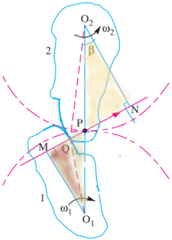


Fig. 28.7. Law of gearing.

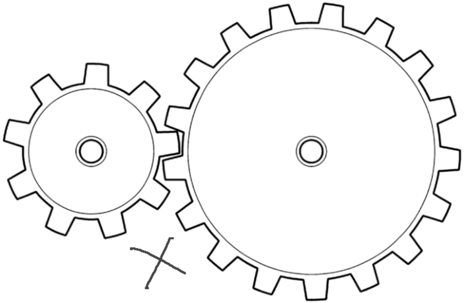
$\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P}$
angular velocity ratio of the wheels is proportional to the distace of P from the centers.

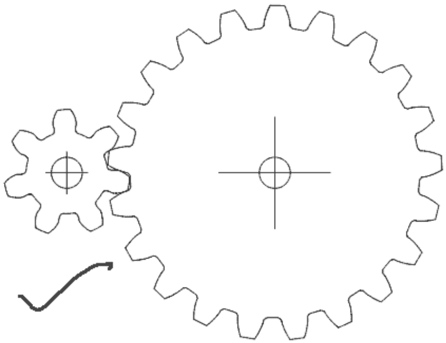
Normal to the surface at the point of contact intersects the line connecting the centers at P, which divides the center distance inversely as the ratio of angular velocities.

Therefore to have a constant velocity ratio, P must be a fixed point.
Remember P is defined on the line connecting the centers-(line $O_1 O_2$ is fixed) and the common normal-(varies based on tooth profile geometry -changes with contact point location.)

Tooth profile geometry which satisfy this geometrical constraint can give constant angular velocity ratio. It turns out that only involute and cycloidal curves satisfies the fundamental law of gearing.

Gear tooth profile cannot be arbitrary!

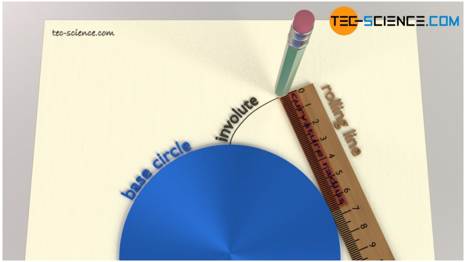


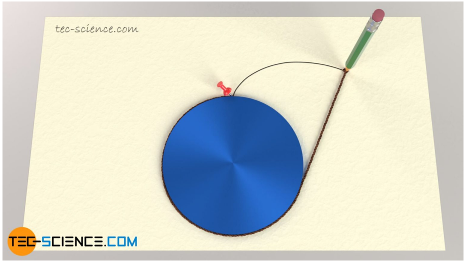


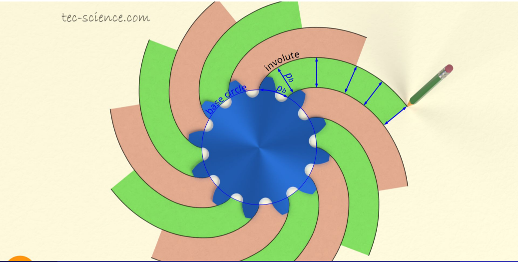
Tooth profile geometry which satisfies the geometrical constraint provided by the law of gearing can give a constant angular velocity ratio. If there is a slip at the contact point, the gear tooth in the mesh will slide(rub) over the other tooth, generating frictional forces and wear. The proof for an involute or cycloidal profile satisfying the law of gearing will be dealt with in a detailed course on gears.

Involute gear tooth profile

Involute is a curve traced by a point on a line as the line rolls without slipping on a circle.







Force analysis in gears

According to fundamental law of gearing the normal force P_N always acts along the pressure line.

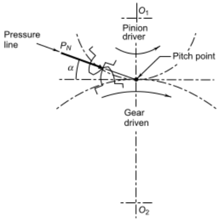
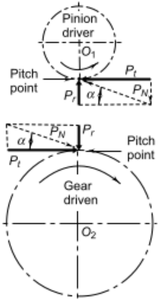


Fig. 17.20 *Gear Tooth Force*



ig. 17.21 *Components of Tooth Force*

Assumptions:

- Only a pair of teeth in contact
- Static conditions

The force exerted by the pinion on the gear P_N can be resolved as P_r- seperating the gears and P_t- tangential component. The tangential component is useful because it determines the magnitude of torque transmitted.

$Power = \frac{2\pi n M_t}{60}$;

The tangential force acting on the pinion is $P_t = \frac{2M_t}{d_p'}$ As P_N is resolved as radial

$P_r = P_N \sin(\alpha)$ and $P_t = P_N \cos(\alpha)$, from the power information we know only the tangential force, thus if we want to know the radial component, first we need to find $P_N = \frac{P_r}{\cos(\alpha)}$ and then the radial component as

$P_r = P_N \sin(\alpha) \implies P_t \tan(\alpha)$

Force analysis in gears: problem

[1]Pitch circle of a train of spur gears are shown in the figure: Gear A receives 3.5 kW of power @ 700 rpm. Gear B is the idlerwhile gear C is drive. The number of teeth on gears A, B and C are 30, 60 and 40 respectively, module is 5 mm,

Calculate (a) Torque on each gears and (b) components of gear tooth forces. Draw the FBD, assume $\alpha = 20^o$.

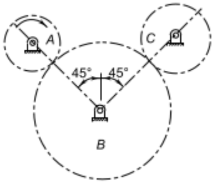


Fig. 17.23

Pitch circle dia$=$ $module * numberofteeth$

Idler gear B does not transmit any torque to its shaft

$M_{tA} * n_A = M_{tC} * n_C$

Find individual torque acting on gears M_{tA} and M_{tC}- find individual tangential forces P_{tA} and P_{tC}

Find individual radial forces $P_r = P_t \tan(\alpha)$

If reactions on the gear pivot is asked- use FBD of the particular gear.