Week 3: Heuristic Search

COMP90054 – AI Planning for Autonomy

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Key concepts

- Heuristic Functions and their properties and relations
- Heuristic search algorithms
- State-space model and size of the problem

Problem 1: Task 1

Heuristic function

h(s) estimates the distance from the current state **s** to the <u>*closest*</u> goal state

$\boldsymbol{h}^*(\boldsymbol{s})$ is a **perfect heuristic,** the optimal cost from the current state to the goal state

Problem 1: Task 1

- *safe* if $h^*(s) = \infty$ for all $s \in S$ with $h(s) = \infty$;
- *goal-aware* if $h(s) = 0$ for all goal states $s \in S^G$;
- *admissible* if $h(s) \leq h^*(s)$ for all $s \in s$;
- *consistent* if $h(s) \leq h(s') + c(a)$ for all transitions $s \xrightarrow{a} s'$.

Heuristic function's properties

4 properties:

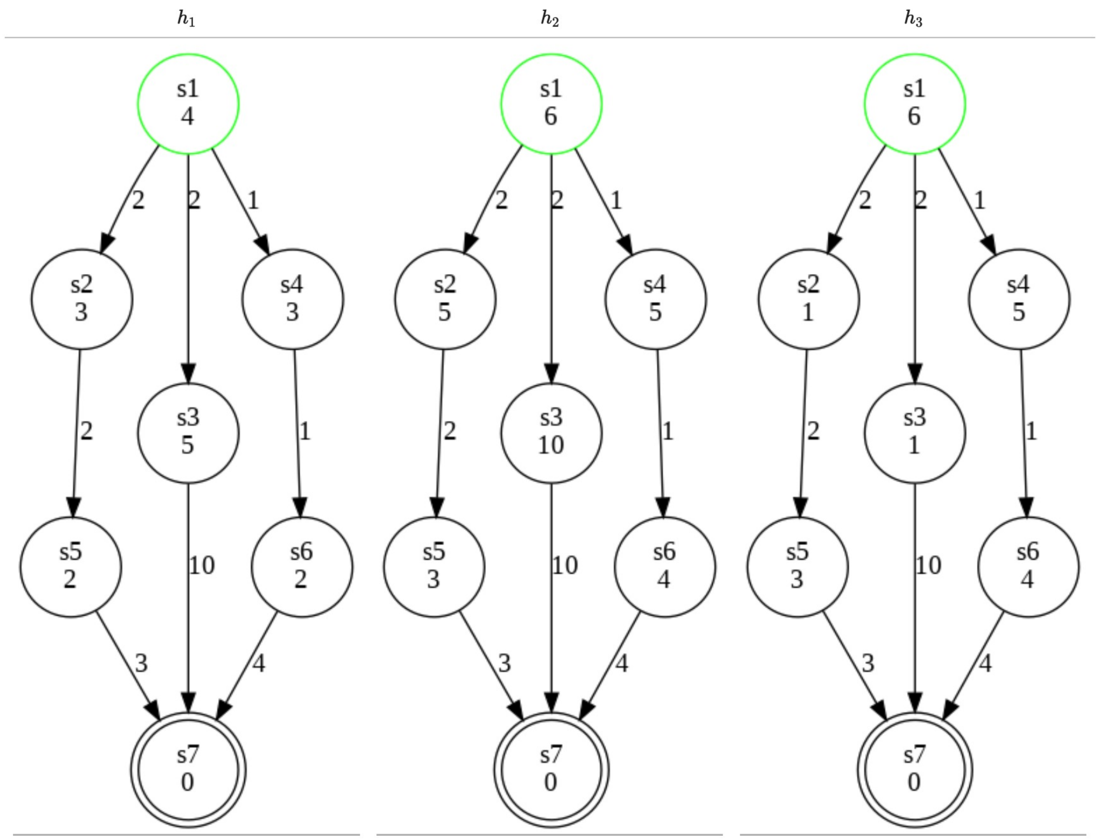
- **Safe**: if a solution exists from state s, then $h(s) < \infty$
- **Goal-aware**: All goal states have a heuristic h = 0
- **Admissible**: never over-estimate the cost
- **Consistent**: the cost diff between the parent and the child heuristics is never larger than the actual cost

