Summer of Science Report– Cosmology

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By

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Introduction

This is the report for the Summer of Science, an initiative by the Maths and Physics club. This report contains a brief abstract of the topics covered over the summer. The first chapter discusses the early universe and the origin of the cosmic microwave background. The following chapters explore the effect of mass on the curvature of space and its expansion. Chapters 5 and 6 explore inflation, dark energy and Dark matter. The last chapter briefly discusses the anisotropies in the cosmic microwave background.

Chapter 1 – The early universe and the cosmic microwave background radiation.

The first chapter describes the universe as it was shortly after the big bang. The contents of the universe are described as they change with temperature and time. The story starts from about a hundredth of a second after the big bang.

The temperature of the universe is about 10^{11}K, and time is about a hundredth of a second past the big bang. The universe is filled with a homogenous 'soup' of photons and matter, and is rapidly expanding with a characteristic time of 0.02 seconds.

Since characteristic photon energy is the temperature times the Boltzmann constant, it follows that for a particle to be produced out of radiation(pair of particles, rather – conserving Baryon number, Lepton number, charge, spin and other conserved quantities), the temperature has to be at least of the order of rest mass mc^2 divided by Boltzmann constant. Over this threshold temperature, particles are produced from colliding photons. For example, electrons and positrons are formed from radiation at temperatures above 6×10^9 K. However, they also annihilate each other producing more radiation – the rates of production and annihilation are so high that the concentration of electrons and positrons are in equilibrium. It also follows that at this temperature, the concentration of photons must roughly equal the concentration of electrons and positrons together, as two photons are produced in an annihilation reaction.

$$e^+ + e^- \to \gamma + \gamma$$

Since the temperature is far above the characteristic temperatures of electrons, neutrinos and their anti-particles and the frequent collisions with photons, they behave much like radiation themselves. This was hence a 'radiation dominated' era. Despite the rapid expansion of the universe, the matter and radiation is in thermal equilibrium.

The temperature is far below the threshold for protons, neutrons, muons, etc. The abundant particles, at this stage are, therefore, electrons, positrons, photons, neutrinos and anti-neutrinos. The equivalent mass density of all these particles together (for photons, using the relation $E = mc^2$) is 3.8 billion Kg L^{-1}.

The heavier particles have annihilated with their anti-particles long back – but the small excess of matter over anti matter was left over. This left over Baryonic matter, which was spared annihilation was destined to become the stuff we are familiar with. The reason for the excess of matter over anti matter is not well understood to this day.

There is one proton or neutron for every billion photon. Since the energy of electrons, positrons and so on are much higher than the mass difference between the proton and the neutron, they convert to each other freely.

$$n + v \leftrightarrow p + e^{-}$$
$$p + \bar{v} \leftrightarrow e^{+} + n$$

The temperature is too high to form heavier nuclei at this stage.

The universe cooled down to $3 \times 10^{10} K$ after another 0.11 seconds. The decreasing temperature has shifted the neutron – proton equilibrium in favour of proton, and the ratio has gone down to 38% neutrons. The rate of expansion has gone down and the characteristic time of expansion has gone to 0.2 seconds.

1.1 seconds after the big bang, the temperature drops to $10^{10} K$. The mean free path of the weakly interacting neutrinos have increased so much that they no longer behave like radiation. They are no longer in thermal equilibrium with the rest of the matter or radiation. The energy density falls of as the fourth power of temperature, and the equivalent density is 380,000 Kg L^{-1}. Characteristic time of expansion has also increased to 2 seconds. The neutron – proton balance has shifted to 24% neutrons.

Nearly 14 seconds after the big bang, temperature hit 3 million kelvin – well below the threshold temperature for electrons. Electrons and positrons are rapidly annihilated, and it is cool enough for stable nuclei like He^4 to form. However, Helium is not formed in large quantities due to the especially unstable Deuterium nuclei which is involved in one of the intermediate steps. The neutron proton balance has shifted to 17% neutron. The heat released in the recombination of electrons has slowed down the lowering of the temperature.

After nearly 3 minutes and two seconds, temperature drops to $10^9 K$, and it's cool enough for tritium nuclei to form. However, due to the instability of the Deuterium nuclei, large quantities of heavy nuclei are not produced. The decay of neutrons is becoming important at this stage, as the half-life of the free neutron is about 10 minutes.

$$n \rightarrow p + e^{-} + \bar{v} + \gamma$$

Soon after, temperature drops down to a point where Deuterium nuclei can hold together. The neutron – proton ratio gets locked to 13% neutrons, as free neutrons rapidly convert to He^4.

