Computation with Polytopal Uncertainty in d-Dimensional Space

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1 Introduction

A measurement is a quantitative description of the transformation required to carry one state to another state. For example, measuring the position of an object involves determining what translation and rotations that will move the axes at the origin to the object's present configuration. In an ideal world, making a measurement between two states u and v yields precisely one transformation. In such a setting, information can be easily consolidated: a measurement from u to v can be composed with a measurement from v to w—all that is needed is vector algebra. In the real world, however, errors and limitations in the sensitivities of the measurement apparatus make it impossible to precisely determine the transformation that will carry u to v. At best, we can determine a small set of candidates transformations, and assert that one of these carries u to v. We accept that the size of this set of candidates will never be 1, or equivalently the "uncertainty" in our measurement will never be zero. In this situation, naive vector algebra does not suffice to allow us to compose measurements. The process by which we consolidate information from several measurements poses a difficult computational problem. We will describe and address several aspects of the problem in this paper.

The principal applications of these results lie in automated consolidation of information derived from sensor measurements in robot planning [5] and in automated registration in augmented reality systems [4]. In recent years, many approaches to this research area have shied away from analytic solutions, and favored the use of Bayesian networks [1, 3] and other statistical schemes [6, 2] to consolidate measurements and minimize uncertainty in robot planning. One notable exception is the work of Rajan and Taylor which provided closed form solutions [7] in low-dimensional settings.

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4 Conclusion and Future Work

In this paper, we derived linear approximations for composition and inversion of measurements in spaces of arbitrary dimension. By doing so, we close the class of d-dimensional measurements in the polytopal error model under the operations of composition and inversion. The derivation of these closed form expressions opens the door to many applications in automated consolidation of information derived from sensor measurements in robot planning as well as in automated registration in augmented reality systems. In future, we intend to investigate applications of these results in the design of proactive measurement and sensing strategies which seek to minimize uncertainty in robot motion planning.

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