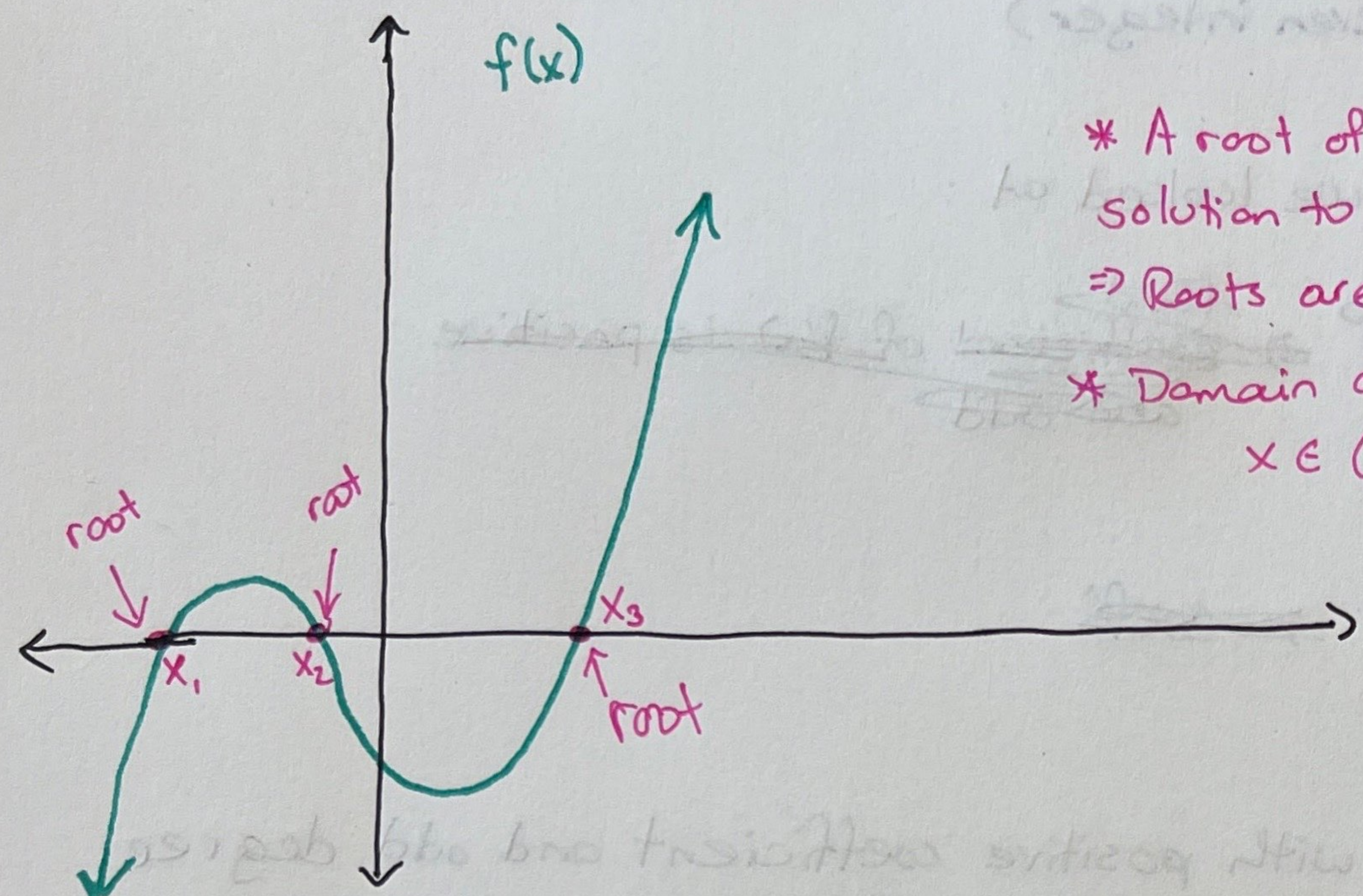
Polynomial Functions

Consider the graph of a polynomial function $f(x)$:



* A root of a polynomial is a solution to the equation $f(x) = 0$

$\Rightarrow$ Roots are $x = x_1, x = x_2, x = x_3$

* Domain of any polynomial: $x \in (-\infty, \infty)$

Q: For what values of x is $f(x) > 0$? Justify your answer

Q: For what values of x is $f(x) \leq 0$? Justify your answer

Create a number line and mark the roots

$$\underset{}{-} \quad \underset{x_1}{\circ} \quad + \quad \underset{x_2}{\circ} \quad - \quad \underset{x_3}{\circ} \quad +$$

Look at graph and see where graph is above or below the x-axis.

