Problem 1
To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution

Let: A have Hansen's disease.
B have no Hansen's disease.
C has a positive test result.

We have:

$$\begin{aligned} P(C) &= P(C|A) * P(A) + P(C|B) * P(B) \\ P(C) &= 0.98 * 0.05 + 0.03 * 0.95 \\ P(C) &= 0.0775 \\ P(A|C) &= \frac{P(C|A) * P(A)}{P(C)} \\ P(A|C) &= \frac{0.98 * 0.05}{0.0775} \\ &\approx 0.632 \end{aligned}$$