PROBABILITY

Table of Contents

Introduction

- Branch of mathematics that deals with randomness and uncertainty
- Probability is a measure of the likelihood of an event.

- **Mathematically,**

$$\textbf{Probability} = \frac{\textbf{Total favourable cases}}{\textbf{Total possible cases}}$$

- **Example:** Rolling a fair 6 sided Die, What is the probability of getting 6?

  Probability (6) = $\frac{1}{6}$

- Probability lies in between 0 to 1,
  - 0 indicates impossible to occur,
  - 1 indicates full certainty

Terminologies

1. **Experiment**
    - Experiment is a process that generates an outcome.
    - Example: Tossing a coin is an experiment

2. **Outcomes**
    - Outcomes are the possible results of an experiment.
    - Example: In the case of a coin toss, Outcomes are **head** and **tail**.

3. **Trial**
    - Trial is a single performance of an experiment.
    - Example:
        - Single coin flip is a trial,
        - In multiple coin flip, each flip is a separate trial.

4. **Sample Space**
    - Sample Space (S) is set of all possible outcomes of an experiment.
    - Example: For a coin toss, **Sample space = {head, tail}**

Terminologies (CONTD)

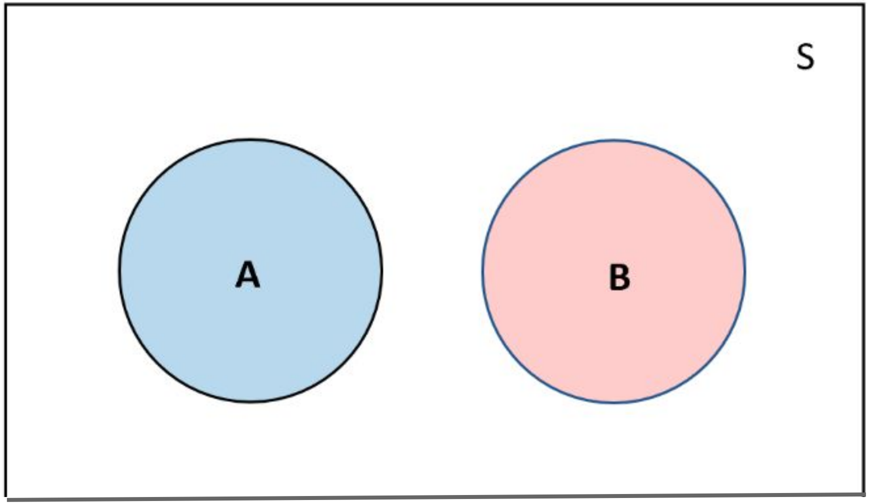
4. **Events:**
    - Events are the subset of sample space that consists one or more outcomes.

    - Example: When rolling a die,
        - Event of **rolling a “6”** consists of only one outcomes,
        - Event of **rolling a “even numbers”** consists of the outcomes {2, 4, 6}

- **Rolling a die** is an experiment
- **1, 2, 3, 4, 5, 6** are an outcomes of rolling a die
- **Single roll** of an dice is a trial
- **{1, 2, 3, 4, 5, 6}** are the sample space
- **Rolling a 6** is an event

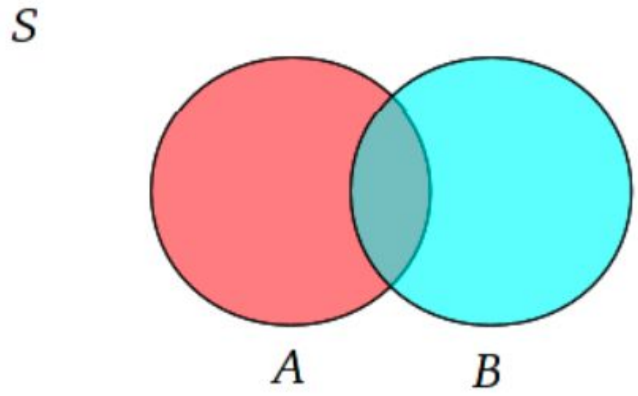
Mutually Exclusive Events

- Two events are mutually exclusive, if they cannot occur at the same time.
- Example:
  - When **rolling a Dice**, occurrence of event 1 and 2 at the same time is not possible.
    - It means we can either get 1, or, 2, or, 3, or, 4, or, 5, or 5
  - When **Tossing a coin**, we may either get Head or Tail but not both.



