IE598OUU: Project

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1 Routing Problems with Uncertain Reward

Routing problems under uncertainty are prevalent in various transportation and logistics scenarios. Such problems may include vehicle routing problems with uncertain travel times, or orienteering problems where rewards at nodes are uncertain. This project focuses on scenarios where agents navigate through networks collecting uncertain rewards. Our primary objective is to formulate a problem to minimize the routing cost while maximizing the overall revenue for all agents.

More specifically, we can apply and solve such problem formulation and apply it to the scenario of parking patrols. Inspired by [Lei et al., 2017], we can use parking enforcement as a concrete scenario. As such, we can assume the role of an agency in routing parking enforcement vehicles through parking lots, in which the probability of collecting a reward can be equated to the probability of a driver underpaying their parking time. From the perspective of drivers, there are close-form solutions to their optimal strategy because parking enforcement can be formulated as a variant of The Newsvendor Problem studied in class, given the frequency of patrolling. From the agency's perspective, the decision is the routing and frequency of visits at each parking lot.

In this project, we reformulate the parking patrol as a two-stage problem (as opposed to the one stage used in the paper) and make the required assumptions for an optimal schedule. We will first introduce the model formulation and variables, then we will apply the Monte Carlo method to approximate an optimal solution and derive optimality bounds to check the quality of the solution.

Given that the routing problem is an NP-hard problem, we are limited in the instance size we can solve efficiently to optimality, so we consider only relatively small networks that MOSEK can handle. The following section provides the required assumptions and problem setup.

2 Formulation

In this section, we propose an alternate approach to model the routing problem for parking patrols to the one presented by [Lei et al., 2017]. We consider the objective to minimize routing costs while maximizing revenue from parking fees collected by overstaying parking times.

Let $\mathcal{N}$ be the set of parking lots modeled as discrete nodes with capacity P_i. Let these lots be connected via a set of arcs $\mathcal{A}$. We can assume all lots are reachable from each other. Let $c_{ij}, (i, j) \in \mathcal{A}$ be the routing cost vector between lots.

We consider a service shift of length T such that we want to maintain the operations within this horizon. If operations are longer than T, we penalize them as patrol drivers need to be paid overtime.

When drivers visit a lot i, the number of violations found, ξ_i, is a random variable ranging from 0 to P_i (i.e., all parked cars are violating their parking duration).

2.1 Routing Problem

The agency has a set of patrol vehicles, $\mathcal{K}$. The agency aims to route these vehicles in a manner that maximizes profit or enforcement effectiveness while using the least amount of resources. For such decisions, let x_{kij} be a binary variable where $x_{kij} = 1$ if vehicle k travels from parking lot i to parking lot j, and 0 otherwise. Define u_{kj} as an integer variable indicating the order in which lot j is visited within the route of vehicle k (i.e., $u_{kj} = 1$ if parking lot j is visited first, $u_{kj} = 2$ if visited second, and so on). Then, the first-stage patrol routing problem can be expressed as follows:

$$\min \sum_{k \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{kij} + c_{\text{veh}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}} x_{k0j} + \mathbb{E}[Z(\boldsymbol{x}, \boldsymbol{\xi})] \tag{1}$$

$$\text{s.t.} \tag{2}$$

$$\sum_{j \in \mathcal{N}} x_{kOj} = \sum_{j \in \mathcal{N}} x_{kjS} \leq 1, \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{j \in \mathcal{N}} x_{kij} = \sum_{j \in \mathcal{N}} x_{kji}, \quad \forall i \in \mathcal{N}, k \in \mathcal{K}, \tag{4}$$

$$u_{kj} \geq u_{ki} + 1 - |\mathcal{N}|(1 - x_{kij}), \quad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K}, \tag{5}$$

$$x_{kij} \in \{0, 1\}, \quad \forall k \in \mathcal{K}, (i,j) \in \mathcal{A} \tag{6}$$

$$u_{ki} \in \mathbb{N}, \quad \forall k \in \mathcal{K}, i \in \mathcal{N} \tag{7}$$

Where the objective (1) seeks to minimize the total number of vehicles deployed and the routing cost for each deployed vehicle. Equation (3) ensures that each vehicle leaves and arrives at the depot at most once. Equation (4) ensures the conservation of vehicles at parking lots. Equation (5) is used for subtour elimination for each vehicle, while Equations (6) and (7) define the binary and integer variables.

