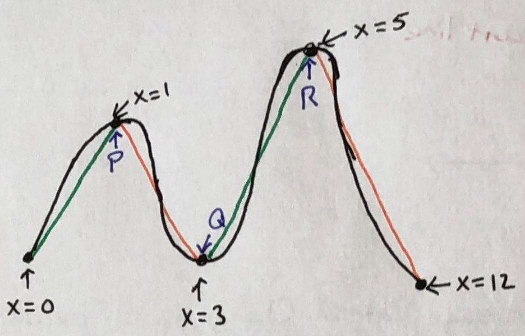
Consider the graph of a continuous function $f(x)$ on $[0,12]$ and the table of selected values of $f(x)$:



x	f(x)
0	100
1	102
3	100
5	103
12	99

* On $[0,12]$, the min value of f is $f(12)=99$ and the max value is $f(5)=103$

* Not all functions have a min. or max. value.

*P, Q and R are turning points → points where the graph changes from increasing to decreasing (or vice versa)

* For any ~~domain~~ interval I in the domain of f and $\forall a,b \in I$:
  - If $a<b$ and $f(a)<f(b)$, then f is increasing on I
  - If $a<b$ and $f(a)>f(b)$, then f is decreasing on I.

On $[0,12]$, f is increasing on $[0,1]$ and $[3,5]$ and decreasing on $[1,3]$ and $[5,12]$

Q: How do we determine where f is increasing (or decreasing) faster?
A: Slope, or average rate of change.

Def: The average rate of change of f on $[a,b]$ is the slope of the line joining $(a, f(a))$ and $(b, f(b))$

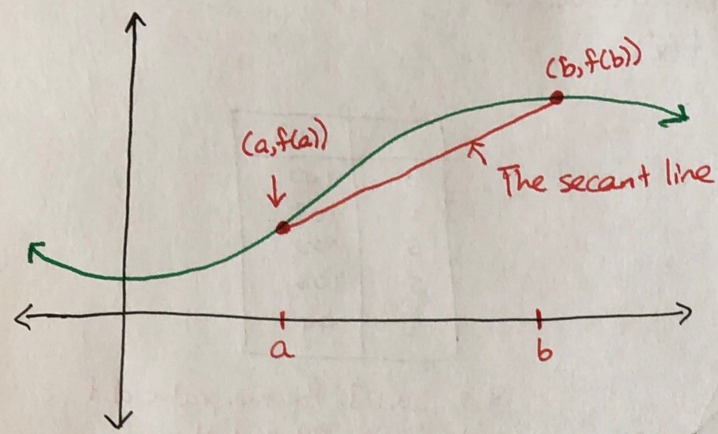
$\Rightarrow$ A.R.C. $= \dfrac{f(b)-f(a)}{b-a}$ (A.R.C. $= \dfrac{\Delta f}{\Delta x}$) * Go from right end point of interval to left end point of interval.

$[0,1]$: $\dfrac{\Delta f}{\Delta x} = \dfrac{3-1}{1-0} = 2$, $[3,5]$: $\dfrac{\Delta f}{\Delta x} = \dfrac{3}{2}$

$[1,3]$: $\dfrac{\Delta f}{\Delta x} = -1$, $[5,12]$: $\dfrac{\Delta f}{\Delta x} = -\dfrac{4}{7}$

$\Rightarrow$ f increases at a faster rate on $[0,1]$ and decreases at a faster rate on $[1,3]$

General Overview:

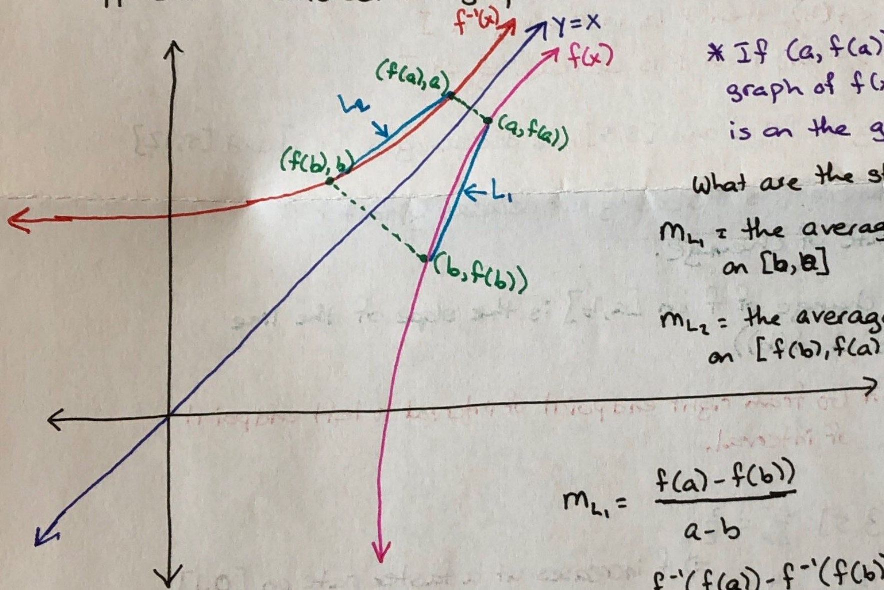


On $[a,x]$, $\dfrac{\Delta f}{\Delta x} = \dfrac{f(x)-f(a)}{x-a}$

On $[x,x+h]$, $\dfrac{\Delta f}{\Delta x} = \dfrac{f(x+h)-f(x)}{h}$

* Average Rate of Change is an example of a difference quotient, which is very important when studying rates of change in calculus.

Application: Consider the graphs of $f(x)$ and $f^{-1}(x)$



* If $(a, f(a))$ is on the graph of $f(x)$, then ~~(f(a))~~ $(f(a), a)$ is on the graph of $f^{-1}(x)$.

What are the slopes of L_1 & L_2?

m_{L_1} = the average rate of change of f on $[b,a]$

m_{L_2} = the average rate of change of f^{-1} on $[f(b),f(a)]$

$$m_{L_1} = \frac{f(a)-f(b)}{a-b}$$

$$m_{L_2} = \frac{f^{-1}(f(a))-f^{-1}(f(b))}{f(a)-f(b)}$$

$$= \frac{a-b}{f(a)-f(b)} = \frac{1}{m_{L_1}}$$

∴ The average rate of change of f^{-1} on $[f(b),f(a)]$ is the reciprocal of the average rate of change of f on $[b,a]$