Binomial Theorem

1) Gen. Term: $\binom{7}{k} x^k (2)^{7-k}$

$k=4$: $\binom{7}{4}(2)^3 = 35 \cdot 8$
$= \boxed{280}$

2) Gen. Term: $\binom{6}{k}(x^2)^k\left(\frac{1}{x}\right)^{6-k}$

$= \binom{6}{k} x^{2k} \cdot x^{k-6}$

$= \binom{6}{k} x^{3k-6}$

$k=0$ (constant): $3k-6=0$
$\Rightarrow k=2$

$\therefore \binom{6}{2} = \boxed{15}$

3) Gen. Term: $\binom{10}{k}(2a)^k(-b)^{10-n}$

$k=4$: $\binom{10}{4}(2)^4 = \frac{10 \cdot \cancel{9}^{3} \cdot \cancel{8} \cdot 7}{\cancel{4 \cdot 3 \cdot 2}} \cdot 16$

$= 10 \cdot 7 \cdot 3 \cdot 16$

$= \boxed{3360}$

4) $(1-\frac{1}{a})^6$ Gen. Term: $\binom{6}{k}(1)^k(-\frac{1}{a})^{6-k}$

$k=4$: $\binom{6}{4} = 15$

$k=5$: $\binom{6}{5}(-1) = -6$

$k=6$: $\binom{6}{6} = 1$

$\therefore 15-6+1 = \boxed{10}$

5) Gen. Term: $\binom{8}{k}\left(\frac{1}{2}x^2\right)^k(-2x^{-1})^{8-k}$

$= \binom{8}{k}(2)^{-k}(-2)^{8-k}x^{3k-8}$

$3k-8 = 7 \Rightarrow k=5$

$\therefore \binom{8}{5}(2)^{-5}(-2)^3 = 56\left(\frac{1}{32}\right)(-8)$
$= \boxed{-14}$

6) Gen. Term: $\binom{8}{k}(x^{1/2})^k(-1)^{8-k}$

x^3: $\frac{1}{2}k = 3 \Rightarrow k=6$

$\therefore \binom{8}{6}(-1)^2 = \boxed{28}$

$x^{5/2}$: $\frac{1}{2}k = \frac{5}{2} \Rightarrow k=5$

$\therefore \binom{8}{5}(-1)^3 = \boxed{-56}$