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Application of Nonlinear Slosh Damping to Launch Vehicle Control Analysis

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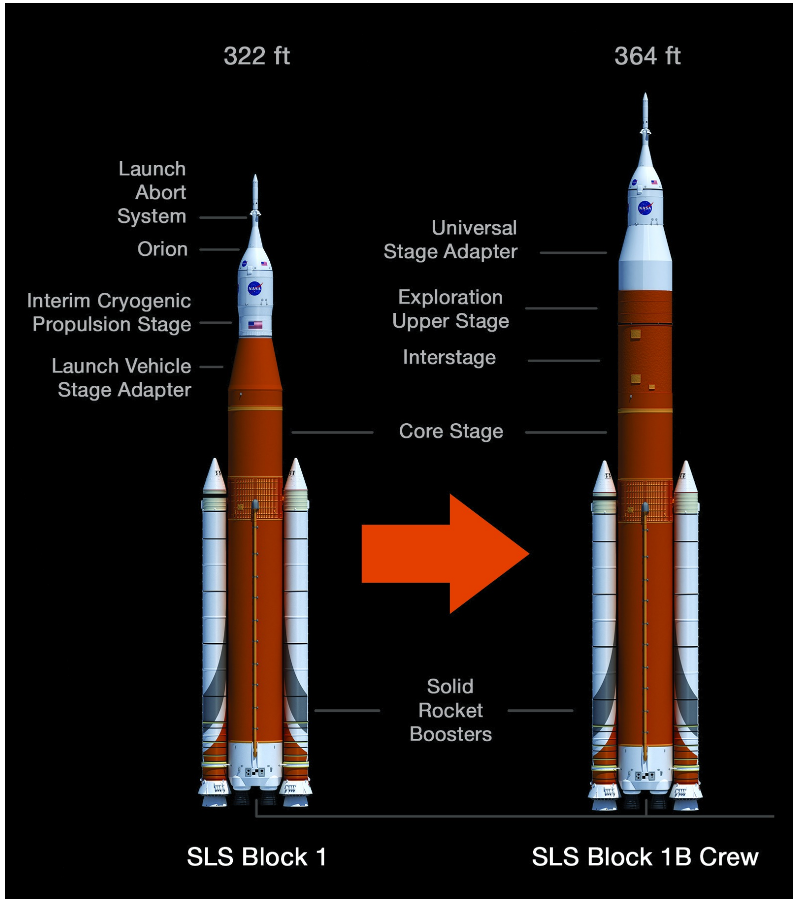
Preliminaries

- This presentation is based on techniques developed for the SLS Block 1 and Block 1B vehicles.
    - However, no SLS data is included this presentation
- Today’s presentation is based on the conference paper:
    - *J. Ottander, R. Hall, J. Powers, “Practical Methodology for the Inclusion of Nonlinear Slosh Damping in the Stability Analysis of Liquid-propelled Space Vehicles”, AIAA Guidance Navigation and Control Conference, 2018*
    - Would like to acknowledge Rob Hall (Blue Origin), Joey Powers (NASA), Tannen VanZwieten (NASA), and John Wall (Dynamic Concepts), Jeb Orr (McClaurin)



Overview

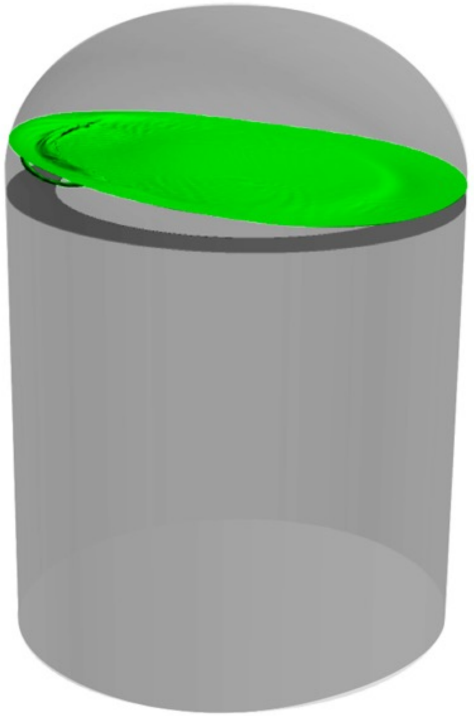
- Why Can Slosh be De-stabilizing?
  - Introduction of a minimal model to demonstrate.
- Typical Issues with Mitigation of Slosh-Control Instability
- The Use of Non-Linear Damping
- Discussion

Why Can Slosh be De-stabilizing?

Introduction of a minimal model to demonstrate.

