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Heterogeneity and its Impact on Thermal Robustness and Attractor Density

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Abstract

There is considerable research relating the structure of Boolean networks to their state space dynamics. In this paper, we extend the standard model to include the effects of thermal noise, which has the potential to deflect the trajectory of a dynamical system within its state space, sending it from one stable attractor to another. We introduce a new "thermal robustness" measure, which quantifies a Boolean network's resilience to such deflections. In particular, we investigate the impact of structural homogeneity on two dynamical properties: thermal robustness and attractor density. Through computational experiments on cyclic Boolean networks, we ascertain that as a homogeneous Boolean network grows in size, it tends to underperform most of its heterogeneous counterparts with respect to at least one of these two dynamical properties. These results strongly suggest that during an organism's growth and morphogenesis, cellular differentiation is required if the organism seeks to exhibit *both* an increasing number of attractors *and* resilience to thermal noise.

Keywords: Boolean networks, cellular automata, dynamical systems, noise, robustness

1. Introduction

Since the seminal work of Von Neumann [1], the subject of cellular automata has received considerable and continued attention (see [2, 3] for brief surveys). Understanding how the structure of a cellular network impacts its behavior as a dynamical system is crucial to determining how networks should be built, how they evolve over time, and how they can be made to grow while still exhibiting desired dynamical properties.

Biological networks (e.g. neural networks) are typically subject to a Darwinian preferential selection process, and are seen to exhibit evolution over long time scales. It is reasonable to expect that this selection process would be based not only on the structural properties [4] of networks, but on their dynamical properties as well [5]. Previous researchers have consideredmeasures such as landscape ruggedness [6, 7] and redundancy [8] in evaluating dynamical systems. In this work, the dynamical property we consider is the robustness of a dynamical system's attractors against thermal noise from the environment. We refer to this property, formally defined in Section 3, as *thermal robustness*. Thermal or Johnson-Nyquist noise man-

References

[1] J. V. Neumann, *Theory of Self-Reproducing Automata,* University of Illinois Press, Champaign, IL, USA, 1966.

[2] P. Sarkar, A brief history of cellular automata, *ACM Computing Surveys* 32 (2000) 80–107. doi:10.1145/349194.349202.

[3] N. Ganguly, B. K. Sikdar, A. Deutsch, G. Canright, P. P. Chaudhuri, A survey on cellular automata, Tech. rep. (2003).

[4] D. W. Thompson, *On Growth and Form,* canto Edition, Cambridge University Press, 1992.

[5] M. Ebner, M. Shackleton, R. Shipman, How neutral networks influence evolvability, *Complex*. 7 (2001) 19–33.

[6] T. Malloy, G. Jensen, T. Song, Mapping knowledge to Boolean dynamic systems in Batesons epistemology, in: *Nonlinear Dynamics, Psychology, and Life Sciences,* Vol. 9, 2005, pp. 37–60.

[7] T. Malloy, G. Jensen, Dynamic constancy as a basis for perceptual hierarchies, in: *Nonlinear Dynamics, Psychology, and Life Sciences,* Vol. 1(2), 2008, pp. 191–203.

[8] C. Gershenson, S. A. Kauffman, I. Shmulevich, The role of redundancy in the robustness of random boolean networks, Tech. Rep. nlin.AO/0511018. ECCO-2005-08 (Nov 2005).

[9] S. A. Kauffman, *The Origins of Order: Self-Organization and Selection in Evolution,* 1st Edition, Oxford University Press, USA, 1993.

[10] C. Gershenson, Classification of random boolean networks (2002).

[11] K. Sutner, Linear cellular automata and the Garden-of-Eden, *The Mathematical Intelligencer* 11 (1989) 49–53.

[12] C. R. Shalizi, K. L. Shalizi, Quantifying self-organization in cyclic cellular automata, in: in *Noise in Complex Systems and Stochastic Dynamics*, Lutz Schimansky-Geier and Derek Abbott and Alexander Neiman and Christian Van den Broeck, Proceedings of SPIE, vol 5114, 2003.

[13] C. L. Nehaniv, *Evolution in asynchronous cellular automata,* MIT Press, 2002, pp. 201–209.

[14] T. Lundh, Cellular automaton modeling of biological pattern formation: Characterization, applications, and analysis, *Genetic Programming and Evolvable Machines* 8 (2007) 105–106.