Similarity measures for tonal models

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ABSTRACT

Multidimensional-scaling models of tonal hierarchy encode cognitive relationships between chords as euclidean distances. Fred Lerdahl's tonal pitch space (TPS) model approximates cognitive perceptual relations between chords by providing a combinatorial procedure for computing the distance value between two chords. Because of the influence of experimental data on the TPS model, we would expect a high correlation between experimental data and analyses of chord progressions generated by the TPS model. The value of such a comparison is clear. If the TPS model posits a hypothesized model of perception, then we would like to know if and by how much it differs from experimental data it claims to approximate. In this paper, we focus on the intra-regional relation descriptions of TPS. We achieve two important goals. First, we develop a similarity measure that allows us to accurately compare the TPS model with a model of perceived chord relations created by Bharucha, et al. Second, this paper applies the similarity measure to normalized canonical representations of each model, thereby avoiding comparisons affected by arbitrary design choices. The greatest obstacle inherent in quantifying a comparative procedure is negotiating the manner in which each constituent model codes its data. The similarity measure and the method of normalization are applicable to any model with formal properties described herein and has the potential to focus experimental design and strengthen the relationship between experimental data and analytic systems.

I. INTRODUCTION

The search for compelling representations of tonal hierarchy and its constituent harmonic relations has a long-standing history. Fred Lerdahl describes geometric approaches to this problem [1] that involve the collection and modelling of data from experiments in music cognition. The multidimensional-scaling models of Bharucha and Krumhansl [2], and Deutsch [3], for example, seek to encode cognitive relationships between chords *within a single key area* (region) as Euclidean distances. The work of Heinichen [4], Kellner [5], and Weber [6], is more speculative and, when formalized, develops geometric representations of relationships between different key areas (regions) through their placement within a multidimensional space.

The first class of representations informs us about perceived tonal relationships within a region, while the second presents geometric representations of inter-regional relationships. Both representations can be integrated into an analytical program, facilitating analysis and prescriptive understanding.

Fred Lerdahl's tonal pitch space (TPS) model [1] approximates the cognitive perceptual relation between chords by providing a combinatorial procedure for computing the distance value between two arbitrary chords. The procedure employed by the TPS model is informed by experimental data and plausible hypotheses about how we perceive tonal relations. The TPS model is able to describe relations both between chords within a region (e.g. Bharucha, et al. [2]) and between regions themselves (e.g. Heinichen, et al.); TPS thus bridges these two representational classes.

Because of the influence of experimental data on the TPS model, we would expect a high correlation between experimental data and analyses of intra-regional chord progressions generated by the TPS model. The value of such a comparison is clear. If the TPS model posits a hypothesized model of perception, then we would like to know if and by how much it differs from the experimental data it claims to approximate. In this paper, we shall focus on the intra-regional relation descriptions of TPS and their counterparts in Bharucha, et al.

VI. CONCLUSION

This paper has achieved four important music-analytic goals. First, we showed how certain tonal models are also metric models, meeting the requirements of a metric space. Second, we defined a distance measure between pairs of tonal models. Third, we showed how to derive canonical representatives from tonal models. This allowed us to compare models without the interference of arbitrary design choices. Comparing canonical representatives allowed us to critique analytic claims made by L against BK. Finally, these two tonal models are members of two different equivalence classes, whose representatives come from a metric space of tonal models, and since every subset of a metric space is a metric space, the two models form a metric space.

[1] F. Lerdahl, *Tonal Pitch Space* (Oxford University Press, New York, 2001).

[2] J. Bharucha and C. Krumhansl, Cognition **13**, 63 (1983).

[3] D. Deutsch, in *The Psychology of Music*, edited by D. Duetsch (Academic Press, New York, 1999), chap. 10.

[4] J. D. Heinichen, *Der Generalbass in Der Composition* (Dresden, 1728).

[5] D. Kellner, *Treulicher Unterricht im General-Bass* (Christian Herold, Hamburg, 1737).

[6] G. Weber, *Versuch Einer Geordenten Theorie Der Tonsetzkunst* (B. Schotts Sohne, Mainz, 1821-1824).