Minimizing Communication Costs in Hierarchical Multi-Agent Systems

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ABSTRACT- In this paper, we consider several mathematical and algorithmic problems which arise naturally in the context of optimizing the performance of a network management system (NMS) whose architecture consists of a *distributed hierarchy* of cooperating intelligent agents.

Key Words: Distributed Hierarchy, Network Management, Multi-agent System.

1 Introduction

A network management system (NMS) must be able to coordinate thousands of network devices, and *scale* to perform well as the number of devices increases. To achieve this goal, the designers of the Optiprism [4] NMS drew upon the compelling analogies between scalable distributed computer systems and distributed human organizations, as presented by Fox [2] and others. Indeed, the architecture of Optiprism has its roots in the theory of human organizational hierarchies, as developed in the works of J. R. Galbraith [3], Mount and Reiter [5], Radner [7], Patrick and Dewatripont [6].

The Optiprism NMS is a hierarchy of (software) *managers*, each of which may act as a *supervisor* of several *subordinate* managers. State information about the network is aggregated and flows recursively upwards at each node of the hierarchy, facilitating decisions and actions which are then executed by recursive delegation of subtasks to subordinates [1, pp. 12]. In this way, decentralized information processing facilitates decentralized decision-making [8, pp. 1].

In most NMS' the human network administrator interacts with the management system using a *browser* application. For scalability, however, such a browser should not communicate with *all* of the NMS' managers at any point in time. In Optiprism, the browser application is itself a leaf node in the management hierarchy. The browser acts as "consultant", changing its position within the hierarchy, and—following Galbraith's mechanistic

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model [3]—establishing *lateral relationships* with a small subset of the managers based on its present position in the tree. The set of *visible* managers for a given browser consists of: the browser's peers, together with the browser's supervisor's peers, its supervisor's supervisor's peers, and so on. Optiprism thus provides *scalable connectivity* between each browser and the managers that are logically nearby. Each browsers logical position within the hierarchy can be changed by the administrator using it. Specifically, the browser can be either (i) *promoted*, causing it to become a peer of its current supervisor, or (ii) *demoted*, causing it to become subordinate to a manager that is presently its peer. This *logical movement* of the browser, causes the set of visible managers to change in a manner corresponding to (i) zooming out from and (ii) zooming into particular regions of the network. While managers can be moved between machines, a browsers physical location within the computational environment cannot be changed since it needs to display information on the administrator's machine, and so must reside there. In this paper, we consider the problem of how best to distribute the managers across a set of machines so to optimize the responsiveness of browsers and the multi-agent NMS as a whole.

2 Mathematical Model

The computational environment in which the NMS resides is modeled as a set of machines M, connected by a set of links $L \subseteq M \times M$ on a network $G = (M, L)$. In this paper we assume that G is a complete graph.

We model the state of the NMS at any instant in time as a rooted tree $T = (V, E)$, with root node denoted by $r^* \in V$. Denote the leaves of T by $\ell(T) \subset V$. A fixed subset $U \subseteq \ell(T)$ of these leaves are identified as browser agents; the remaining nodes of the hierarchy $V \backslash U$ are manager agents. Each of the browser agents $U = \{u_1, u_2, \ldots, u_k\}$ is operated by a network administrator. We define $m_i \in M$ to be the machine at which the administrator operating browser u_i resides. The initial assignment of browsers to machines is then defined in terms of a map, $g : U \to M$ where $g(u_i) = m_i$.