Slosh Introduction

- This presentation focus on lateral slosh in a high-G regime and its interaction with vehicle attitude control.
- Typically, high-G slosh is modelled with mechanical analogue models of either the pendulum model or the spring-mass model.
- Slosh mechanical model parameters (including damping) derived from fluids theory, test, and CFD.
- Slosh frequency scales with the diameter of the tank and the acceleration the tank is experiencing



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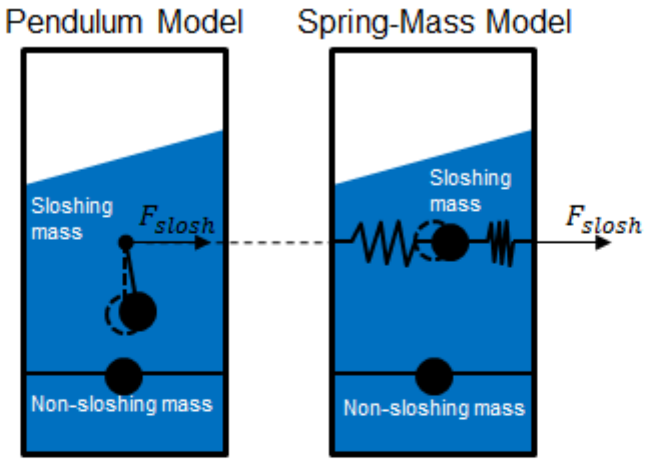
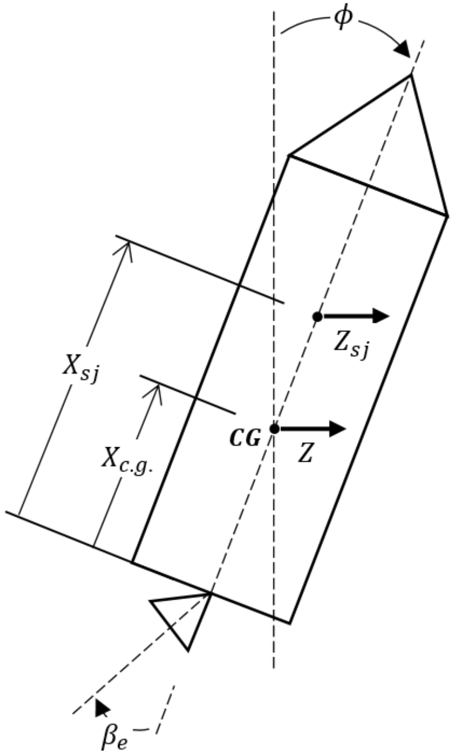


Equation of Motion

- Need a minimal model to demonstrate slosh-control interaction.
- The equations from the seminal paper Frosch and Vallely[7] (1967) includes everything we need.
- Provides linear EOMs of rocket with rigid body, flex, tail wag dog, and slosh.



Z	displacement of vehicle c.g. normal (m)
Z_{zj}	sloshing fluid displacement in jth tank (m)
β_E	engine angle (rad)
ϕ	angle of vehicle centerline (rad)
a_0	attitude control gain (-)
a_1	attitude-rate control gain (-)
c_2	$R'X_{c.g.}/I$ $(1/s^2)$
D	drag force (N)
F	total engine thrust (N)
I	pitch-yaw vehicle moment of inertia with engines and sloshing fluid (kg-m2)
k_3	F/M (m/rad-s2)
k_4	R'/M (m/rad-s2)
l_{sj}	c.g.-to-slosh mass distance $= X_{c.g.} - X_{sj}$ (m)
$l_{c.p.}$	center of percussion $= I/(MX_{c.g.})$ (m)
M	vehicle mass with engines and sloshing fluid (kg)
m_{sj}	slosh mass, jth tank (kg)
R'	vectored engine thrust (N)
$X_{c.g.}$	center of gravity measured from gimbal (m)
X_{sj}	slosh mass location measured from gimbal (m)
ξ_{sj}	slosh equivalent linear damping, jth tank (-)
ω_{sj}	slosh natural frequency, jth tank (rad/s)
α_{sj}	slosh damping slope, jth tank (1/m)
s	complex Laplace variable

Equations of Motions (2)

- Reduce the EOMs to coupled rotational, translation, and slosh dynamics.

$$\phi s^2 = -c_2\beta_e + \frac{1}{I}\sum_{j=1}^{n} m_{sj}(l_{sj}s^2 + k_3)Z_{sj}$$ Rotational Dynamics

$$Zs^2 = k_4\beta + k_3\phi - \frac{1}{M}\sum_{j=1}^{n} m_{sj}Z_{sj}s^2$$ Translational Dynamics

$$(s^2 + 2\xi_{sj}\omega_{sj}s + \omega_{sj}^2)Z_{sj} = -Zs^2 + (l_{sj}s^2 + k_3)\phi$$ Slosh Dynamics (spring-mass-damper)

$$x = \begin{bmatrix} z \\ \phi \\ z_{s1} \\ \dot{z} \\ \dot{\phi} \\ \dot{z_{s1}} \end{bmatrix}$$

$$u = \beta_e$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{m_{s1}}{M} \\ 0 & 0 & 0 & 0 & 1 & -\frac{m_{s1}l_{s1}}{I} \\ 0 & 0 & 0 & 1 & -ls1 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & k_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{s1}k_3/I & 0 & 0 & 0 \\ 0 & k_3 & -\omega_{s1}^2 & 0 & 0 & -2\xi_{s1}\omega_{s1} \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_4 \\ -C_2 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The dynamics can be converted to state space form for easier analysis (here assuming a single slosh mass, n=1).

