

$$1) (5-3i)^2 + (7+4i)^2 = 25 - 30i - 9 + 49 + 56i - 16 = 49 + 26i$$

$$\therefore a+b = \boxed{75}$$

$$2) z_1 = \frac{1-i}{1+i}, z_2 = \frac{1+2i}{1-i}$$

$$\begin{aligned} a) z_1 &= \frac{(1-i)(1-i)}{(1+i)(1-i)} \\ &= \frac{1-2i-1}{1+1} \\ &= -i \end{aligned}$$

$$\therefore \bar{z}_1 = \boxed{i}$$

$$\begin{aligned} b) z_1 \cdot z_2 &= \frac{1+2i}{1+i} \\ &= \frac{(1+2i)(1-i)}{(1+i)(1-i)} \\ &= \frac{1+i+2}{1+1} \\ &= \frac{1}{2}(3+i) \end{aligned}$$

$$\therefore |z_1 z_2| = \boxed{\frac{1}{2}\sqrt{10}}$$

$$3) a+bi + \sqrt{a^2+b^2} = 2+8i$$

$$\Rightarrow b=8$$

$$\begin{aligned} a + \sqrt{a^2+b^2} &= 2 \\ \Rightarrow a + \sqrt{a^2+64} &= 2 \\ \Rightarrow \sqrt{a^2+64} &= 2-a \\ \Rightarrow a^2+64 &= 4-4a+a^2 \\ \Rightarrow 60 &= -4a \\ \Rightarrow a &= -15 \\ \therefore |z| &= \sqrt{(-15)^2+8^2} \\ &= \boxed{17} \end{aligned}$$

$$4) z = a+164i$$

$$\Rightarrow \frac{a+164i}{(a+n)+164i} = 4i$$

$$\begin{aligned} \Rightarrow a+164i &= 4i(a+n) = 656i \\ \Rightarrow a &= -656, 164 = 4(-656+n) \\ \Rightarrow -656+n &= 41 \\ \therefore n &= \boxed{697} \end{aligned}$$

$$5) \operatorname{Im}(z^2) = 18b, \operatorname{Im}(z^3) = \operatorname{Im}((9+bi)^3), b \in \mathbb{N}$$

$$\begin{aligned} &= \operatorname{Im}(\mu^3) = \operatorname{Im}(p+pq^2-9q) \\ &= 3(9)^2b - b^3 \\ &= 243b - b^3 \end{aligned}$$

$$\Rightarrow -b^3 + 243b = 18b$$

$$\Rightarrow -b^2 + 243 = 18$$

$$\Rightarrow b^2 = 225$$

$$\boxed{8} = \frac{274}{274} = \frac{1}{1} \therefore \boxed{b=15}$$

$$6) z = 3i^3 - 2ai^2 + (1-a)i + 5, a \in \mathbb{R}$$

$$\Rightarrow z = -3i + 2a + (1-a)i + 5$$

$$\operatorname{Im}(z) = 0 \Rightarrow -3 + (1-a) = 0$$

$$\therefore \boxed{a=-2}$$

$$7) x^2 + 4ix - 13 = 0$$

$$\Rightarrow (x^2 + 4ix - 4) - 13 + 4 = 0$$

$$\Rightarrow (x+2i)^2 - 9 = 0$$

$$\Rightarrow (x+2i)^2 = 9$$

$$\Rightarrow x+2i = \pm 3$$

$$\therefore \boxed{x = 3-2i, -3-2i}$$

$$8) \text{ roots are } r, 2r$$

$$\Rightarrow 3r = -b, 2r^2 = 1$$

$$\Rightarrow r = \pm \frac{1}{\sqrt{2}} \therefore \boxed{b = \pm \frac{3}{\sqrt{2}}}$$

$$9) a+b = -\frac{14}{5}, ab = -\frac{18}{5}$$

$$\begin{aligned} \Rightarrow (a-2)(b-2) &= ab - 2(a+b) + 4 \\ &= -\frac{18}{5} + \frac{28}{5} + \frac{20}{5} \\ &= \boxed{6} \end{aligned}$$

$$10) 4x^2 + (a+8)x + 9 = 0 \text{ exactly 1 real root}$$

$$\Rightarrow (a+8)^2 - 4(4)(9) = 0$$

$$\Rightarrow (a+8)^2 - 144 = 0$$

$$\Rightarrow (a+8)^2 = 144$$

$$\Rightarrow a+8 = \pm 12$$

$$\therefore \boxed{a=4, -20}$$

$$11) x^2 - 26x + N = (x-r)(x-s)$$

$$\Rightarrow r+s = 26, N = rs$$

$$\Rightarrow 19(26) + 75s = 1994$$

$$\Rightarrow 75s = 1994 - 494 = 1500$$

$$\Rightarrow s = 20, r = 6$$

$$\therefore \boxed{N=120}$$

