D/BSE/19/0014
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Assignment II

Problem - 1

X = Number of particles withdrawn $\quad X \sim Poisson(\lambda)$

50 particles $\rightarrow$ 1ml

250 particles $\rightarrow$ 5ml

$\lambda = 250$

Since $\lambda > 10$ we can use normal approximation

$X \sim N(250, 250)$

a) $P(235 < X < 265) = P\left(\frac{235-250}{\sqrt{250}} < \frac{X-250}{\sqrt{250}} < \frac{265-250}{\sqrt{250}}\right)$

$= P(-0.94868 < Z < 0.94868)$

$= P(Z < 0.94868) - P(Z < -0.94868)$

$= P(Z < 0.94868) - [1 - P(Z < 0.94868)]$

$= 2(0.82639) - 1$

$= \underline{0.65278}$

(b) Average number per 1ml in 5ml sample.

Since the average number of particles for 1ml will be between 48 and 52.

The average number of particles for 5ml will be between $48(5) = 240$ and $52(5) = 260$.

$\lambda = 250 > 10$

we use normal approximation.

$$P(240 < \bar{x} < 260) = P\left(\frac{240-250}{\sqrt{250}} < \frac{\bar{x}-250}{\sqrt{250}} < \frac{260-250}{\sqrt{250}}\right)$$

$$= P(-0.63245 < Z < 0.63245)$$

$$= P(Z < 0.63245) - P(Z < -0.63245)$$

$$= P(Z < 0.63245) - [1 - P(Z < 0.63245)]$$

$$= 2P(Z < 0.63245) - 1$$

$$= 2(0.73565) - 1$$

$$= \underline{\underline{0.4713}}$$

c) Average number per 1ml in a 10ml sample,

Since the Average number of particles for 1ml will be between 48 and 52

The average number of particles for 10ml will be between 480 and 520

λ for 10 ml sample is $500 > 10$ $\quad \bar{X} \sim N(500, 500)$

$$P(480 < \bar{x} < 520) = P\left(\frac{480-500}{\sqrt{500}} < \frac{\bar{x}-500}{\sqrt{500}} < \frac{520-500}{\sqrt{500}}\right)$$

$$= P(-0.8944 < Z < 0.8944)$$

$$= P(Z < 0.8944) - P(Z < -0.8944)$$

$$= 2P(Z < 0.8944) - 1$$

$$= 2(0.81327) - 1$$

$$= \underline{\underline{0.62654}}$$

d) Let n be the sample size.

The λ is $50n$.

The average number of particles for n ml is between $48n$ and $52n$.

$$0.95 = P(48n < \bar{x} < 52n) = P\left(\frac{48n - 50n}{\sqrt{50n}} < \frac{\bar{x} - 50n}{\sqrt{50n}} < \frac{52n - 50n}{\sqrt{50n}}\right)$$

$$0.95 = P\left(\frac{-2n}{\sqrt{50n}} < Z < \frac{2n}{\sqrt{50n}}\right)$$

$$0.95 = 2P\left(Z < \frac{2n}{\sqrt{50n}}\right) - 1$$

$$P\left(Z < \frac{2n}{\sqrt{50n}}\right) = \frac{1.95}{2}$$

$$= 0.975$$

$$Z = 1.96 = \frac{X - \mu}{\sigma}$$

$$1.96 = \frac{48n - 50n}{\sqrt{50n}}$$

$$1.96 = \frac{-2n}{\sqrt{50n}}$$

$$(1.96)^2 = \frac{4n^2}{50n}$$

$$4n^2 = (1.96)^2 \times 50n \quad ; \quad n > 0$$

$$n = \frac{(1.96)^2 \times 50}{4}$$

$$= 48.02$$

$$\simeq 48$$

$\therefore$ The sample size will be 48 ml //

Problem - 2

(a) X - no of defective parts in a shipment.

$p = 0.2$

$n = 400$

$X \sim Bin(400, 0.2)$

By CLT,

$X \sim N(np, npq)$

$X \sim N(80, 64)$

$P(X > 90) = P(X > 90 + 0.5)$

$= P\left(Z > \frac{90.5 - 80}{8}\right)$

$= P(Z > 1.31)$

$= 1 - 0.9049$

$= 0.0951$

(b) X - no of returned shipments in a day.

$p = 0.0951$ $\qquad X \sim Bin(500, 0.0951)$

$n = 500$.

