The Foundations of Probability

Understanding the Language of Chance

What is Probability?

Probability is a measure of how likely an event is to occur. It is the foundation of statistics and data analysis.

It is a number between 0 and 1, where:

- **0** means the event is **impossible**. (e.g., rolling a 7 on a standard six-sided die)
- **1** means the event is **certain**. (e.g., the sun rising in the east)
- **0.5** means a **50/50 chance**. (e.g., a fair coin toss landing on heads)

Abstract graphic representing chance or probability

The Language of Probability

Event

A specific outcome or set of outcomes you are interested in.

Example: "Rolling an even number."

Outcome

The result of a single trial of an experiment.

Example: The die lands on a '4'.

Sample Space

The set of *all* possible outcomes of an experiment.

Example: {1, 2, 3, 4, 5, 6}

Calculating Basic Probability

1/6

P(Rolling a 5)

The Basic Formula

For events with equally likely outcomes, the formula is:

P(Event) = (Number of Favorable Outcomes) / (Total Number of Possible Outcomes)

Example: Rolling a '5' on a die.
Favorable Outcomes = 1 (the '5')
Total Outcomes = 6 (the numbers 1-6)

Types of Events

1. Independent Events

One Event Doesn't Affect the Next

The outcome of a previous event has no impact on the probability of the next event.

- **Example: Coin Flips**
  Tossing a coin twice. The first toss doesn't change the 50/50 probability of the second toss.
- **Example: Dice Rolls**
  Rolling a die. The probability of rolling a 6 is always 1/6, no matter what you rolled before.

2. Dependent Events

One Event *Does* Affect the Next

The outcome of one event changes the probabilities for the next event. This often happens in problems "without replacement."

- **Example: Drawing Cards**
  You draw an Ace from a deck (P=4/52). If you don't replace it, the probability of drawing another Ace is now 3/51. The sample space has changed.

Basic Probability Rules

The Multiplication Rule (for 'AND')

The Rule

Finds the probability of two events *both* happening. For independent events:

P(A and B) = P(A) × P(B)

The Question

What is the probability of:
Tossing 'Heads' (P=0.5)
AND
Rolling a '6' (P=1/6)?

The Calculation

P(Heads and 6) = P(Heads) × P(6)

0.5 × (1/6) = 1/12

(Approx. 8.3%)

The Addition Rule (for 'OR')

The Rule

Finds the probability of *either* of two events happening. For mutually exclusive events*:

P(A or B) = P(A) + P(B)

The Question

What is the probability of:
Rolling a '1' (P=1/6)
OR
Rolling a '6' (P=1/6)?

The Calculation

P(1 or 6) = P(1) + P(6)

(1/6) + (1/6) = 2/6

(or 1/3, 33.3%)

Advanced Concepts

Conditional Probability & Bayes' Theorem

Why Learn This?

- **Make Better Decisions:** These concepts move beyond simple chance and help you make informed choices based on new evidence.
- **Understand Real-World Risk:** They allow you to quantify risk more accurately. (e.g., "What is the risk of a market crash, *given* that interest rates just rose?")
- **Find Hidden Connections:** They are the tools for understanding how events are linked, forming the basis of all modern data analysis and science.
- **Power Modern Technology:** This is the logic that powers AI spam filters, medical diagnostic tools, and self-driving car sensors.