After another 378,000 years, the universe cools down enough to allow recombination of electrons and protons/nuclei to form neutral atoms. The universe suddenly becomes transparent to radiation. The photons and matter particles are no longer in equilibrium.

Just before recombination, the universe was in thermal equilibrium. The frequency of radiation and the temperature was related by the black body radiation equation. When the ions combined to form neutral atoms, the universe suddenly became transparent to radiation – neutral atoms are not as effective as ions in scattering photons. The universe was no longer in equilibrium.

However, when the radiation was set free, the average distance between photons simply expanded in proportion with the size of the universe. In addition, the cosmological red shift pulled out the wavelength of any ray of light in the universe, that too in proportion with the size of the universe. The photons thus remain one wavelength apart, similar to black bogy radiation. Hence even though the universe is not in equilibrium, the cosmic background radiation behaves exactly like black body radiation, corresponding to a temperature of 2.7K. This radiation, which peaks at the microwave range, was detected by Penzias and Wilson in 1964. This discovery won them the 1978 Nobel Prize for physics.

Chapter 2 – The Robertson Walker metric.

In Euclidean geometry, the metric of normal space is

$$dS^2 = a(t)^2(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\emptyset^2)$$

In layman's terms, this gives the effective 'distance' between two points in space, i.e. the distance you have to walk to go from one point to another through a given path is the line integral of this function from the first point to the second through the given path.

Now if we assume a uniform space (one in which no point is in any way 'special'), it so happens that the only possible metric is the Robertson Walker metric, given by –

$$dS^2 = a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\emptyset^2\right)$$

The Robertson Walker metric allows space to 'Shrink' or 'Expand' with time by the parameter a(t). Effectively, it means that the distance between any two points (with given coordinates) in space can increase or decrease with time. The parameter k changes the geometry of space itself – making it spherical, hyperbolic or flat. The flat universe follows Euclidian geometry and has k = 0. The others are said to be curved geometries.

In a universe with non-zero value of k, the ratio of the circumference to the diameter of a circle can deviate from the familiar value of pi. To make it a bit more rigorous, define π_0 as the smallest positive real number which satisfies the equation $e^{i\,x} + 1 = 0$.

Consider a circle centred at the origin and passing through the point $\left(r_0, \frac{\pi}{2}, 0\right)$ and $\left(r_0, \frac{\pi}{2}, \frac{\pi}{2}\right)$ in spherical coordinates. Let its circumference be C, and the radius be r. Interestingly, r_0 and r need not be equal.

$$C = \int_0^{2\pi} a(t) r_0 d\theta$$

$$C = 2\pi_0 r_0 a(t)$$

$$r = \int_0^{r_0} \frac{a(t)dr}{\sqrt{1-kr^2}}$$

While k can be any real number, for the ease of evaluating the integral, assume k to be +1, 0 or -1. The nature and physical interpretations associated with the value of k depend only on its sign.

$$r = a(t) sin^{-1}(r_0), \qquad if\ k = 1$$

$$r = a(t) sinh^{-1}(r_0), \qquad if\ k = -1$$

$$r = a(t)r_0, \qquad if\ k = 0$$

The value of circumference to diameter of the circle works out to be, (Let's call it π)

$$\pi = \frac{\pi_0 r_0}{\sin^{-1}(r_0)}, \qquad k = 1$$

$$\pi = \frac{\pi_0 r_0}{\sinh^{-1}(r_0)}, \qquad k = -1$$

$$\pi = \pi_0 \qquad k = 0$$

For the more general case, where k is any real number,

$$\pi = \frac{\pi_0 r_0 \sqrt{k}}{\sin^{-1}(r_0\sqrt{k})}, k > 0$$

$$\pi = \frac{\pi_0 r_0 \sqrt{|k|}}{\sinh^{-1}(r_0\sqrt{|k|})}, k < 0$$

The value of π deviates from the usual Euclidian value in a Robertson Walker metric. Since the universe is assumed to be homogenous (there is no 'special' point in space), and the observed value of pi is fairly close to the theoretical value of pi in Euclidian space, we can safely conclude that the value of k is close to zero.

Similarly, the angles of a triangle need not add up to 180^0 in a non-Euclidean space. Chapter 4 picks up the topic further.