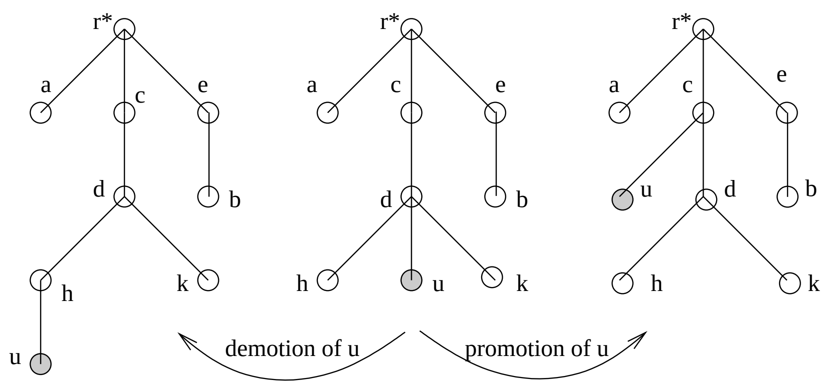
We will assume for simplicity that g is onto, which is to say that there is at least one browser running on each machine. The computational environment G, the browser agents U, the set of manager agents $V \backslash U$, and the initial assignment of browsers g, will all be assumed to remain unchanged over time, In contrast, however, the structure of the tree T may change over time, because each browser may be logically moved by the administrator using it. Our research begins with the observation, that while g specifies the location of the browsers $U \subseteq V$, it leaves open the question of where the managers $V \backslash U$ should be placed. More formally, we would like to extend g to a function, $f : V \to M$ so that $f|U \equiv g$, and f is optimal in the sense that it minimizes total network communications.

2.1 Properties of Optimal Mappings

Let us begin by considering the behavior of f on just two NMS nodes $u \in U$, $v \in V$. If f determines that u should be assigned to machine $f(u) \equiv g(u)$ and v should be assigned to machine $f(v)$, then whenever u and v communicate, they must do so over the physical network G connecting machines $f(u)$ and $f(v)$. Each act of communication between u and v incurs cost proportional to the distance from $f(u)$ to $f(v)$ in G, which we denote $d_G(f(u), f(v))$. The question remains as to whether browser u and manager v may communicate at all. As noted earlier, for reasons of scalability, each browser $u \in U$ is only permitted to communicate with a restricted subset of so-called *visible* managers, defined as

$$Vis(u) \equiv \bigcup_{k=1}^{h(u)} Vis^k(u)$$

where for each $k \in \mathbb{N}$, $Vis^k(u)$—the k^{th} visibility of u— is the set of $v \in V \backslash U$ that are peers of the $(k-1)^{\text{th}}$ ancestor[*] of u, and $h(u)$ is the distance from u to the root of the tree. The table and figure below are an example of how the visibility structure of a browser u changes when a browser is promoted or demoted.



$Vis^1(u)$	$\emptyset$
$Vis^2(u)$	k
$Vis^3(u)$	$\emptyset$
$Vis^4(u)$	a, e

$\leftarrow$

$Vis^1(u)$	h, k
$Vis^2(u)$	$\emptyset$
$Vis^3(u)$	a, e
$Vis^4(u)$	$\emptyset$

$\rightarrow$

$Vis^1(u)$	d
$Vis^2(u)$	a, e
$Vis^3(u)$	$\emptyset$
$Vis^4(u)$	$\emptyset$

We now define a *local cost function* $C_{v,u} : M \to \mathbb{R}^+$, for each $v \in V$ and $u \in U$, whose value $C_{v,u}(m)$ is interpreted as the cost that would be incurred during pairwise communication between the nodes u and v if v were placed on machine vertex m in the network G, while u was placed at machine $f(u) \equiv g(u)$. Later we will propose candidate expressions for $C_{v,u}$ that are tailored to address particular concerns.

[*] By $(k)^{\text{th}}$ ancestor of u we mean the $(k+1)^{\text{th}}$ node on the path from u to r^*. For example, $Vis^1(u)$ consists simply of the peers of u, since the 0^{th} ancestor of u is u itself.