The function $Z(\boldsymbol{x}, \boldsymbol{\xi})$ is the second-stage cost which includes, overtime pay for drivers, and the revenue from collecting fines. Since the patrolling vehicles do not know in advance how many violations they will encounter, their rewards depend on the number of violations per lot, ξ_i. Let ϕ be the time it takes a vehicle to process a fine and let γ be the fine charged per violation. The second-stage cost can be formulated as follows:

$$Z(\boldsymbol{x}, \boldsymbol{\xi}) = \min c_{\text{ot}} \sum_{k \in \mathcal{K}} \tau_k - \gamma \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} y_{ki} \xi_i \tag{8}$$

$$y_{ki} = \sum_{j \in \mathcal{N}} x_{kij}, \quad \forall k \in \mathcal{K}.i \in \mathcal{N} \tag{9}$$

$$\sum_{(i,j \in \mathcal{A}} x_{kij} t_{ij} + \phi \xi_i \sum_{i \in \mathcal{N}} y_{ki} = T + \tau_k - \tau'_k, \quad \forall k \in \mathcal{K} \tag{10}$$

$$\tau_k, \tau'_k \geq 0, \quad \forall k \in \mathcal{K} \tag{11}$$

$$\tag{12}$$

The second stage objective is to minimize the overtime paid to policemen while maximizing the revenue collected by all patrolling vehicles. Equation (9) defines the parameter y_{ki} (defined only for convenience) which equals 1 if vehicle k visits lot i and 0 otherwise. Equation (10) computes the total patrol time for each vehicle, ensuring that the variable τ_k captures the overtime. Finally, Equation (11) defines the auxiliary variables for the second-stage problem.

The underlying assumption in the objective value is that each vehicle visit is independent of one another regardless of the time between two consecutive vehicle visits. For this, we assume that ξ_i is a Poisson variable with a parameter λ_i representing the mean number of violations per lot based on historical data.

3 Solution Approaches

3.1 Monte Carlo Method

We consider an instance of the parking vehicle routing problem with fines defined above defined by $c_{ij}\ U(1, 5), c_{veh} = 7, t_{ij} = 2 * c_{ij}, \gamma = 40, c_{ot} = 30, \phi = 2, T = 60$. We assume ξ_i to be distributed Poisson with rate parameter $\lambda_i = 5$.

We consider that the number of available vehicles is $|\mathcal{K}| = 5$ and the total number of lots is $|\mathcal{N}| = 25$. We solve for the wait-and-see and expected value objectives, defined as:

$$WS = \mathbb{E}[\min_{x \in X} Z(x, \tilde{\xi})],$$

$$EV = \min_{x \in X} Z(x, \mathbb{E}[\tilde{\xi}])$$

respectively. For the wait-and-see objective, we generate 100 samples of ξ_i for each $i \in \mathcal{N}$ and compute the optimal solution for each of these samples as if the parking operator knew the number of violations before dispatching the parking enforcement vehicles. For the expected value objective, we simply assume $\xi_i = \lambda_i = 5$ for each $i \in N$ and solve for the problem of finding the cost if all violation numbers were exactly at their expected values.

The mean wait-and-see objective is -4741.17 with a 95% confidence interval at $[-4756.4, -4725.9]$. The expected value objective is -4599.23. In our formulation, the technology matrix associated with ξ is not multiplied by x either in the objective or in the constraints. So, we would expect $WS \leq EV$ and we can verify this inequality at 95% confidence.

For the Monte Carlo method, we get candidate solutions by solving for sample average from $50, 100, 150, ..., 500$ samples of ξ and testing them on $L = 50$ batches of $S = 50$ samples each. The results are presented in Table 1 and summarized in Figure 1.

Table 1: Result Table for Monte Carlo sampling

Sample size	Mean Objective	5% objective	95% objective
50	-4144.92	-4154.63	-4135.22
100	-4152.62	-4161.65	-4143.59
150	-4150.08	-4158.94	-4141.23
200	-4153.97	-4163.62	-4144.31
250	-4153.43	-4163.35	-4143.51
300	-4151.95	-4161.61	-4142.29
350	-4153.97	-4163.62	-4144.31
400	-4153.97	-4163.62	-4144.45
450	-4151.95	-4161.61	-4144.29