Chapter 3 – The Friedmann Equation and Hubble's constant.

The Friedmann equation, arguably the most equation in cosmology, relates the Hubble's constant and the density of the universe to the geometry of the universe. It should, in all respects, be derived using the principles of general relativity. However, the same equation can also be derived loosely using classical Newtonian mechanics and Hubble's Law. A derivation of the Friedmann Equation using the principles of Newtonian mechanics is presented here.

Hubble's Law.

According to the Hubble's law, any two bodies in space recede away from each other, with a speed proportional to the distance between the two bodies, in a radial direction.

Let a body be at a position $\vec{x}$ with respect to another. The velocity of the first body as observed from the second would be $\vec{v} = H\vec{x}$, where H is the Hubble's constant. If the distance as a function of time is $a(t)x$, the Hubble's constant is given by $\frac{1}{a}\frac{da}{dt}$.

Consider a homogenous distribution of mass, spread through all of space (This is a simplified model of our universe). Pick an arbitrary point and two perpendicular direction vectors to be the reference of the coordinate frame (Spherical polar coordinates).

Consider a mass m somewhere at a radial distance a(t)*x from the origin.

Since the mass distribution was assumed to be infinite, the potential energy would be

$$P = -\frac{G\left(\frac{4}{3}\pi r^3 \rho\right) m}{r} = -\frac{4}{3} G\pi r^2 \rho m = -\frac{4}{3} G\pi (ax)^2 \rho m$$

Since the velocity of the mass is proportional to its displacement from the origin, the direction of velocity is radial. Hence its velocity (w.r.t origin) would be the time derivative of the radial distance from the origin. This follows from the Hubble's Law.

Hence the kinetic energy would be

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{d(ax)}{dt}\right)^2 = \frac{1}{2} m x^2 (\dot{a})^2, where\ \dot{a} = \frac{da}{dt}$$

Total energy, *U* would be,

$$U = P + K$$

With some rearrangement, this equation becomes,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

Where $k = -\frac{2U}{mc^2 x^2}$

This is the Friedmann equation.

When the same equation is derived using general relativity, it so happens that the parameter k in this equation equals the k in the Robertson Walker metric. As discussed before, the Hubble's constant is given by $\left(\frac{\dot{a}}{a}\right)$. Hence the Friedmann Equation relates the Hubble's constant, density and the geometry of the universe.

If the universe has to be truly flat, k has to be zero. The value of density at which k becomes zero is called the critical density – given by

$$\rho_{critical} = \frac{3H^2}{8\pi G}$$

Plugging in the value of Hubble's constant, the critical density comes out to be about $9.47 \times 10^{-27}\ kgm^{-3}$. In accordance with our expectations, the measured value of density of the universe comes out to be very close to this value.

This is indeed a very puzzling problem. *Why is our universe so flat?* Chapter 5 seeks to find some answers to this and some other fascinating questions.

The critical density is also important in another respect – a universe with a super-critical density will collapse under its own gravity, just as one with sub-critical density will expand forever. Due to this reason, a universe with super-critical density is called closed universe. Eventually, all mass is going to fall back on itself, resulting in a 'Big Crunch'. On the other hand, a universe with sub-critical density is called open universe, for it expands forever.

This is slightly reminiscent of a stone thrown straight up from earth – if the velocity is large enough, the stone goes on for ever, falls into the earth otherwise. The gravity or mass of the earth is analogous to the density of the universe, and the Hubble's constant analogous to the speed of the stone.

Chapter 4 – The geometry of the universe.

More about the geometry of the universe.

A universe with k > 0 is a 'spherical' universe, while one with k < 0 is a 'hyperbolic' universe. A hyperbolic universe is infinite in nature, meaning it simply goes on and on.

On the other hand, a spherical universe, is finite in extend. A space ship, if it tries to travel in a straight line, after covering a sufficient distance, will end up exactly where it started.

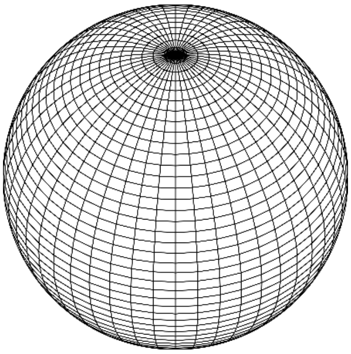
Before going further, it is worthwhile to look at the relation between density and the geometry of the universe. As is apparent from the Friedmann equation, if the average density of the universe is greater than the critical density, the value of k is positive, and hence a closed universe. Similarly, an average density less than the critical density leads to an open universe.

