Problem 1
Tự biến đổi lại thuật toán logistic regression, từ xây dựng công thức, likelihood, maximize likelihood, đạo làm negative log likelihood theo ma trận hệ số.

Solution

The model logistic regression is:

$$P(C_1|\phi) = y(\phi) = \sigma(w^T\phi)$$

$$P(C_2|\phi) = y(\phi) = 1 - P(C_1|\phi)$$

For a data set ϕ_n, t_n where $t_n \in 0, 1$ and $\phi_n = \phi(x_n)$ with $n = 1, ..., N$, the likellihood function can be written :

$$p(t|w) = \prod_{n=1}^{N} y_n^{t_n}(1 - y_n)^{1-t_n}$$

where:

$$t = (t_1, ...t_N)^T \quad and \quad y_n = p(C_1|\phi_n)$$

We have:

$$L = -log\, p(t|w) = -\sum_{n=1}^{N}[\, t_n log\, y_n + (1 - t_n)log(1 - y_n)\,]$$

where $y_n = \sigma(a_n)$ and $a_n = w^T\phi_n$

Using chain rule we have:

$$\frac{\partial L}{\partial w_0} = \frac{\partial L}{\partial y_n}.\frac{\partial y_n}{\partial a_n}.\frac{\partial a_n}{\partial w_0}$$

$$= -(\frac{t_n}{y_n} - \frac{1 - t_n}{1 - y_n}).y_n(1 - y_n)$$

$$= y_n - t_n$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial y_n}.\frac{\partial y_n}{\partial a_n}.\frac{\partial a_n}{\partial w_1}$$

$$= -(\frac{t_n}{y_n} - \frac{1 - t_n}{1 - y_n}).y_n(1 - y_n).\phi_1$$

$$= (y_n - t_n)\phi_1$$

$$\Rightarrow \frac{\partial L}{\partial w} = (y - t)\phi$$

Problem 2
Tìm hàm f(x), biết f'(x) = f(x)(1-f(x))

Solution

We have:

$$f'(x) = f(x)(1 - f(x))$$

$$\Leftrightarrow \frac{df(x)}{dx} = f(x)(1 - f(x))$$

$$\Leftrightarrow \frac{df(x)}{f(x)(1 - f(x))} = dx$$

$$\Leftrightarrow \int \frac{df(x)}{f(x)(1 - f(x))} = \int dx$$

$$\Leftrightarrow \int [\frac{1}{f(x)} + \frac{1}{1-f(x)}]df(x) = \int dx$$
$$\Leftrightarrow ln|f(x)| - ln|1-f(x)| = x + C$$
$$\Leftrightarrow ln|\frac{f(x)}{1-f(x)}| = x + C$$
$$\Leftrightarrow \frac{f(x)}{1-f(x)} = e^{x+C}$$
$$\Leftrightarrow 1 - f(x) = \frac{1}{1+e^{x+C}}$$
$$\Leftrightarrow f(x) = \frac{e^{x+C}}{1+e^{x+C}}$$