~~Problem~~

Combo Problem Set

1) # of apples: 0-6 → 7
# of bananas: 0-8 → 9

$\therefore 7 \cdot 9 - 1 = \boxed{62}$

2) 13 balls (9 green, 4 yellow)

a) $\boxed{\binom{13}{6} \text{ or } {}_{13}C_6}$ subsets

b) 4 green, 2 yellow $\Rightarrow \binom{9}{4} \cdot \binom{4}{2}$

$\Rightarrow \binom{9}{4} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{5!\,4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{24} = 126$
$= 63 \cdot 2$

$\binom{4}{2} = \frac{4 \cdot 3 \cdot 2!}{2!\,2!} = 6$

$\binom{13}{6} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}\,6!}$

$= \frac{13 \cdot 12 \cdot 11 \cdot \cancel{10 \cdot 9 \cdot 8}}{\cancel{6!}}$

$= 13 \cdot 12 \cdot 11$

$\therefore \frac{63 \cdot 2 \cdot 6}{13 \cdot 12 \cdot 11} = \boxed{\frac{63}{143}}$

3) $\binom{n}{5} = \frac{n!}{(n-5)!\,5!}, \binom{n}{3} = \frac{n!}{(n-3)!\,3!}$

$\Rightarrow \frac{\cancel{n!}}{(n-5)!\,5!} \cdot \frac{3!\,(n-3)!}{\cancel{n!}} = \frac{3}{2}$

$\Rightarrow \frac{(n-3)(n-4)}{4 \cdot 5} = \frac{3}{2}$

$\Rightarrow (n-3)(n-4) = \frac{3}{2} \cdot 20 = 30$

$\Rightarrow n^2 - 7n + 12 - 30 = 0$

$\Rightarrow n^2 - 7n - 18 = 0 \Rightarrow (n-9)(n+2) = 0$

$\therefore \boxed{n = 9}$

4) $\binom{n}{4} = 2\binom{n}{3}$

$\Rightarrow \frac{n!}{(n-4)!\,4!} = \frac{2 \cdot n!}{(n-3)!\,3!}$

$\Rightarrow \frac{(n-3)!\,6}{(n-4)!\,24} = 2$

$\Rightarrow \frac{n-3}{4} = 2$

$\Rightarrow n - 3 = 8$

$\therefore \boxed{n = 11}$

5) $\binom{9}{2}\binom{n}{2} = 360$

$\Rightarrow 36\left(\frac{n(n-1)}{2}\right) = 360$

$\Rightarrow n(n-1) = 10 \cdot 2$

$\Rightarrow n^2 - n - 20 = 0 \Rightarrow (n-5)(n+4) = 0$

$\therefore \boxed{n = 5}$

6) $\_\ \_\ 5\ \_\ \_$ last digit is 1, 3, 7, 9

Case 1: last digit is 3 (1st digit is 2, 4, 6)

$\underline{3}\ \underline{7}\ \underline{1}\ \underline{6}\ \underline{1} = 126$

Case 2: last digit is not 3

$\underline{4}\ \underline{7}\ \underline{1}\ \underline{6}\ \underline{3} = 504$

$\therefore 504 + 126 = \boxed{630}$

7) ways to pack 3 books in same box: $\binom{12}{3}$

Case 1: 3 math books in small box: $\binom{3}{3} = 1$

Case 2: 3 math books in med. box: $\binom{4}{3} = 4$

Case 3: 3 math books in large box: $\binom{5}{3} = 10$

$\Rightarrow p = \frac{15}{\binom{12}{3}}$

$\binom{12}{3} = \frac{12!}{9!\,3!}$

$= \frac{12 \cdot 11 \cdot 10}{6} = 220$

$= \frac{15}{220} = \boxed{\frac{3}{44}}$

8) $P(1 \text{ head}) = P(2 \text{ heads})$, h = prob. of flipping heads

$\Rightarrow \binom{5}{1} h (1-h)^4 = \binom{5}{2} h^2 (1-h)^3$

$\Rightarrow 5h(1-h)^4 = 10h^2(1-h)^3$

$\Rightarrow 1 - h = 2h$

$\Rightarrow 3h = 1$

$\Rightarrow h = \frac{1}{3}$

$\therefore P(3 \text{ heads}) = \binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$

$= 10\left(\frac{1}{27}\right)\left(\frac{4}{9}\right)$

$= \boxed{\frac{40}{243}}$