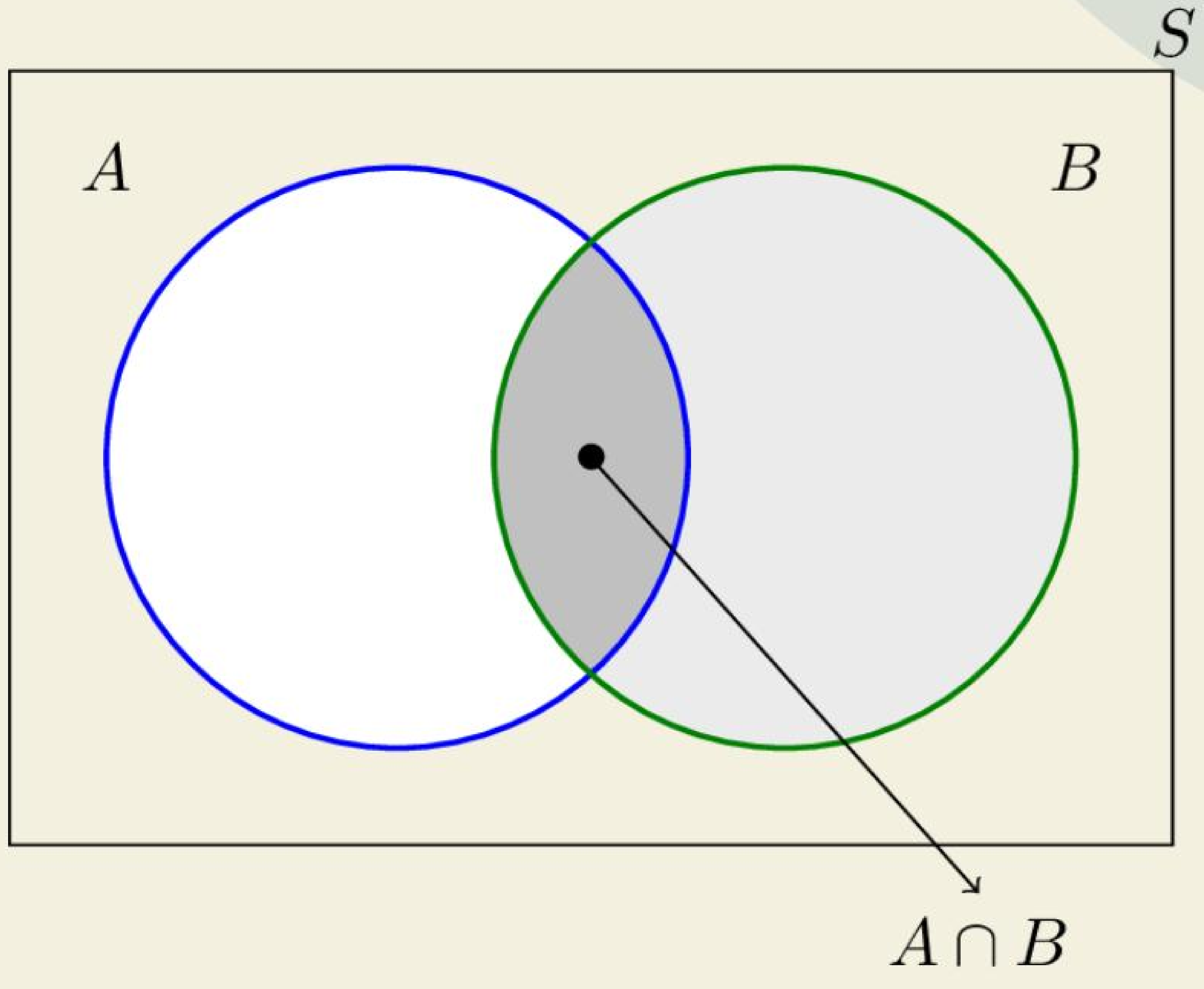
3. Conditional Probability

The "Given That" Probability

Conditional probability is the likelihood of an event (A) occurring, *given that* another event (B) has already happened.

This is crucial for understanding how events influence each other.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example: Conditional Probability

The Problem

In a class, 60% of students pass Math (P(M)=0.6), 50% pass Science (P(S)=0.5), and 30% pass *both* (P(M and S)=0.3).

The Question

What is the probability that a student passed Science, *given that* they passed Math?

Find P(S | M)

The Calculation

P(S | M) = P(M and S) / P(M)

= 0.3 / 0.6

= 0.5 (or 50%)

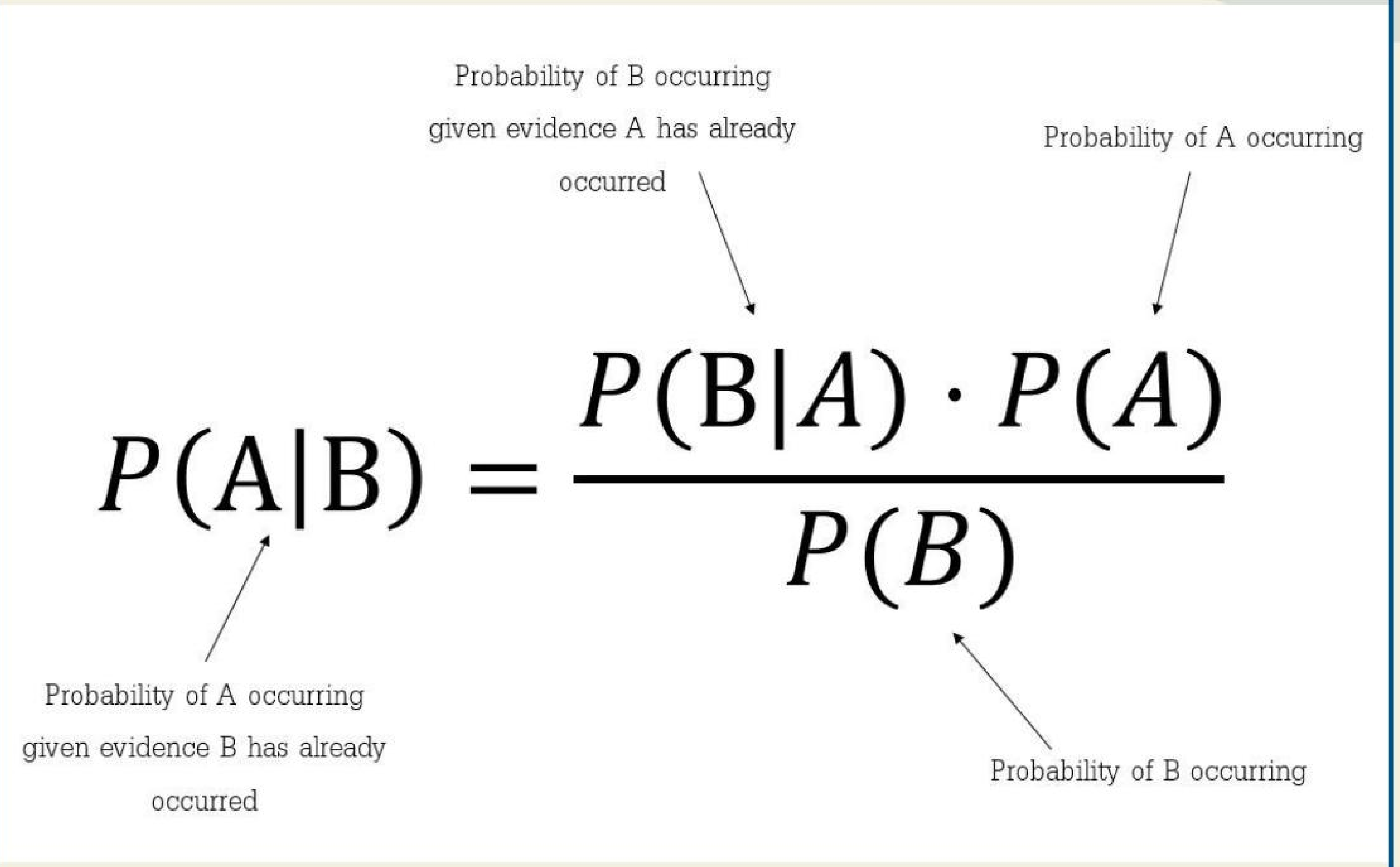
4. Bayes' Theorem

Updating Your Beliefs

Bayes' Theorem is a powerful formula that describes how to update the probability of a hypothesis (A) based on new evidence (B).

It "flips" conditional probability to find P(A|B) using P(B|A).

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$



Example: Bayes' Theorem

The Problem

1% of a population has a disease. A test is 95% accurate (95% true positive, 5% false positive).

You test positive.

The Question

What is the probability you *actually* have the disease?

Find P(Disease | Positive)

The Answer

Your intuition says 95%, but... ...the actual probability is only **16%**!

Bayes' theorem accounts for the rare disease and the 5% of healthy people who *also* test positive.

Weather

Meteorologists use probability to create forecasts, like a "70% chance of rain," helping you plan your day.

Finance

Investors use probability to assess the risk of an investment and model the likely returns of a stock portfolio.

Insurance

Companies calculate premiums based on the probability of an event (like a car accident) occurring in a given group.

Questions?

Thank you for your attention.

Image Sources

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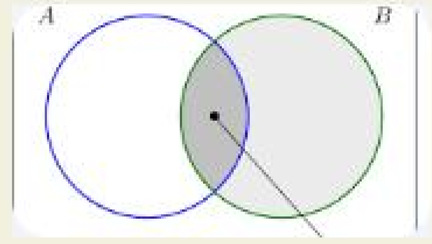

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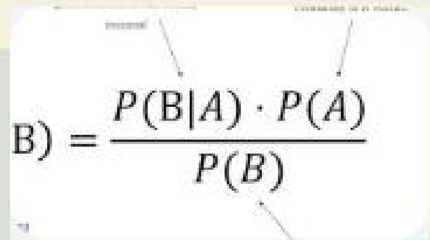

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