

Analysis and Design of Missile Two Loop Autopilot

A. Chowdhury¹ and S. Das²

¹*Department of Electronics & Communication Engineering, NIT Agartala, India.*

²*Department of Electrical Engineering, NIT Agartala, India.*

Abstract

In this paper, a flight path rate demand two loop autopilot in pitch plane called lateral autopilot is considered where the steady state gain of the autopilot system is non-unity. The present work is an attempt to achieve a unity steady state gain of the autopilot system which generates a flight path rate output for a flight path rate command. For this, a PI controller is introduced in the reduced order model of autopilot system obtained through Routh Approximation method in order to eliminate the static error and to obtain a unity steady state gain of the two loop autopilot system. A model based PI design method based on plants parameter is used for the calculation of PI gain values. A numerical example has been considered and the simulation results are obtained through MATLAB software.

Keywords: Missile, Two-loop Autopilot, Pitch plane, rate gyro, Accelerometer, Reduced order model, PI Controller.

1. Introduction

A Guided missile is one which receives steering commands from the guided system to improve its accuracy. Guidance system actually gives command to the autopilot to activate the controls to achieve the correction necessary. Autopilot is an automatic control mechanism for keeping the spacecraft in desired flight path. An autopilot in a missile is a close loop system and it is a minor loop inside the main guidance loop. If the missile carries accelerometer and rate gyros to provide additional feedback into the missile servos to modify the missile motion then the missile control system is usually called an autopilot. When the autopilot controls the motion in the pitch and the yaw plane, they are called lateral autopilot. For a symmetrical cruciform missile pitch and the yaw autopilots are identical. The guidance system detects whether the missile is

flying too high or too low, or too much to the left or right. It measures the deviation or errors and sends signals to the control system to minimize the errors. The lateral autopilot of a guided missile is a servo system delivering lateral acceleration (latax) according to the demand from the guidance computer. For aerodynamically controlled skid to run missile the autopilot activates to move the control surfaces suitably for orienting the missile body with respect to the flight path. This action generates angle of attack and consequently latax for steering the missile in the desired path [4]. In this work, Flight path rate demand Two loop autopilot in pitch plane obtained from the conventional configuration of Two loop lateral autopilot in pitch plane with one Accelerometer and one rate Gyro [1] has been considered and shown in Fig.1

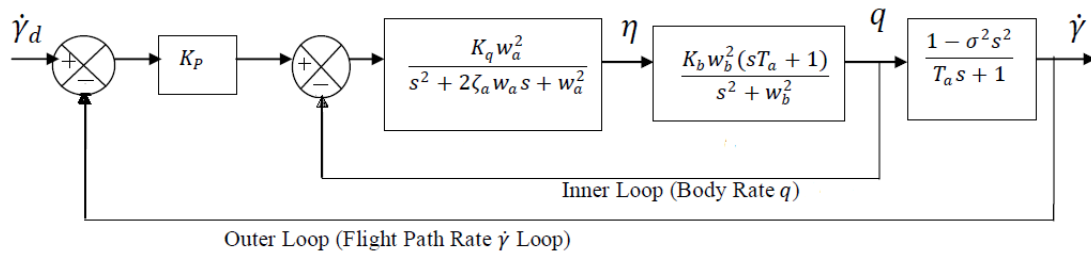


Figure 1: Flight Path Rate Demand Two Loop Autopilot in pitch plane.

Nomenclature

k_p is lateral autopilot control gain of outer loop ; k_q is fin servo gain ; k_b is airframe aerodynamic gain ; q is missile body rate in pitch ; T_a is the incidence lag of the air frame ; η is Elevator deflection ; $\dot{\gamma}$ is flight path rate ; ω_a is natural frequency of actuator ; ω_b is weather cock frequency ; ζ_a is damping ratio of actuator ; σ is a quantity whose inverse determines the locations of non-minimum phase zeros .

Numerical Values

The following numerical data for a class of guided missile have been considered [1] for MATLAB simulation,

Table1: Autopilot System Design Parameters.

Control gain		T_a s	ω_b rad / Sec	σ^2 s^2	ω_a rad / Sec	ζ_a	k_b s^{-1}
k_p	k_q						
5.69	-0.07	0.36	11.77	0.00029	180	0.6	-9.91

2. Mathematical Model of Two Loop Autopilot In Pitch Plane

The closed loop transfer function of two loop autopilot as shown in fig.1 is

$$\frac{\dot{\gamma}}{\gamma_d} = \frac{-k_q k_b k_p \omega_a^2 \omega_b^2 T_a \sigma^2 s^3 - k_q k_b k_p \omega_a^2 \omega_b^2 \sigma^2 s^2 + k_q k_b k_p \omega_a^2 T_a s + k_q k_b k_p \omega_a^2 \omega_b^2}{T_a s^5 + (2\zeta_a \omega_a T_a + 1)s^4 + (\omega_a^2 T_a + \omega_b^2 T_a + 2\zeta_a \omega_a - k_q k_b k_p \omega_a^2 \omega_b^2 T_a \sigma^2)s^3 + (\omega_a^2 + \omega_b^2 + 2\zeta_a \omega_a \omega_b T_a + k_q k_b \omega_a^2 \omega_b^2 T_a^2 - k_q k_b k_p \omega_a^2 \omega_b^2 \sigma^2)s^2 + (k_q k_b \omega_a^2 \omega_b^2 T_a + 2\zeta_a \omega_a \omega_b^2 + k_q k_b \omega_a^2 \omega_b^2 T_a + k_q k_b k_p \omega_a^2 \omega_b^2 T_a)s + (k_q k_b \omega_a^2 \omega_b^2 + k_q k_b k_p \omega_a^2 \omega_b^2)}$$

Using the values of autopilot system design parameters, the closed loop transfer function of two loop autopilot becomes

$$\frac{\dot{\gamma}}{\gamma_d} = \frac{-1849.62s^3 - 5137.8424s^2 + 6377998.27s + 17716661.9}{0.36s^5 + 78.76s^4 + 10080.2523s^3 + 441701.9085s^2 + 8649748.545s + 20830310.72}$$

3. Reduced Order Model of Two Loop Autopilot

As the analysis and synthesis of higher order systems are difficult and generally not desirable on economic and computational consideration, the reduced order model of the original autopilot system is obtained so that the obtained reduce order system maintains the characteristics of the original system. The reduced order model of the original autopilot system is obtained through Routh Approximation method [2] by the following steps:

Step1- The Routh array is constructed for the denominator polynomial of the closed loop transfer function of two loop autopilot starting with the first entry as the constant term and then a new Routh array is formed using the first three entries as the first column to obtain a reduced model.

Step2- The transfer function of the closed loop transfer function of two loop autopilot system is expanded into a power series.

Step3- The reduced order model of two loop autopilot is obtained by matching initial time moments.

The Reduced-order transfer function as obtained by Routh Approximation method is

$$\frac{15.278s + 42.44}{s^2 + 20.72s + 49.9}$$

4. PI Design Methodology for Two Loop Autopilot

The standard block diagram of PI controller is shown in Fig.2

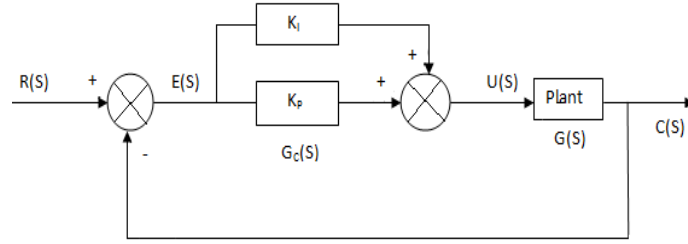


Figure 2: Block diagram of PI Controller.

Control signal $u(t)$ is a linear combination of error $e(t)$ and its integral. Mathematically,

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau \right]$$

Where K_p =proportional gain, T_i =integral time,

The PI controller transfer function is given as

$$G_c(S) = \frac{U(S)}{E(S)} = K_p \left[1 + \frac{1}{T_i S} \right]$$

The above equation can be rewritten as

$$G_c(s) = K_p + \frac{K_i}{s}$$

Where K_i represents the integral and derivative gain values of the controller.

5. Calculation of PI controller Gain values

A simple and efficient model based PI design method based on plants parameter is applied for tuning PI controller [3]. The PI controller gain values K_p and K_i are obtained in terms of reduced order autopilot system's damping ratio ξ and undamped natural frequency ω_n from the formula as shown in Table 2.

Table 2: PI Gains Calculation.

Plant parameter		PI controller gain values	
		K_p	K_i
ζ_a	ω_a	1	$\frac{\omega_a}{2\zeta_a}$
1.466	7.064	1	2.4083

6. Simulation Results

The simulations are carried out in MATLAB environment and the results obtained are shown in fig.3, fig.4, fig.5 and fig.6

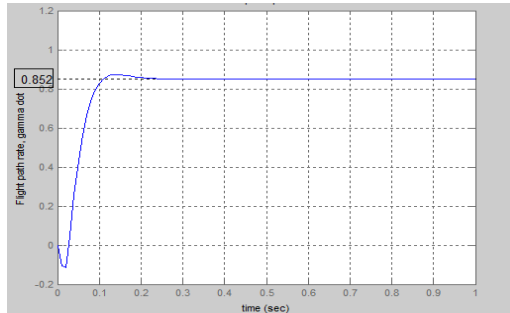


Figure 3: Step response of original Two-loop autopilot

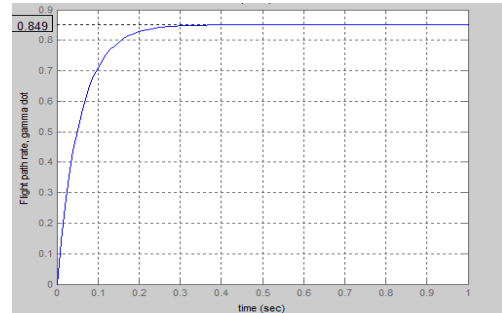


Figure 4: Step response of reduced order model of Two-loop autopilot

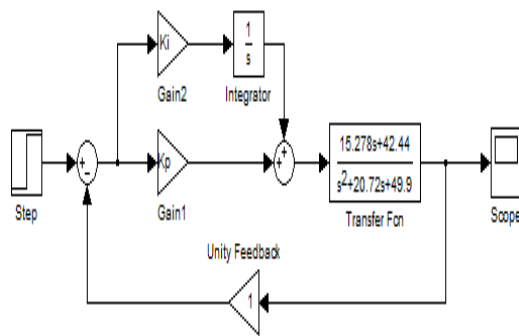


Figure 5: Simulink model of reduced order Auto pilot system with PI controller.

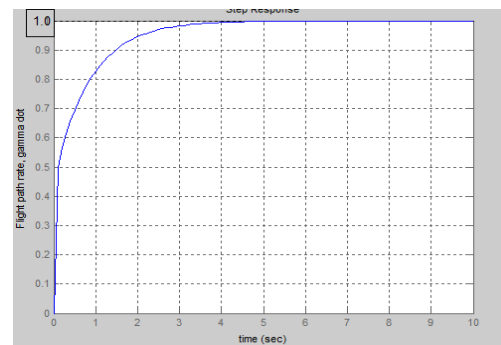


Figure 6: Step response of reduced order model of Two-loop autopilot with PI controller.

7. Conclusion

In this paper, the steady state performance of the flight path rate demand two loop Auto pilot system has been improved. It has been shown that the steady state response of reduced order autopilot system obtained by Routh Approximation method is exactly matching with that of the original autopilot system (Fig.3 & Fig.4). The PI controller introduced in the reduced order model of autopilot system eliminates the system static error. MATLAB simulation results show that the steady state gain of the reduced order autopilot system with PI controller is unity which generates a flight path rate output for a flight path rate command (Fig.6).

References

- [1] G. Das, K. Dutta, T K. Ghosal, and S. K. Goswami, 'Structured Design Methodology of Missile Autopilot', Institute of Engineers (I) journal – EL, Kolkata, India, November. pp 49-59 1996.
- [2] S. Panda, S. K. Tomar, R. Prasad, C. Ardil, 'Reduction of Linear Time-Invariant Systems Using Routh-Approximation and PSO', World Academy of Science, Engineering and Technology 33 2009
- [3] Farhan A. Salem, 'New efficient model-based PID design method', European Scientific Journal, May 2013, Vol.9, No.15
- [4] P. Garnell and D. J. East, "Guided Weapon Control Systems", Pergamon press, 1977
- [5] A L Greensite "Analysis and Design of Space Vehicle Flight Control System". Sparton Books. Vol-2, 1970.
- [6] B Etkins 'Dynamics of flight stability and control' Jhon wiley and sons, 1982.
- [7] J.H Blakelock.' Automatic control of aircraft and missiles' Jhon wiley and sons, 1965
- [8] K. Ogata, "Modern Control Engineering", 5th Edition, New Delhi, Prentice Hall of India Pvt. Ltd, 2010
- [9] M. Gopal, "Modern Control System Theory", 2nd Edition, New Delhi, Wiley Eastern Limited, April, 1993.
- [10] B.C. Kuo, 'Automatic control system', prentice hall; 1983