Week 6: Delete Relaxation

COMP90054 – AI Planning for Autonomy

Key concepts

- Delete relaxation heuristic h^+
- The relationship between h^{max}, h^{add} and h^+

Problem 1

once true → forever true.

O : $\begin{cases} \text{prec:} \\ \text{add:} \\ \cancel{\text{del:}} \end{cases}$

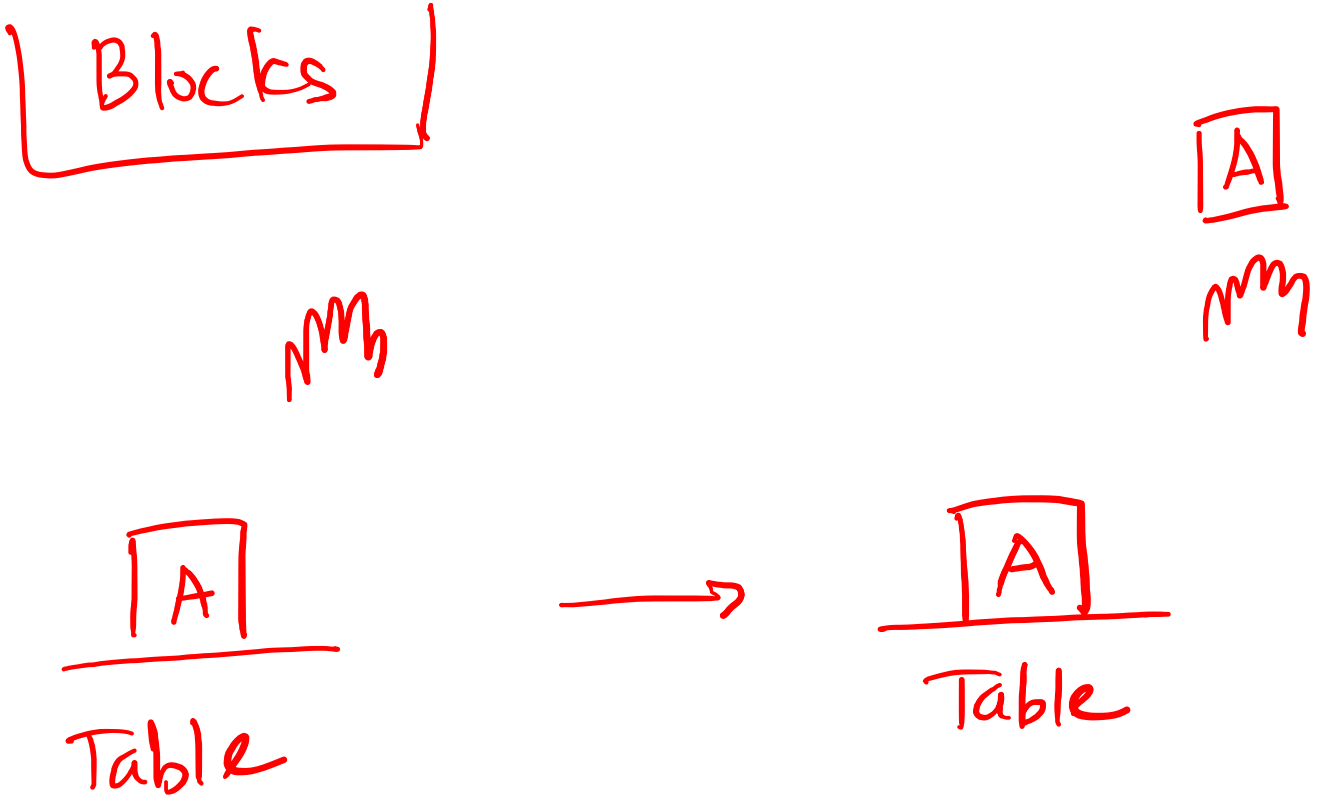
What is the (optimal) delete relaxation heuristic h^+?