Known as a sign chart (not suff. justification by itself)

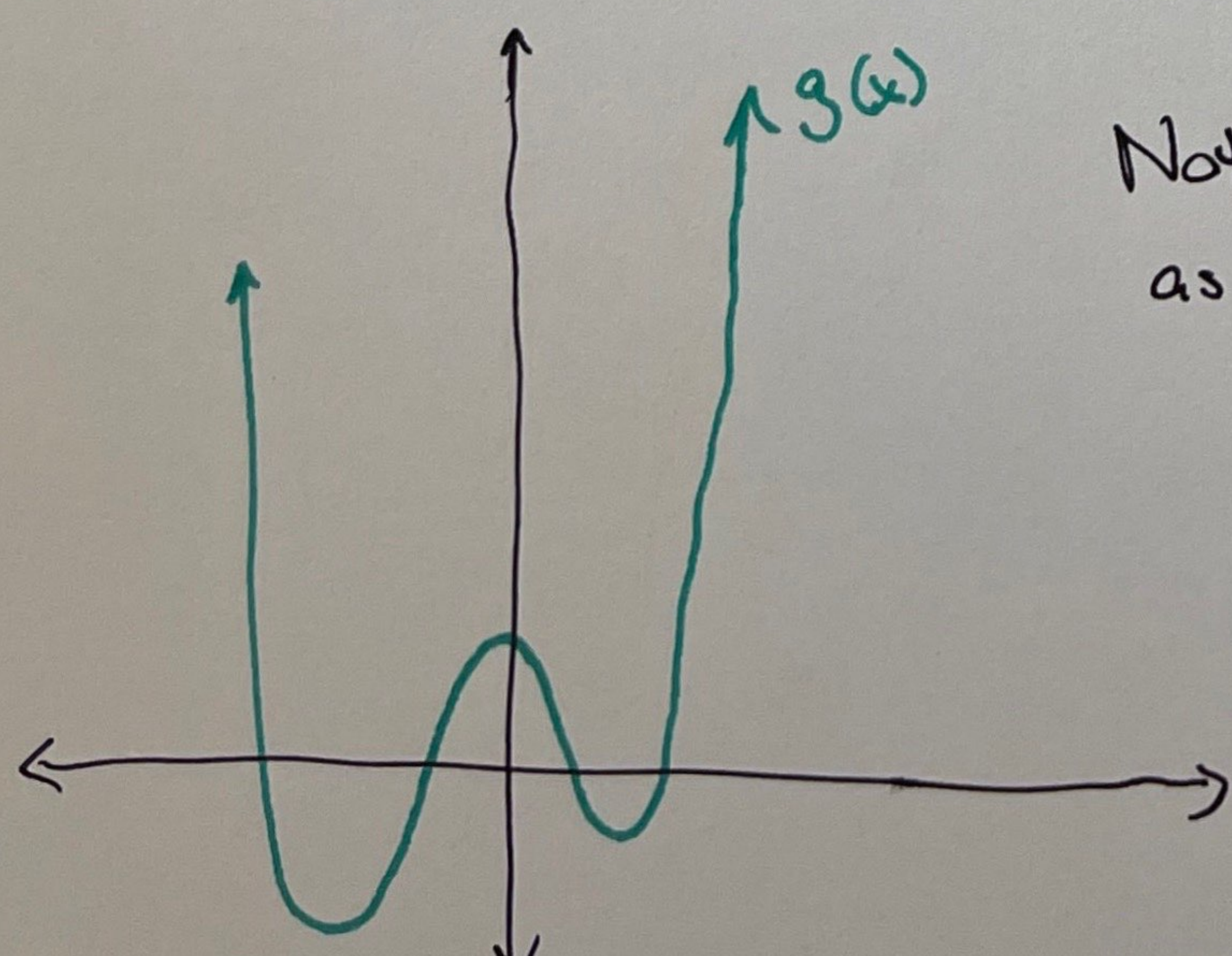
$\therefore f(x) > 0$ on $x \in (x_1, x_2) \cup (x_3, \infty)$

$\therefore f(x) \leq 0$ on $x \in (-\infty, x_1] \cup [x_2, x_3]$

Note as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$

(Known as end behavior of function)

Consider the graph of the polynomial function $g(x)$:



Note as $x \to \infty$, $g(x) \to \infty$, and as $x \to -\infty$, $g(x) \to \infty$

* The end behavior of a polynomial function is based on the lead term (the term with the highest degree)

⇒ We have to look at both the coefficient (pos. or neg.) and the degree (odd or even integer)

Consider the two graphs we looked at:

$f(x)$: $x \to -\infty$, $f(x) \to -\infty$ ⇒ ~~lead coefficient of f(x) is positive and odd~~

$x \to \infty$, $f(x) \to \infty$

$g(x)$: $x \to -\infty$, $g(x) \to \infty$ ⇒ ~~coeff~~

$x \to \infty$, $g(x) \to \infty$

⇒ $f(x)$ has a lead term with positive coefficient and odd degree

$g(x)$ has a lead term with positive coefficient and even degree.

Describe the end behavior of $h(x) = -10x^5 + 7x^3 + 8x^2 + 3$

Lead term: odd degree & neg. coefficient

$$\therefore \boxed{\begin{array}{l} x \to \infty, h(x) \to -\infty \\ x \to -\infty, h(x) \to \infty \end{array}}$$