Gini Index

$$Gini = 1 - \sum_{i=1}^{C} (p_i)^2$$

Gini Index is a metric to measure how often a randomly chosen element would be incorrectly identified. It means an attribute with lower gini index should be preferred.

Example: Construct a Decision Tree by using "gini index" as a criterion

	A	B	C	D	E
1	4.8	3.4	1.9	0.2	positive
2	5	3	1.6	0.2	positive
3	5	3.4	1.6	0.4	positive
4	5.2	3.5	1.5	0.2	positive
5	5.2	3.4	1.4	0.2	positive
6	4.7	3.2	1.6	0.2	positive
7	4.8	3.1	1.6	0.2	positive
8	5.4	3.4	1.5	0.4	positive
9	7	3.2	4.7	1.4	negative
10	6.4	3.2	4.5	1.5	negative
11	6.9	3.1	4.9	1.5	negative
12	5.5	2.3	4	1.3	negative
13	6.5	2.8	4.6	1.5	negative
14	5.7	2.8	4.5	1.3	negative
15	6.3	3.3	4.7	1.6	negative
16	4.9	2.4	3.3	1	negative

We are going to use same data sample that we used for information gain example. Let's try to use gini index as a criterion. Here, we have 5 columns out of which 4 columns have continuous data and 5th column consists of class labels.

A, B, C, D attributes can be considered as predictors and E column class labels can be considered as a target variable. For constructing a decision tree from this data, we have to convert continuous data into categorical data.

We have chosen **some random values** to categorize each attribute:

A	B	C	D
>= 5	>= 3.0	>=4.2	>= 1.4
< 5	< 3.0	< 4.2	< 1.4

$$Gini = 1 - \sum_{i=1}^{C} (p_i)^2$$

Gini Index for Var A

Var A has value >=5 for 12 records out of 16 and 4 records with value <5 value.

- For Var A >= 5 & class == positive: 5/12
- For Var A >= 5 & class == negative: 7/12
    - gini(5,7) = 1- ($(5/12)^2 + (7/12)^2$) = 0.4860
- For Var A <5 & class == positive: 3/4
- For Var A <5 & class == negative: 1/4
    - gini(3,1) = 1- ($(3/4)^2 + (1/4)^2$) = 0.375

By adding weight and sum each of the gini indices:

$$\text{gini}(\text{Target, A}) = (12/16) * (0.486) + (4/16) * (0.375) = 0.45825$$

Gini Index for Var B

Var B has value >=3 for 12 records out of 16 and 4 records with value <5 value.

- For Var B >= 3 & class == positive: 8/12
- For Var B >= 3 & class == negative: 4/12
    - gini(8,4) = 1- ($(8/12)^2 + (4/12)^2$) = 0.446
- For Var B <3 & class == positive: 0/4
- For Var B <3 & class == negative: 4/4
    - gin(0,4) = 1- ($(0/4)^2 + (4/4)^2$) = 0

$$\text{gini}(\text{Target, B}) = (12/16) * 0.446 + (4/16) * 0 = 0.3345$$

Gini Index for Var C

Var C has value >=4.2 for 6 records out of 16 and 10 records with value <4.2 value.

- For Var C >= 4.2 & class == positive: 0/6
- For Var C >= 4.2 & class == negative: 6/6
    - gini(0,6) = 1- ($(0/8)^2 + (6/6)^2$) = 0
- For Var C < 4.2& class == positive: 8/10
- For Var C < 4.2 & class == negative: 2/10
    - gin(8,2) = 1- ($(8/10)^2 + (2/10)^2$) = 0.32

$$\text{gini}(\text{Target, C}) = (6/16) * 0 + (10/16) * 0.32 = 0.2$$

Gini Index for Var D

Var D has value >=1.4 for 5 records out of 16 and 11 records with value <1.4 value.

- For Var D >= 1.4 & class == positive: 0/5
- For Var D >= 1.4 & class == negative: 5/5
    - gini(0,5) = 1- ($(0/5)^2 + (5/5)^2$) = 0
- For Var D < 1.4 & class == positive: 8/11
- For Var D < 1.4 & class == negative: 3/11
    - gin(8,3) = 1- ($(8/11)^2 + (3/11)^2$) = 0.397