By CLT,

$X \sim N(47.55, 43.03)$

$P(X \geq 60) = P(X > 60 - 0.5)$

$= P\left(Z > \dfrac{59.5 - 47.55}{6.56}\right)$

$= P(Z > 1.82)$

$= 1 - 0.9656$

$= 0.0344$ //

(c) $0.01 = P\left(Z > \dfrac{90.5 - 400p}{\sqrt{400p(1-p)}}\right)$

$0.01 = P(Z > Y)$

$0.01 = P(Z > Y)$

$0.01 = 1 - P(Z < Y)$

$P(Z < Y) = 0.99 \Rightarrow Y = 2.33$

$$2.33 = \frac{90.5 - 400p}{20\sqrt{p(1-p)}}$$

$$46.6 = \frac{90.5 - 400p}{\sqrt{p - p^2}}$$

$2171.56(p - p^2) = 8190.25 + 160\,000p^2 - 72400p$

$(157{,}828.44)p^2 - (74571.56)p + 8190.25 = 0$

$p = 0.29 \; / \; p = 0.17$

To reduce the probability that a shipment is returned the percentage of defective parts should be less than 20%.

∴ 17% defective parts needs to reach this goal.

problem - 3

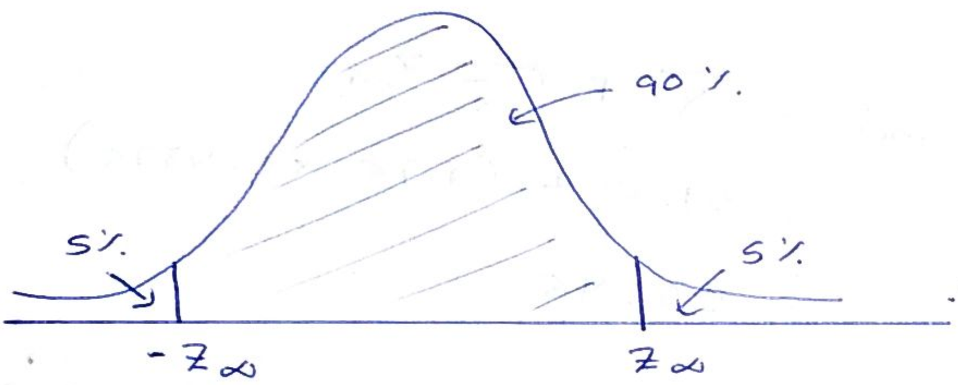
$n = 53$ $\bar{x} = 21.6$ $\sigma = 3.2$

The standard error of the mean, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$= \frac{3.2}{\sqrt{53}}$

$= 0.4395$

(a)



$\alpha = \frac{1+0.9}{2}$

$= 0.95$

$\therefore Z_\alpha = 1.64$

Upper confidence bound $= \bar{x} + \sigma_{\bar{x}} \cdot Z_\alpha$

$= 21.6 + (0.4395 \times 1.64)$

$= 22.32.$

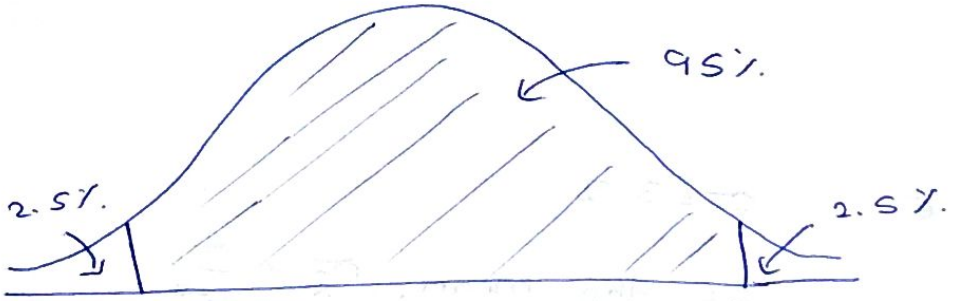
Lower confidence bound $= \bar{x} - \sigma_{\bar{x}} \cdot Z_\alpha$

$= 21.6 - (0.4395 \times 1.64)$

$= 20.87$

$\therefore$ 90% confidence Interval is $(20.87, 22.32)$ //

(b)



$\alpha = \frac{1+0.95}{2} = 0.975$

$\therefore Z_{\alpha} = 1.96$

upper confidence bound $= \bar{X} + \sigma_{\bar{X}} Z_{\alpha}$

$= 21.6 + (1.96 \times 0.4395)$

$= 22.46$

Lower confidence bound $= \bar{X} - \sigma_{\bar{X}} Z_{\alpha} = 21.6 - (1.96 \times 0.4395)$