$$E\dot{x} = A'x + B'u$$

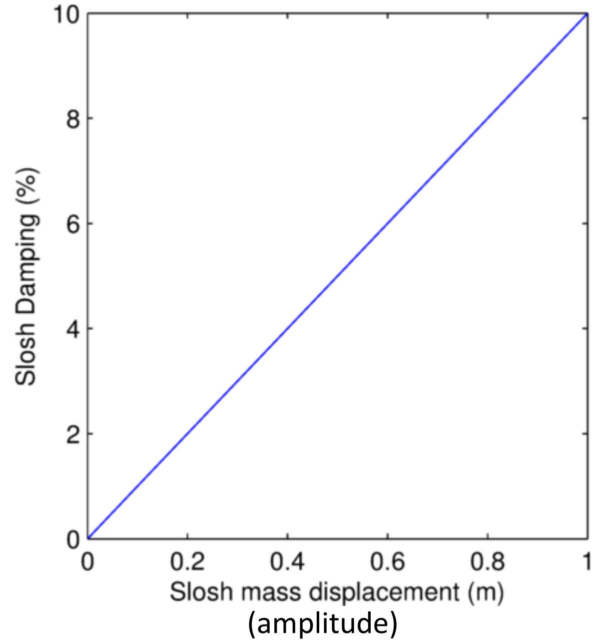
$$\dot{x} = (E^{-1}A')x + (E^{-1}B')u$$

Example System Parameters

- An example configuration was created in order to illustrate results.
- Configuration represents a large upper stage with a diameter on the order of 10 m
  - Somewhat larger than SLS Exploration Upper Stage or the Saturn S-IVB, but demonstrates the same issues faced on real-world stages.
- Control gains result in bandwidth like previously flown upper stages.

Name	Value	Units
$F(=R')$	$9x10^5$	N
M	$1.6x10^4$	kg
I	$9x10^5$	kg-m^2
$X_{c.g.}$	6	m
X_{s1}	9	m
m_{s1}	$1x10^3$	kg
ω_{s1}	2	rad/s
α_{s1}	0.1	1/m
a_0	0.08	rad/rad
a_1	0.14	rad/(rad/s)

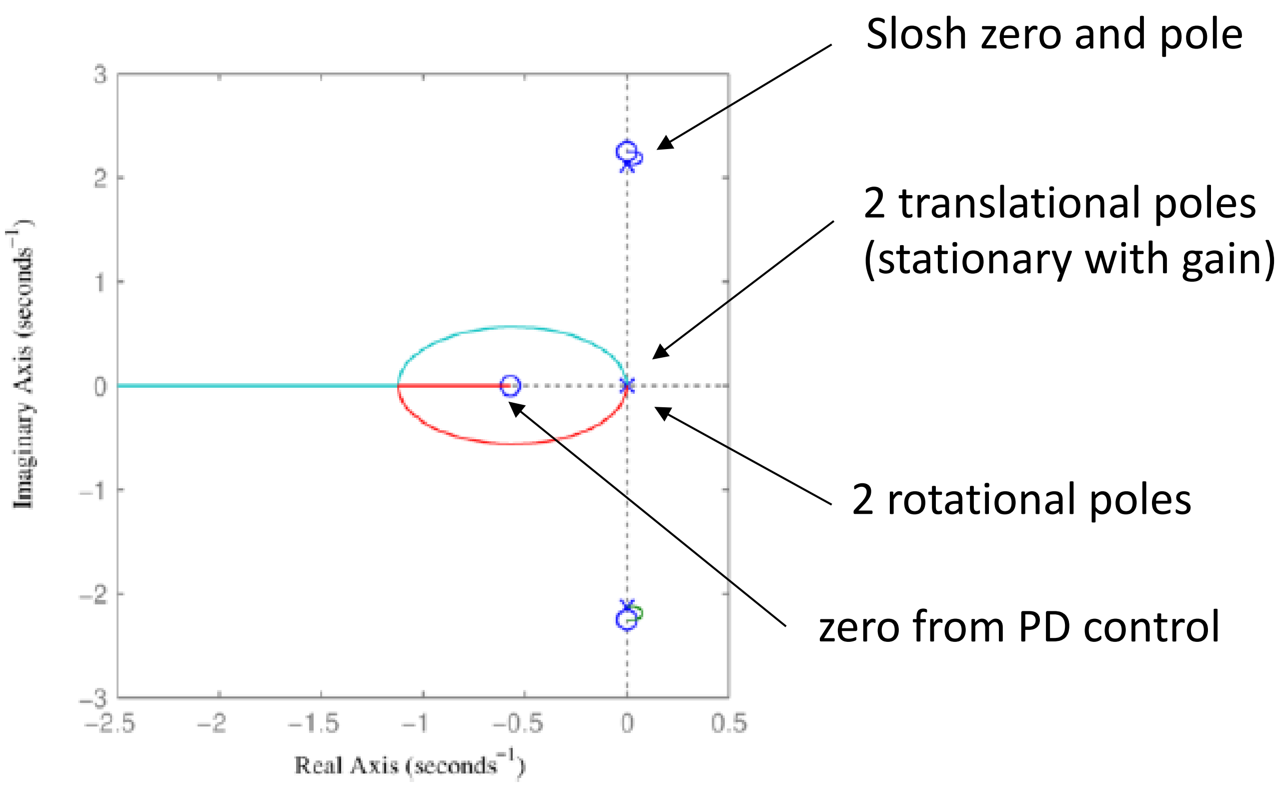
<u>Assume slosh damping is a linear function of slosh amplitude</u>