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \qquad \rho_{critical} = \frac{3H^2}{8\pi G}$$

Consider a two dimensional analogy.

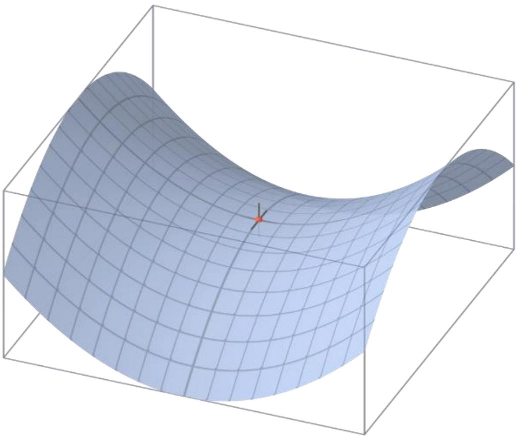
The flat universe is analogous to a plane in two dimension. Euclidean geometry holds true, and the angles of a triangle add up to 180^0.

The closed universe is analogous to a spherical surface.



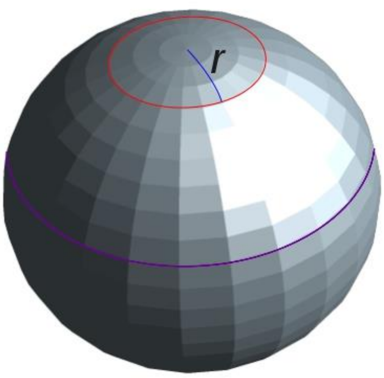
Any two dimensional beings in the 2D surface will not be able to directly observe the curvature. If any such being starts moving in one direction, it will eventually end up where it started.

The open universe, on the other hand is analogous to a ‘saddle shape’.



Again, two dimensional beings in the surface will not be able to see the curvature. Any 2 dimensional being, moving in a ‘straight line’ will keep on going without meeting any boundary or repetition.

It is easy to imagine the deviation of pi from its intuitive value in these geometries.

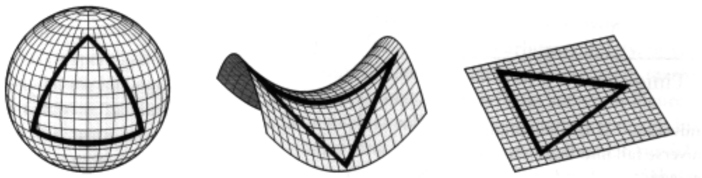


Since all distances are measured *in the plane*, the radius is longer than in a flat geometry. Hence the value of pi (I am using the term pi very loosely here. By definition, pi is the ratio of circumference to diameter of a circle in a Euclidian geometry. It is a fundamental constant which cannot change) is less than its value in Euclidean geometry.

Similarly, in a hyperbolic space, because of the curvy nature of the circle, the circumference is larger than in a flat geometry. Hence the value of pi is larger.

In both these cases, it is critical to define the circle properly. Think of it like this – a bunch of bugs trained to stop after a certain distance are released in different directions from a particular point. The final locations of the bugs are joined to form the circle.

Similarly, the angles of a triangle need not add up to 180^0. It is more than 180^0 for a closed geometry and less than that for an open one. The closed universe is said to have 'Positive' curvature and the open one has 'Negative' curvature. The flat universe has zero curvature. The angles of a triangle sum up to exactly 180^0 in that case.



Positive Curvature Negative Curvature Flat Curvature

As discussed before, the measured density of the universe is extremely close to the critical density. So close, that with current estimates of average mass of the universe, it is not possible to determine if the universe is open or closed. The best estimates for density parameter (ratio of actual density of the universe to critical density) to date is about 1.002 ± 0.005

Chapter 5 – Inflation.

At this stage modern cosmology faced several major problems.

The flatness problem – why is the density of the universe so close to the critical density? And the interesting thing is, any deviation from critical density tends to get magnified over time. I.e. even if the early universe had a density that deviated even a tiny fraction from the critical density, it would now have differed by hundreds of orders of magnitude. By the accepted standard model, the density shortly after the big bang had to agree with the then critical density to a fantastic amount of precision.