Non-mutually Exclusive Events

- Non-mutually exclusive events are events that can occur at the same time.
- If event A happens, event B can still happen, and vice-versa.
- **Example:**
  - In Deck of cards, the events "drawing a red card" and "drawing a face card" are non-mutually exclusive events. You can draw red face card (e.g. the Queen of Hearts), satisfying both conditions.



Addition Rule in Probability

- **Mutually Exclusive Events**
  - If A and B are two mutually exclusive events, then the probability of getting either A or B denoted by **P(A or B)** is given by,

$$P(A \text{ or } B) = P(A) + P(B)$$

- **Non-Mutually Exclusive Events**
  - If A and B are two non-mutually exclusive events, then the probability of getting either A or B denoted by P(A or B) is given by,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

Problems

1. **If a fair Dice is Roll, What is the probability of getting 1 or 3 or 6 ?**

   - **Hint:** mutually exclusive → p(1) + p(2) + p(3) → ($\frac{1}{6}$ + $\frac{1}{6}$ + $\frac{1}{6}$)

2. **If a fair Coin is Toss, What is the probability of getting Head or Tail?**

   - **Hint:** Non-mutually exclusive → p(Head) + p(Tail) → ($\frac{1}{2}$ + $\frac{1}{2}$)

Problems

3. **You are picking a card randomly from a Deck of 52 cards. What is the probability of choosing a card that is Queen or Heart i.e P(Queen or Heart)?**

- Non-mutually exclusive events
- P(A or B) = P(A) + P(B) - P(AnB)

- **P(Queen)** → 4/52
- **P(Heart)** → 13/52
- **P(Queen n Heart)** → 1/52

- From addition theorem in probability (for Non mutually exclusive events)

- P(Queen or Heart) = P(Queen) + P(Heart) - P(Queen n Heart)
    - 4/52 + 13/52 - 1/52
    - 16/52

Independent Events

- Two events, A and B are independent event if the occurrence of one event does not affect the probability of the other event happening.

- Independent events are not affected by previous events.

- **Example:**
  - Consider flipping a coin twice. The outcomes of the first flip does not affect the outcome of the second flip. These events are independent.

Q. When you toss a coin and it comes up "Heads" three times. What is the chance that the next toss will also be a "Head"?

>> Since Tossing a coin is an independent event, probability will still be 1/2

Dependent Events

- Two events A and B are considered dependent if the occurrence (or non-occurrence) of one event affects the probability of the other event happening.

- In other words, outcome of one event influences the outcome of the other event.

- **Example:**
  - Suppose you draw two cards from a deck without replacement.
  - The probability of drawing a black card on the second draw depends on the outcome of the first draw because there’s one less card in the deck after the first draw.

“Drawing a card with replacement is Independent Events”

Multiplication Rule

- **Independent Events**
  - The probability of simultaneous happening of two independent events A and B is given by their product of individual probabilities

$$P(A \text{ and } B) = P(A \cap B) = P(A) * P(B)$$

- **Dependent Events**
  - The probability of simultaneous happening of two dependent events A and B is given by,

    i. the product of probability of event A and the conditional probability of event B given that A has already occured

$$P(A \text{ and } B) = P(A \cap B) = P(A) * P(B|A)$$

    ii. the product of probability of event B and the conditional probability of event A given that B has already occured

$$P(A \text{ and } B) = P(A \cap B) = P(B) * P(A|B)$$

Conditional Probability

- For any two dependent variables the probability of event given that other has already occurred is called conditional probability.