Relaxing by **ignoring delete lists** **"What was once true remains true forever"**

Definition **(Delete Relaxation).**

For a STRIPS action a, by a^+ we denote the corresponding delete relaxed action, or short relaxed action, defined by $pre_{a^+} := pre_a$, $add_{a^+} := add_a$, and $del_{a^+} :=$

P = <F, O, I, G> P = <F, O^+, I, G>



pickup(A).

state 1

state 2.

delete relaxation → once true, forever true

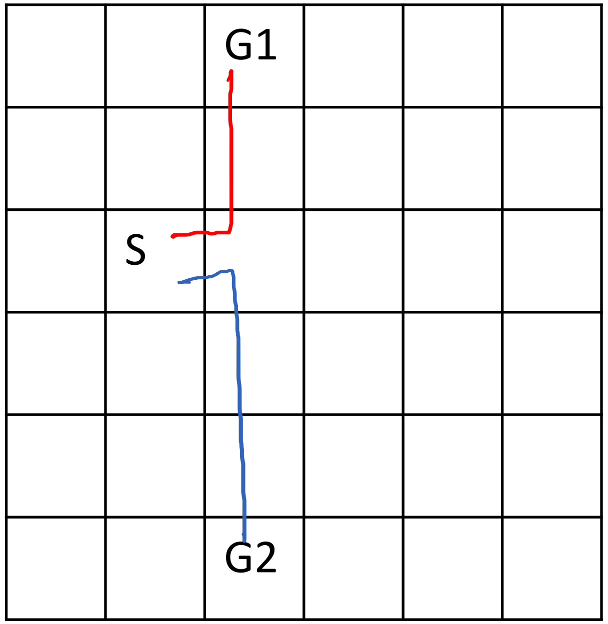
Problem 1

$h(s) = d(s,G1) + d(s,G2)$

How would it be interpreted in pacman?

$= 3 + 4 = 7$

(1)

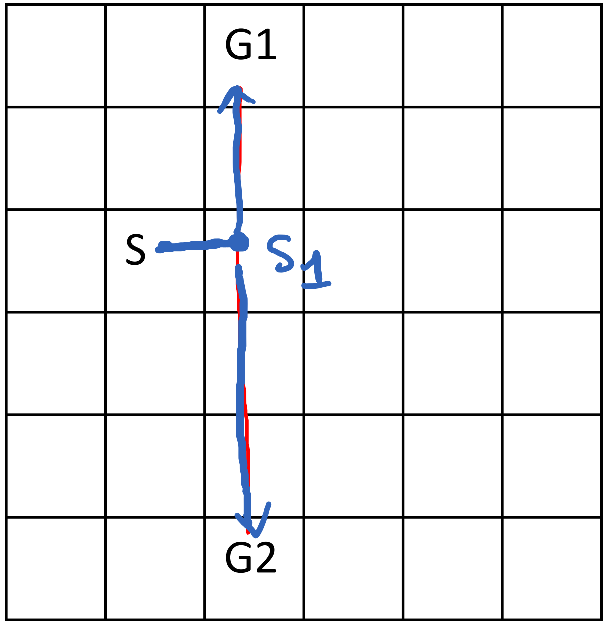


Minimum spanning tree:
Admissible, Not consistent

$h^+(s) = d(s,G1) + d(s_1,G2)$
$= 3 + 3 = 6$

(2) delete relaxation.

not easy to compute



Minimum Steiner tree:
Admissible, consistent

Problem 1

approximation of h^+

What is the relationship between $\boldsymbol{h^{max}}$, $\boldsymbol{h^{add}}$ and $\boldsymbol{h^+}$? What about $\boldsymbol{h^*}$?

h* is the perfect heuristic (the optimal cost from the current state to the goal state)

h^+ is the **optimal delete relaxation** heuristic (not easy to compute)
h^+ is admissible

h^{max} is an approximation of h^+
h^{max} is admissible. h^{max} is very small.
h^{max} <= h^+ <= h*

h^{add} is an approximation of h^+
h^{add} is not admissible
h^{add} >= h^+

Problem 2: Computing h^{max} and h^{add}

Definition (h^{add}). *Let* $\Pi = (F, A, c, I, G)$ *be a STRIPS planning task. The additive heuristic* h^{add} *for* Π *is the function* $h^{\text{add}}(s) := h^{\text{add}}(s, G)$ *where* $h^{\text{add}}(s, g)$ *is the point-wise greatest function that satisfies* $h^{\text{add}}(s, g) =$