Problem 1: Task 1

Admissible: for all $s \in S,\ h(s) \leq h^*(s)$

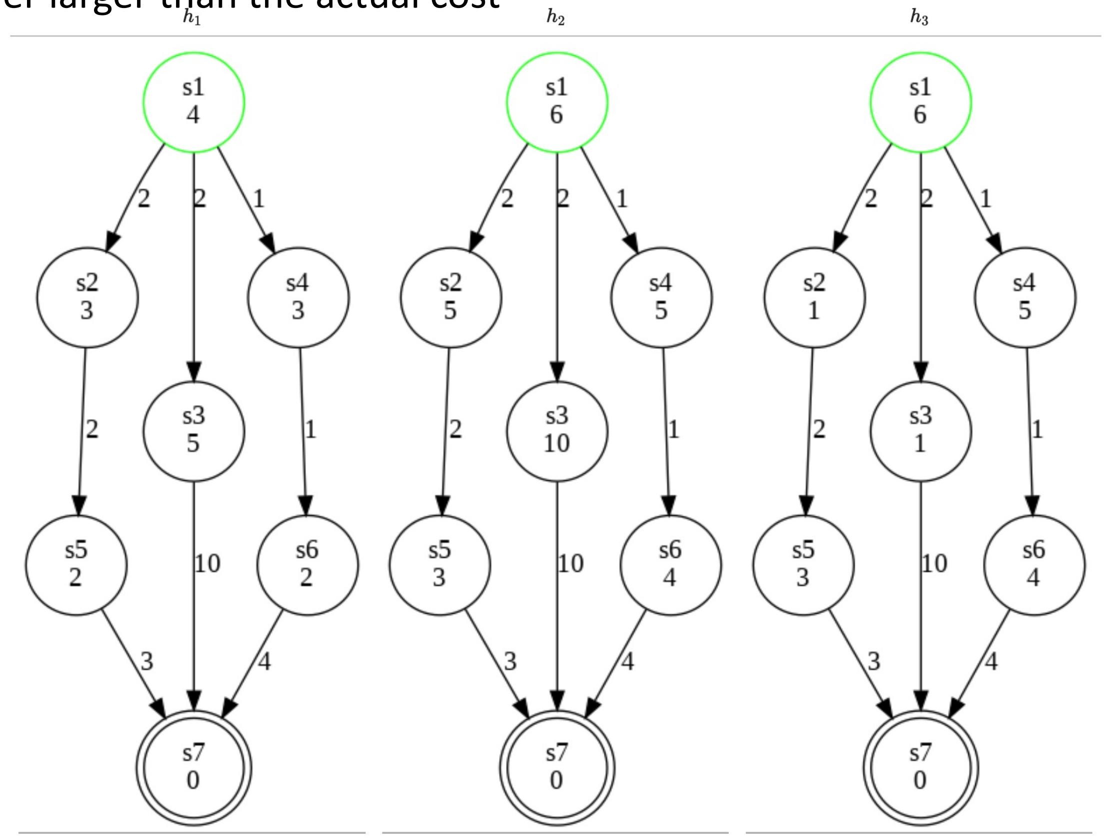
Which Heuristics are admissible?

h1, h2, h3



Problem 1: Task 1

Consistent: the cost diff between the parent and the child heuristics is never larger than the actual cost



Which heuristics are consistent?

