Decentralised Coverage Control of Autonomous Mobile Robots

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***Abstract*—A decentralised coverage control algorithm is proposed for optimal sensing using multiple mobile robots with single integrator mechanics. It is assumed that the density function is of unknown nature and that only local information is available to the agents. The stability and convergence of the agents are discussed using a Lyapunov-type proof. The behaviour of the system as various parameters are changed is studied using numeric simulations.**

***Keywords*—Coverage Control, decentralised control, multi-agent, spiral phyllotaxis**

I. Introduction

The paper discusses a control algorithm for optimal sensing of a density function using multiple agents with single integrator dynamics.

The controller can be used for environmental monitoring, for example, cleaning up of oil spills (Schwager et al., 2007 [1]), sensing nuclear radiation in a region, optimal distribution of mobile phone towers in a region with varying population (Schwager et al., 2007), and the like. The proposed controller can be used in many environments where action is to be taken using multiple agents based on some function over the position space of agents.

A. Problem Description

We consider a compact convex region $Q \subset \mathbb{R}^2$ in which N agents are deployed to cover the region. The position of each agent is denoted by $y_i \in \mathbb{R}^2$ and the corresponding velocities $\dot{y_i} \in \mathbb{R}^2$. The coverage problem is formulated as in Cortes et al. [2] and Rihab et al. [3]

We consider a continuously differentiable bounded density function $\Psi : \Omega \to \mathbb{R}_+$ over Ω, where Ω is an open and connected domain which contains Q. Ψ describes the relative importance of various regions of Q with respect to coverage objective i.e. the regions where Ψ has higher values are more important than the regions with lower values of Ψ and in the optimal coverage configuration, the agents should cover the region in proportion to the value of Ψ.

The agents are assumed to be able to sense the value of Ψ in a small neighbourhood around them and know their position in a common global coordinate system. It is also assumed that each agent has a unique 'address', an integer ranging from 1 to N.

Agents are to aggregate at the peaks of the density function. In a region of uniform density function, the agents should be distributed as evenly as possible. The objective is to minimise a cost function in the space of agent positions, taking into account the evenness of the distribution and the density function to be sensed.

II. Proposed Solution

We assume that the agents have single integrator dynamics i.e. the agent dynamics are given by

$$\dot{y}_i = u_i \qquad i = 1, 2 \dots n \tag{1}$$

where u_i is the control input for the ith agent.
Define ∂Q as the set of boundary points of Q.

$$\partial Q = \{x \in Q \,|\, x \text{ is a boundary point of } Q\}$$

We also assume that the gradient of Ψ at $y \in \partial Q$ points along some $x \in Q$. (i.e. $\nabla \Psi = c(x - y); x \in Q, y \in \partial Q, c \in \mathbb{R} \geq 0$) This is the case in many realistic scenarios, for example, Ψ is the linear superposition of one or more positive Gaussian distributions centered at points in the interior of Q. (Appendix A)

The final distribution of the agents must be a compromise between the evenness of the distribution and the aggregation near the local maxima of Ψ. To facilitate this, each agent is assigned a default position based on its index number. The default positions are distributed evenly across the region Q.

For a circular region, a very good choice for the even distribution of the agents' default position is the *spiral phyllotaxis pattern*, the distribution of seeds in a sunflower plant.

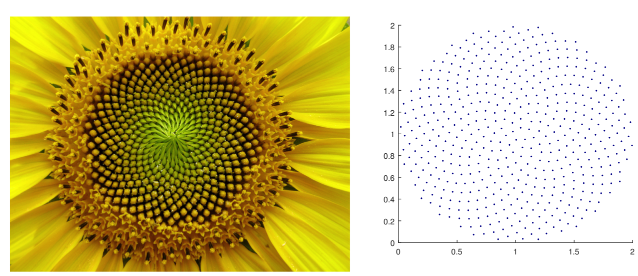


Fig. 1. Spiral Phyllotaxis pattern

If N agents were to be distributed in a unit circle as per the Spiral Phyllotaxis pattern, the coordinates of the i^{th} agent would be, (in polar coordinates)(Prusinkiewicz et al. [4])

$$\left(\sqrt{\frac{i}{N}}, \frac{2\pi i}{\phi^2}\right)$$

where ϕ is the golden ratio, $\frac{\sqrt{5}+1}{2}$

This can be extended to any convex region by considering a circle which superscribes the region and taking the default positions in the intersection of both. The number of points in the superscribing circle is increased one by one until the intersection region contains the requisite number of points.

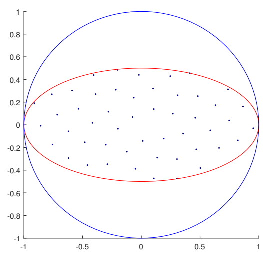


Fig. 2. Default positions for elliptical Q with 50 agents

Since the final distribution of agents has to be a compromise between the evenness of the distribution and the aggregation near the local maxima of Ψ, we propose to minimise the cost function -

$$\Theta_i : \Omega \rightarrow \mathbb{R}$$
$$\Theta_i(y_i) = -\alpha\Psi(y_i) + \beta\left(y_{0i} - y_i\right)^2$$

for each $i = 1, 2 \ldots n$. where y_{0i} is the default position of agent i and y_i the current position, α and β are two non-negative parameters.