$$\text{gini}(\text{Target}, \text{D}) = (5/16) * 0 + (11/16) * 0.397 = 0.273$$

		wTarget	
		Positive	Negative
A	>= 5.0	5	7
	<5	3	1

Ginin Index of A = 0.45825

		Target	
		Positive	Negative
B	>= 3.0	8	4
	< 3.0	0	4

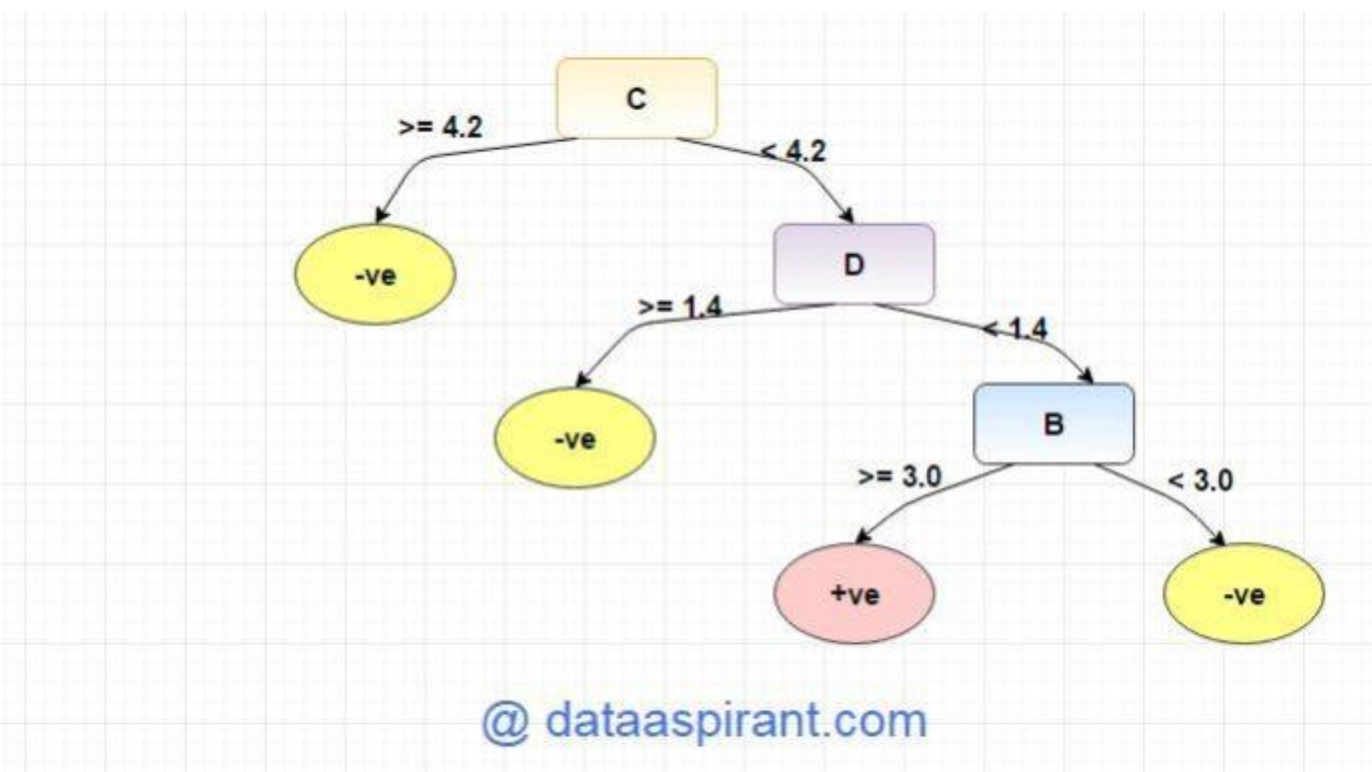
Gini Index of B= 0.3345

		Target	
		Positive	Negative
C	>= 4.2	0	6
	< 4.2	8	2

Gini Index of C= 0.2

		Target	
		Positive	Negative
D	>= 1.4	0	5
	< 1.4	8	3

Gini Index of D= 0.273



Entropy

$$Entropy = \sum_{i=1}^{C} -p_i * \log_2(p_i)$$

$$Gain(T, X) = Entropy(T) - Entropy(T, X)$$

Example: Construct a Decision Tree by using "information gain" as a criterion

	A	B	C	D	E
1	4.8	3.4	1.9	0.2	positive
2	5	3	1.6	0.2	positive
3	5	3.4	1.6	0.4	positive
4	5.2	3.5	1.5	0.2	positive
5	5.2	3.4	1.4	0.2	positive
6	4.7	3.2	1.6	0.2	positive
7	4.8	3.1	1.6	0.2	positive
8	5.4	3.4	1.5	0.4	positive
9	7	3.2	4.7	1.4	negative
10	6.4	3.2	4.5	1.5	negative
11	6.9	3.1	4.9	1.5	negative
12	5.5	2.3	4	1.3	negative
13	6.5	2.8	4.6	1.5	negative
14	5.7	2.8	4.5	1.3	negative
15	6.3	3.3	4.7	1.6	negative
16	4.9	2.4	3.3	1	negative

We are going to use this data sample. Let's try to use information gain as a criterion. Here, we have 5 columns out of which 4 columns have continuous data and 5th column consists of class labels.

