Part I of Lecture 15

Probability

Basics

Probability = Chance or likelihood something will happen

- **Lowest value:** 0 (or 0%)
  - Chance of an event that is impossible
- **Highest value:** 1 (or 100%)
  - Chance of an event that is certain

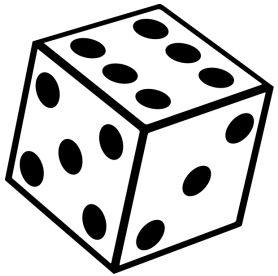
Example:

The chance of getting a heads or tails when tossing a coin is 1, because we are guaranteed one of these outcomes.

The chance of rolling a 7 on a 6-sided die is 0, because this outcome is not possible.

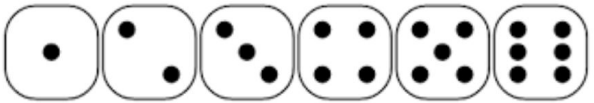
Basics

Assuming all outcomes are **equally likely**, the chance of an event *A* is...



$$P(A) = \frac{\text{Number of outcomes that make } A \text{ happen}}{\text{Total number of outcomes}}$$

Example: Let's roll a die.



Outcomes: {1, 2, 3, 4, 5, 6}

- *P*(we roll an even number) = 3/6
- P(we roll a 1 or a 2) = 2/6
- P(we roll a 3 or greater) = 4/6

Basics

- Two events are **complementary** if one of the two must occur, and if they can't both occur at the same time.

- If events A and B are complementary, then P(A) + P(B) =1.

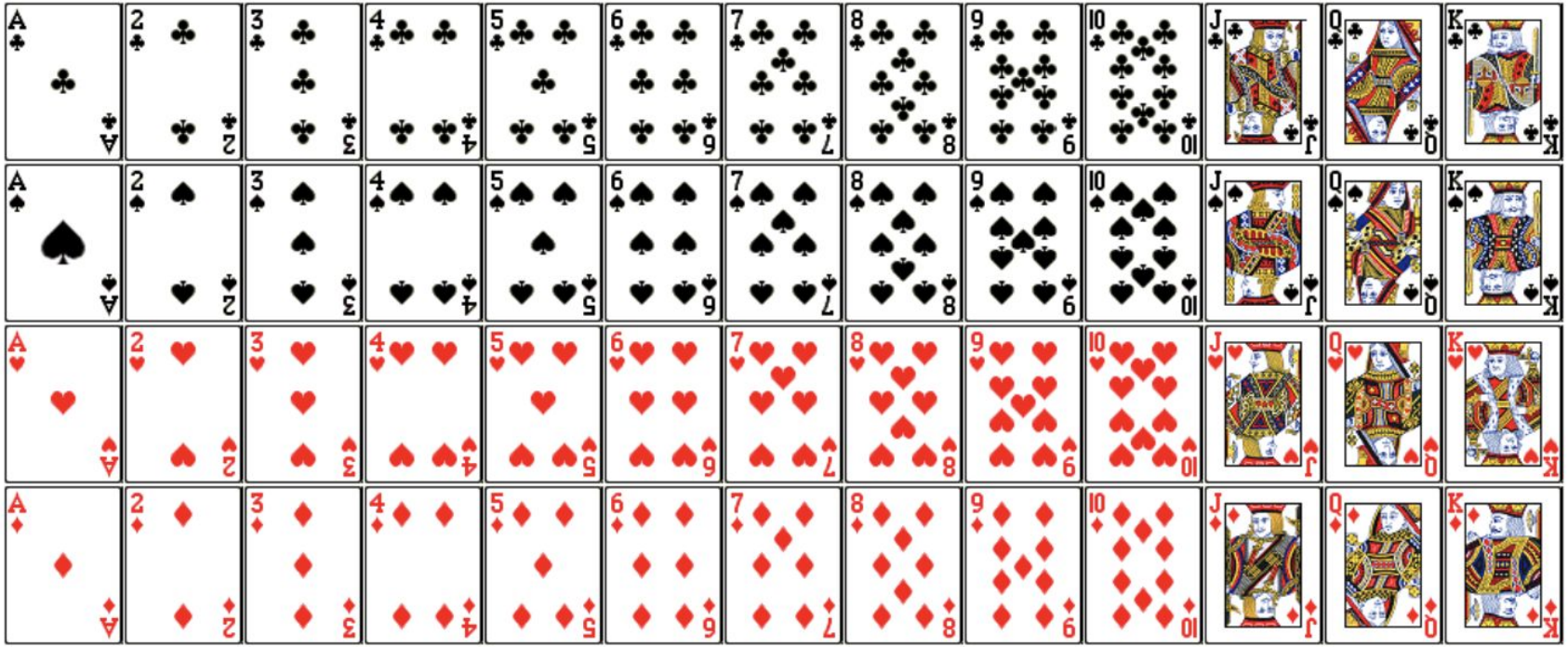
Example: Let's roll a 6-sided die.

