Distance Between Graphs Using Graph Labelings

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Abstract

In [?], Fan Chung Graham investigates notion of graph labelings and related bandwidth and cut-width of such labelings when the host graph is a path graph. Motivated by problems presented in [?], and our investigation of designing efficient virtual path layouts for communication networks, we investigate in this note, labeling methods on graphs where the host graph is not restricted to a particular kind of graph. In [?], authors introduced a metric on the set of connected simple graphs of a given order which represents load on edges of host graph under some restrictions on bandwidth of such labelings. In communication networks this translates into finding mappings between guest graph and host graph in a way that minimizes the congestion while restricting the delay. In this note, we present optimal mappings between special n-vertex graphs in $\mathcal{G}_n$ and compute their distances with respect to the metric introduced in [?]. Some open questions are also presented.

1 Background

We recall some definitions from Bhutani and Khan [?]. Given an undirected graph $G = (V, E)$, recall that a **path** of length l in G is a sequence of

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Subject Classification: 05C99, 05C12

$P_n, C_n, S_n, K_n \in \mathcal{G}_n$ and whose edges are weighted by the distance between its endpoints in $(\mathcal{G}_n, d_n^*)$. In the above figure α is given by Proposition 2.6.

Open Questions

(1) If s_G for any $G \in \mathcal{G}_n$ denotes a selected subset of k vertices of a graph G, then is it possible to find an optimal map ϕ from $V(G)$ into $V(H)$ such that $\phi(s_G) = s_H$?

(2) If f_1 and f_2 are optimal maps for (H_1, K_1) and (H_2, K_2) in $\mathcal{G}_n$ and $\mathcal{G}_m$ respectively, then is $f_1 \times f_2$ an optimal map for $(H_1 \times H_2, K_1 \times K_2)$ in $\mathcal{G}_{mn}$?

(3) Given a graph G and a natural number n, can we find a graph H possessing a special property (e.g. Hamilton, Euler) with $d_n^*(G, H) \leq log_2 n$?

Acknowledgements

The authors gratefully acknowledge the Center for Computational Science at the Naval Research Laboratory, Washington DC, where this work began, in the context of virtual path layout for high-speed computer networks. The authors would like to thank Professor Mikhail Ostrovskii for helpful discussions with proof of Proposition 2.6.

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