A GRAPHIC GENERALIZATION OF ARITHMETIC

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Abstract

In this paper, we introduce a natural arithmetic on the set of all *flow graphs*, that is, the set of all finite directed connected multigraphs having a pair of distinguished vertices. The proposed model exhibits the property that the natural numbers appear as a submodel, with the directed path of length n playing the role of the standard integer n. We investigate the basic features of this model, including associativity, distributivity, and various identities relating the order relation to addition and multiplication.

1 Introduction

The *language of arithmetic* $\mathcal{L}$ consists of two constants $\mathbf{0}$ and $\mathbf{1}$, one binary relation $\leqslant$, and two binary operations $+$ and $\times$. In this paper, we generalize classical arithmetic defined over the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$, to the set F consisting of all *flow graphs*: finite directed connected multigraphs [1] in which a pair of distinguished vertices is designated as the *source* and *target* vertex. We give natural interpretation for $\mathcal{L}$ on the set F. To avoid confusion with the standard model of arithmetic, the corresponding operations in F are denoted with a circumscribed circle. The new model $\mathcal{F} = \langle F, 0, 1, \leqslant, +, \times \rangle$ is a natural extension of the

[1] By multigraph we mean graphs in which parallel and loop edges are permitted.

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