Overview of Inequalities

Polynomial Inequalities: 1) $x^3 - 4x^2 + 4x + 16 \leq 0$ 2) $(x-1)^3(x+2)^4(3x-10)^2 > 0$

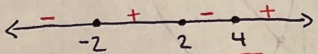
Rational Inequality: 3) $\dfrac{2x-11}{x^2-4x+3} < 0$

Solutions:

1) Factor by grouping: $x^2(x-4) - 4(x-4) \leq 0$

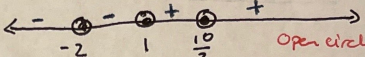
$\Rightarrow (x-4)(x^2-4) \leq 0$

$\Rightarrow (x+2)(x-2)(x-4) \leq 0$



$\therefore \boxed{(-\infty, -2] \cup [2, 4]}$

2) Roots are $1, -2, \frac{10}{3}$ (watch repeated roots)

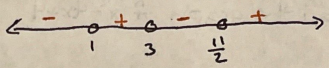


Open circles, not closed circles (note the inequality)

$\therefore \boxed{(1, \frac{10}{3}) \cup (\frac{10}{3}, \infty)}$

* Can't write a single interval here (Why not?)

3) Factor the denominator: $\dfrac{2x-11}{(x-1)(x-3)} < 0$



$\therefore \boxed{(-\infty, 1) \cup (3, \frac{11}{2})}$

Absolute Value Inequalities:

Solve: $5|2x-9| \geq 15$

$\Rightarrow |2x-9| \geq \frac{15}{5} = 3$

$\Rightarrow 2x-9 \geq 3$ or $-(2x-9) \geq 3$

$\Rightarrow 2x-9 \leq -3$

$\Rightarrow x \geq 6$ $\quad x \leq 3$

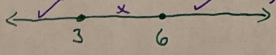
$\therefore \boxed{(-\infty, 3] \cup [6, \infty)}$

or

$|2x-9| = \frac{15}{5} = 3$

$\Rightarrow 2x-9 = 3, \; 2x-9 = -3$

$\Rightarrow x = 6, \; x = 3$



Solve: $4|2x-1| \leq 28$

$\Rightarrow |2x-1| \leq 7 \rightsquigarrow 2x-1 \leq 7$ or $2x-1 \geq -7$

$\Rightarrow -7 \leq 2x-1 \leq 7$

$\Rightarrow -6 \leq 2x \leq 8$

$\therefore \boxed{x \in [-3, 4]}$

Solve: $\big||2x-3| - 9\big| > 4$

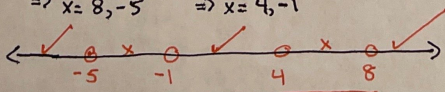
While you can write all the possible inequalities, it is more efficient to find the solutions to $||2x-3|-9| = 4$, then setup a number line for the inequality

$\Rightarrow |2x-3| - 9 = 4$ or $|2x-3| - 9 = -4$

$\Rightarrow |2x-3| = 13, \; |2x-3| = 5$

$\Rightarrow 2x-3 = \pm 13, \; 2x-3 = \pm 5$

$\Rightarrow x = 8, -5$ $\quad \Rightarrow x = 4, -1$



$\therefore \boxed{(-\infty, -5) \cup (-1, 4) \cup (8, \infty)}$

Solve $|x| \leq x^2-4x+4$

$\Rightarrow x \leq x^2-4x+4$ or $x \geq -(x^2-4x+4)$

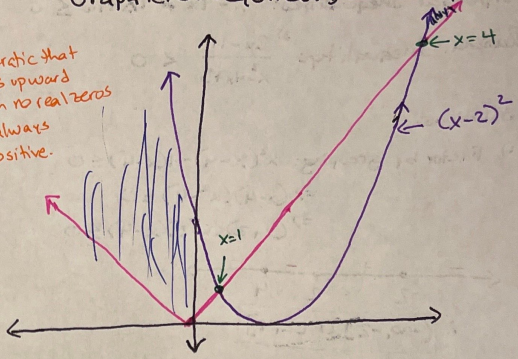
$\Rightarrow x^2-5x+4 \geq 0$ or $x^2-3x+4 \geq 0$

$\downarrow$ $(-\infty, 1] \cup [4, \infty)$

$\downarrow$ $(-\infty, \infty)$ → Quadratic that opens upward with no real zeros is always positive.

$\therefore \boxed{x \in (-\infty, 1] \cup [4, \infty)}$

Graph: (not required), but is helpful



The solution is wherever the graph of $|x|$ meets or is below the graph of x^2-4x+4