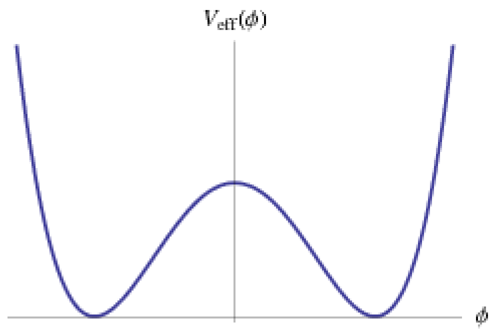
The horizon problem – distant parts of the universe, separated by more than the distance light could travel in the age of the universe, seem to have the same temperature and large scale structures. The cosmic microwave background radiation is isotropic in all directions. Apparently, these widely scattered parts of the universe were in contact sometime in the distant past. But then the transfer of information cannot occur at more than the speed of light, and these parts of the universe are separated by more than the distance that can be covered by light in the age of the universe.

The small lumps and bumps in the density that led to the clumping of matter, eventually forming stars and galaxies. The same patterns of irregularities are observed in vastly separated parts of the universe.

Magnetic monopoles, as predicted by the grand unified theories have not been observed.

All of these problems can be addressed by inflationary models of the universe. Many models have been proposed with subtle differences, but all of them predict that the universe was once stuck in a state of ‘False vacuum’, a state in which a high energy field permeates space and provides an additional term to the Friedmann equation, similar to Einstein’s cosmological constant.

Inflationary models assume an inflationary field with an associated Mexican hat potential. This is the inflation field, believed by many to be closely associated with the Higgs field (some argue this field to be the Higgs field itself).



Before 10^{-36}seconds had elapsed after the big bang, the energy of the universe was above the central peak of the Mexican hat, and the expectation value of the field was zero. When the energy level of the universe matched the peak of the energy diagram, the value of the field was stuck at an unstable position. The universe was stuck there for a brief period of time, after which a spontaneous symmetry breaking happened, and universe moved down to a stable position.

In the short while when the universe was stuck in the top of the potential hill, it was said to be in a state of ‘False vacuum'. The universe had a gigantic energy density when it was stuck there, and this translates into a huge positive addition to the density in the Friedmann equation. The effect of the false vacuum is not unlike the cosmological constant introduced by Einstein in 1917.

The Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

Unlike normal matter or radiation density, the energy density due to false vacuum does not depend on the expansion of the universe. As a good qualitative approximation, the right side of the equation does not change appreciably throughout the era of inflation. It immediately follows that the universe expanded exponentially during this period.

$$\frac{\dot{a}}{a} = \, large\, positive\, quantity$$

The inflationary epoch lasted from 10^{-36}s after the big bang to sometime between 10^{-33} and 10^{-32}s, after which the universe ‘rolled over’ to one side of the Mexican hat due to quantum fluctuations.

The inflationary model naturally explains the horizon problem – the contents of the universe was squeezed together before the period of inflation – close enough that they could be in thermal equilibrium. The expansion of space carried away different parts of the universe further apart, to the point where they could never be in contact again – not for several billion years. The universe was indeed expanding faster than the speed of light – but no information was transmitted from one part to another, and hence no law of physics is being violated.

As far as the flatness problem is concerned, the inflationary period stretched out any curvature in space to a point where it appears flat locally. The universe does appear flat to us, but that might be only because we are sampling a tiny portion of the universe, expanded to such extends that the curvature is no longer observable (with current measurements, anyway).

Magnetic monopoles which had formed before the inflationary period would have spread out to such a large extend that it is quite probable the visible universe does not have any. The universe was just not hot enough for them to form post inflation.

Quantum fluctuations in the density before the period of inflation could've been stretched out to produce the inhomogeneity that led to the formation of stars and galaxies.

Despite its success at explaining these observations, inflation has not yet been fully accepted by the scientific community, because it has not yet made any predictions which could be tested against observation. For example, the general theory of relativity was universally accepted only after the bending of light from a distant star by the sun's mass, as predicted by the theory, was experimentally observed.

Chapter 6 – Dark matter and Dark Energy.

In the 1920s and 1930s, it was observed that the observed matter in the universe, i.e. stars, planets, and pretty much everything that interacts with electromagnetic radiation just could not account for all the gravitation that holds our universe together. By the 1980s it was recognised that only about one sixth of the mass of the universe as predicted by gravitational effects was visible to us, i.e. interacted with electromagnetic radiation. The rest was dubbed dark matter. Their effects are observed in gravitational lensing, galaxy rotation curves and the expansion rate of the universe. The widely accepted hypothesis is that dark matter is made of weakly interacting massive particles (WIMPs) that interact only via gravitation and weak nuclear force.