Why Can Slosh be Destabilizing?

- With PD attitude control, $\beta_e = -a_0\phi - a_1\dot{\phi}$

- For zero slosh damping, the following open-loop root locus will result when breaking the loop at the gimbal command.



- From the root locus angle of departure rule, if the slosh zero is above the slosh pole, then the angle of departure is towards the right-half plane
- This is an example of unstable slosh phasing. Damping is required to be added to the slosh mode to stabilize system.

Where is slosh destabilizing?

- Looking at the gimbal angle to vehicle attitude angle transfer function,

$$\frac{\phi(s)}{\beta_e(s)} = \frac{(c_2 - (c_2 m_{s1})/M + (k_4 l_{s1} m_{s1})/I)s^2 + c_2\omega^2 + (k_3 k_4 m_{s1})/I}{(m_{s1}/M + (l_{s1}^2 m_{s1})/I - 1)s^4 + ((k_3 l_{s1} * m_{s1})/I - \omega^2)s^2}$$

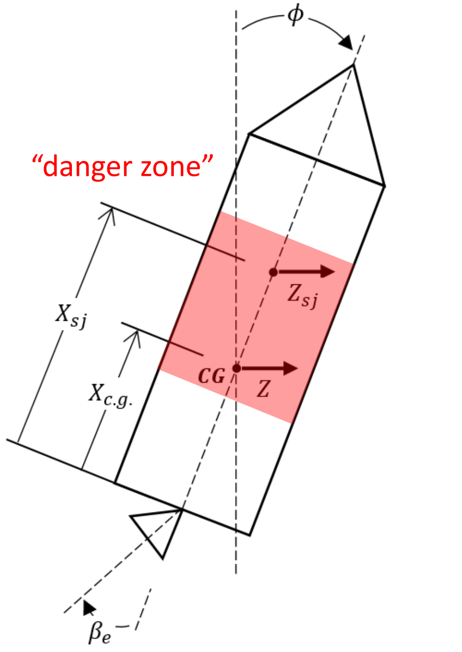
l_{sj} c.g.-to-slosh mass distance $= X_{c.g.} - X_{sj}$ (m)

$$l_{s1} > \frac{-I}{M x_{c.g.}}$$

(Center of Percussion)

$$l_{s1} < \frac{F(M - m_{s1})}{M^2 \omega_{s1}^2}$$

- Can determine at what slosh mass location the slosh zero is above the slosh pole by assuming k3=k4 (all thrust is gimballed, drag is negligible).
  - This is the so called “danger zone” for slosh described by Bauer[8] and Greensite[9].
  - This zone extends from the “center of percussion” ahead of the CG to a point behind the CG.
- With multi-tank coupling or slosh-flex or slosh-aero coupling, things can get more complicated.
  - Pei, J., “Analytical Investigation of Propellant Slosh Stability Boundary on a Space Vehicle,” Aerospace Research Central. Published April 23, 2021. https://doi.org/10.2514/1.A35024.