- The conditional probability of event A given that B has already happened is given by,

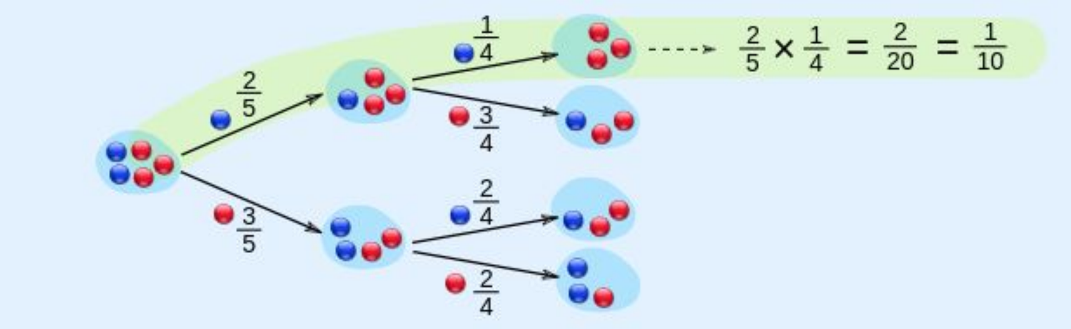
$$P(A|B) = P(A \cap B) / P(B) \qquad P(B) \neq 0$$

- The conditional probability of event B given that A has already happened is given by,

$$P(B|A) = P(B \cap A) / P(A) \qquad P(A) \neq 0$$

Problems

4. 2 blue and 3 red marbles are in a bag. What are the chances of drawing 2 blue marbles without replacement i.e P(blue_marble and blue_marble) ?



P(blue_marble and blue_marble) = P(blue_marble) * P(blue_marble | blue_marble)
= 2/5 * 1/4
= 2/20

Image source: math is fun

Bayes Theorem

P(A|B) = P(A n B) / P(B) → P(A n B) = P(B) * P(A|B)

P(B|A) = P(B n A) / P(A) → P(B n A) = P(A) * P(B|A)

P(A n B) == P(B n A)

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$ → **BAYES THEOREM**

P(A|B) → Probability of event A occurring given B. Posterior probability
P(B|A) → Probability of event B occurring given A. Likelihood probability
P(A) → Probability of observing A without any conditions. Prior Probability
P(B) → Probability of observing B without any conditions. Marginal Probability

Bayes Theorem (Application)

Suppose we have a dataset of weather conditions and corresponding target variable "Play". So using this dataset we need to decide that whether we should play or not on a particular day according to the weather conditions.

Problem: If the weather is sunny, then the Player should play or not?

	Outlook	Play
0	Rainy	Yes
1	Sunny	Yes
2	Overcast	Yes
3	Overcast	Yes
4	Sunny	No
5	Rainy	Yes
6	Sunny	Yes
7	Overcast	Yes
8	Rainy	No
9	Sunny	No
10	Sunny	Yes
11	Rainy	No
12	Overcast	Yes
13	Overcast	Yes

Solution

1. Create frequency table for the weather conditions:

Weather	Yes	No
Overcast	5	0
Rainy	2	2
Sunny	3	2
Total	10	5

2. Generate Likelihood table by finding the probabilities of given features.

Weather	No	Yes	
Overcast	0	5	5/14= 0.35
Rainy	2	2	4/14=0.29
Sunny	2	3	5/14=0.35
All	4/14=0.29	10/14=0.71	

Solution

3. Apply Bayes Theorem to calculate the posterior probability

- P(Yes|Sunny)= P(Sunny|Yes)*P(Yes)/P(Sunny)
  - P(Sunny|Yes)= 3/10= 0.3
  - P(Sunny)= 0.35
  - P(Yes)=0.71

- P(Yes|Sunny) = 0.3*0.71/0.35= 0.60

- P(No|Sunny)= P(Sunny|No)*P(No)/P(Sunny)
  - P(Sunny|NO)= 2/4 = 0.5
  - P(No)= 0.29
  - P(Sunny)= 0.35
- P(No|Sunny)= 0.5*0.29/0.35 = 0.41

P(Yes|Sunny) > P(No|Sunny), Hence on a Sunny day, Player can play the game.

References

- https://www.mathsisfun.com/data/probability.html
- https://www.youtube.com/watch?v=LZzq1zSL1bs&t=10488s
- https://www.javatpoint.com/machine-learning-naive-bayes-classifier