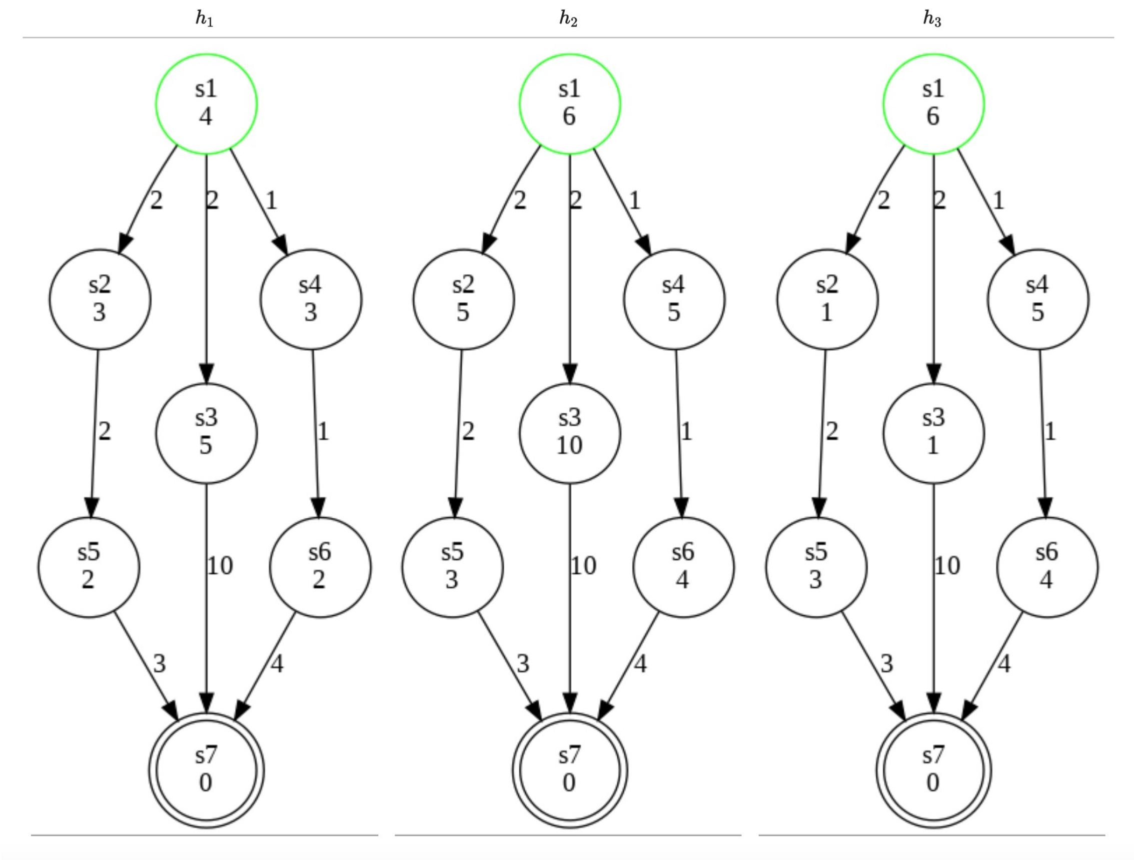
h1, h2

Problem 1: Task 1

Dominate relation
h1 dominates h2 if
- both heuristics are admissible
- h2 <= h1 <= h* for all s in S

Does any of the heuristic dominate any other?

h2 dominates h1
h2 dominates h3



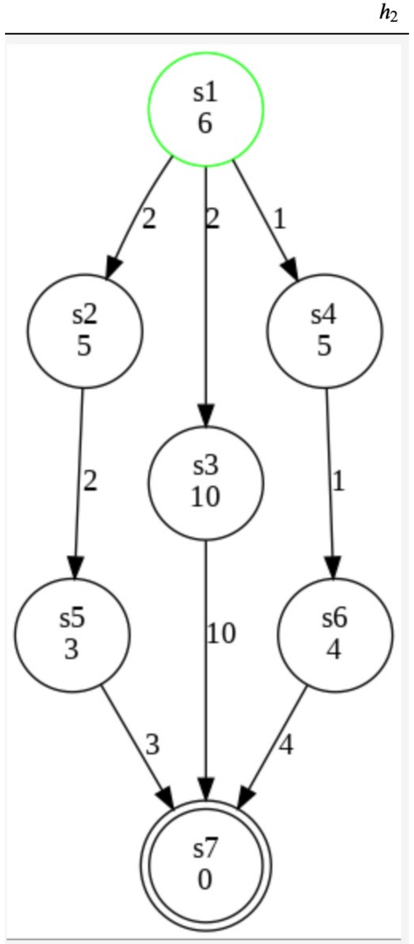
Problem 1: Task 2

Heuristic search algorithms

Search node: n = <s, f(n), g(n), parent n>
f(n) is a priority value for node in the priority queue

DS: priority queue

- Uniform-cost search (Dijkstra): f(n) = g(n)
- Greedy best-first search: f(n) = h(s)
- A*: f(n) = h(s) + g(n)
- WA*: f(n) = W * h(s) + g(n)