The family of local cost functions

$$\mathcal{C} = \{C_{v,u} \mid v \in V \backslash U, u \in U\}$$

can then be used to define the *global cost function* C_*, whose minimization would in turn be used as the criterion for finding a globally optimal f.

$$C_*(f,T,U,g) = \sum_{v \in V \backslash U} \sum_{u \in U} C_{v,u}(f(v))$$

NMS Communication Minimization Problem. *Given a network management system consisting of a hierarchy of nodes $T = (V,E)$ distributed over a physical network of machines $G = (M,L)$, where the NMS contains both browsers agents $U \subseteq V$ whose physical location is given by the map $g : U \to M$ and manager agents $V \backslash U$ whose physical location is unknown. Determine the optimal physical locations for the all nodes of the NMS, including manager agents. In other words extend g to a function $f : V \to M$ in a manner that minimizes $C_*(f,T,U,g)$.*

3 Mathematical Preliminaries

We now define some functions which will be used to specify the local cost functions, and for describing the algorithm for constructing the optimal agent assignment function f.

For each $v \in V \backslash U$, let $q_v : U \to \{0,1\}$ be defined as

$$q_v(u) = \begin{cases} 1 & \text{if } v \in Vis(u) \\ 0 & \text{if } v \notin Vis(u) \end{cases}$$

and let $p_v : U \to \mathbb{N}$ be given by

$$p_v(u) = \begin{cases} k & \text{if } v \in Vis^k(u) \\ \infty & \text{if } v \notin Vis^k(u) \text{ for any } k. \end{cases}$$

p_v and q_v are closely related to the visibility relation.

For each $v \in V$ the set of all browsers which can see v is called $q_v^* = \{w \in U \mid q_v(w) = 1\}$; of these the subset which see v with smallest visibility are denoted

$$p_v^* = \{w \in U \mid p_v(w) = min_{x \in U}(p_v(x)) \text{ } and \text{ } p_v(w) \neq \infty\}$$

For each $u \in q_v^*$ we denote the set of all browsers that can see v *and* are mapped to the same machine as u by the initial mapping g as $q_v^*(g,u) = \{w \in q_v^* \mid g(w) = g(u)\} \subseteq q_v^*$. Analogously, for $u \in p_v^*$ the set of browsers that see v with smallest visibility *and* are mapped by g to the same machine as u, is denoted

$$p_v^*(g,u) = \{w \in p_v^* \mid g(w) = g(u)\} \subseteq p_v^*$$

Finally $q_v^{\oplus} = \{g(w) \in M | \ w \in q_v^* \text{ and all } g(w) \text{ distinct}\}$ is the set of all machines that have been assigned at least one browser from q_v^*, and the set of all machines that have been assigned at least one browser from p_v^* is

$$p_v^{\oplus} = \{g(w) \in M | \ w \in p_v^* \text{ and all } g(w) \text{ distinct}\}$$

4 Local Communication Cost Functions

We turn now to defining appropriate local cost functions (see subsection 2.1)—a necessary pre-requisite before an optimal f can be found. If the network G is a complete graph, then we might consider the BINARY local cost function below

$$C_{v,u}(m = f(v)) = \begin{cases} 0 & \text{if } m = g(u) \text{ or if } v \notin Vis(u) \\ 1 & \text{if } m \neq g(u) \text{ and } v \in Vis(u) \end{cases} \tag{1}$$

To see how such a cost function would affect C_* fix $v \in V \backslash U$. Clearly, if for all $u \in U$, $v \notin Vis(u)$ then f can map v to any machine $m \in M$ without affecting $C_*(f, T, U, g)$. If there exists a unique $u \in U$ for which $v \in Vis(u)$, then f must map v to the machine $f(u) \equiv g(u)$ in order to optimize $C_*(f, T, U, g)$. The remaining, interesting case occurs when the set of browsers observing v has size greater than 1, that is, $|q_v^*| > 1$.

In this case, the cumulative contribution to C_* by observers $u \in q_v^*$ can grow to be as large as the size of $|q_v^*|$. This biases the importance of $f(v)$ in C_*, so to counteract this bias, we consider the following UNIFORMLY BOUNDED local cost function

$$C_{v,u}(m) = \begin{cases} 0 & \text{if } m = g(u) \text{ or if } v \notin Vis(u) \\ \frac{1}{|q_v^*|} & \text{if } m \neq g(u) \text{ and } v \in Vis(u) \end{cases} \tag{2}$$

This new cost function ensures that the total contribution to C_* by browsers that can see v remains bounded by 1.