A steepest descent method can be used to reach the optimal position which minimises the value of Θ_i for each $i = 1, 2 \ldots n$.

$$\dot{y_i} = -\nabla\Theta_i$$
$$\dot{y_i} = \alpha\nabla\Psi(y_i) + 2\beta\left(y_{0i} - y_i\right)$$

III. Convergence

Lemma 1: The agent indexed i *converges to a point where* $\nabla\Theta_i = 0$*. The bias between peak aggregation and evenness of distribution is decided by the ratio between* α *and* β

Proof:

We now prove the above lemma. Since $\Psi(y_i)$ and $\left(y_{0i} - y_i\right)^2$ are both continuously differentiable functions on Ω, $\Theta_i(y_i) = -\alpha^2\Psi(y_i) + \beta^2\left(y_{0i} - y_i\right)^2$ is continuously differentiable in $Q \subset \Omega$. Further, since Q is a closed set, Θ_i is bounded.

Note that Q is positively invariant with respect to $\dot{y} = -\nabla\Theta_i$.(Appendix B)

Consider the function $V : \Omega^n \rightarrow \mathbb{R}$

$$V = \sum_{i=1}^{N} \Theta_i(y_i)$$
$$\dot{V} = \sum_{i=1}^{N} \nabla\Theta_i(y_i).\dot{y_i} \tag{2}$$
$$\dot{V} = -\sum_{i=1}^{N} \{\nabla\Theta_i(y_i)\}^2$$

Let E be the set of all points in Q where $\dot{V} = 0$. Since $\dot{V} = -\sum_{i=1}^{N}\{\nabla\Theta_i(y_i)\}^2 = 0$ forces all $\nabla\Theta_i(y_i) = -\dot{y_i}$ to be zero, E is an invariant set with respect to the system dynamics.

Since $\dot{V}$ is non-positive, by the LaSalle's Invariance Principle, the system converges to the largest invariant set in E, which is E itself.

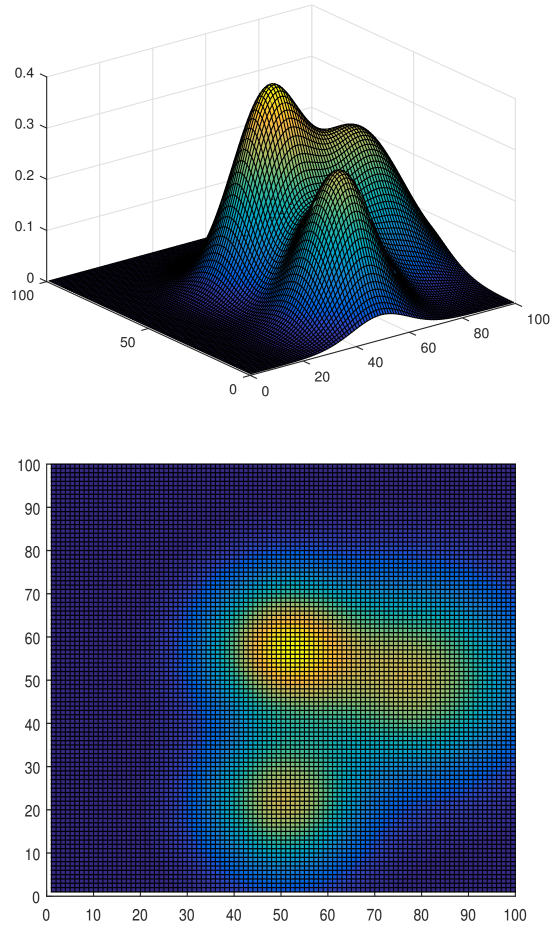
Hence every agent eventually converges to a position where $\nabla\Theta_i = 0$. This implies

$$\alpha\Psi = 2\beta\left(y_i - y_{0i}\right) \tag{3}$$

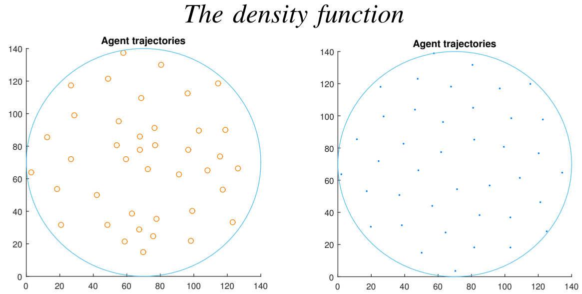
Hence the ratio of α and β decides the bias between the peak aggregation and evenness of distribution. For example, if α = 0, then $y_i = y_{0i}$, which means all the agents will be evenly distributed over R.

