The Thesis committee for Tharit Tangkijwanichakul
Certifies that this is the approved version of the following thesis:

Thesis Title

APPROVED BY

SUPERVISING COMMITTEE:

_____
Dr. Sergey Fomel, Supervisor

_____
Dr. Stephen Grand

_____
Dr. Kyle Spikes

Thesis Title

by

Tharit Tangkijwanichakul

THESIS

Presented to the Faculty of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

BACHELOR OF SCIENCE WITH HONORS

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2021

Acknowledgments

This is acknowledgement chapter

THARIT TANGKIJWANICHAKUL

The University of Texas at Austin

April 2021

Thesis Title

Tharit Tangkijwanichakul, B.S.

The University of Texas at Austin, 2021

Supervisor: Dr. Sergey Fomel

We approximate the inverse Hessian operator by a chain of weights in time/space and frequency domains. Tests on synthetic data shows that this approach provides an effective approximation while having the minimal cost of forward and inverse FFTs (Fast Fourier Transforms). The method can be applied either for compensating migrated images or in the form of a preconditioner inside iterative least-squares reverse-time migration (LSRTM). As demonstrated by experiments with synthetic data, the latter significantly accelerates the convergence of LSRTM and achieves high-quality imaging results in fewer iterations.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Intrp to problem of seismic imaging

Chapter 2

Chain of operator: Theory

DEVELOPMENT IN INVERSE HESSIAN ESTIMATION

Least-squares migration (LSM) is a well-established technique for improving the quality of seismic imaging through inversion (Nemeth et al., 1999; Ronen and Liner, 2000). By formulating seismic imaging as a linear estimation problem, least-squares migration utilizes the power of iterative inversion for recovering the reflectivity of subsurface structures. In recent years, least-squares imaging, in particular LSRTM (least-square reverse-time migration) has found many successful applications (Dai et al., 2012; Wang et al., 2016; Wong et al., 2011).

The least-squares formulation involves the inverse Hessian operator. The exact computation of such inverse is prohibitively expansive. In the conventional approach, the inverse Hessian is approximated by iterative methods, such as conjugate gradients (Tarantola, 1984; Sun et al., 2016; Xue et al., 2016). This raises the cost of LSRTM to the cost of migration and modeling multiplied by the number of iterations. Only a small number of iterations can be affordable in practice.

To reduce this cost or, alternatively, to reduce the number of iterations by accelerating the iterative convergence, a number of methods have been proposed in the literature for approximating the inverse Hessian using relatively inexpensive computations. Rickett (2003) estimated a simple scaling operator to approximate the inverse Hessian with a diagonal matrix. Guitton (2004) and Greer et al. (2018) extended this

approach to matching filters, approximating the inverse Hessian with a nonstationary convolution operator. This approach is also known as *migration deconvolution* (Hu et al., 2001; Yu et al., 2006). Aoki and Schuster (2009) approximated inverse Hessian as a deblurring filter for regularization and precondition scheme. Kaur et al. (2020) adopt the deep learning technique and use generative adversarial networks in a conditional setting (CycleGANs) to approximate inverse Hessian.

An asymptotic theory (Miller et al., 1987; Bleistein, 1987) shows that the inverse Hessian can be represented as a combination of weights in the time domain and the frequency domain. The result extends to the case of LSRTM (Hou and Symes, 2015, 2016). Motivated by this theory, we propose to approximate the inverse Hessian by reformulate by simultaneously estimating a chain of weights in the space and frequency domains from initial images. We design this chain to be symmetric to preserve the symmetric property of the Hessian. Once the weight matrices are estimated, they can be efficiently approximate the inverse Hessian. Alternatively, the approximate inverse Hessian can be incorporated in the form of a preconditioner to accelerate the convergence of iterative LSRTM.

The paper is organized as follows: We first present the theory, problem formulation, and an estimation algorithm. Next, we test the accuracy of the proposed approach using zero-offset synthetic data from the Marmousi model (Versteeg, 1994). Subsequently, we use prestack Marmousi data to evaluate the effectiveness of the proposed approximation as a migration-deconvolution filter and as a preconditioner for LSRTM.

