Concavity and Polynomial Functions

a) roots are $x=-4$, $x=0$ (triple root), $x=4$, $h(3)=-3$, $h(-3)=3$

$\Rightarrow h(x) = ax^3(x-4)(x+4)$ ← Don't assume the lead coefficient is 1

$x=3$: $a(27)(-1)(7) = -3$

$\Rightarrow a = \frac{1}{63}$ $\therefore \boxed{h(x) = \frac{1}{63}x^3(x-4)(x+4)}$

b) $h(-3)=3$, $h(4)=0$

$\therefore \frac{\Delta h}{\Delta x} = \frac{0-(3)}{4-(-3)} = \boxed{-\frac{3}{7}}$

c) h changes from inc. to dec.

$\therefore$ $\boxed{\text{Rel. Max at } x=-3}$

h changes from dec. to inc.

$\therefore$ $\boxed{\text{Rel. Min at } x=3}$

d) Triple root at $x=0$

$\Rightarrow$ P.O.I. at $x=0$

One P.O.I. about halfway between $x=-3$ & $x=0$

$\Rightarrow x \approx \frac{-3+0}{2} = -\frac{3}{2}$

One P.O.I. about halfway between $x=0$ & $x=3$

$\Rightarrow x \approx \frac{0+3}{2} = \frac{3}{2}$

$\therefore$ $\boxed{\text{P.O.I. at } x = 0, \pm\frac{3}{2}}$

e) h is concave up when the rate of change of h is increasing.

$\therefore \boxed{x \in (-\frac{3}{2}, 0) \cup (\frac{3}{2}, \infty)}$

f) h is dec. when the rate of change of h is negative

$\therefore \boxed{x \in (-3, 3)}$

g) h is decreasing and concave up when the rate of change is negative and increasing. ↑ Intersection of 2 answers in parts e) & f.)

$\therefore \boxed{x \in (-\frac{3}{2}, 0) \cup (\frac{3}{2}, 3)}$