Event A: rolling a number less than 3. P(A) = 2/6 or 1/3

Event B: rolling number greater than or equal to 3. P(B) =4/6 or 2/3

Observe that A and B are complementary, and P(A)+P(B)=1

A deck of 52 cards:



Example: drawing a card

In a standard deck of 52 cards, what's the chance of drawing an ace?

The event = drawing an ace

Number of outcome that make the event happens = 4

Total number of outcomes = 52

P(draw an ace) = 4/52 = 1/13

What's the probability that we *do not* draw an ace?

P(we do not draw an ace) = 1 - P(draw an ace) = 1 - 1/13 = 12/13

A Question about M&M's:

- I have four M&M's: a green, yellow, blue, and red.
- I choose two *at random without replacement.*
- **What is the chance that I select the green, followed by the red?**

Possible outcomes: { GY, GB, GR, YG, YB, YR, BG, BY, BR, RG, RY, RB}

Number of ways to select first the green, followed by the red: 1

P(First green, then red) = 1/12

Multiplication Rule

Chance that two events, *A* and *B,* both happen

= P(*A* happens) x P(*B* happens, *given that A* happened)

- The answer will be *less than or equal to* each of the two chances being multiplied
- The more conditions we satisfy, the less likely we are to satisfy them all

A Question about M&M's, again:

- I have four M&M's: a green, yellow, blue, and red.
- I choose two *at random without replacement.*
- What is the chance that I select the green, followed by the red?

P(First green, then red) = P(green) x P(red, given a green was selected)
= 1/4 x 1/3
= 1/12

Addition Rule

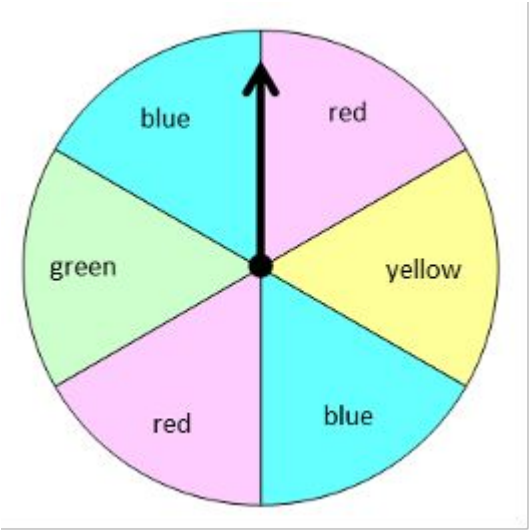
If event A can happen in exactly one of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

The answer is greater than or equal to the chance of each individual way.

Addition rule: example

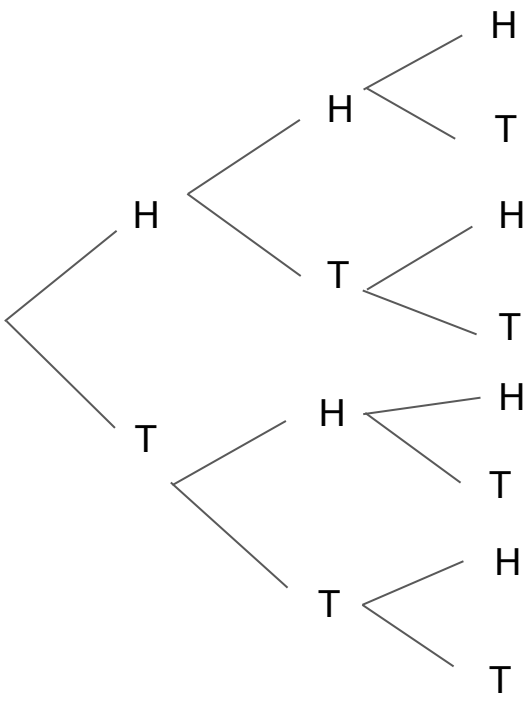
If the spinner is spun once, what's the probability that the pointer lands in a yellow or blue region?



P(Yellow or Blue) = P(Yellow) + P(Blue)

= 1/6 + 2/6 = 3/6 = 1/2

Possible outcomes when tossing a coin 3 times



Outcomes = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Example: in 3 coin tosses…
What's the probability that we get at *least* one head?

Outcomes: { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

The event of getting at least one head occurs with any outcome ***except*** **TTT**

P(at least one head) = 1 - P(TTT)

But P(TTT) = (1/2) x (1/2) x (1/2) = (½)**3 =1/8

So P(at least one head) = 1 - P(TTT) = 1 - (1/8) = 87.5%

Same question, but with 10 tosses:

1 - (1/2)**10 ≅ 99.9%