THEORY

Problem formulation

Given surface seismic data $\mathbf{d}$, the first migrated image $\mathbf{m}_1$ is obtained by applying seismic migration as the adjoint of forward linear modeling operator $\mathbf{L}$:

$$\mathbf{m}_1 = \mathbf{L}^T \mathbf{d} \ . \tag{2.1}$$

Next, we can model and migrate again, generating the second image

$$\mathbf{m}_2 = \mathbf{L}^T \mathbf{L} \mathbf{m}_1 \tag{2.2}$$

If we can find an effective approximation for the Hessian operator from equation (2.2)

$$\mathbf{H} \approx \mathbf{L}^T \mathbf{L} \tag{2.3}$$

and its inverse, then applying this inverse to the initial image $\mathbf{m}_1$ will provide an effective approximation for the desired least-squares inverse

$$\mathbf{m}_3 = \mathbf{H}^{-1} \mathbf{m}_1 \approx \left(\mathbf{L}^T \mathbf{L}\right)^{-1} \mathbf{L}^T \mathbf{d} \ . \tag{2.4}$$

Chain operator

We propose to approximate the connection between $\mathbf{m}_1$ and $\mathbf{m}_2$ in equation (2.2) using a chain of weights alternating between the original domain and the Fourier domain, as follows:

$$\mathbf{m}_2 \approx \mathbf{W}\mathbf{F}^{-1}\mathbf{W_f}\mathbf{F}\mathbf{W}\mathbf{m}_1 \ , \tag{2.5}$$

where $\mathbf{W}$ and $\mathbf{W_f}$ are diagonal matrices, and $\mathbf{F}$ represents the Fourier transform. Our goal is to estimate $\mathbf{W}$ and $\mathbf{W_f}$ from the given pairs of $\mathbf{m}_1$ and $\mathbf{m}_2$,

4

In the digital filtering terminology, the operation of $\mathbf{F}^{-1}\mathbf{W_f}\mathbf{F}$ is equivalent to convolution. Therefore, the proposed chain represents a combination of scaling and convolution in a symmetric setting. The symmetry is important because the Hessian operator $(\mathbf{L^T L})$, which we are approximating using this representation, is symmetric by definition.

Once we obtain $\mathbf{W}$ and $\mathbf{W_f}$, inverting the chain is straightforward. The compensated migrated image $\mathbf{m}_3$ from equation (2.4) becomes

$$\mathbf{m}_3 = \mathbf{W}^{-1}\mathbf{F}^{-1}\mathbf{W_f}^{-1}\mathbf{F}\mathbf{W}^{-1}\mathbf{m}_1 \; . \tag{2.6}$$

Solving for W and $\mathbf{W_f}$

To solve the nonlinear representation of chain of operators in equation (2.5), we introduce intermediate vectors $\mathbf{x_1}$ and $\mathbf{x_2}$ and treat them as additional unknown variables, splitting the initial condition into three linear equations, as follows:

$$\mathbf{x_1} \approx \mathbf{W}\mathbf{m_1}$$

$$\mathbf{x_2} \approx \mathbf{F}^{-1}\mathbf{W_f}\mathbf{F}\mathbf{x_1} \tag{2.7}$$

$$\mathbf{m_2} \approx \mathbf{W}\mathbf{x_2}$$

In the vector notation,

$$\mathbf{r} = \begin{bmatrix} \mathbf{W} & -\mathbf{I} & 0 & 0 \\ 0 & \mathbf{F}^{-1}\mathbf{W_f}\mathbf{F} & -\mathbf{I} & 0 \\ 0 & 0 & \mathbf{W} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{m_1} \\ \mathbf{x_1} \\ \mathbf{x_2} \\ \mathbf{m_2} \end{bmatrix} \approx \mathbf{0} \; , \tag{2.8}$$

and the solution is found by minimizing the residual vector $\mathbf{r}$ while updating $\mathbf{w}$, $\mathbf{w_f}$, $\mathbf{x_1}$, and $\mathbf{x_2}$. Linearizing equation (2.8) with respect to perturbations in unknown

variables, we arrive at

$$
\begin{bmatrix}
-\mathbf{I} & \mathbf{0} & \mathbf{diag}[\mathbf{m_1}] & \mathbf{0} \\
\mathbf{F}^{-1}\mathbf{W_f}\mathbf{F} & -\mathbf{I} & \mathbf{0} & \mathbf{F}^{-1}\mathbf{diag}[\mathbf{Fx_1}] \\
\mathbf{0} & \mathbf{W} & \mathbf{diag}[\mathbf{x_2}] & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{\Delta x_1} \\
\mathbf{\Delta x_2} \\
\mathbf{\Delta W} \\
\mathbf{\Delta W_f}
\end{bmatrix}
\approx -\mathbf{r}
\tag{2.9}
$$

We solve (2.9) using the conjugate-gradient method with shaping regularization that enforces the smoothness of estimated variables (Fomel, 2007b), then update $\mathbf{W}$, $\mathbf{W_f}$, and recalculate residual $\mathbf{r}$ in (2.8) iteratively until convergence. This corresponds to a regularized Gauss-Newton approach for solving the original nonlinear system.

