

# Assignment 1

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```
# load packages
if(!require(pacman)){install.packages("pacman")}

p_load(devtools,tidyverse,dplyr,ggplot2,latex2exp,cowplot,tseries,sampleSelection)
```

```
There is a binary version available but the source version is later:
      binary source needs_compilation
devtools  2.4.4  2.4.5                FALSE
```

```
#load data
dfData = read.csv("assignment1_2023.csv")
attach(dfData)
```

## 1 Question 1

### 1.1 (i)

```
lm_model = lm(logwage ~ age + agesq + schooling, data = dfData)
summary(lm_model)
```

Call:

```
lm(formula = logwage ~ age + agesq + schooling, data = dfData)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.3224	-1.1782	0.0024	1.2208	3.1957

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	26.409280	8.057036	3.278	0.00113	**
age	-0.341890	0.521078	-0.656	0.51211	
agesq	-0.011142	0.008374	-1.331	0.18408	
schooling	0.215996	0.031534	6.850	2.71e-11	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.499 on 412 degrees of freedom

(250 observations deleted due to missingness)

Multiple R-squared: 0.8148, Adjusted R-squared: 0.8135

F-statistic: 604.3 on 3 and 412 DF, p-value: < 2.2e-16

Looking at our OLS results, firstly, our F-statistic is significant, which means that there is an association between at least of one of the predictor variables and logwage. Thus, we can move on to interpret the coefficients. The adjusted R-square of 0.8135 means that 81.35% of the variance in the dependent variable can be explained by the model. Looking at our OLS estimate, only the OLS estimate of schooling is significant. Thus, we can only interpret the effect of the variable schooling. There is an association between the years of schooling and the salary of a person. Holding other variables constant, a year of schooling is associated with around 0.2160 units increase in log salary of an individual.

## 1.2 (ii)

The sample selection problem here is to choose observations of the non-employed, which are those who have no income. The selection equation is then:

$$I_i = \begin{cases} 1 & \text{if logwage} > 0 \\ 0 & \text{otherwise,} \end{cases}$$

and the second regression equation is:

$$Y_i^* = \mathbf{X}_i' \boldsymbol{\beta} + U_i.$$

We select a sample consisting of:

$$Y_i = \begin{cases} Y_i^* & \text{if } I_i = 1 \\ \text{missing} & \text{if } I_i = 0, \end{cases}$$

An OLS may fail in this context because the dependent variable (logwage) is missing for the non-employed sample, thus, it is not possible to derive an estimate of this variable for the non-employed

## 1.3 (iii)

The exclusion restriction variable is one that is included in  $\mathbf{Z}_i$  but excluded from  $\mathbf{X}_i$ , I would choose 'married' as a suitable candidate for the sample selection model. My motivation is that married people tends to have stable income, and thus, employed.

```

# Create I variable:
dfData = mutate(dfData, vI = if_else(logwage > 0, TRUE, FALSE))
dfData["vI"][is.na(dfData["vI"])] <- FALSE

# Heckman model with restriction
heckman_rest = heckit( vI ~ married+age + agesq + schooling, logwage ~
  ↪ age + agesq + schooling, data = dfData)
summary(heckman_rest)

```

```

-----
Tobit 2 model (sample selection model)
2-step Heckman / heckit estimation
666 observations (250 censored and 416 observed)
12 free parameters (df = 655)
Probit selection equation:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.315285    5.293574  -1.004    0.316
married      0.432572    0.100338   4.311 1.87e-05 ***
age          0.332077    0.342618   0.969    0.333
agesq       -0.005141    0.005512  -0.933    0.351
schooling    0.018246    0.022309   0.818    0.414
Outcome equation:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) 27.209400    8.517748   3.194 0.00147 **
age         -0.385453    0.541932  -0.711 0.47718
agesq       -0.010459    0.008692  -1.203 0.22932
schooling    0.214536    0.031874   6.731 3.69e-11 ***
Multiple R-Squared:0.8148, Adjusted R-Squared:0.813
Error terms:
      Estimate Std. Error t value Pr(>|t|)
invMillsRatio -0.1737    0.6148  -0.283    0.778
sigma          1.4971         NA      NA      NA
rho           -0.1160         NA      NA      NA
-----

```

```

# Heckman model without restriction
heckman_unrest = heckit( vI ~ married + age + agesq + schooling, logwage
  ↪ ~ age + agesq + schooling + married, data = dfData)
summary(heckman_unrest)

