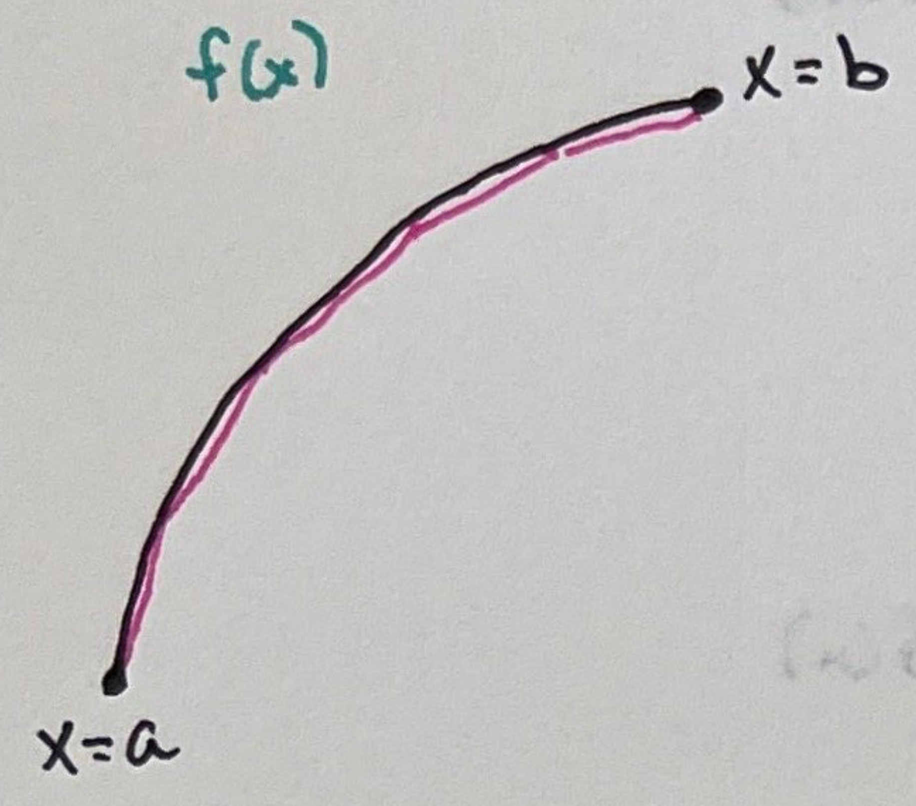
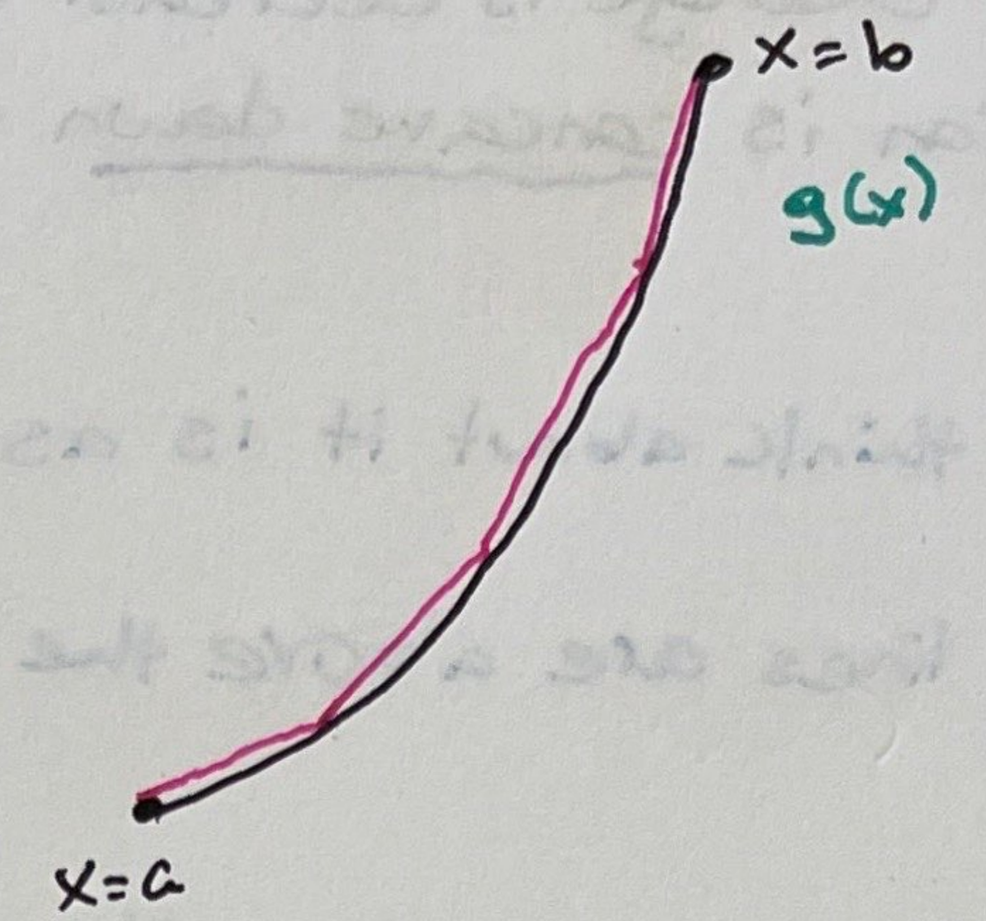
Given the graphs of 2 different functions on $x \in [a, b]$