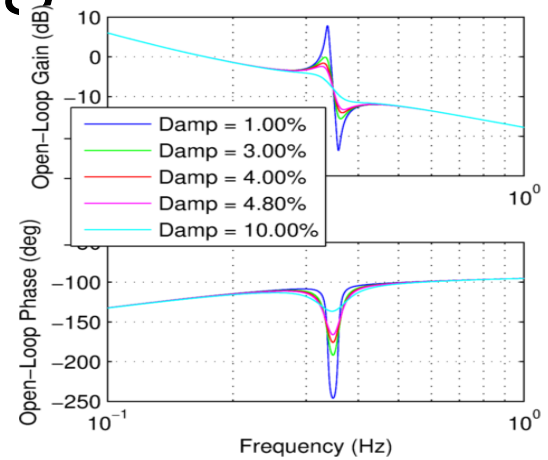


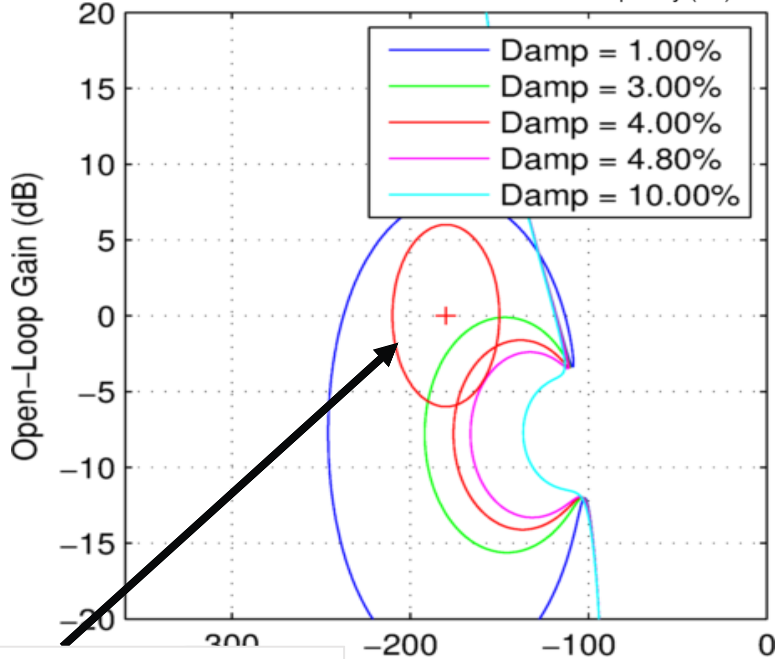
Impact of Slosh Damping on Stability Margins

- For launch vehicles a typical gain/phase margin requirement is 6db/30 degrees.
- Using disc margin which is the composite of phase and gain margin, normalized to the desired gain/phase margin of 6db/30deg.

$$DM = \sqrt{\left(\frac{20log_{10}(|G(\omega)|)}{GMd}\right)^2 + \left(\frac{(\angle G(\omega)mod2\pi) - \pi}{PMd}\right)^2}$$

- In our example system, the control margins exhibit strong sensitivity to the assumed slosh damping.
  - Neutral stability occurs near 3% damping.
  - In order to achieve margins, need 4.8% damping.





Ellipse corresponding to DM = 1.

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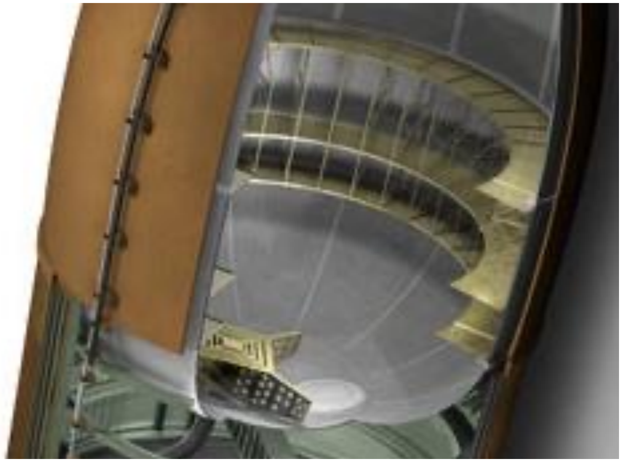
Typical Mitigations to Slosh-Control Instability

Typical Mitigations and Issues

- If the slosh frequency falls near the rigid body frequency (which is often the case with launch vehicle stages) then filtering is not an option.
- Damping must be added to the slosh mode.
- Typically done through baffling inside the tank.
  - Ring baffles typically but there are other options.
- Due to the mass/manufacturing impact, the size and number baffles is often a contentious issue.
- In addition, baffle requirements are often set early in a program before mature modeling is available.
  - Mature modeling can show insufficient damping to achieve full margins later in a program.

Example: Shuttle SLWT LOX Tank Ring Baffles



Typical Mitigations and Issues

- As a result of the baffle impact, many systems have resorted to other types of mitigations.
  - Allowing for reduced stability margins for slosh.
  - Flying through an unstable part of the trajectory.
    - Making sure the part of the trajectory under which slosh instability is predicated, is sufficiently small relative to the time to double of the unstable mode.
    - Assumes no unpredicted external forcing will excite mode.
  - There is no industry standard practice for this
- The Falcon 1 Flight 2 is an example were slosh baffles were omitted due to the belief that slosh would not be excited.
  - Resulted in a large TVC limit cycle oscillation (LCO) which ultimately led to pre-mature shutdown of the stage.

SpaceX Falcon 1 Flight 2 LCO

Wave Amplitude Dependence on Damping

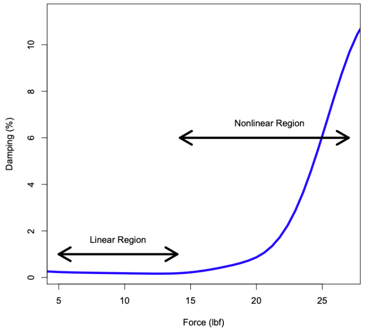
- One of the hidden variables in control-slosh analysis is that the damping increases as a function of the slosh amplitude.
  - Often the wave height used for slosh parameters is picked from heritage instead of engineering analysis.
- For a given analyzed slosh amplitude/wave height
  - If stable, often a limit cycle may occur at a lower wave height.
  - If unstable, the motion may increase until a stable limit cycle is reached.
- There is historical flight data showing low amplitude slosh limit cycles.
- The key question then becomes, what is the size of the limit cycle and is that size acceptable?

Use of Non-linear Damping

Limit Cycle Prediction With Non-linear Damping

Inputs

Damping vs Slosh Amplitude Curve
- Assume damping increases with slosh amplitude.



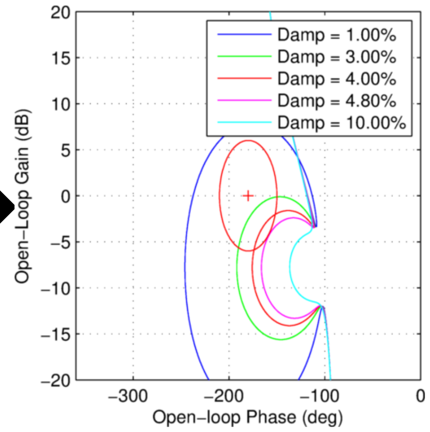
Linearized Equation of Motion and Control System Model
- Ability to determine Disc Margin
- Ability to determine the ratio of TVC amplitude to slosh amplitude at a given frequency

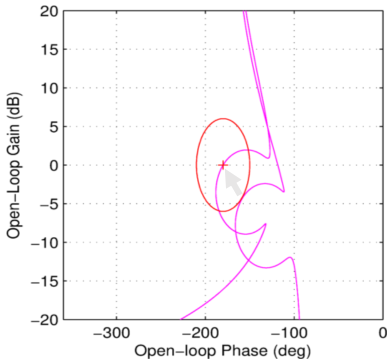
$$\phi s^2 = -c_2\beta_e + \frac{1}{I}\sum_{j=1}^{n} m_{sj}(l_{sj}s^2 + k_3)Z_{sj}$$

$$Zs^2 = k_4\beta + k_3\phi - \frac{1}{M}\sum_{j=1}^{n} m_{sj}Z_{sj}s^2$$

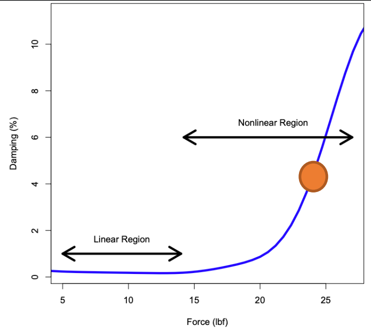
$$(s^2 + 2\xi_{sj}\omega_{sj} + \omega_{sj}^2)Z_{sj} = -Zs^2 + (l_{sj}s^2 + k_3)\phi$$

Using a desired Disc Margin (DM) to protect against, determine the damping needed to achieve it.
- Assumption that degradation occurred which caused this point to become neutrally stable.





Determine slosh amplitude corresponding to the damping value.



Determine system limit cycle amplitude from slosh amplitude and frequency surrounding disc margin close approach.
- TVC motion amplitude
- Rigid body angle/rate amplitude
- Etc.

Check if limit cycle amplitude is acceptable.

Linear process for predicting non-linear limit cycle amplitude

Simulation in Non-Linear Damping in Non-Linear Simulations

- Slosh damping acceleration in non-linear simulation modelled with second order damping

$$\text{Specific force due to } slosh\ damping = -2\zeta\omega\dot{z}_s$$

- If damping is a function of amplitude, question arises how to compute the effective linear damping.
  - One idea is to analyze peaks of the previous slosh motion to determine the current damping
- Another way is to assume the damping acceleration can be approximated with an odd-square law
  - Many energy dissipation processes product a force proportional to velocity squared

$$Acc_{d,nonlinear} = -b_{sj}\dot{z}_{sj}|\dot{z}_{sj}|$$

Odd-Square Law Usage

- Know from literature (Gelb[15]) that an odd square law has a describing function (DF) for sinusoidal input.

