Review of Complex Numbers

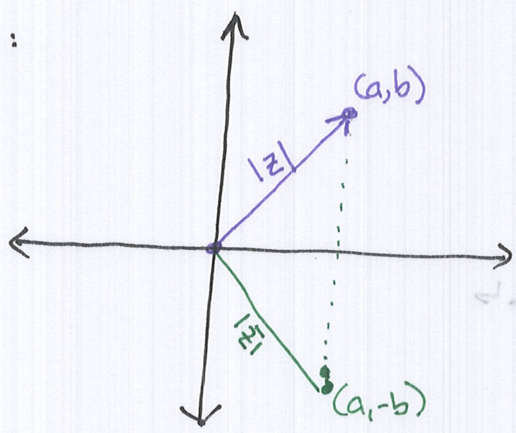
$z = a + bi, \quad a, b \in \mathbb{R}$

$a = Re(z)$ - the real part of z, $b = Im(z)$ - the imaginary part of z.

$|z| = \sqrt{a^2+b^2}$ (the magnitude or norm of z)

$\bar{z} = a - bi$ (the conjugate of z)

Geometrically: z is the vector from the origin to (a,b)



* $|z|$ is the length of the segment from $(0,0)$ to (a,b)

* $\bar{z}$ is a reflection of z in the x-axis

* $|z| = |\bar{z}|$

* $z + \bar{z} = (a+bi) + (a-bi)$
$= 2a$
$= 2\,Re(z)$

* $z \cdot \bar{z} = (a+bi)(a-bi)$
$= a^2 - b^2 i^2 \quad (i^2 = -1)$
$= a^2 + b^2$
$= |z|^2$

* $z + \bar{z}$ and $z\bar{z} \in \mathbb{R}$

$\cancel{z_1 z_2 =}$

Let $z_1 = a_1 + b_1 i$ & $z_2 = a_2 + b_2 i \Rightarrow \bar{z}_1 = a_1 - b_1 i, \ \bar{z}_2 = a_2 - b_2 i$

$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$

$\cancel{z_1 z_2 = (\ldots)}$

$z_1 z_2 = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i$

$\overline{z_1 + z_2} = (a_1 + a_2) + (b_1 + b_2)i$
$= (a_1 + a_2) - (b_1 + b_2)i$
$= (a_1 - b_1 i) + (a_2 - b_2 i)$
$= \bar{z}_1 + \bar{z}_2$

(The conjugate of a sum is the sum of the conjugates)

$\cancel{\overline{z_1 z_2} = (\ldots) - (a_1 b_2 + a_2 b_1)i}$

$\overline{z_1 z_2} = (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1)i$
$= a_1 a_2 - b_1 b_2 - a_1 b_2 i - a_2 b_1 i$
$= a_1(a_2 - b_2 i) - b_1 i(a_2 - b_2 i)$
$= (a_1 - b_1 i)(a_2 - b_2 i)$
$= \bar{z}_1 \cdot \bar{z}_2$

(The conjugate of a product is the product of the conjugates)

Quadratic Equations and Roots

$f(x) = ax^2+bx+c$.

roots of $f(x)=0$ are r_1 & r_2

$$\Rightarrow f(x) = ax^2+bx+c$$
$$= a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$
$$= a(x-r_1)(x-r_2)$$
$$= a(x^2-(r_1+r_2)x+r_1r_2)$$

Equate Coefficients:

$$-\frac{b}{a} = r_1+r_2, \quad \frac{c}{a} = r_1r_2$$

Ex: $g(x) = 2x^2-9x+17$.

The roots of $g(x)=0$ are a & b

$$\boxed{\begin{aligned} a+b &= \frac{9}{2} \\ ab &= \frac{17}{2} \end{aligned}}$$

- Find a^2+b^2.

$$\Rightarrow a^2+b^2 = (a+b)^2-2ab$$
$$= \left(\frac{9}{2}\right)^2-2\left(\frac{17}{2}\right)$$
$$= \frac{81}{4}-17$$
$$= \boxed{\frac{13}{4}}$$

- Find $(a+2)(b+2)$ w/o finding a & b

$$\Rightarrow (a+2)(b+2) = ab+2(a+b)+4$$
$$= \frac{17}{2}+9+4$$
$$= \frac{17}{2}+13$$
$$= \boxed{\frac{43}{2}}$$

Find the roots of

- ~~Solve~~ $P(x) = 2x^2+4ix-52$ w/o using the quadratic formula

$$2x^2+4ix-52=0$$
$$\Rightarrow x^2+2ix-26=0 \quad \text{Complete the Square}$$
$$\Rightarrow (x^2+2ix-1)+1-26=0$$
$$\Rightarrow (x+i)^2 = 25$$
$$\Rightarrow x+i = \pm 5$$
$$\therefore \boxed{x = 5-i, -5-i}$$