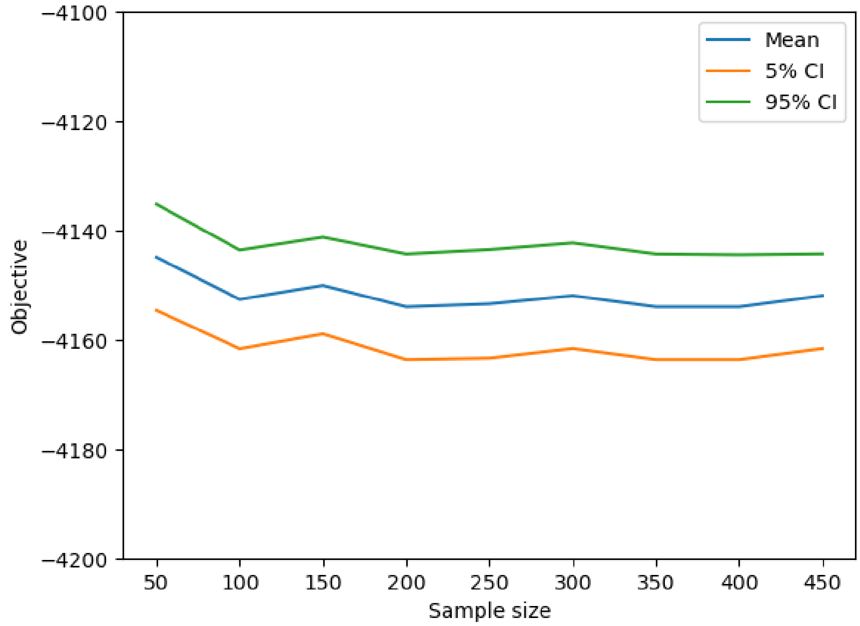

Figure 1: Result Plot for Monte Carlo sampling

Based on the Monte Carlo results in Table 1 and Figure 1, it is clear that a sample size of about 200 would suffice to produce high-quality solutions. The difference between the 200 and 450 samples is minimal, showing the algorithm's convergence. The Figure shows the solution stabilizing after a sample size¿150.

We can observe the hierarchy of the solutions is maintained as the one we observed in class, that is $z^* \geq EV \geq WS$, indicating the "modeler's regret" effect.

4 Bender's Decomposition

Here we attempted to implement the bender's decomposition algorithm. Let μ_k be the dual for constraint (10). Since y_{ki} is not a variable, it's simply for convenience, we don't need to add an extra dual constraint. The dual problem for Z_d as defined by only τ_k and τ_k' is the following:

$$
\begin{aligned}
Z_d(\boldsymbol{x}, \boldsymbol{\xi}) = \max \sum_{k \in \mathcal{K}} \mu_k & \left(\sum_{(i,j) \in \mathcal{A}} x_{kij} t_{ij} + \phi \sum_{i \in \mathcal{N}} \xi_i \sum_{j \in \mathcal{N}} x_{kij} - T \right) - \gamma \sum_{i \in \mathcal{N}} \xi_i \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}} x_{kij} && (13) \\
& \text{s.t.} && (14) \\
& c_{\text{ot}} - \mu_k \geq 0, \quad \forall k \in \mathcal{K} && (15) \\
& - \mu_k \geq 0, \quad \forall k \in \mathcal{K} && (16) \\
& && (17)
\end{aligned}
$$

Here we are only trying to determine the μ_k variables that maximize Z_d. Given the objective is only multiplied by μ_k in the first term, this has a close-form solution where $mu_k = c_{\text{ot}}$ if $\sum_{(i,j) \in \mathcal{A}} x_{kij} t_{ij} + \phi \sum_{i \in \mathcal{N}} \xi_i \sum_{j \in \mathcal{N}} x_{kij} > 0$ and $\mu_k = 0$ otherwise. Nonetheless, this approach was shown to be inefficient as the number of scenarios to solve (for each subproblem) grow too quickly with the capacity of lots being greater than 5. We need to solve a subproblem considering that each lot can have $\{0, 1, ..., P_i\}$ violations which equated to $P_i^{|\mathcal{N}|}$ (assuming all P_i are equal). After attempts to make the code efficient, we quickly realized that only extremely small cases could be solved with this approach.

For that, we determined that Monte Carlo is the most appropriate methodology for this type of problem.

5 Conclusion

This term project addresses routing problems with uncertain rewards, focusing on scenarios such as parking patrols where agents navigate through lots collecting fines for overstaying vehicles. The project proposes a formulation for minimizing routing costs while maximizing overall revenue, considering factors such as vehicle routing, patrol scheduling, and fine collection formulated as a two-stage stochastic optimization problem.

The first stage deals with routing vehicles to parking lots while the second stage involves optimizing patrol schedules and revenue collection based on uncertain reward outcomes. Mathematical models and solution approaches, including Monte Carlo simulation and Bender's decomposition, are discussed for addressing the problem.

Monte Carlo simulation is identified as the most suitable approach due to its ability to produce high confidence bounds with a relatively low number of samples. The modeler's regret effect is observed in the results as the $z^* \geq EV \geq WS$. The bender's decomposition is discussed, although no solutions were reached for reasonable problem sizes due to the number of scenarios growing exponentially with the number of lots and their capacity.

References

[Lei et al., 2017] Lei, C., Zhang, Q., and Ouyang, Y. (2017). Planning of parking enforcement patrol considering drivers' parking payment behavior. *Transportation Research Part B: Methodological*, 106:375–392.