

# COMPRESSION AND EXPANSION IN GRAPHS USING OVERLAYS

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**Abstract.** In this paper, we describe how to construct physical computer network topologies which can support the establishment of overlays that reduce or increase the distances between nodes. Reducing pairwise distances (i.e., *compression*) implies that the overlay enjoys significantly lower inter-node latencies compared to the ambient physical network; such an overlay can be used to implement a “high-performance mode” for disaster situations in which network responsiveness is of critical importance. On the other hand, increasing pairwise distances (i.e., *expansion*) implies that the overlay exhibits significantly higher inter-node latencies compared to the ambient physical network; such an overlay can be used to implement a brief “dilated state” in networks that have been infected by a malicious worm, where slowing down the infection spread allows greater time for antidote generation. We show that it is possible to design physical networks which support overlays whose logical link bandwidth is *equal* to the physical link bandwidth while providing arbitrarily high compression or expansion. We also show that it is possible to “grow” such networks over time in a scalable way, that is to say, it is possible to retain the compression/expansion properties while augmenting the network with new nodes, by making relatively small adjustments to the physical and overlay network structure.

**Key words.** graphs embeddings, diameter, expansion, compression, overlays

**1. Introduction.** In the last few years we have witnessed a renewed interest in applications of peer-to-peer overlay networks. Within an overlay network, each logical link is implemented as a multi-hop connection over a path in an underlying physical network. The overlay’s configuration and protocols ensure that data can travel along a logical link without undergoing expensive processing at intermediate physical nodes. Overlays thus enable network designers to superimpose alternate topologies (i.e., “virtual networks”) whose properties diverge considerably from those of the ambient physical network. The malleability thus obtained is very valuable, since modifications to the physical network topology are both time consuming and expensive, while alterations to the overlay topology are, by comparison, implementable inexpensively and quickly. The mathematical theory of overlays has been developed and applied in many areas, including the simulation of one parallel architecture on another [4, 8, 10, 11], the design of active networks [7], the layout of virtual paths in circuit switched networks [1, 6, 9], scalable distributed content-addressable storage [13, 18], and construction of efficient multicast trees [3]. Today, the design and implementation of network overlays is made commonplace by readily available tools such as Xbone, 6Bone, and Dynabone [14, 17].

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