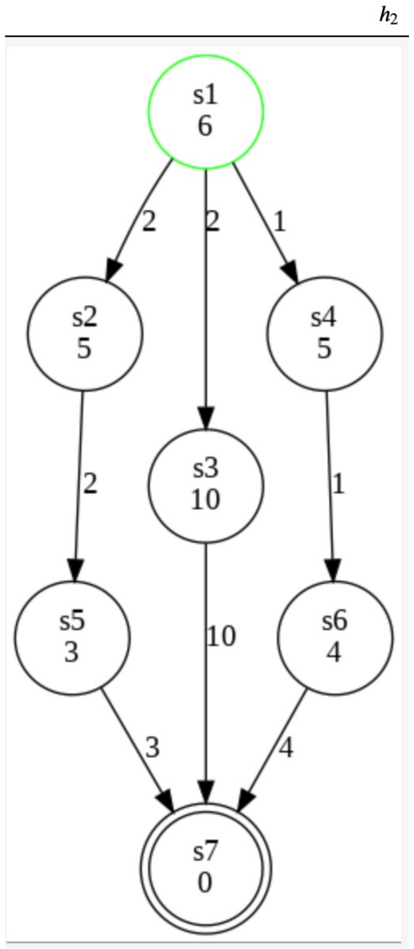
Problem 1: Task 2

Search node: n = <s, f(n), g(n), parent n>
f(n) = h(s)

Greedy best-first search

Step	Open (Priority Queue)	Close (Visited)
1	n0 = <s1, 6, 0, None>	
2	n1 = <s2, 5, 2, 0> n2 = <s3, 10, 2, 0> n3 = <s4, 5, 1, 0>	n0
3	n1 = <s2, 5, 2, 0> n2 = <s3, 10, 2, 0> n4 = <s6, 4, 2, 3>	n0, n3
4	n1 = <s2, 5, 2, 0> n2 = <s3, 10, 2, 0> n5 = <s7, 0, 6, 4>	n0, n3, n4
5	n1 = <s2, 5, 2, 0> n2 = <s3, 10, 2, 0>	n0, n3, n4, n5

Solution: s1 -> s4 -> s6 -> s7

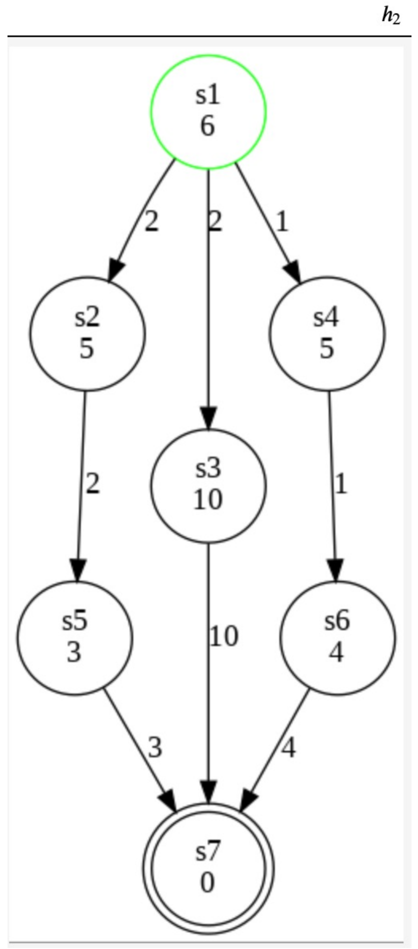


Problem 1: Task 2

Search node: n = <s, f(n), g(n), parent n>
f(n) = h(s) + g(n)

A*

Step	Open (Priority Queue)	Close (Visited)
1	n0 = <s1, 6, 0, None>	
2	n1 = <s2, 7, 2, 0> n2 = <s3, 12, 2, 0> n3 = <s4, 6, 1, 0>	n0
3	n1 = <s2, 7, 2, 0> n2 = <s3, 12, 2, 0> n4 = <s6, 6, 2, 3>	n0, n3
4	n1 = <s2, 7, 2, 0> n2 = <s3, 12, 2, 0> n5 = <s7, 6, 6, 5>	n0, n3, n4
5	n1 = <s2, 7, 2, 0> n2 = <s3, 12, 2, 0>	n0, n3, n4, n5



