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Batch: B3

Expt-3: Linear Regression: Parameter Estimation using OLS, MLE, and Gradient Descent

```
In [87]: # Step 1: Import Libraries
import numpy as np
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score
```

```
In [88]: # Step 2: Input Data
z = np.array([1, 2, 3]).reshape(-1,1) # input
y = np.array([2, 3, 5]) # output
```

```
In [89]: # Step 3: Create model and fit
model = LinearRegression()
model.fit(z,y)
```

```
Out[89]: ▼ LinearRegression ⓘ ?
```

Parameters	
fit_intercept	True
copy_X	True
tol	1e-06
n_jobs	None
positive	False

```
In [90]: # Step 4 : Get coefficients (MLE estimates)
w = model.coef_[0]
b = model.intercept_

print(f"w(slope) = {w}")
print(f"b(intercept) = {b}")
```

```
w(slope) = 1.4999999999999993
b(intercept) = 0.33333333333333348
```

```
In [91]: y_pred = model.predict(z)
```

```
print (z)
```

```
[[1]  
[2]  
[3]]
```

```
In [92]: print("\nPrediction for training data:")  
        for zi, yi, ypi in zip(z.flatten(), y, y_pred):  
            print(f"x={zi}, Actual sales ={yi}, Predicted Sales = {ypi:2f}")
```

Prediction for training data:

x=1, Actual sales =2, Predicted Sales = 1.833333

x=2, Actual sales =3, Predicted Sales = 3.333333

x=3, Actual sales =5, Predicted Sales = 4.833333

```
In [93]: mse = mean_squared_error(y, y_pred)  
        r2 = r2_score(y, y_pred)  
  
        print(f"\nMean Squared Error (MSE) = {mse:.4f}")  
        print(f"R^2 Score = {r2:.4f}")
```

Mean Squared Error (MSE) = 0.0556

R^2 Score = 0.9643

MLE

```
In [94]: import numpy as np  
        from sklearn.linear_model import LinearRegression  
        from sklearn.metrics import mean_squared_error, r2_score  
        from scipy.optimize import minimize
```

```
In [95]: # Step 2: Input data  
        X = np.array([1,2,3])  
        y = np.array([2,3,5])
```

```
In [96]: # Step 3: Negative Log Likelihood function  
        def neg_log_likelihood(parms):  
            w, b = parms  
            sigma2 = 1 #assume variance = 1  
            y_pred = w*X + b  
            nll = 0.5*np.sum((y-y_pred)**2 / sigma2)  
            return nll
```

```
In [97]: #Initial values for w (weight) and b (bias)
```

```
In [98]: # Step 4: Minimize nll  
        result = minimize(neg_log_likelihood, initial_guess)  
        w_mle, b_mle = result.x  
        print(f"Slope is: {w_mle}\nIntercept is: {b_mle}")
```

Slope is: 1.500000003897125

Intercept is: 0.3333333918730798

```
In [99]: # Step 5: Prediction
y_pred = w_mle*X + b_mle
print("Predictions for training data\n")
for xi, yi, ypi in zip(X.flatten(), y, y_pred):
    print(f"X = {xi}, Actual y = {yi}, Predicted y = {ypi}")
```

Predictions for training data

X = 1, Actual y = 2, Predicted y = 1.833333395770205
X = 2, Actual y = 3, Predicted y = 3.33333339966733
X = 3, Actual y = 5, Predicted y = 4.833333403564454

```
In [100]: # Step 6: Error Calculation
mse = mean_squared_error(y, y_pred)
r2 = r2_score(y, y_pred)
print(f"Mean Squared Error is: {mse}")
print(f"R2_Score is: {r2}")
```

Mean Squared Error is: 0.055555555555556004
R2_Score is: 0.9642857142857114

Gradient Descent

```
In [101]: # Step 1: Imports
import numpy as np
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score
```

```
In [102]: # Step 2: Input data
X = np.array([1,2,3])
y = np.array([2,3,5])
```

```
In [103]: # Step 3:
w, b = 0, 0
alpha = 0.001
n_iter = 100000
n = len(X)

for i in range(n_iter):
    y_pred = w*X.flatten() + b
    dw = (-2/n)*np.sum(X.flatten()*(y-y_pred))
    db = (-2/n)*np.sum(y-y_pred)
    w -= alpha*dw
    b -= alpha*db

print(f"Slope is: {w}\nIntercept is: {b}")
```

Slope is: 1.499999999999562
Intercept is: 0.333333333334329107

```
In [104]: # Step 6 :Make Predictions
# Make predictions using estimated parameters
y_pred = w *X + b
```

```
print("Predictions for training data\n")
for xi, yi, ypi in zip(X.flatten(), y, y_pred):
    print(f"X = {xi}, Actual y = {yi}, Predicted y = {ypi}")
```

Predictions for training data

```
X = 1, Actual y = 2, Predicted y = 1.8333333333338911
X = 2, Actual y = 3, Predicted y = 3.3333333333334531
X = 3, Actual y = 5, Predicted y = 4.8333333333330152
```

```
In [105... # Step 7: Calculate metrics
mse = mean_squared_error(y, y_pred)
r2 = r2_score(y, y_pred)
print(f"Mean Squared Error is: {mse}")
print(f"R2_Score is: {r2}")
```

```
Mean Squared Error is: 0.055555555555555553
R2_Score is: 0.9642857142857143
```