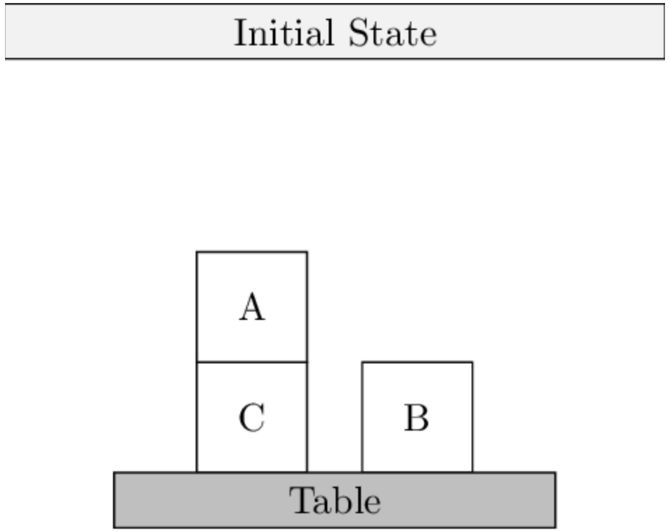
$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{add}}(s, pre_a) & |g| = 1 \\ \sum_{g' \in g} h^{\text{add}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Definition (h^{max}). *Let* $\Pi = (F, A, c, I, G)$ *be a STRIPS planning task. The max heuristic* h^{max} *for* Π *is the function* $h^{\text{max}}(s) := h^{\text{max}}(s, G)$ *where* $h^{\text{max}}(s, g)$ *is the point-wise greatest function that satisfies* $h^{\text{max}}(s, g) =$

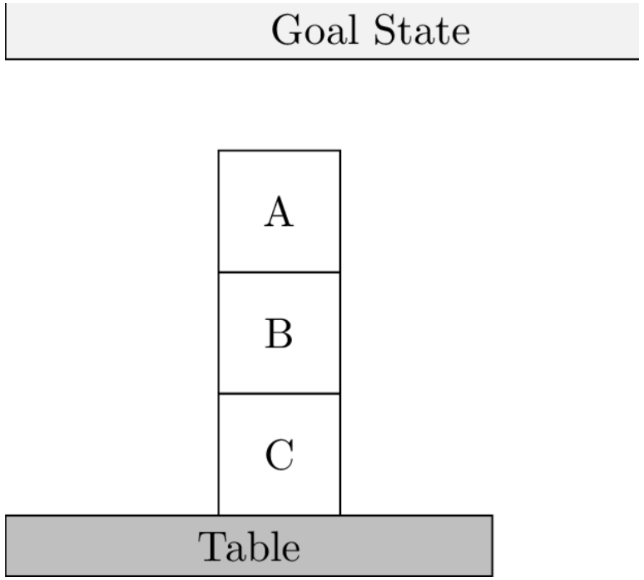
$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{max}}(s, pre_a) & |g| = 1 \\ \max_{g' \in g} h^{\text{max}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Problem 2: Computing h^{max} and h^{add}

Initial state
I = {on(A, C), onTable(C), onTable(B), clear(A), clear(B), handFree}



Goal state
G = {on(A,B), on(B,C), onTable(C)}



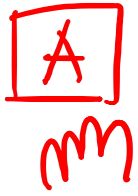
Problem 2: Computing h^{max} and h^{add}

clear(x), handFree, holding(x) ...