Solution: s1 -> s4 -> s6 -> s7

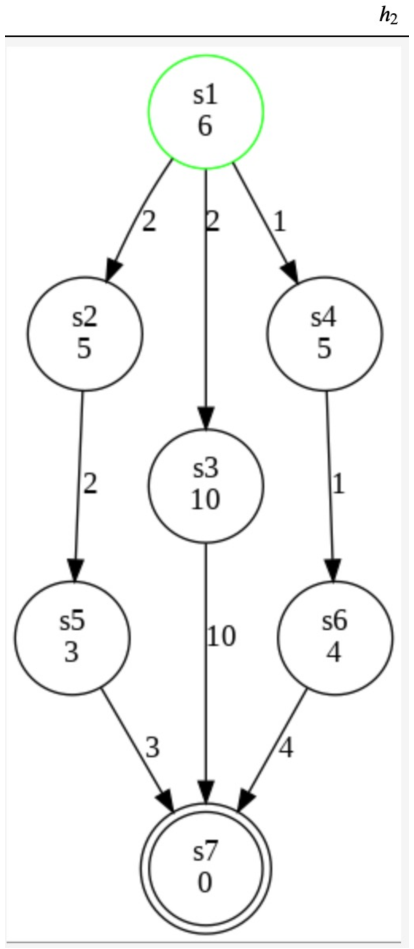
Problem 1: Task 2

Search node: n = <s, f(n), g(n), parent n>
f(n) = W * h(s) + g(n)

WA* (W = 2)

Step	Open (Priority Queue)	Close (Visited)
1	n0 = <s1, 12, 0, None>	
2	n1 = <s2, 12, 2, 0> n2 = <s3, 22, 2, 0> n3 = <s4, 11, 1, 0>	n0
3	n1 = <s2, 12, 2, 0> n2 = <s3, 22, 2, 0> n4 = <s6, 10, 2, 3>	n0, n3
4	n1 = <s2, 12, 2, 0> n2 = <s3, 22, 2, 0> n5 = <s7, 6, 6, 4>	n0, n3, n4
5	n1 = <s2, 12, 2, 0> n2 = <s3, 22, 2, 0>	n0, n3, n4, n5

Solution: s1 -> s4 -> s6 -> s7



Problem 1: Task 2

Heuristic algorithms

Which is the path returned as solution? (using h2 and A* as example)

s1 -> s4 -> s6 -> s7

Is this the optimal plan? Has the algorithm proved this? (using h2 and A* as example)

Yes. h2 is both admissible and consistent

Problem 2

Consider an $m \times m$ **Manhattan Grid**, and a set of coordinates G to visit in any order.

Hint: Consider a set of coordinates V' remaining to be visited, or a set of coordinates V already visited. What's the difference between them

Formulate a state-based search problem to find a tour to all the desired points

(x,y)		

State space model:
P = $<s_0, S, S_G, A, f, c>$

a state = <current coordinate, a set of remaining coordinates>

Initial state $s_0 = <(0,0), G \backslash \{(0,0)\}>$

Goal state $S_G = \{<(x,y), \{\}> \mid x, y \in \{0, \dots, m-1\}\}$

State $S \quad = \{<(x,y), V'> \mid x, y \in \{0, \dots, m-1\} \wedge V' \subseteq G\}$

Action A$(<(x,y), V'>) = \{(dx, dy) \mid dx, dy \in \{-1, 0, 1\}$
$\wedge\ |dx| + |dy| = 1$
$\wedge\ x + dx, y + dy \in \{0, \dots, m-1\}\}$

Transition f$(<(x,y), V'>, (dx, dy)) = <(x+dx, y+dy),\ V' \backslash \{(x+dx, y+dy)\}>$

Cost c(a) **= 1**

Problem 2

Consider an $m \times m$ **Manhattan Grid**, and a set of coordinates G to visit in any order.

Hint: Consider a set of coordinates V' remaining to be visited, or a set of coordinates V already visited. What's the difference between them

(x,y)		

What is the branching factor of the search?

4 (branching factor = max number of child nodes)

Problem 2

Consider an $m \times m$ **Manhattan Grid**, and a set of coordinates G to visit in any order.

Hint: Consider a set of coordinates V' remaining to be visited, or a set of coordinates V already visited. What's the difference between them

What is the size of the state space in terms of m and G?

(x,y)		

If using V’ (remaining to be visited), then $m^2 \times 2^{|G|}$