```

```

-----
Tobit 2 model (sample selection model)
2-step Heckman / heckit estimation
666 observations (250 censored and 416 observed)
13 free parameters (df = 654)
Probit selection equation:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.315285    5.293574  -1.004    0.316
married      0.432572    0.100338   4.311 1.87e-05 ***
age          0.332077    0.342618   0.969    0.333
agesq       -0.005141    0.005512  -0.933    0.351
schooling    0.018246    0.022309   0.818    0.414
Outcome equation:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) 94.14571    30.04111   3.134   0.0018 **
age         -3.68541     2.91861  -1.263   0.2071
agesq        0.04058     0.04759   0.853   0.3941
schooling     0.03527     0.19887   0.177   0.8593
married     -4.30249         NaN         NaN         NaN
Multiple R-Squared:0.8153, Adjusted R-Squared:0.8131
Error terms:
      Estimate Std. Error t value Pr(>|t|)
invMillsRatio -17.976         NaN         NaN         NaN
sigma          13.399         NA          NA          NA
rho            -1.342         NA          NA          NA
-----

```

Looking at the outcomes of the two model, we can see that the unrestricted model have a much higher standard error, this is because the unrestricted model run into the problem of multicollinearity (i.e., the Inverse Mill Ratio is almost perfectly collinear to the rest of the explanatory variables). Because of multicollinearity, the variable schooling is no longer statistically significant in the model. Thus, we are more unsure of our estimates.

## 1.4 (iv)

```

# Maximum likelihood estimator, restricted
ML_rest = selection(vI ~ married+age + agesq + schooling,
                    logwage ~ age + agesq + schooling, data = dfData)
summary(ML_rest)

```

```

-----
Tobit 2 model (sample selection model)
Maximum Likelihood estimation
Newton-Raphson maximisation, 2 iterations
Return code 8: successive function values within relative tolerance limit (reltol)
Log-Likelihood: -1186.617
666 observations (250 censored and 416 observed)
11 free parameters (df = 655)
Probit selection equation:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.347695    5.290476  -1.011    0.312
married      0.432671    0.100314   4.313 1.86e-05 ***
age          0.334151    0.342394   0.976    0.329
agesq       -0.005174    0.005508  -0.939    0.348
schooling    0.018294    0.022308   0.820    0.412
Outcome equation:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) 27.091220    8.430218   3.214 0.00138 **
age         -0.378997    0.537729  -0.705 0.48118
agesq       -0.010560    0.008627  -1.224 0.22139
schooling    0.214749    0.031784   6.757 3.13e-11 ***
Error terms:
      Estimate Std. Error t value Pr(>|t|)
sigma  1.49568    0.06006  24.902  <2e-16 ***
rho    -0.09931    0.37382  -0.266   0.791
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
-----

```

```

# Maximum likelihood estimator, unrestricted
ML_unrest = selection(vI ~ married + age + agesq + schooling,
                      logwage ~ age + agesq + schooling + married,
                      data = dfData)

summary(ML_unrest)

```

```

-----
Tobit 2 model (sample selection model)
Maximum Likelihood estimation
Newton-Raphson maximisation, 4 iterations
Return code 3: Last step could not find a value above the current.
Boundary of parameter space?

```

Consider switching to a more robust optimisation method temporarily.

Log-Likelihood: -1486.155

666 observations (250 censored and 416 observed)

12 free parameters (df = 654)

Probit selection equation:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-5.315355	Inf	0	1
married	0.432747	Inf	0	1
age	0.331045	Inf	0	1
agesq	-0.005158	Inf	0	1
schooling	0.018152	Inf	0	1

Outcome equation:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	94.10254	Inf	0	1
age	-4.29437	Inf	0	1
agesq	0.05000	Inf	0	1
schooling	0.07999	Inf	0	1
married	-3.79444	Inf	0	1

Error terms:

	Estimate	Std. Error	t value	Pr(> t )
sigma	7.6483	Inf	0	1
rho	-0.9989	Inf	0	1

-----

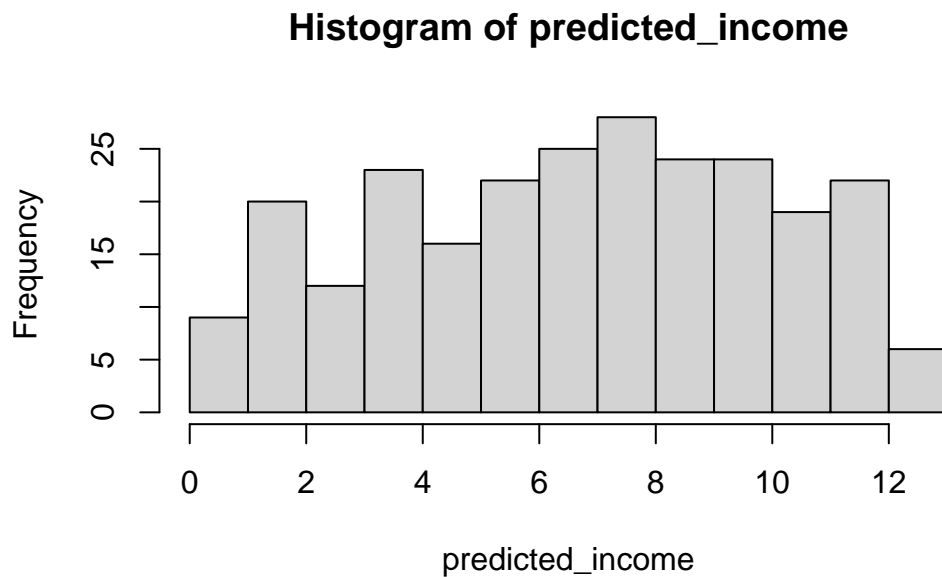
Similar to the situation in (iii), we can see that the unrestricted model have a much higher standard error, this is because the unrestricted model run into the problem of multicollinearity (i.e., the Inverse Mill Ratio is almost perfectly collinear to the rest of the explanatory variables). Because of in increase in standard errors, the variable schooling is no longer statistically significant in the model. Thus, we are unsure of our estimates.

## 1.5 (v)

To specify the distribution of potential earnings for the non-employed, we first get a subsample of the unemployed individuals. Then, we use one of the restricted models in (iii) or (iv) to predict potential income of the non-employed and draw a histogram.

```
# Get subsample of unemployed individuals
dfUnEmployed = dfData[dfData$vI == FALSE, ]

predicted_income = predict(ML_rest, newdata = dfUnEmployed)
hist(predicted_income)
```



The histogram does not give an apparent normal distribution. However, we can say that most predictions lies between 6 and 8, and the distribution is slightly left-skewed.

## 2 Question 2

### 2.1 (i)

```
# Get subsample of employed individuals
dfEmployed = dfData[dfData$vI == TRUE, ]

model0 = lm(logwage ~ schooling + age + agesq, data = dfEmployed)
summary(model0)
```

Call:

```
lm(formula = logwage ~ schooling + age + agesq, data = dfEmployed)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.3224	-1.1782	0.0024	1.2208	3.1957



Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	26.409280	8.057036	3.278	0.00113	**
schooling	0.215996	0.031534	6.850	2.71e-11	***
age	-0.341890	0.521078	-0.656	0.51211	
agesq	-0.011142	0.008374	-1.331	0.18408	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.499 on 412 degrees of freedom

Multiple R-squared: 0.8148, Adjusted R-squared: 0.8135

F-statistic: 604.3 on 3 and 412 DF, p-value: < 2.2e-16

Looking at our OLS results, firstly, our F-statistic is significant, which means that there is an association between at least of one of the predictor variables and logwage. Thus, we can move on to interpret the coefficients. The OLS estimate of schooling is significant. Thus, we can only interpret the effect of the variable schooling. There is an *association* between the years of schooling and the salary of a person. Holding other variables constant, a year of schooling is associated with around 0.2160 increase in log salary of an individual.

However, we CANNOT discuss the causal effect of schooling on income, because association is different from causation.

Regarding whether it is plausible that regularity conditions for applying OLS are satisfied. We believe it is plausible that some conditions such as homoskedasticity, X non-random, the error terms are normally distributed and has mean zero, no-auto correlation are satisfied, the condition that model is linear is also satisfied. However we are not sure if there is any multicollinearity between the explanatory variables yet. For example, the distance to school and the regional subsidy for school expenses might be correlated with the years of schooling of an individual. Specifically, people who have more subsidy and shorter distance to school could spend more years in school.

## 2.2 (ii)

```
# Using distance as instrument variable
model1 = lm(schooling ~ distance, data = dfEmployed)
X.hat.1 = fitted.values(model1)

# Fit Linear regression model again using the fitted values of first
↪ step
```

```
model2 =lm(logwage ~ X.hat.1 + age + agesq, data = dfEmployed)
summary(model2)
```

Call:

```
lm(formula = logwage ~ X.hat.1 + age + agesq, data = dfEmployed)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.4199	-1.2578	-0.0541	1.2115	3.5095

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	26.429485	8.687422	3.042	0.0025 **
X.hat.1	0.457857	0.284959	1.607	0.1089
age	-0.460586	0.547910	-0.841	0.4010
agesq	-0.009026	0.008804	-1.025	0.3059

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.577 on 412 degrees of freedom

Multiple R-squared: 0.795, Adjusted R-squared: 0.7935

F-statistic: 532.6 on 3 and 412 DF, p-value: < 2.2e-16

```
# Using subsidy as instrument variable
model3 = lm(schooling ~ subsidy , data = dfEmployed)
X.hat.3 = fitted.values(model3)

# Fit Linear regression model again using the fitted values of first
↪ step

model4 =lm(logwage ~ X.hat.3 + age + agesq , data = dfEmployed)
summary(model4)
```