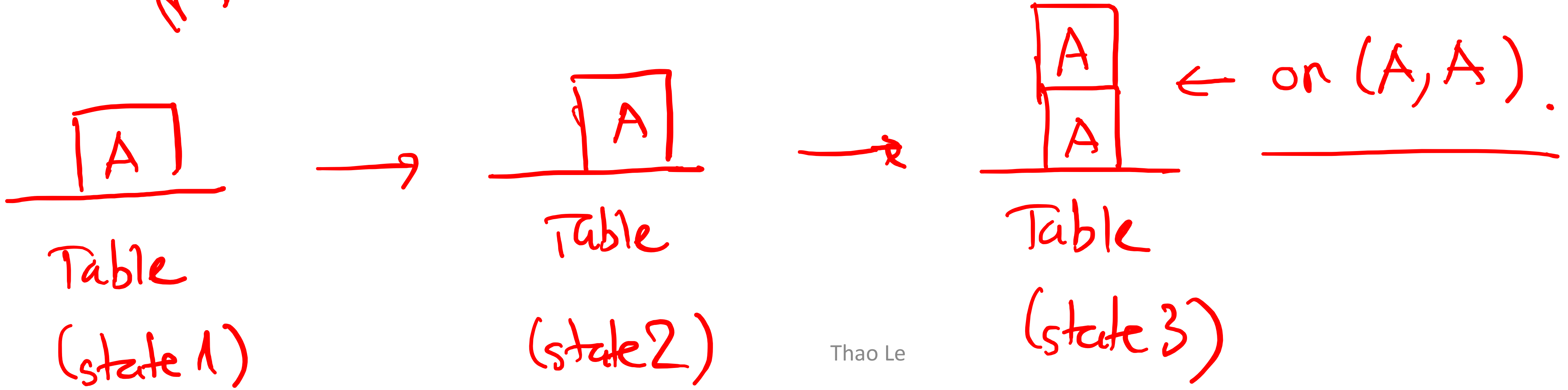
A, B, C

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1																			
2																			

I = {on(A, C), onTable(C), onTable(B), clear(A), clear(B), handFree}



c(A) = clear(A)
onTable(A) = onT(A)
hold(A) = holding(A)



Problem 2: Computing h^{max} and h^{add}

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	?	0						0								0	0

$h^{add}(\text{clear } C) = ?$ $h^{max}(\text{clear } C) = ?$

Which actions can we take to make **clear(C)** True?

Problem 2

$x, y \in \{A, B, C\}$

Which actions can we take to make **clear(C)** True?

- unstack (x, C)
- putdown (C)
- stack (C, y)

putdown(C)
stack(C, A)
stack(C, B)
unstack(A, C)
unstack(B, C)
stack(C, C)
unstack(C, C)

7

Define Operators 4

O = {
pickup(x)
- Prec: onTable(x), clear(x), handFree
- Add: holding(x)
- Del: onTable(x), clear(x), handFree

unstack(x, y) $y = C$
- Prec: on(x, y), clear(x), handFree
- Add: holding(x), clear(y)
- Del: on(x, y), clear(x), handFree

putdown(x)
- Prec: holding(x)
- Add: clear(x), onTable(x), handFree
- Del: holding(x)

stack(x, y) $x = C$
- Prec: holding(x), clear(y)
- Add: clear(x), on(x,y), handFree
- Del: clear(y), holding(x)

}

Problem 2

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	?	0						0								0	0

h^{add} = action cost + ***sum***(heuristic of preconditions)

h^{max} = action cost + ***max***(heuristic of preconditions)

$$h^{add}(stack(C,A)) = 1 + holding(C) + clear(A) = 1 + \infty + 0 = \infty$$

$$h^{max}(stack(C,A)) = 1 + \max(holding(C), clear(A)) = 1 + \infty = \infty$$

putdown(C)
stack(C, A)
stack(C, B)
unstack(A, C)
unstack(B, C)
stack(C, C)
unstack(C, C)

unstack(x, y)
- Prec: on(x, y), clear(x), handFree
- Add: holding(x), clear(y)
- Del: on(x, y), clear(x), handFree

putdown(x)
- Prec: holding(x)
- Add: clear(x), onTable(x), handFree
- Del: holding(x)

stack(x, y)
- Prec: holding(x), clear(y)
- Add: clear(x), on(x,y), handFree
- Del: clear(y), holding(x)

Problem 2

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	?	0						0								0	0

h_{add}

h_{max}

putdown(C) = 1 + hold(C) = 1 + ∞ = ∞
1 + hold(C) = ∞

stack(C, A) = 1 + hold(C) + clear(A) = 1 + ∞ + 0 = ∞
1 + max(hold(C), clear(A)) = 1 + ∞ = ∞

stack(C, B) = 1 + hold(C) + clear(B) = 1 + ∞ + 0 = ∞
1 + max(hold(C), clear(B)) = 1 + ∞ = ∞

unstack(A, C) = 1 + on(A, C) + clear(A) + handFree = 1 + 0 + 0 + 0 = 1
1 + max(on(A, C), clear(A), handFree) = 1

unstack(B, C) = 1 + on(B, C) + clear(B) + handFree = 1 + ∞ + 0 + 0 = ∞
1 + max(on(B, C), clear(B), handFree) = ∞

stack(C, C) = 1 + hold(C) + clear(C) = 1 + ∞ + ∞ = ∞
1 + max(hold(C), clear(C)) = 1 + ∞ = ∞

unstack(C, C) = 1 + on(C, C) + clear(C) + handFree = 1 + ∞ + ∞ + 0 = ∞
1 + max(on(C, C), clear(C), handFree) = 1 + ∞ = ∞

unstack(x, y)
- Prec: on(x, y), clear(x), handFree
- Add: holding(x), clear(y)
- Del: on(x, y), clear(x), handFree

putdown(x)
- Prec: holding(x)
- Add: clear(x), onTable(x), handFree
- Del: holding(x)

stack(x, y)
- Prec: holding(x), clear(y)
- Add: clear(x), on(x,y), handFree
- Del: clear(y), holding(x)