A, B, C, D attributes can be considered as predictors and E column class labels can be considered as a target variable. For constructing a decision tree from this data, we have to convert continuous data into categorical data.

We have chosen some random values to categorize each attribute:

A	**B**	**C**	**D**
>= 5	>= 3.0	>= 4.2	>= 1.4
< 5	< 3.0	< 4.2	< 1.4

There are **2 steps for calculating information gain** for each attribute:

1. Calculate entropy of Target.
2. Entropy for every attribute A, B, C, D needs to be calculated. Using information gain formula we will subtract this entropy from the entropy of target. The result is Information Gain.

The entropy of Target: We have 8 records with negative class and 8 records with positive class. So, we can directly estimate the entropy of target as 1.

Variable E	
Positive	Negative
8	8

Calculating entropy using formula:

E(8,8) = -1*((p(+ve)*log(p(+ve)) + (p(-ve)*log(p(-ve)))
= -1*((8/16)*$\log_2$(8/16)) + (8/16) * $\log_2$(8/16))
= 1

Information gain for Var A

$$Gain(T,X) = Entropy(T) - Entropy(T,X)$$

Var A has value >=5 for 12 records out of 16 and 4 records with value <5 value.

- For Var A >= 5 & class == positive: 5/12
- For Var A >= 5 & class == negative: 7/12
    - Entropy(5,7) = -1 * ((5/12)*log2(5/12) + (7/12)*log2(7/12)) = 0.9799
- For Var A <5 & class == positive: 3/4
- For Var A <5 & class == negative: 1/4
    - Entropy(3,1) = -1 * ((3/4)*log2(3/4) + (1/4)*log2(1/4)) = 0.81128

Entropy(Target, A) = P(>=5) * E(5,7) + P(<5) * E(3,1)
= (12/16) * 0.9799 + (4/16) * 0.81128 = 0.937745

Information Gain(IG) = E(Target) - E(Target,A) = 1- 0.9337745 = 0.062255

Information gain for Var B

Var B has value >=3 for 12 records out of 16 and 4 records with value <5 value.

- For Var B >= 3 & class == positive: 8/12
- For Var B >= 3 & class == negative: 4/12
    - Entropy(8,4) = -1 * ((8/12)*log2(8/12) + (4/12)*log2(4/12)) = 0.39054
- For VarB <3 & class == positive: 0/4
- For Var B <3 & class == negative: 4/4
    - Entropy(0,4) = -1 * ((0/4)*log2(0/4) + (4/4)*log2(4/4)) = 0

Entropy(Target, B) = P(>=3) * E(8,4) + P(<3) * E(0,4)
= (12/16) * 0.39054 + (4/16) * 0 = 0.292905

Information Gain(IG) = E(Target) - E(Target,B) = 1- 0.292905= 0.707095

Information gain for Var C

Var C has value >=4.2 for 6 records out of 16 and 10 records with value <4.2 value.

- For Var C >= 4.2 & class == positive: 0/6
- For Var C >= 4.2 & class == negative: 6/6
  - Entropy(0,6) = 0
- For VarC < 4.2 & class == positive: 8/10
- For Var C < 4.2 & class == negative: 2/10
  - Entropy(8,2) = 0.72193

Entropy(Target, C) = P(>=4.2) * E(0,6) + P(< 4.2) * E(8,2)
= (6/16) * 0 + (10/16) * 0.72193 = 0.4512

Information Gain(IG) = E(Target) - E(Target,C) = 1- 0.4512= 0.5488

Information gain for Var D

Var D has value >=1.4 for 5 records out of 16 and 11 records with value <5 value.

- For Var D >= 1.4 & class == positive: 0/5
- For Var D >= 1.4 & class == negative: 5/5
  - Entropy(0,5) = 0
- For Var D < 1.4 & class == positive: 8/11
- For Var D < 14 & class == negative: 3/11
  - Entropy(8,3) = -1 * ((8/11)*log2(8/11) + (3/11)*log2(3/11)) = 0.84532

Entropy(Target, D) = P(>=1.4) * E(0,5) + P(< 1.4) * E(8,3)
= 5/16 * 0 + (11/16) * 0.84532 = 0.5811575

