Absolute Value Functions

Recall that $f(x) = |x|$ is a piecewise function : $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Write $g(x) = |8x| + x + 5$ and $h(x) = |2x+3| + |x-8| + 6$ as piecewise functions.

Solutions: First, find the key values for each function. The key values are the values that make the quantity inside | | equal to 0

$g(x)$: $x = 0$

$h(x)$: $x = -\frac{3}{2}, 8$

$g(x)$:

$(-\infty, 0)$: $-8x + x + 5 = -7x + 5$

$[0, \infty)$: $8x + x + 5 = 9x + 5$

$\therefore \boxed{g(x) = \begin{cases} 9x+5, & x \geq 0 \\ -7x+5, & x < 0 \end{cases}}$

$h(x)$: $(-\infty, -\frac{3}{2})$: $(-2x-3) + (8-x) + 6 = 11 - 3x$ (or $-3x+11$)

$[-\frac{3}{2}, 8)$: $(2x+3) + (8-x) + 6 = x + 17$

$[8, \infty)$: $(2x+3) + (x-8) + 6 = 3x + 1$

$\therefore \boxed{h(x) = \begin{cases} -3x+11, & x < -\frac{3}{2} \\ x+17, & -\frac{3}{2} \leq x < 8 \\ 3x+1, & x \geq 8 \end{cases}}$

- Solve $h(x) = 18$

Solution: Set each piece of $h(x)$ equal to 18 and solve, but make sure the value of x falls in the interval for that particular piece.

$\Rightarrow -3x + 11 = 18$
$\Rightarrow x = -\frac{7}{3} < -\frac{3}{2}$ ✓

$x + 17 = 18$
$\Rightarrow x = 1$ ✓

$3x + 1 = 18$
$\Rightarrow x = \frac{17}{3} < 8$ ⊗

$\therefore \boxed{x = -\frac{7}{3}, 1}$

Solve $\left| \frac{x+2}{3x-1} \right| = 5$

Solution: Either $\frac{x+2}{3x-1} = 5$ or $\frac{x+2}{3x-1} = -5$

$\Rightarrow x + 2 = 15x - 5, \; x + 2 = -15x + 5$

$\Rightarrow 14x = 7, \; 16x = 3$

$\therefore \boxed{x = \frac{1}{2}, \frac{3}{16}}$

Solve : $|5x - 1| = |2x + 3|$

Solution: Either $5x - 1 = 2x + 3$ or $5x - 1 = -(2x+3)$

$\Rightarrow 3x = 4$ or $7x = -2$

$\therefore \boxed{x = \frac{4}{3}, -\frac{2}{7}}$

Solve $|x^2-4x-5| = 7$

Either $x^2-4x-5 = 7$ or $x^2-4x-5 = -7$

$\Rightarrow x^2-4x-5-7=0$ $\qquad x^2-4x+2=0$

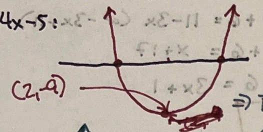
$\Rightarrow x^2-4x-12=0$ $\qquad \Rightarrow x = \frac{4\pm\sqrt{16-8}}{2}$

$\Rightarrow x = 6, -2$ $\qquad = \frac{4\pm 2\sqrt{2}}{2}$

$= 2\pm\sqrt{2}$

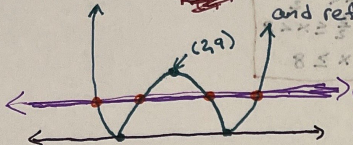
Graphical Interpretation: Note $x^2-4x-5=0$ has solutions $x=5,-1$

x^2-4x-5:



But absolute value functions can never fall below x-axis.

$\Rightarrow$ To graph $|x^2-4x-5|$, take the portion between -1 & 5 and reflect in the x-axis. Now draw the line $y=7$.



~~Now~~ Since $y=7$ passes through the graph 4 times, the equation must have 4 solutions.

Q: How many solutions are there to $|x^2-4x-5|=9$? $|x^2-4x-5|=16$? Refer to the graph.

Solve $\big||2x+5|-3\big| = 8$ ← "Nested" absolute value equation, meaning we will have extra cases to deal with.

$\Rightarrow |2x+5|-3 = 8$ or $|2x+5|-3=-8 \Rightarrow |2x+5| = -5$

Impossible

$\Rightarrow |2x+5| = 11$

$\Rightarrow 2x+5 = 11$ or $2x+5=-11$

$\therefore \boxed{x = 3, -8}$

Solve $\big||2x+5|-9\big| = 8$

$\Rightarrow |2x+5|-9=8$ or $|2x+5|-9=-8$

$\Rightarrow |2x+5| = 17$ or $|2x+5| = 1$ (Both are possible)

$\Rightarrow 2x+5 = \pm 17$ or $2x+5=\pm 1$

$\therefore \boxed{x = 6, -11, -2, -3}$