Since nodes in the kth visibility of a browser u are, in a sense, encapsulating state and control information that is $k-1$ resolution quanta coarser than the resolution of information held by peers of u, it might be desirable to emphasize the relative importance of mapping agents that are within smaller visiblity of u to $g(u)$. While the previous cost function does not do this, it could be modified to take visibility into account.
One such VISIBILITY-GRADED local cost function is

$$C_{v,u}(m) = \begin{cases} 0 \text{ if } m = f(v) = g(u) \text{ or if } v \notin Vis(u) & \\ \frac{1}{d^{((k(k+1)/2)-1}} & \text{if } p_v(u) = k \end{cases} \tag{3}$$

where d is the maximum degree in the tree T.

Observe that if $w \in p_v^*$ and $p_v(w) = k$, then there are at most

$d(d-1) + d^2(d-1) + d^3(d-1) + ... + d^{k-1}(d-1) + d^k(d-2) = d(d^{k-1} - 1) + d^k(d-2)$

browsers which can see v with visibility $k+1$. These are the browsers at depth $d_T(w, r^*)+1$ in the tree, that is one level below the level of browser w. This VISIBILITY-GRADED local cost function has the property that the cost a manager pays to a single browser w in p_v^* where $p_v(w) = k$ is more than the total cost it pays to all possible browsers which can see v with visibility $k+1$.

5 Optimal Mappings

We now answer the problem of determining an assignment f of NMS nodes to machines that minimizes the global communication cost C_* for each of the local cost functions described in the previous section.

5.1 The BINARY and UNIFORMLY BOUNDED Case

We assign the NMS nodes to machines via $f : V \to M$ defined as follows:

$$f(v) = \begin{cases} g(v) & \text{if } v \in U \\ g(w) & \text{if } |q_v^*| = 1 \text{ or if } |q_v^{\oplus}| = 1, w \in q_v^* \\ g(w) & \text{if } |q_v^{\oplus}| > 1 \text{ and } |q_v^*(g, w)| = \max_{u \in q_v^*} |q_v^*(g, u)| \\ m_1 & \text{if } q_v^* = \emptyset \end{cases}$$

where m_1 is a fixed machine. If there is more than one vertex w that satisfies the conditions, then we simply use the leftmost vertex among all possibilities—which w is used does not effect the optimality of the assignment function.

The assignment f described above is an optimal solution to the NMS Communication Minimization Problem. To see this we prove the following:

Lemma 5.1. *Given any $t : V \to M$ such that $t|U = g$ and any $v \in V \backslash U$, define a function $t' : V \to M$ as:*

$$t'(w) = \begin{cases} t(w) & \textit{if } w \neq v \\ f(v) & \textit{otherwise} \end{cases}$$

Then $C_*(t', V, U, g) \leqslant C_*(t, V, U, g)$

Proof. It is enough to show that

$$\sum_{u \in U} C_{v,u}(t'(v)) \leqslant \sum_{u \in U} C_{v,u}(t(v)) \tag{4}$$

There are two cases to consider,

(1) If $t(v) = f(v)$ or if $q_v^* = \emptyset$, then the (4) becomes an equality, so the total cost does not get worse with this change, that is, $C_*(t', V, U, g) \leqslant C_*(t, V, U, g)$

(2) Let $t(v) \neq f(v)$, and since in this case $q_v^* \neq \emptyset$, it follows either $|q_v^{\oplus}| = 1$ or $|q_v^{\oplus}| > 1$.

In each of these cases one can show that $C_*(t', V, U, g) \leqslant C_*(t, V, U, g)$, hence the function f minimizes C_* for the BINARY and UNIFORMLY BOUNDED local cost functions. ☐

5.2 The VISIBILITY-GRADED Case

To describe the function f which minimizes C_* in the case of the VISIBILITY-GRADED local cost function, we need to introduce the *choice* function $* : V\backslash U \to U$. Intuitively, the choice function assigns to each manager v a corresponding browser $v^* \in U$ which most significantly influences the physical location of v.

Assuming one has a choice function $*$, we can now define an assignment f of NMS nodes to machines for the case of a VISIBILITY-GRADED local cost function as follows:

$$f(v) = \begin{cases} g(v) & \text{if } v \in U \\ m_1 & \text{if } p_v^* = \emptyset \\ g(v^*) & \text{otherwise,} \end{cases}$$

where v^* is the *choice* of v, and m_1 is a fixed machine.

The following lemma shows that this extension function is optimal in the sense that it minimizes the total communication cost C_*.

Lemma 5.2. *Given any $t : V \to M$ such that $t|U = g$ and any $v \in V\backslash U$, define a function $t' : V \to M$ as:*

$$t'(w) \quad = \quad \begin{cases} t(w) & \text{if } w \neq v \\ f(v) & \text{otherwise} \end{cases}$$

Then $C_*(t', V, U, g) \leqslant C_*(t, V, U, g)$

Proof. The change in total cost $C_*(t, V, U, g)$ due to the new function t' occurs only when $v = w$. Thus the only term in the total cost that can change with the new mapping is the term $\sum_{u \in U} C_{v,u}(t(v))$. It is enough to show $\sum_{u \in U} C_{v,u}(t'(v)) \leqslant \sum_{u \in U} C_{v,u}(t(v))$

There are two cases to consider,

(1). If $t(v) = f(v)$ or if $p_v^* = \emptyset$, then the two sides of the above inequality are equal and so the total cost does not get worse with this change.

(2). Let $t(v) \neq f(v)$, and since in this case $p_v^* \neq \emptyset$, it follows either $|p_v^{\oplus}| = 1$ or $|p_v^{\oplus}| > 1$.

In each of these cases one can show that $C_*(t', V, U, g) \leqslant C_*(t, V, U, g)$, hence the function f minimizes C_* for the VISIBILITY-GRADED local cost function. $\square$

6 Conclusion

In this paper, we have developed algorithms for distributing a hierarchy of network management agents in a manner which minimizes the network communication required between browsers and managers. We showed that for three different natural local cost functions, it is possible to compute a globally optimal assignment. In this work, we considered the NMS agents to be distributed across a network of machines modelled as a complete graph, and charged unit cost for communication between any two distinct machines.

Acknowledgements: The first author thanks the Center for Computational Sciences at the Naval research Lab Washington D.C. for providing funding for this work during the summer of 2000. We would like to thank Dardo Kleiner and David Talmage for many spirited discussions about hierarchical network management and the Optiprism NMS.

References

[1] M. M. Cheikhrouhou, P. Conti, and J. Labetoulle. Intelligent agents in network management, a state-of-the-art. *Networking and Information Systems*, 1(1):9–38, 1998.

[2] M. S. Fox. An organizational view of distributed systems. *IEEE Transactions on Systems, Man, and Cybernetics*, 11(1):70–80, 1981.

[3] J. R. Galbraith. *Organization Design*. Addison-Wesley Publishing Company, 1977.

[4] B. Khan, D. D. Kleiner, and D. Talmage. Towards an agent-based distributed hierarchical network management system for all-optical networks. In *Proceedings of 3rd International Workshop on Mobile Agents in Telecomunications Applications*, Montreal, Canada, August 15-18, 2001.

[5] K. Mount and S. Reiter. A model of computing with human agents. Technical Report 890, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, Evanston, IL, 1990.

[6] B. Patrick and M. Dewatripont. The firm as a communication network. *Quarterly Journal of Economics*, 109(4):809–839, 1994.

[7] R. Radner. The organization of decentralized information processing. *Econometrica*, 61:5, 1993.

[8] T. V. Zandt. Real-time hierarchical resource allocation. Technical report, Kellogg Graduate School of Management, Northwestern University, 1997.