

Communications in Contemporary Mathematics
© World Scientific Publishing Company

On positive theories of groups with regular free length functions

BILAL KHAN

*John Jay College of Criminal Justice, CUNY,
Department of Mathematics and Computer Science
899 Tenth Avenue, New York, NY, 10019, USA
bilal@bilalkhan.org*

ALEXEI G. MYASNIKOV

*McGill University, Department of Mathematics and Statistics,
805 Sherbrooke W., Montreal QC H3A 2K6, Canada
alexeim@att.net*

DENIS E. SERBIN

*McGill University, Department of Mathematics and Statistics,
805 Sherbrooke W., Montreal QC H3A 2K6, Canada
zloidyadya@yahoo.com*

Received (Day Month Year)

Revised (Day Month Year)

In this paper we discuss a general approach to positive theories of groups. As an application we get a robust description of positive theories of groups with regular free Lyndon length function. Our approach combines techniques of infinite words (see [27], [5]), cancellation diagrams introduced in [21], and Merzlyakov's method [22].

Contents

1	Introduction	2
2	Preliminaries	6
2.1	Lyndon length functions	6
2.2	Infinite words	7
2.3	Examples	9
3	Properties	11
4	Cancellation diagrams	14
5	Equations	19
6	Main results	22

- [17] O. Kharlampovich, A. Myasnikov, and D. Serbin, *Free regular length functions on fully residually free groups*. Preprint 2004.
- [18] V. Kopytov and N. Medvedev, *Right-ordered groups*. Siberian School of Algebra and Logic. Consultants Bureau, New York, 1996.
- [19] R. Lyndon, *Groups with parametric exponents*. Trans. Amer. Math. Soc., **9** no. 6 (1960), 518–533.
- [20] R. Lyndon and P. Schupp, *Combinatorial group theory*. Ergebnisse der Mathematik und ihrer Grenzgebiete, **89**, Springer-Verlag, Berlin, Heidelberg, New York, 1977.
- [21] G. S. Makanin, *Equations in a free group*. (Russian) Izv. Akad. Nauk SSSR, Ser. Mat., **46** (1982), 1199–1273.
- [22] Ju. I. Merzlyakov, *Positive formulae on free groups*. Algebra i Logika, **5** no. 4 (1966), 25–42.
- [23] A. Myasnikov and V. Remeslennikov, *Exponential groups I: foundations of the theory and tensor completion*. Siberian Math. J., **35** no. 5 (1994), 1106–1118 (Russian).
- [24] A. Myasnikov and V. Remeslennikov, *Exponential groups II: extensions of centralizers and tensor completion of CSA-groups*. Internat. J. Algebra and Comput., **6** no. 6 (1996), 687–711.
- [25] A. Myasnikov and V. Remeslennikov, *Length functions on free exponential groups*. Proc. Internat. Conf. on Groups in Analysis and Geometry, Omsk, 1995, 59–61.
- [26] A. Myasnikov and V. Remeslennikov, *Length functions on free exponential groups*. Proc. IITPM SO RAN, Omsk, 1996, no. 26, 1–34.
- [27] A. Myasnikov A., V. Remeslennikov and D. Serbin, *Regular free length functions on Lyndon's free $\mathbb{Z}[t]$ -group $F^{\mathbb{Z}[t]}$* , to appear in *Computational and Experimental group theory*, Editors A. Borovik and A. Myasnikov, Contemp. Math., Amer. Math. Soc., 2004.
- [28] A. Myasnikov, V. Remeslennikov and D. Serbin, *Fully residually free groups and graphs labeled by infinite words*, to appear in Internat. J. Algebra and Comput.
- [29] W. Szmielew, *Elementary properties of abelian groups*, Fund. Math., 1955, 41, 203–271.
- [30] A. Tarski, *Decision method of elementary Algebra and geometry*, Berkeley: University California Press, 1951.