Problem 1
Biến đổi lại posterior trên lớp ra latex, từ $p(w|D)->w=(X^TX+\alpha*I)^{-1}X^Tt$

Solution

Bayes theorem

$$p(w|D)=\frac{p(D|w)*p(w)}{p(D)}$$

$$\Leftrightarrow posterior=\frac{likehood*prior}{evidence}$$

$$\Rightarrow p(w|x,t,\alpha,\beta)=\frac{p(t|x,w,\beta)*p(w|\alpha)}{p(x,t,\alpha,\beta}$$

Suppose $p(w|\alpha)$ is a normal distribution. We have:

$$p(w|\alpha)=N(w|0,\alpha^{-1}I)$$

$$p(t|x,w)=\prod_{i=1}^{N}N(t_i|y(x_i,w),\beta^{-1})$$

Then we take:

$$p(w|\alpha)*p(t|x,w)$$

And maximize we get:

$$=log(p(t|(x,w))+ln(p(w|\alpha))$$

$$=\sum_{i=1}^{N}log(N(y(x_i,w),\beta^{-1}+log(p(w|\alpha^{-1}I)$$

$$=\sum_{i=1}^{N}log(\frac{1}{\beta^{-1}\sqrt{2\pi}}e^{\frac{(t_i-y(x_i,w))^2\beta}{2}})+log(\frac{1}{\sqrt{(2\pi)^D}\mid\alpha^{-1}I\mid}e^{\frac{-1}{2}w^T(\alpha^{-1}I)^{-1}w})$$

$$=\frac{-\beta}{2}\sum_{i=1}^{N}(t_i-y(x_i,w))^2+\frac{-1}{2}\alpha w^Tw$$

$$\Leftrightarrow minimize:\sum_{i=1}^{N}(t_i-y(x_i,w))^2+\frac{\alpha}{\beta}w^Tw$$

Let:

$$\frac{\alpha}{\beta}=\lambda(\lambda>0)$$

and then :

$$L=\sum_{i=1}^{N}(t_i-y(x_i,w))^2+\lambda w^Tw$$

$$X,w,t\to L=\|Xw-t\|_2^2+\lambda\|w\|_2^2$$

$$\frac{\partial L}{\partial w}=2X^T(Xw-t)+2\lambda w=0$$

$$\Leftrightarrow w(X^T X + \lambda I) = X^T t$$

$$\Leftrightarrow w = (X^T X + \lambda I)^{-1} X^T t$$