Call:

```
lm(formula = logwage ~ X.hat.3 + age + agesq, data = dfEmployed)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.4083	-1.2306	-0.0321	1.3012	3.6294

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	25.889824	8.403013	3.081	0.002201	**
X.hat.3	0.410382	0.108613	3.778	0.000181	***
age	-0.413512	0.540517	-0.765	0.444691	
agesq	-0.009649	0.008684	-1.111	0.267186	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.555 on 412 degrees of freedom

Multiple R-squared: 0.8006, Adjusted R-squared: 0.7992

F-statistic: 551.5 on 3 and 412 DF, p-value: < 2.2e-16

```
# Using subsidy and distance as instrument variable
model5 = lm(schooling ~ subsidy+distance, data = dfEmployed)
X.hat.5 = fitted.values(model3)

model6 =lm(logwage ~ X.hat.5 + age + agesq, data = dfEmployed)
summary(model6)
```

Call:

```
lm(formula = logwage ~ X.hat.5 + age + agesq, data = dfEmployed)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.4083	-1.2306	-0.0321	1.3012	3.6294

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	25.889824	8.403013	3.081	0.002201	**
X.hat.5	0.410382	0.108613	3.778	0.000181	***
age	-0.413512	0.540517	-0.765	0.444691	
agesq	-0.009649	0.008684	-1.111	0.267186	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.555 on 412 degrees of freedom

Multiple R-squared: 0.8006, Adjusted R-squared: 0.7992

F-statistic: 551.5 on 3 and 412 DF, p-value: < 2.2e-16

After using 3 options of instrument variables, we can see that 'distance' is not a good instrument variable as the part of 'schooling' not explained by 'distance' (stored in 'X.hat.1') is not

statistically significant in the linear model. On the other hand, after adding 'subsidy' as an instrument variable, the part of 'schooling' not correlated with 'subsidy' is statistically significant in explaining the changes in 'logwage'. We will only use 'subsidy' as the instrument variable, not a combination of both 'subsidy' and 'schooling', because using both variables can lead to the issue of over-identification.

### 2.3 (iii)

```
# OLS outcomes  
model0$coefficients
```

(Intercept)	schooling	age	agesq
26.4092796	0.2159957	-0.3418898	-0.0111417

```
# IV outcomes  
model4$coefficients
```

(Intercept)	X.hat.3	age	agesq
25.889824395	0.410381734	-0.413511745	-0.009648645

Above are the OLS and IV estimates, in which, X.hat.3 in the IV estimates is the part of 'schooling' that is not correlated with subsidy.

Looking at the outcomes, we can see that the IV estimate of schooling coefficient is higher than the OLS estimate. This might be because the effect of subsidy on schooling is eliminated, making the number of years of schooling smaller, and thus it needs higher weights to predict the income. Both OLS and IV estimates yielded significant effect from the schooling variable.

We would prefer OLS in the case where there is no correlation between explanatory variables and the error terms.

To decide between OLS and IV, we first perform a t-test to check the relevance of the instrument (i.e., checking whether 'subsidy' and 'schooling' are correlated).

```
summary(model3)
```

Call:

```
lm(formula = schooling ~ subsidy, data = dfEmployed)
```

Residuals:

Min	1Q	Median	3Q	Max
-5.5382	-1.5382	0.0031	1.7324	4.8151

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	4.83150	0.30298	15.947	< 2e-16 ***
subsidy	0.27067	0.04217	6.419	3.77e-10 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.238 on 414 degrees of freedom

Multiple R-squared: 0.09051, Adjusted R-squared: 0.08831

F-statistic: 41.2 on 1 and 414 DF, p-value: 3.769e-10

We can see that there is statistically significant evidence that there is an association between 'subsidy' and 'schooling'. Thus, 'subsidy' is a relevant instrument. Moreover, we also need to check the validity of the instrument using the Sargan test:

```
sargan_test = lm(model3$residuals ~ subsidy + age + agesq, data =  
  ↪ dfEmployed)  
  
test_statistics <- summary(sargan_test)$r.squared*nrow(dfEmployed)  
  
print(1-pchisq(test_statistics,1)) # prints p-value
```

```
[1] 0.01679377
```

We can see that the p-value is 0.0167793, which is significant at  $\alpha = 0.05$ . This means that our instrument variable is valid.

Using the result of both the t-test and the Sargan test, we choose to use the IV estimate instead of the OLS one as the IV estimate utilises a relevant and valid instrument variable.