$= 20.74$

$\therefore$ 95% confidence interval is $(20.74, 22.46)$ //

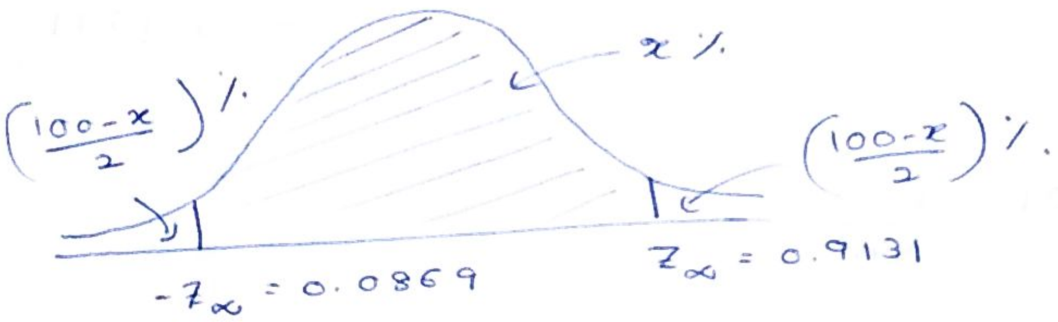
(c) $(21.0, 22.2)$

$$21 = 21.6 - (Z_\alpha \times 0.4395)$$

$$Z_\alpha = \frac{0.6}{0.4395} = 1.36$$

$$22.2 = 21.6 + (Z_\alpha \times 0.4395)$$

$$Z_\alpha = \frac{+0.6}{0.4395} = +1.36$$



According to above normal curve,

$$\left(\frac{100-x}{2}\right)\% = 0.0869$$

$$\frac{100-x}{100} = 0.1738$$

$$100 - x = 17.38$$

$$x = 100 - 17.38$$

$$x = 82.62$$

$\therefore$ confidence level $= \underline{\underline{82.62\%}}$

(d) The Standard error of the mean $= \frac{0.3}{1.64}$

$= 0.1829$

$$0.1829 = \frac{3.2}{\sqrt{n}}$$

$$\sqrt{n} = \frac{3.2}{0.1829} = 17.49$$

$$n = 305.9001$$

∴ 305 Specimens must be sampled //

(90% CI)

(e) The Standard error of the mean $= \frac{0.3}{1.96}$

$= 0.1531$

$$0.1531 = \frac{3.2}{\sqrt{n}}$$

$$\sqrt{n} = 20.9$$

$$n = 436.86$$

∴ 436 specimens must be sampled //

(95% CI).

Problem 4

(a) number of born out fuses $X = 17$

Sample size $n = 75$

$$\hat{p} = \frac{x}{n} = \frac{17}{75} = 0.2267$$

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.2267 \times 0.7733}{75}}$$

$$= 0.0483$$

for a 95% confidence interval the value of

$$\frac{\alpha}{2} = 0.025$$

$$\therefore Z_{\frac{\alpha}{2}} = 1.96$$

Upper confidence bound $= 0.2267 + (1.96 \times 0.0483)$
$= 0.3214$

Lower confidence bound $= 0.2267 - (1.96 \times 0.0483)$
$= 0.1320$

$\therefore$ 95% confidence interval is $(0.1320, 0.3214)$

(b) for a 98% confidence interval the value of $\frac{\alpha}{2} = 0.01$

$\therefore Z_{\frac{\alpha}{2}} = 2.32$

Upper confidence bound $= 0.2267 + (2.32 \times 0.0483)$

$= 0.3387$

Lower confidence bound $= 0.2267 - (2.32 \times 0.0483)$

$= 0.1146$

$\therefore$ 98% confidence interval is $(0.1146, 0.3387)$

(c) $0.05 = 1.96 \times \sqrt{\frac{0.2267 \times 0.7733}{n}}$

$0.0255 = \sqrt{\frac{0.1753}{n}}$

$0.0007 = \frac{0.1753}{n}$

$n = 250.43$

$n = 250$

d) $0.05 = 2.32 \sqrt{\dfrac{0.2267 \times 0.7733}{n}}$

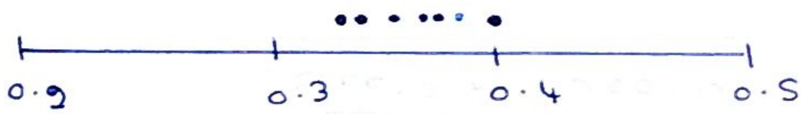
$0.0216 = \sqrt{\dfrac{0.1753}{n}}$

$n = 350.6$

$n = 350$

problem 5

(a)



Corrosion Loss (mm)

(b) The plot is not strongly asymmetric and there are no outliers.