To initialize the initial time/space weight $\mathbf{W}$, we use a smooth division of $\mathbf{m_2}$ by $\mathbf{m_1}$ (Fomel, 2007a) and then take a square root. We set the initial frequency weight $\mathbf{W_f}$ to 1.

Preconditioning with chain weights

To improve the results further, we propose to apply the weights $\mathbf{W}$ and $\mathbf{W_f}$ to form a preconditioner to speed up the convergence of iterative least-square migration. The original problem is to solve for $\mathbf{m}$ where $\mathbf{d} \approx \mathbf{Lm}$. We apply the change of variables from $\mathbf{m}$ to $\mathbf{y}$ according to $\mathbf{m} = \mathbf{Py}$ where $\mathbf{P}$ is the preconditioning matrix. Then the problem becomes

$$
\mathbf{d} \approx \mathbf{LPy}
\tag{2.10}
$$

We then solve this equation for $\mathbf{y}$ with the least-square objective function

$$
J(\mathbf{y}) = \frac{1}{2}\|(\mathbf{LP})\mathbf{y} - \mathbf{d})\|_2^2 \,,
\tag{2.11}
$$

which corresponds to the analytical solution

$$
\mathbf{y} = (\mathbf{P^T L^T L P})^{-1}\mathbf{P^T L^T d} \,.
\tag{2.12}
$$

After iteratively inverting for $\mathbf{y}$, we recover the original variable $\mathbf{m} = \mathbf{Py}$.

According to the approximation of Hessian operator $\mathbf{L^T L} \approx \mathbf{WF^{-1}W_f FW}$

$$\mathbf{L^T L} \approx \mathbf{WF^{-1}W_f FW}$$
$$= (\mathbf{PP^T})^{-1} , \tag{2.13}$$

the preconditioner $\mathbf{P}$ can be constructed from chain weights as

$$\mathbf{P} = \mathbf{W^{-1}F^{-1}W_f^{-1/2}F} . \tag{2.14}$$

This makes the inverted operator in equation (2.12) close to unitary,

$$(\mathbf{LP})^T \mathbf{LP} \approx \mathbf{I} , \tag{2.15}$$

which accelerates the convergence when the inversion is carried out by an iterative method.

Chapter 3

Chain of operators: Experiments

NUMERICAL EXPERIMENT: POST-STACK DECONVOLUTION

use chain in post stack sense

NUMERICAL EXPERIMENT: PRECONDITIONED LEAST-SQUARE RTM
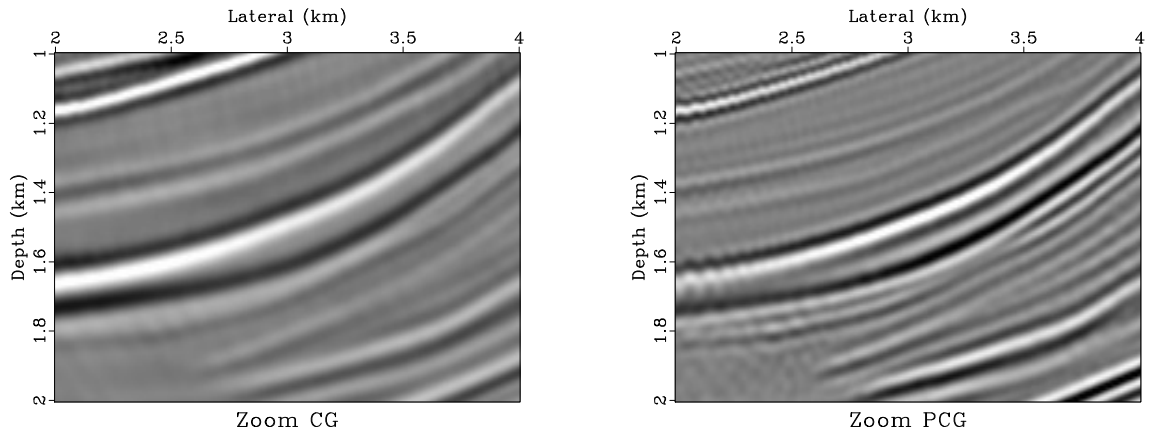
use chain to accelerate LSRTM

Figure 3.1: Zoom-in Comparison1 chapter-lsrtm/../chapter-lsrtm/pre z-ls

---

Chapter 4

Conclusions

In this thesis, I discussed

FUTURE WORK

In the future,

Appendix

CODE

Code

Bibliography

Aoki, N., and G. T. Schuster, 2009, Fast least-squares migration with a deblurring filter: Geophysics, **74**, WCA83–WCA93.

Bleistein, N., 1987, On the imaging of reflectors in the earth: Geophysics, **52**, 931–942.

Dai, W., P. Fowler, and G. T. Schuster, 2012, Multi-source least-squares reverse time migration: Geophysical Prospecting, **60**, 681–695.

Fomel, S., 2007a, Local seismic attributes: Geophysics, **72**, A29–A33.

——, 2007b, Shaping regularization in geophysical-estimation problems: Geophysics, **72**, R29–R36.

Greer, S., Z. Xue, and S. Fomel, 2018, Improving migration resolution by approximating the least-squares hessian using nonstationary amplitude and frequency matching, *in* SEG Technical Program Expanded Abstracts 2018: Society of Exploration Geophysicists, 4261–4265.

Guitton, A., 2004, Amplitude and kinematic corrections of migrated images for nonunitary imaging operators: Geophysics, **69**, 1017–1024.

Hou, J., and W. W. Symes, 2015, An approximate inverse to the extended born modeling operatoran approximate inverse operator: Geophysics, **80**, R331–R349.

——, 2016, Accelerating extended least-squares migration with weighted conjugate gradient iteration: Geophysics, **81**, S165–S179.

Hu, J., G. T. Schuster, and P. A. Valasek, 2001, Poststack migration deconvolution: Geophysics, **66**, 939–952.

Kaur, H., N. Pham, and S. Fomel, 2020, Improving resolution of migrated images by approximating the inverse hessian using deep learning: Geophysics, **85**, 1–62.

Miller, D., M. Oristaglio, and G. Beylkin, 1987, A new slant on seismic imaging: Migration and integral geometry: Geophysics, **52**, 943–964.

Nemeth, T., C. Wu, and G. T. Schuster, 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, 208–221.

Rickett, J. E., 2003, Illumination-based normalization for wave-equation depth migration: Geophysics, **68**, 1371–1379.

Ronen, S., and C. L. Liner, 2000, Least-squares dmo and migration: Geophysics, **65**, 1364–1371.

Sun, J., S. Fomel, T. Zhu, and J. Hu, 2016, Q-compensated least-squares reverse time migration using low-rank one-step wave extrapolation: Geophysics, **81**, S271–S279.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266.

Versteeg, R., 1994, The marmousi experience: Velocity model determination on a synthetic complex data set: The Leading Edge, **13**, 927–936.

Wang, P., A. Gomes, Z. Zhang, and M. Wang, 2016, Least-squares rtm: Reality and possibilities for subsalt imaging, *in* SEG Technical Program Expanded Abstracts 2016: Society of Exploration Geophysicists, 4204–4209.

Wong, M., S. Ronen, and B. Biondi, 2011, Least-squares reverse time migration/inversion for ocean bottom data: A case study, *in* SEG Technical Program Expanded Abstracts 2011: Society of Exploration Geophysicists, 2369–2373.

Xue, Z., Y. Chen, S. Fomel, and J. Sun, 2016, Seismic imaging of incomplete data and simultaneous-source data using least-squares reverse time migration with shaping regularization: Geophysics, **81**, S11–S20.

Yu, J., J. Hu, G. T. Schuster, and R. Estill, 2006, Prestack migration deconvolution: Geophysics, **71**, S53–S62.

Vita

This is my vita

Permanent address: 908 Poplar St. Austin, TX USA 78705

This thesis was typeset with LATEX[†] by the author.

[†]LATEX is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's TEX Program.