Information Gain(IG) = E(Target) - E(Target,D) = 1- 0.5811575 = 0.41189

		Target	
		Positive	Negative
A	>= 5.0	5	7
	<5	3	1

Information Gain of A = 0.062255

		Target	
		Positive	Negative
B	>= 3.0	8	4
	< 3.0	0	4

Information Gain of B= 0.7070795

		Target	
		Positive	Negative
C	>= 4.2	0	6
	< 4.2	8	2

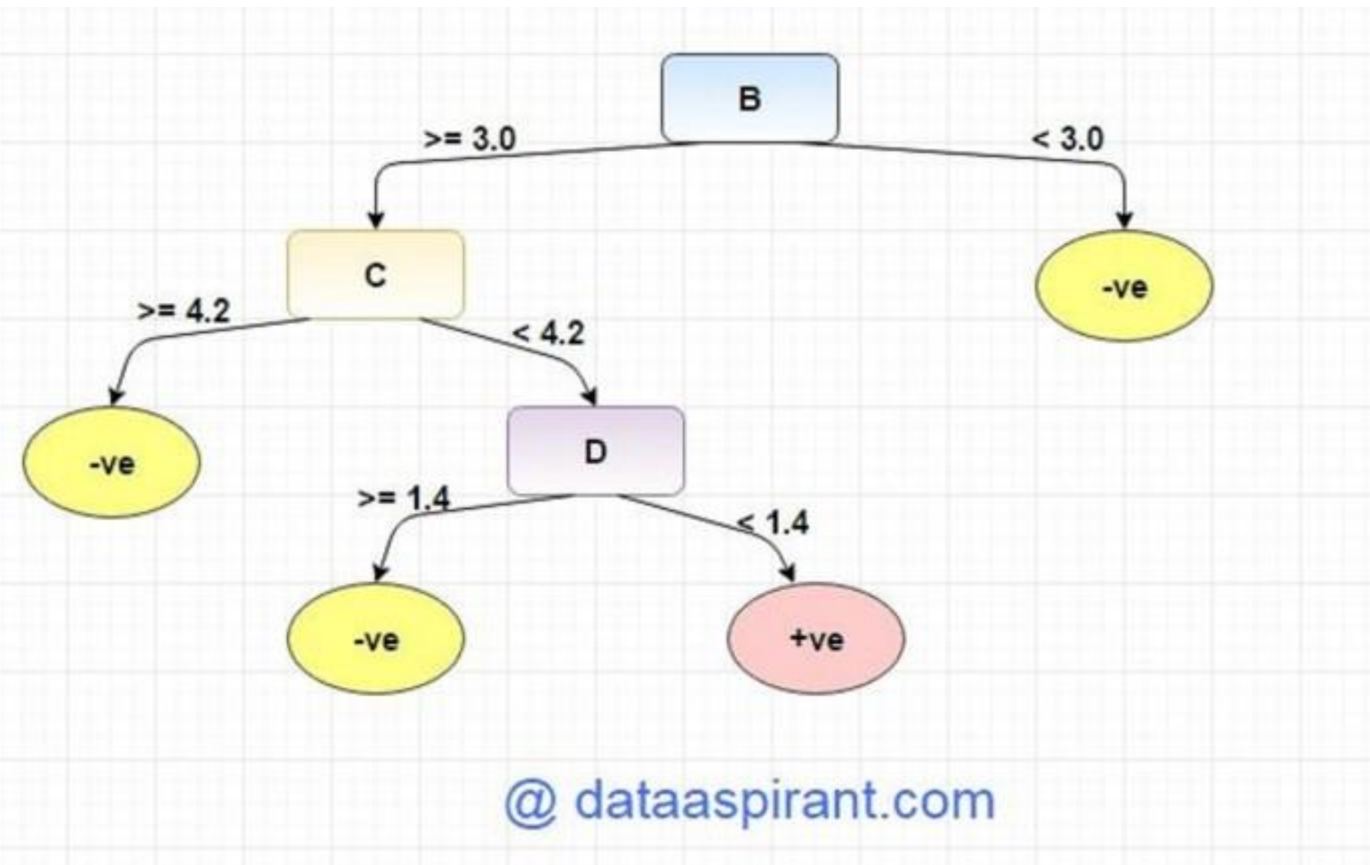
Information Gain of C= 0.5488

		Target	
		Positive	Negative
D	>= 1.4	0	5
	< 1.4	8	3

Information Gain of D= 0.41189

From the above Information Gain calculations, we can build a decision tree. We should place the attributes on the tree according to their values.

An Attribute with better value than other should position as root and A branch with entropy 0 should be converted to a leaf node. A branch with entropy more than 0 needs further splitting.



Tham khảo: https://dataaspirant.com/2017/01/30/how-decision-tree-algorithm-works/