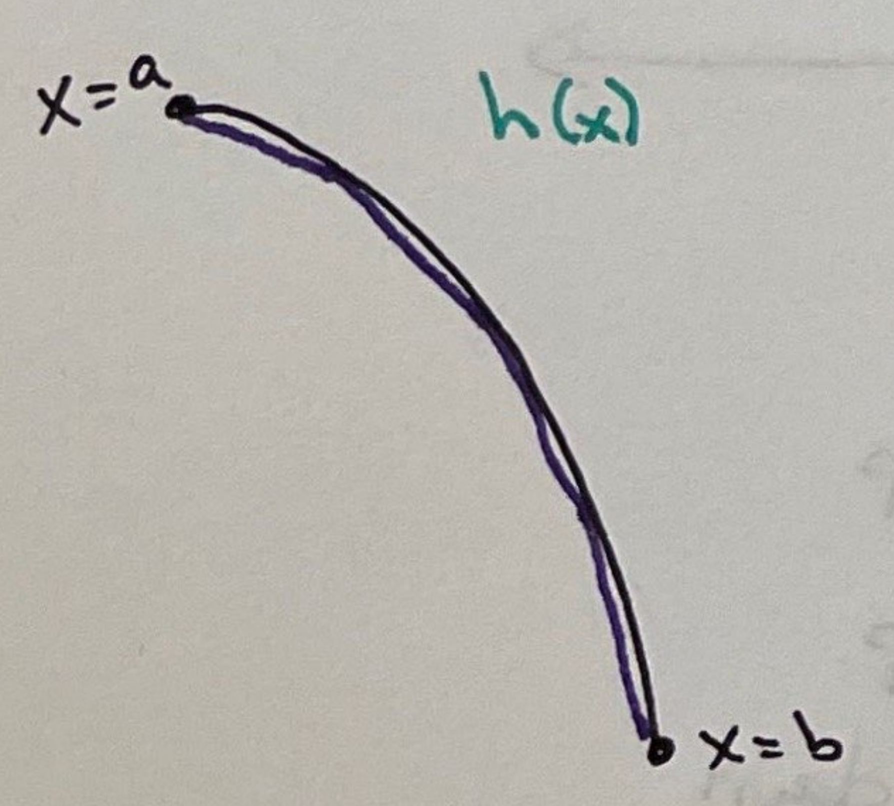


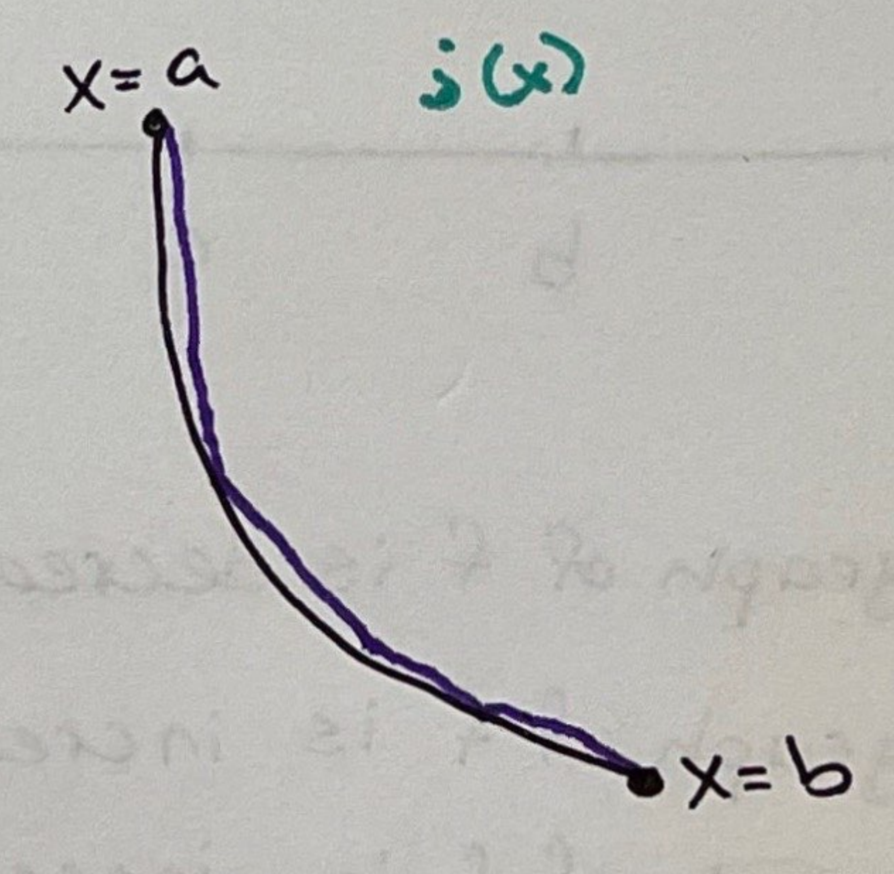
In both cases, we can see that both $f(x)$ and $g(x)$ are increasing
$(a < b,\ f(a) < f(b))$

Let's look at the avg. rate of change for both functions, but we will draw multiple secant lines between $x = a$ and $x = b$

* For the graph of $f(x)$, notice that while all the secant lines have a positive slope, they are getting less steep $\Rightarrow$ the rate of change is decreasing
* For the graph of $g(x)$, note that the secant lines are getting steeper $\Rightarrow$ the rate of change is increasing.

Now consider the graphs of $h(x)$ and $j(x)$:



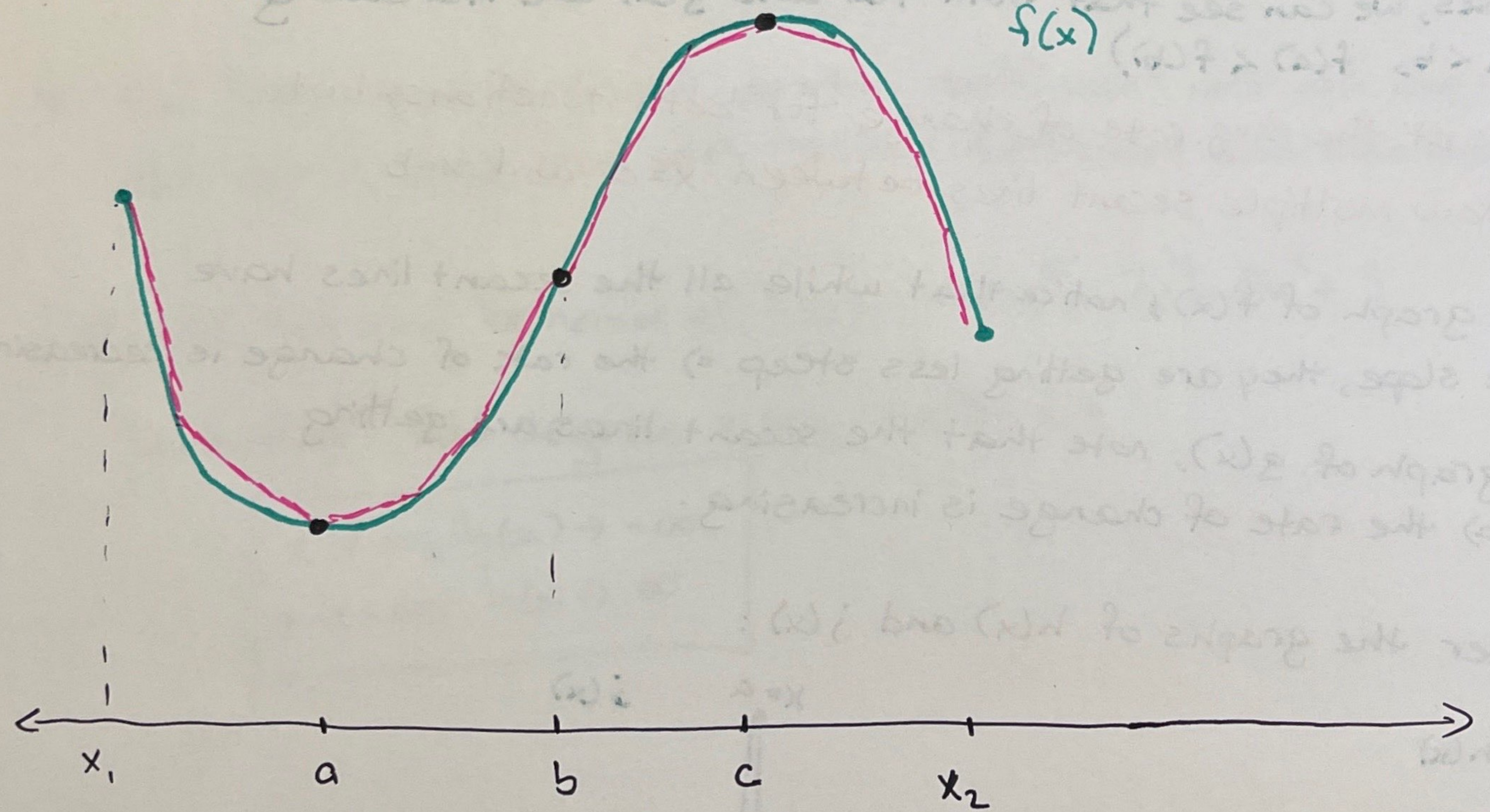


~~Her~~ Both $h(x)$ and $j(x)$ are decreasing $(a < b,\ f(a) > f(b))$
$\Rightarrow$ All the secant lines will have neg. slope, but in the graph of $h(x)$, they are getting steeper $\Rightarrow$ rate of change is decreasing, while in the graph of $j(x)$, they are getting less steep $\Rightarrow$ the rate of change is increasing.

* If the rate of change is increasing on a given interval, then the function is concave up on that interval

* If the rate of change is decreasing on a given interval, then the function is concave down on that interval.

* Another way to think about it is as follows:
  - If the secant lines are above the graph of f, then f(x) is concave up.
  - If the secant lines are below the graph of f, then f(x) is concave down.



On (x_1, a): the graph of f is decreasing and concave up

On (a,b): the graph of f is increasing and concave up

On (b,c): the graph of f is increasing and concave down

On (c,x_2): the graph of f is decreasing and concave down.

At $x=a$, f changes from dec. to inc. $\Rightarrow$ there is a relative minimum at $x=a$

At $x=c$, f changes from inc. to dec. $\Rightarrow$ there is a relative maximum at $x=c$

The concavity of f changes at $x=b$ $\Rightarrow$ there is a point of inflection at $x=b$