IV. Numerical Simulations

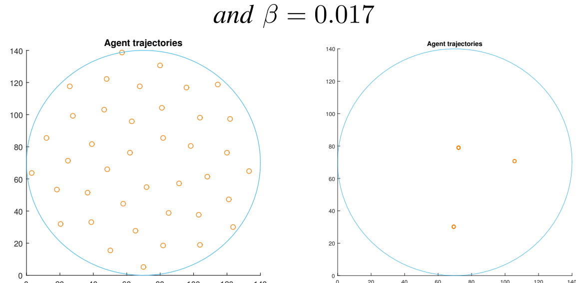
The numerical simulations are presented below for a given density function. Q is taken to be a circular region of radius 50 units. The agents are initially scattered randomly over Q.



The density function



Final agent positions (left) and default positions with $\alpha = 15$ and $\beta = 0.017$



Final agent positions with $\alpha = 0$ (left) and $\beta = 0$ (right)

Though the simulations were done for a circular region of interest, the results can be extended to any convex region.

V. Conclusion

We have seen a decentralised coverage control algorithm for optimally distributing agents and sensing a density function of largely unknown nature. The agents were found to make a good compromise between an even distribution and aggregation near the peaks of the density function.

We have overlooked the case where the gradient of the density function does not lie along any point from the region of interest at the boundary, which might include cases where the agents might move away from the region of interest. For simulation purposes, we have limited ourselves to cases where the region of interest is circular. The results were extended to any arbitrary convex region.

Appendix A
Gradient of Ψ for $x \in \partial Q$

Lemma 2: Let Q be a compact convex set. Let $A_i \in \mathbb{R} > 0$ for $i = 1, 2 \ldots n$. Let $x \in Q$, and $x_i \in Q$ for $i = 1, 2 \ldots n$.

Then, the vector $\sum_{i=1}^{n} A_i(x_i - x)$ can be written as $C(y - x), y \in Q$, where $C \in \mathbb{R}$ and $C \geq 0$

Proof:

Suppose not.

Let $\sum_{i=1}^{n} A_i(x_i - x) \neq 0$
Then there exists no $y \in Q$ for any $C > 0$ such that

$$\sum_{i=1}^{n} A_i(x_i - x) = C(y - x)$$

Let $C = \sum_{i=1}^{n} A_i$. Note that C is strictly positive as $C = 0$ forces all A_i to be zero, which in turn results in $\sum_{i=1}^{n} A_i(x_i - x) = 0$

$$y = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} \tag{4}$$

by the convexity of Q,

$$\frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} \in Q.$$

Which leads to a contradiction. Hence the proof.

Here we consider cases where Ψ is a linear superposition of one or more (finitely many) positive Gaussian distributions, centered at points $x_i \in Q$.

$$\Psi = \sum_{i=1}^{n} A_i e^{-\frac{(x-x_i)^2}{2\sigma_i^2}}$$

$$\nabla\Psi = \sum_{i=1}^{n} A_i e^{-\frac{(x_i-x)^2}{2\sigma_i^2}} . \frac{(x_i - x)}{\sigma_i^2}$$

From lemma 2, it follows that for $y_i \in \partial Q$, $\nabla\Psi$ is of the form $C(z - y_i)$ for some $z \in Q$ and $C \in \mathbb{R} \geq 0$.

Appendix B
Invariance of $\dot{y_i} = -\nabla\Theta_i$ over Q

Here we prove that Q is an invariant set with respect to

$$\dot{y_i} = \alpha\nabla\Psi(y_i) + 2\beta\left(y_{0i} - y_i\right)$$

As Θ_i is continuously differentiable, $-\nabla\Theta = \dot{y_i}$ is necessarily bounded. Hence, $y_i(t)$ is continuous over t. As such, it is sufficient to prove that if $y_i(t) \in \partial Q$, then $y_i(t + \delta t) \in Q$ for $\delta t \to 0^+$.

Let $\delta t \to 0^+$. Then,

$$y_i(t + \delta t) = y_i(t) + \dot{y_i}(t)\delta t \tag{5}$$

As per the starting assumption that the gradient of Ψ at $y \in \partial Q$ points along some $x \in Q$,

$$\nabla\Psi = c(x - y); x \in Q, y \in \partial Q, c \in \mathbb{R} \geq 0$$

Since

$$\dot{y_i} = \alpha\nabla\Psi(y_i) + 2\beta\left(y_{0i} - y_i\right)$$

is of the form $\alpha c(x - y_i) + 2\beta\left(y_{0i} - y_i\right); c > 0$ for $y_i \in \partial Q$, as per lemma 2, $\dot{y_i}$ is of the form $C(z - y_i)$ for some $z \in Q$ and $C \in \mathbb{R} \geq 0$.

$$\begin{aligned}
y_i(t + \delta t) &= y_i(t) + \dot{y_i}(t)\delta t \\
y_i(t + \delta t) &= y_i(t) + C(z - y_i)\delta t \\
y_i(t + \delta t) &= (1 - C\delta t)y_i(t) + C\delta t z \\
y_i(t + \delta t) &= \frac{(1 - C\delta t)y_i(t) + C\delta t z}{1 - C\delta t + C\delta t}
\end{aligned}$$

By the convexity of Q, $y_i(t + \delta t) \in Q$. Hence, the set Q is invariant with respect to the system dynamics.

Acknowledgment

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