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Reducing Model Ordering using Routh Approximation Method

Mukesh Chand

Assistant Professor, ECE Department, Poornima College of Engineering, Jaipur (Rajasthan)

Abstract— The Higher order systems are frequently too arduous to be used in real time applications. Model order reduction was developed in the areas of systems and control theory, which studied properties of dynamical systems in application for reducing their complexity, while preserving their input-output behavior as much as possible. Routh approximation method (RAM) is one of the most elegant methods of model reduction which preserves the stability of original system in reduced system.

***Keywords*-- Model order reduction, routh approximation method,**

I. INTRODUCTION

The reducing of a high order system into its lower order system is well thought-out important in analysis, synthesis and simulation of practical systems. The task of replacing a given dynamical higher order system by a smaller model (with reduced complexity) is called model order reduction. Generally model order reduction is done in time domain and frequency domain approaches. An extensive range of model order reduction methods have been anticipated by several authors for the time of last few decades. In this paper, first we study the routh approximation method; then modified routh approximation method. A Numerical Analysis is also discussed to understand both methods. There are no. of methods for model order reduction of system given in literature [1]-[13], but routh approximation method, [2] always takes attention because of it stability preserving property of system.

II. ROUTH APPROXIMATION METHOD

Maurice F. Hutton and Bernard Friedland [2] gave a new method of approximating the transfer function of a high-order linear system by one of lower order is called the "routh approximation method" because it is based on an expansion that uses the routh table of the original transfer function, the method has a number of useful properties: if the original transfer function is stable, then all approximants are stable; the sequence of approximants converge monotonically to the original in terms of "impulse response" energy.

Let us consider a linear, time-invariant SISO system having the transfer function

$$H(s)=\frac{b_1S^{n-1}+\dots\dots\dots\dots+b_n}{a_0S^n+a_1S^{n-1}+\dots\dots\dots+a_n} \quad (1)$$

A linear time invariant system by m inputs and *I* outputs can be symbolized by a matrix of transfer functions of the form (1) through the numerator coefficients *bi* being *I* x *m* matrices. As the denominator of a Routh approximant depends merely on the denominator of *H(s)*, a Routh approximant to an I-output, m-input system is computed by computing the Routh approximant for every one term in the matrix of transfer functions [10]. The denominator of the Routh approximant merely has to be computed once.

Alogorithm Of Routh Approximation Method [10]:

A reduced or lower model order of is obtained by following steps for a given transfer function G(s).

Step 1: Apply reciprocal transform of G(s).

Step 2: Construct α and β table form [10].

Step 3: Find α and β coefficient from α and β table (shown in Table I and Table II).

Step 4: For kth order model, determine $R_k(s)$ using recursive equation (10, 11) from [10].

$$R_k(s)=\frac{P_k(s)}{Q_k(s)}$$

Step 5: Again applying reciprocal transform of $R_k(s)=\frac{P_k(s)}{Q_k(s)}$ and find $R_k(s)$.

This is the required reduced order model.

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	$a_0^0 = a_0$ $a_0^1 = a_1$	$a_2^0 = a_2$ $a_2^1 = a_3$	$a_4^0 = a_4$ $a_4^1 = a_5$	$a_6^0 = a_6$ --------	-----
$\alpha_1 = a_0^0 / a_0^1$	$a_0^2 = a_2^0 - \alpha_1 a_2^1$	$a_2^2 = a_4^0 - \alpha_1 a_4^1$	$a_4^2 = a_6^0 - \alpha_1 a_6^1$	--------	
$\alpha_2 = a_0^1 / a_0^2$	$a_0^3 = a_2^1 - \alpha_2 a_2^2$	$a_2^3 = a_4^1 - \alpha_2 a_4^2$	------------		
$\alpha_3 = a_0^2 / a_0^3$	$a_0^4 = a_2^2 - \alpha_3 a_2^3$	$a_2^4 = a_4^2 - \alpha_3 a_4^3$	------------		
$\alpha_4 = a_0^3 / a_0^4$	$a_0^5 = a_2^3 - \alpha_4 a_2^4$	---------			
$\alpha_5 = a_0^4 / a_0^5$	$a_0^6 = a_2^4 - \alpha_5 a_2^5$	---------			
$\alpha_6 = a_0^5 / a_0^6$	----------				

TABLE I
ALPHA (ROUTH) TABLE [10]

	$b_0^1 = b_1$ $b_0^2 = b_2$	$b_2^1 = b_3$ $b_2^2 = b_4$	$b_4^1 = b_5$ $b_4^2 = b_6$	....
$\beta_1 = b_0^1 / a_0^1$	$b_0^2 = b_2^1 - \beta_1 a_2^1$	$b_2^3 = b_2^1 - \beta_1 a_4^1$	---------	
$\beta_2 = b_0^2 / a_0^2$	$b_0^4 = b_2^1 - \beta_2 a_2^2$	$b_2^4 = b_2^1 - \beta_2 a_4^2$	----------	
$\beta_3 = b_0^3 / a_0^3$	$b_0^5 = b_2^1 - \beta_3 a_2^3$	----------		
$\beta_4 = b_0^4 / a_0^4$	$b_0^6 = b_2^1 - \beta_4 a_2^4$	------------		
$\beta_5 = b_0^5 / a_0^5$	-----------			
$\beta_6 = b_0^6 / a_0^6$	-----------			

TABLE II
BETA (ROUTH) TABLE [10]

III. NUMERICAL ANALYSIS

In this paper, we consider the following 4th-order transfer function [10].