Gradient Descent with single Parameter

```
In [106... # Step 2: Data
z = np.array([1, 2, 3])
y = np.array([2, 3, 5])
n = len(X)

# Step 3: Calculate loss function
def loss(w1):
    w0 = np.mean(y) - w1*np.mean(X)
    y_pred = w1*X + w0
    return np.sum((y-y_pred)**2)

# Step 4: Calculate Gradient of J wrt w1
def gradient(w1):
    w0 = np.mean(y) - w1*np.mean(X)
    y_pred = w1*X + w0
    return (-2/n)*np.sum(X*(y-y_pred))

# Step 5: Gradient Descent Algorithm
lr = 0.1
w1 = 4
n_iter = 15
w1_values = []
loss_values = []

for i in range(n_iter):
    w1_values.append(w1)
    loss_values.append(loss(w1))
    grad = gradient(w1)
    w1 -= lr*grad

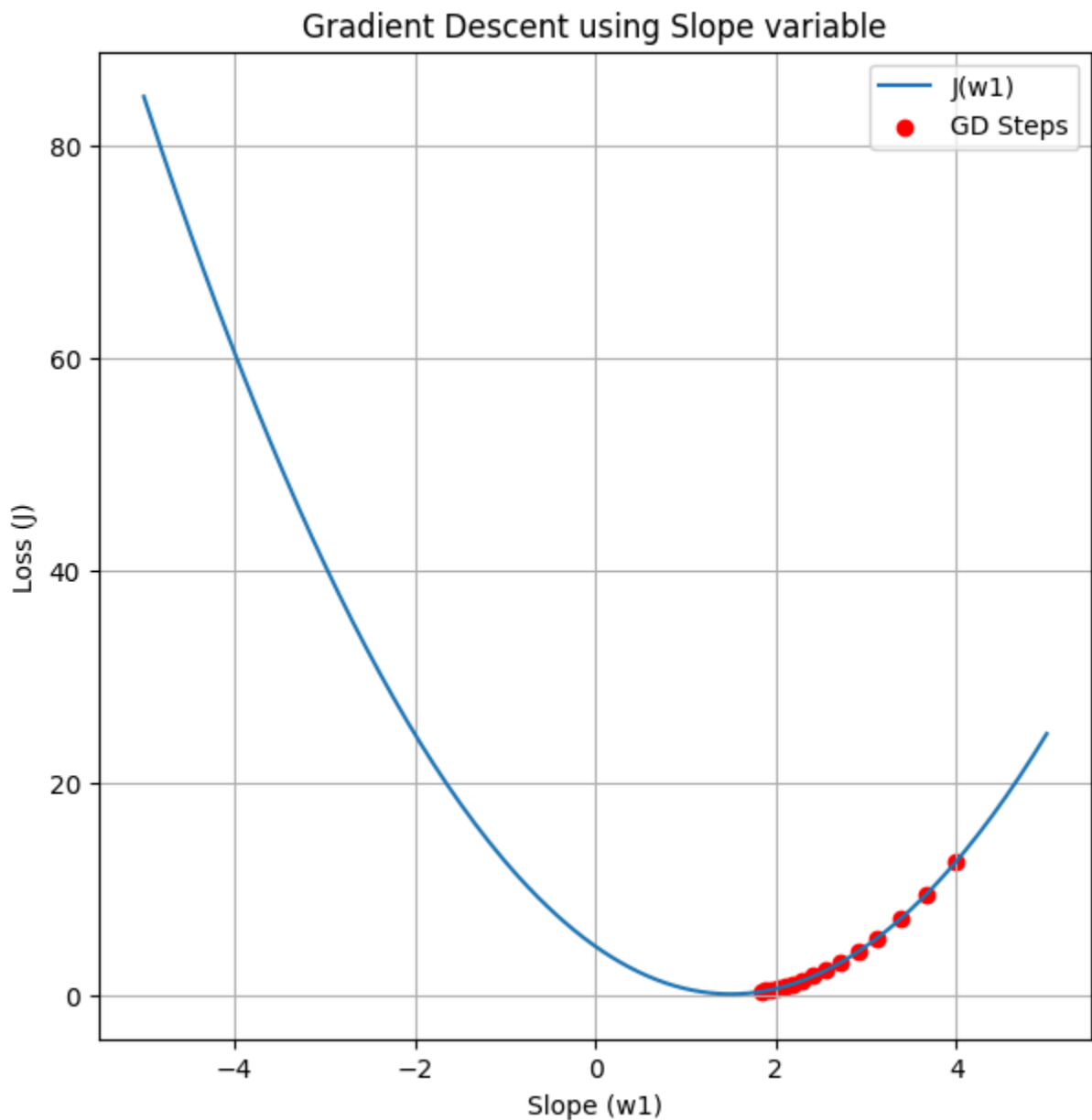
# Step 6: Plot Loss function and GD
```

```

w_space = np.linspace(-5, 5, 200)
loss_space = [loss(w) for w in w_space]

# Plot
plt.figure(figsize=(7, 7))
plt.plot(w_space, loss_space, label="J(w1)")
plt.scatter(w1_values, loss_values, color="red", label="GD Steps")
plt.xlabel("Slope (w1)")
plt.ylabel("Loss (J)")
plt.title("Gradient Descent using Slope variable")
plt.legend()
plt.grid(True)
plt.show()

```



In []:

In []: