Lecture 2 - Potential Outcomes Framework

DS4005 Causal Inference

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21 May, 2024

Potentail Outcomes

- The **potential outcomes (or counterfactuals)** refer to the outcome that would have been observed under each level of the exposure.
  - $Y(1)$ - the outcome that would have been observed if $A = 1$
  - $Y(0)$ - the outcome that would have been observed if $A = 0$

- Note: Some textbooks use the notations as Y^1 and Y^0.

- Neyman(1923)[1] and Rubin (1974)[2] laid the foundations for potential outcomes framework.

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[1] Neyman, J. (1923). On the application of probability theory to agricultural experiments. essay on principles (with discussion). section 9 (translated). reprinted ed. *Statistical Science*, 5:465–472.

[2] Rubin, D. B. (1974). Estimating causal effects of treatments in randomized and nonrandomized studies. *Journal of Educational Psychology*, 66:688–701.

Causal effect

- The individual-level potential outcomes for each unit i are:

$$Y_i(1) \text{ and } Y_i(0)$$

- The **causal effect** of the exposure on the outcome is the difference between the potential outcomes. Individual-level causal effect for unit i is given by,

$$\tau_i = Y_i(1) - Y_i(0)$$

The fundamental problem of causal inference

For an individual we only get to observe one potential outcome.

Activity

- Suppose we consider a group of 8 men having headaches of similar intensity. For each person we randomly assign whether to take asprin ($A = 1$) or not ($A = 0$) and then record the time taken (in minutes) to relieve the headache (Y).

A_i	1	1	0	0	0	1	1	0
Y_i	65	70	85	40	75	30	90	100
$Y_i(0)$	?	?	85	40	75	?	?	100
$Y_i(1)$	65	70	?	?	?	30	90	?

- Can you complete the third and fourth rows of the table?

Assumptions

- Potential outcomes framework is based on the following key assumptions.

Assumption 1 (No Interference)

Unit i's potential outcomes do not depend on other units' treatments.

Assumption 2 (Consistency)

There are no different forms or versions of the treatment. Equivalently, we require that the treatment levels be well-defined.

- The above two assumptions together are commonly denoted as Stable Unit Treatment Value Assumption (SUTVA).

Link between potential outcomes and observed outcome

- Based on the above assumptions we can bridge the potential outcomes and the observed outcome.

$$Y_i = \begin{cases} Y_i(1), & \text{if } A_i = 1 \\ Y_i(0), & \text{if } A_i = 0 \end{cases}$$

- An equivalent expression is,

$$Y_i = A_i Y_i(1) + (1 - A_i) Y_i(0)$$

- The treatment assignment mechanism, i.e., the probability distribution of A, plays an important role when estimating causal effects.

Average treatment effect (ATE)

- Average treatment effect (ATE) in the population is given by,

$$\begin{aligned} ATE(\tau) &= E[Y(1) - Y(0)] \\ &= E[Y(1)] - E[Y(0)] \quad \text{(Linearity of expectation)} \end{aligned}$$

- The expectation $E[.]$ is defined with reference to the population of interest.

Causal estimands for binary outcome

- What we want to estimate is the causal risk difference given by,

$$E[Y(1)] - E[Y(0)] = P[Y(1) = 1] - P[Y(0) = 1]$$

- What we actually observe is the observed counterpart given by,

$$P(Y = 1 \mid A = 1) - P(Y = 1 \mid A = 0)$$

- It is important to note that,

$$E[Y(a)] \neq E[Y \mid A = a]$$

Importance of the treatment mechanism - Activity

- Consider a sample size of $n = 500$. Generate $Y(0)$ randomly from a standard normal distribution. [Use set.seed(2024)]
- Set average causal effect close to -0.5 by generating τ_i as $\tau_i = -0.5 + Y_i(0)$. Then generate $Y_i(1) = Y_i(0) + \tau_i$.

A perfect doctor scenario

- If the doctor already knows that the individual causal effect is positive, he will definitely assign the treatment to the patient. Allocate the treatment assignment accordingly and construct the observed outcome vector Y. Find the difference in means of the observed outcome for the treatment and control groups.

A clueless doctor scenario

- A clueless doctor does not have any information about the individual causal effects and assigns the treatment randomly. Allocate the treatment assignment accordingly and construct the observed outcome vector Y. Find the difference in means of the observed outcome for the treatment and control groups.

Next... **Causal Graphs**

Thank you