The numerical is solved step by step procedure as earlier discussed in section-II RA algorithm.

The 4th-order transfer function G(s) is:

$$G(s) = \frac{81.6012s^3 + 506.6497s^2 + 99.8432s + 5}{s^4 + 105.2s^3 + 521.01s^2 + 101.05s + 5}$$

The poles of $G(s)$ are at s = -0.1; -0.1; -5; -100

STEP 1: Take reciprocal transform of $G(s)$

$$G(s) = \frac{1}{s} G(\frac{1}{s})$$

$$= \frac{5s^3 + 99.8432s^2 + 506.6497s + 81.6012}{5s^4 + 101.05s^3 + 521.01s^2 + 105.2s + 1}$$

STEP: 2 Make $\alpha - \beta$ table [3] for $G(s)$

α **Table**

5	521.01	1
101.05	105.2	
515.804		

β **-Table**

5	506.6497
99.8432	81.6012

STEP 3: Now from above tables, we get α and β coefficients given below:

α_1 = 0.0494; α_2 =0.1959; β_1=0.0494; β_2 =0.1936

STEP 4: Find $H_2(s)$

$$H_2(s) = \frac{0.0097s + 0.1936}{0.0097s^2 + 0.1959s + 1}$$

STEP 5: Take reciprocal transform again

$$H_2(s) = \frac{0.0097 + 0.1936s}{s^2 + 0.1959s + 0.0097}$$

It has poles at $s = -0.098 \pm j0.0099$

The poles of reduced system are s = (-0.098 $\pm$ j0.0099).

The Impulse energy and Integral square error of step response for reduced system is 0.1204 and 2.3806 shown in Table III.

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METHOD/ SPECIFICATIONS	POLES	IMPLUSE ENERGY	ISE OF STEP RESPONSE
G(s) (Original system)	S = (-0.1, -0.1, -5, -100)	33.9986	----
$H_2(s)$(RA method)	S = (-0.098 ± j0.0099)	0.1204	2.3806

TABLE III
(Solution for original system RA method)

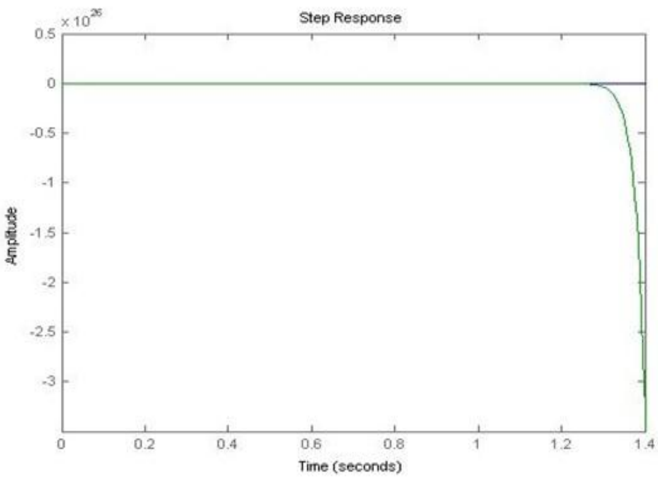

FIGURE I
(Step Response by RA method)

IV. ADVANTEAGS AND DISADVATAGES OF RA METHOD

Advanteags:

Routh approximation method has following advantages-

1. It guarantees that stability of system preserved [10], when the original system is stable.
2. The poles and zeros of the approximants approaches the poles and zeros of the original system function as the order of the approximation is improved.

Disadvantegs:

Routh approximation method has following disadvantages-

1. Generally routh approximation method approximants for the no. of higher order system to produce the same reduced order model from[5] because this method only focus only lower order poles of the system.
2. Only guarantees that the steady state of system preserved and gives no guarantees that transient state of system is preserved.

V. RESULT AND DISCUSSION

The Numerical example for original higher order (4^{th}) system is discussed is section III and solved to reduce the order up to 2^{nd} order .The poles , impulse energy and ISE of step response of each reduced order corresponds to RA is shown in Table III. The Figure I show the step response of original system with step response for reduced order system corresponds to RA method.

The RA method preserves the stability of original system into the reduced order system. The RAM retains some of the initial time moments only, the reduced model obtained by the RAM tends to approximate the steady state response of the original system in the time domain. However, it is often advantageous to retain the initial Markov parameters and the initial time moments of the original system in the reduced model to construct a more correct initial transient response.

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