Dark matter, on the other hand, was introduced because the universe was found to expand at an *accelerating* rate, rather than being slowed down by gravity. The accelerated expansion is believed to have started 5 to 7 billion years ago. This is similar to the idea of the false vacuum associated with the idea of inflation, and reminiscent of Einstein's cosmological constant.

The Friedmann equation with Einstein's cosmological constant is,

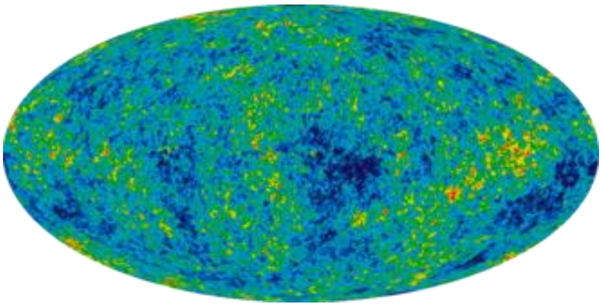
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

This is equivalent to having a mass density of $\frac{\Lambda}{3H^2}$. When this term dominates the Friedmann equation, the universe expands at an accelerating rate. This false vacuum has been associated with another field, possibly a Mexican hat potential, or a simpler curve with just one minima at a small positive value. Whatever be the exact nature of the potential function, the universe seems to be stuck in another false vacuum for the last seven billon years or so. Since the vacuum energy is currently very small compared to that during the era of inflation, the effects are much less aggressive. The best estimates measure dark energy to contribute 69% of the total density of the universe. 26% is dark matter, and 5% amounts to baryonic matter.

The value of this vacuum energy is truly a baffling mystery, as it is expected to be 120 orders of magnitude greater than the present value by particle physics calculations, or zero altogether due to some symmetry. Unlike dark matter or 'normal' baryonic matter, the energy density of Dark energy does not change with the expansion of space – It is rather strange that an energy density that does not vary with the size of the universe and a mass density that varies as the inverse cube of the size of the universe should be of comparable magnitude, right when we're here to ask questions.

Chapter 7 – Acoustic oscillations

The cosmic microwave background was thought to be initially very uniform, but soon peaks in frequencies had been observed in certain directions. Removing the Doppler shifts due to the motion of Earth in the CMB and the other contributions from stars, and other sources, a mottled pattern was observed.



(Source – WMAP data)

Quantum fluctuations prior to inflation were 'frozen over' when inflation ended. This led to slight density fluctuations, which further attracted matter (dark and baryonic) into it, making the fluctuations more pronounced. As the dark matter clumped in, baryonic matter started oscillating between regions of higher and lower density between gravitational attraction and the pressure due to scattering of photons. These are called acoustic oscillations because they behave much like a sound wave – as they are oscillations in pressure of the baryon – photon fluid.

This oscillation continued for another 378,000 years, after which the baryonic ions recombined, and the radiation escaped into space. By this time, the baryonic matter had formed clumps at a characteristic distance from each other. This clumping promises a scale which can be used to measure astronomical distances, and might prove better than the current method of using brightness of Ia supernovae. In addition, this density fluctuations also provide very accurate information on the baryon – dark matter ratio (baryons and dark matter respond differently to the density fluctuations in presence of photons) as well as the density parameter of the universe, since angles and distances scale differently in curved geometries.

As a result of the acoustic oscillations, the galaxies have retained some of the 'clumpiness'. Galaxies form clumps at about 150Mpc. This provides a fairly accurate yardstick for measuring large distances. Then the distance at which this length unit subtends 1 degree is measured experimentally. This distance is compared with the calculated distance for a flat, closed or open universe to determine the value of the density parameter. As expected, the value is close to 1.

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Thasni Shanavas, for putting through all my tantrums, hearing me out in the wee hours of the night, and reading through my drafts. I owe you one!

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- Modern Cosmology – Scott Dodelson
- A brief history of time – Stephen Hawking
- An introduction to astronomy and astrophysics – Pankaj Jain

Suggested Reading

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- The early universe – Kolb and Turner
- An introduction to stellar astrophysics – Leblanc
- High energy Astrophysics – Longair
- Black holes, white dwarves, and neutron stars – Shapiro, Tuekolosky