Problem 2

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1/1	0						0								0	0

putdown(C) = ∞

stack(C, A) = ∞

stack(C, B) = ∞

unstack(A, C) = 1

unstack(B, C) = ∞

stack(C, C) = ∞

unstack(C, C) = ∞

min(putdown(C), stack(C, A), stack(C, B), stack(C, C), unstack(A, C), unstack(B, C), unstack(C, C)) = 1

Problem 2

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1	0						0								0	0

Summary

1. Find all actions that make the predicate become True
2. Calculate h^{add} and h^{max} of all actions

h^{add} = action cost + ***sum***(heuristic of preconditions)
h^{max} = action cost + ***max***(heuristic of preconditions)

3. Get the minimum heuristic value

Definition (h^{add}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The additive heuristic h^{add} for Π is the function $h^{\text{add}}(s) := h^{\text{add}}(s, G)$ where $h^{\text{add}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{add}}(s, g) =$*

$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{add}}(s, pre_a) & |g| = 1 \\ \sum_{g' \in g} h^{\text{add}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Definition (h^{max}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The max heuristic h^{max} for Π is the function $h^{\text{max}}(s) := h^{\text{max}}(s, G)$ where $h^{\text{max}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{max}}(s, g) =$*

$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{max}}(s, pre_a) & |g| = 1 \\ \max_{g' \in g} h^{\text{max}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Problem 2: Computing h^{max} and h^{add}

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1	0	1	1	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
2													?						

pickup(x)
- Prec: onTable(x), clear(x), handFree
- Add: holding(x)
- Del: onTable(x), clear(x), handFree

unstack(x, y)
- Prec: on(x, y), clear(x), handFree
- Add: holding(x), clear(y)
- Del: on(x, y), clear(x), handFree

putdown(x)
- Prec: holding(x)
- Add: clear(x), onTable(x), handFree
- Del: holding(x)

stack(B,C)

stack(x, y)
- Prec: holding(x), clear(y)
- Add: clear(x), on(x,y), handFree
- Del: clear(y), holding(x)

Problem 2: Computing h^{max} and h^{add}

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1	0	1	1	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
2													? 3/2						

h^{add} = action cost + ***sum***(heuristic of preconditions)

h^{max} = action cost + ***max***(heuristic of preconditions)

stack(B,C) = 1 + hold(B) + c(C) = 1 + 1 + 1 = 3

stack(B,C) = 1 + max(hold(B), c(C)) = 1 + 1 = 2

stack(x, y)

- Prec: holding(x), clear(y)
- Add: clear(x), on(x,y), handFree
- Del: clear(y), holding(x)

Problem 2: Computing h^{max} and h^{add}

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1	0	1	1	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
2	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	∞	∞	∞	2	0	0

h^{add} / h^{max}

Problem 2: Computing h^{max} and h^{add}

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1	0	1	1	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
2	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	∞	∞	∞	2	0	0
3	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	3	3	4 / 3	2	0	0
4	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	3	3	4 / 3	2	0	0

h^{add} / h^{max}

STOP.

stop when converge (2 rows have the same values)

Problem 2: Computing h^{max} and h^{add}

Iter	c(A)	c(B)	c(C)	hand Free	h(A)	h(B)	h(C)	on(A, A)	on(A,B)	on(A,C)	on(B,A)	on(B,B)	on(B,C)	on(C,A)	on(C,B)	on(C,C)	onT(A)	onT(B)	onT(C)
0	0	0	∞	0	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
1	0	0	1	0	1	1	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	0	0
2	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	∞	∞	∞	2	0	0
3	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	3	3	4 / 3	2	0	0
4	0	0	1	0	1	1	2	2	2	0	2	2	3 / 2	3	3	4 / 3	2	0	0

h^{add} / h^{max}

G = {on(A,B), on(B,C), onTable(C)}

h^{add}(s0) = 2 + 3 + 0 = 5

$h^{max}(s0) = \max(2, 2, 0) = 2$