The Student's t distribution can be used.

$$\bar{X} = \frac{\sum x_i}{n} = 0.3688 \qquad \sigma = 0.0187$$

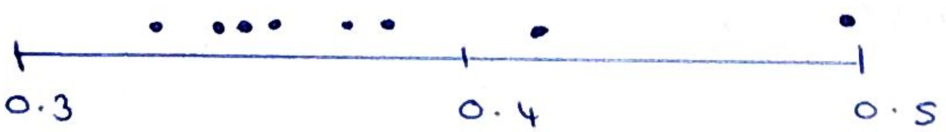
$n = 8 \qquad \frac{\alpha}{2} = 0.005 \rightarrow$ from t table.

$$t_{7, 0.005} = 3.499$$

$$\therefore \; 99\% \; CI = 0.3688 \pm (3.499)\frac{(0.0187)}{\sqrt{8}}$$

$$= (0.3919 , 0.3457) //$$

(c)



Corrosion Loss (mm)

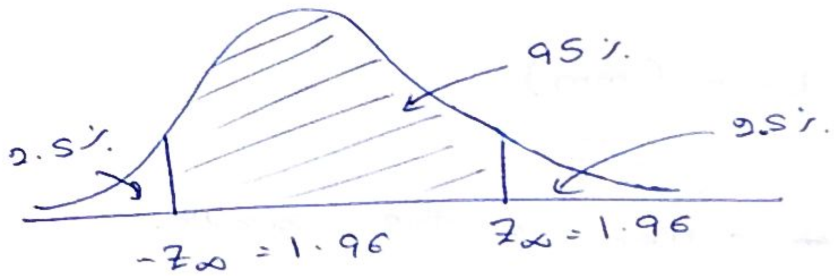
(d) There is an outlier in this sample.

The Student's t distribution should not be used.

Problem 6

$n = 6$, $\bar{x} = 19.35$, $\sigma = 0.577$

(a)



The standard error of the mean $= \frac{0.577}{\sqrt{\ }}$

$= 0.2355$

upper confidence bound $= \bar{x} + Z_\alpha \sigma_{\bar{x}}$

$= 19.35 + (0.2355 \times 1.96)$

$= 19.81$

Lower confidence bound $= \bar{x} - Z_\alpha \sigma_{\bar{x}}$

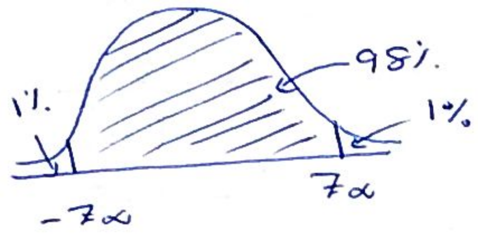
$= 19.35 - (0.2355 \times 1.96)$

$= 18.89$

$\therefore$ 95% confidence interval is (18.89 , 19.81) //

(b)

$$\alpha = \frac{1 + 0.98}{2} = 0.99$$



$$\therefore Z_\alpha = 2.32$$

Upper confidence bound $= \bar{x} + Z_\alpha \sigma_{\bar{x}}$

$= 19.35 + (0.2355 \times 2.32)$

$= 19.89$

Lower confidence bound $= \bar{x} - Z_\alpha \sigma_{\bar{x}}$

$= 19.35 - (2.32 \times 0.2355)$

$= 18.89$

$\therefore$ 98% confidence interval is $(18.89, 19.89)$ //

(c) Sample mean of measurements $= \dfrac{\sum_{i=1}^{6} x_i}{n} = \dfrac{116.1}{6}$

$= 19.35$

Both 95% an 98% confidence interval are valid because mean value is in between the intervals //

problem 7

$n = 100$ $\qquad \bar{x} = 150$ $\qquad \sigma = 25$

The standard error of the mean $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

$$= \frac{25}{\sqrt{100}}$$

$$= 2.5$$

(i) for 95% confidence interval $Z_{\alpha} = 1.96$

Upper confidence bound $= 150 + (1.96 \times 2.5)$

$= 154.9$

Lower confidence bound $= 150 - (1.96 \times 2.5)$

$= 145.1$

$\therefore$ 95% confidence interval is $(145.1, 154.9)$ //

(ii) For 99% confidence interval, $\alpha = \dfrac{1 + 0.99}{2}$

$= 0.995$

$\therefore Z_{\alpha} = 2.57$

Upper confidence bound $= 150 + (2.57 \times 2.5)$

$= 156.425$

Lower confidence bound $= 150 - (2.57 \times 2.5)$

$= 143.575$

$\therefore$ 99% CI is $(143.575, 156.425)$ //

(iii) 95% confidence interval

(iv) $\frac{2}{1.96} = \frac{25}{\sqrt{n}}$

$\sqrt{n} = 24.5$

$n = 600.25$

600 batteries must be sampled. So that a 95% confidence interval will specify the mean to within ± 2 hours.//

(v) $\frac{2}{2.51} = \frac{25}{\sqrt{n}}$

$\sqrt{n} = 32.125$

$n = 1032.01$

1032 batteries must be sampled. So that a 99% confidence interval will specify the mean to with in ± 2 hours.//