Function	*With Input*	*Describing Function*
$y = x\|x\|$	$x = A_x sin(\Omega t)$	$y = x\frac{8}{3\pi}A_x$

- If we have a function of linear equivalent damping as a function of the slosh motion amplitude.

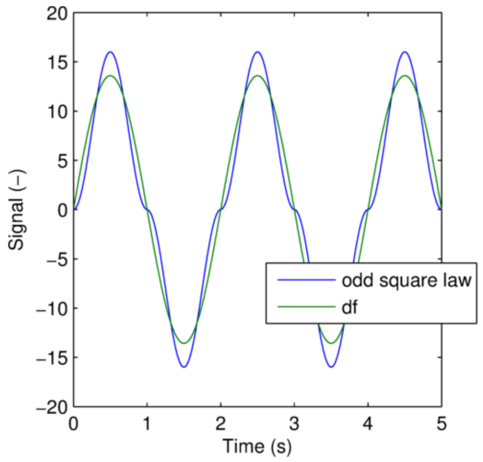
$$(\xi_{sj} = f_{\xi_{sj}}(A_{z_{sj}}))$$

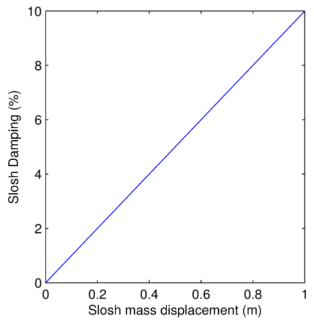
- Then using the describing function, we can produce a non-linear expression for the damping coefficient in damping acceleration

$$\xi_{sj} = \frac{3\pi}{8} f_{\xi_{sj}}\left(\frac{\dot{z}_{sj}}{\omega_{sj}}\right)$$

for use in: $-2\zeta\omega\dot{z}_s$

Match between DF and Odd Square Law

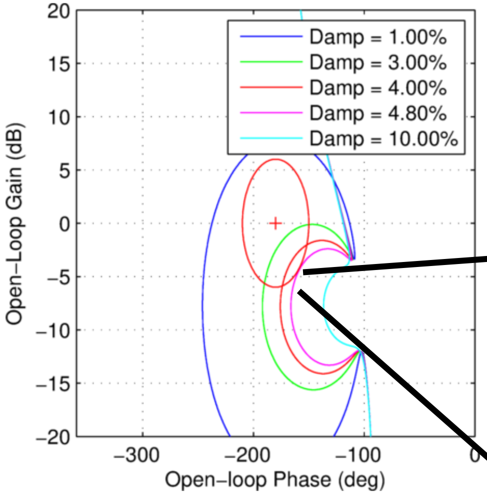




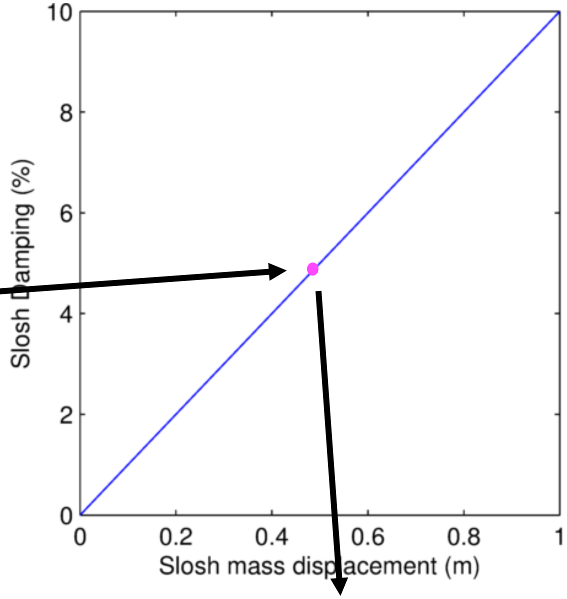
Note that odd square law results in linear damping curve (like example system)

