The project introduces Monte-Carlo simulation for risk-free interest rates, mutual funds, zero-coupon bond and a portfolio. This method considers instantaneous risk-free interest rates, following Hull-White model in which their parameters depends on time:

$$dr(t) = a_r(\theta(t) - r(t))dt + \sigma_r dW_1{}^{\mathbb{Q}}(t)$$

where $\theta(t)$ is a deterministic function of t; representing the interest rate mean in the long-term; a_r is the mean reversion speed, σ_r: interest rate volatility; $dW_1{}^{\mathbb{Q}}(t)$ is a Brownian motion under risk-neutral measure Q.

$\theta(t)$ is chosen as the following in order to replicate the initial term structure:

$$\theta(t) = \frac{1}{a_r}\frac{\partial f^M(0,T)}{\partial T} + f^M(0,T) + \frac{\sigma_r{}^2}{2a_r{}^2}(1 - e^{2a_r.t})$$

where $f^M(0,T)$ is the instantaneous forward rate. After that, the formula for $\theta(t)$ could be further simplified with the support of continuous zero-coupon rates $r_0(t)$, for example in our case following Nelson-Siegel (NS) model.

The obtained instantaneous risk-free interest rates will then be applied to simulate the values of a mutual fund, zero-coupon bonds, and the combination of two of them. For example,

Mutual fund S(t):

$$dS(t) = S(t)[r(t)dt + \sigma_S dW_2{}^{\mathbb{Q}}(t)].$$

We could write the correlated Brownian montions $W_1{}^{\mathbb{Q}}(t)$ and $W_2{}^{\mathbb{Q}}(t)$ with ρ as their correlation parameter as a linear combination of 2 independent Brownian motions $\widetilde{W}_1^{\mathbb{Q}}(t)$ and $\widetilde{W}_2^{\mathbb{Q}}(t)$.

$$dS(t) = S(t)[r(t)dt + \sigma_S(\rho d\widetilde{W}_1^{\mathbb{Q}}(t) + \sqrt{1-\rho^2}d\widetilde{W}_2^{\mathbb{Q}}(t))]$$

Zero coupon bond P(t,T):

$$dP(t,T) = P(t,T)[r(t)dt - \sigma^r.B(t,T)d\widetilde{W}_1^{\mathbb{Q}}(t)]$$

Portfolio as a combination of mutual fund and zero-coupon bond V(t):

$$
\begin{aligned}
dV(t) &= x_S V(t)\frac{dS(t)}{S(t)} + x_P V(t)\frac{dP(t,T)}{P(t,T)} \\
\text{`} &= V^a(t)[r(t)dt + [x_S.\sigma_S\rho - x_P.\sigma_r B(t,T)]d\widetilde{W}_1^{\mathbb{Q}}(t) + x_S.\sigma_S\sqrt{1-\rho^2}d\widetilde{W}_2^{\mathbb{Q}}(t)]
\end{aligned}
$$

where x_S: contribution of mutual fund in the portfolio $V(t)$; and x_P: contribution of zero-coupon bond in the portfolio $V(t)$