Example System Results

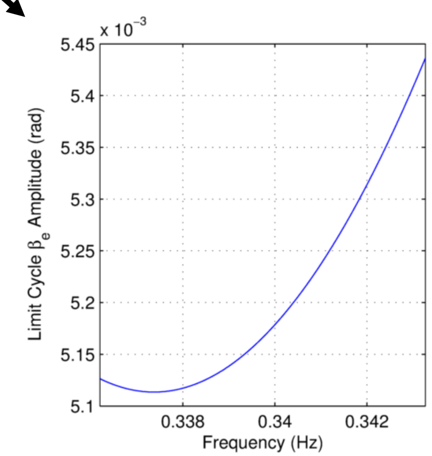
Linear System Sensitivity



Slosh Amplitude At Damping



TVC Amplitude Over Freq. Range

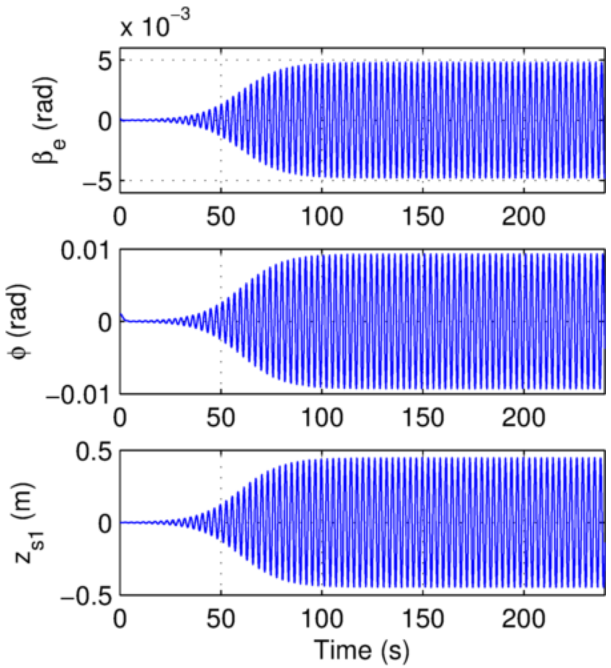


Summary

Parameter	Value
Damping to achieve DM=1	4.8%
Slosh amplitude for needed damping value	0.48 m
Near DM=1 frequency range	0.336 Hz to 0.343 Hz
Predicted TVC Limit Cycle Amplitude Range With Margin Reduction	5.1 to 5.45 mrad

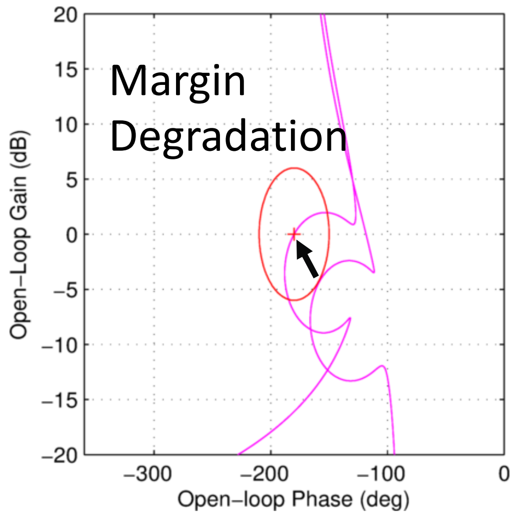
Example System Non-Linear Check

- Check the results in time domain with the linear equations of motion augmented with non-linear damping model.
- Adding gain and time delay associated with a loss of a full 6dB/30deg disc margin to degrade system.
- Time domain sim with small initial perturbation show initial growth which converges to an LCO.



Prediction of 5.1 to 5.45 mrad

Prediction of 0.48 m



LCO from non-linear time domain is close to linear prediction.

Discussion - General

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- Assumption that desired Disc Margin is lost for LCO is conservative.
  - The example assumed a full 6dB/30 deg disc margin was lost but this is more conservative than necessary.
  - Could use a factor of 0.9 or 0.5 and still have some conservatism relative to nominal system.
  - Alternative, could assume zero disc margin, but use a population of dispersed linear models.
- The example assumed odd-square law damping for the whole non-linear damping curve, but method can be used when damping curve vs amplitude has other shapes which will occur from CFD/test.
- In this example the slosh damping was tabulated as a function of slosh mass displacement amplitude, but often the damping is tabulated vs. force or wave height instead. There exists transformations between these quantities.

Discussion - Caveats

- Must make sure the resulting LCO amplitude on TVC is acceptable from a systems perspective (hydraulic usage, loads, etc).
- The limit cycle prediction captures vehicle intrinsic stability, but the amplitude can go higher due to external forcing functions.
  - Guidance maneuvers, gusts, etc can cause a transient response higher than predicted LCO.
  - Important to run dispersed time domain analysis with expected external forcing.
- The non-linear damping LCO prediction is a tool which complements existing methods of analysis of slosh-control interaction but does not replace them.

Discussion - Application

- For SLS, the method is used with a full axis EOM formulation which includes fully coupled flex, slosh, and nozzle dynamics, and a more complex attitude controller. (FRACTAL)
- The SLS program has successfully used this method as part of the justification of reduced slosh baffle requirements (less mass) than would be required to meet full gain/phase margins at heritage wave heights.
- The non-linear damping method does not, generally, allow for the usage of bare walls (no baffles) in unfavorably phased large tanks.
  - Rather it may show that the baffles may not have to be as large or numerous as more traditional analysis may show.
- For very small small slosh masses, the result could be that the control system will have an acceptable small limit cycle, but the tank itself max experience very large wave heights (and crash over) during the LCO which may or may not be acceptable.

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