MPS21XH - More Practice with Rate of Change and Concavity
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x	$1 < x < 2$	$2 < x < 3$	$3 < x < 4$	$4 < x < 5$
Rate of change of $g(x)$	Positive,Increasing	Negative,Increasing	Positive,Decreasing	Negative,Decreasing

1.) Given a table that gives characteristics of the rates of change of the function $g(x)$.

a.) Describe the behavior of the graph of $g(x)$ for all $x \in (3, 4)$. Justify your answer.

On $(3,4)$, the rate of change of $g(x)$ is pos. and dec. $\therefore$ $g(x)$ is increasing and concave down

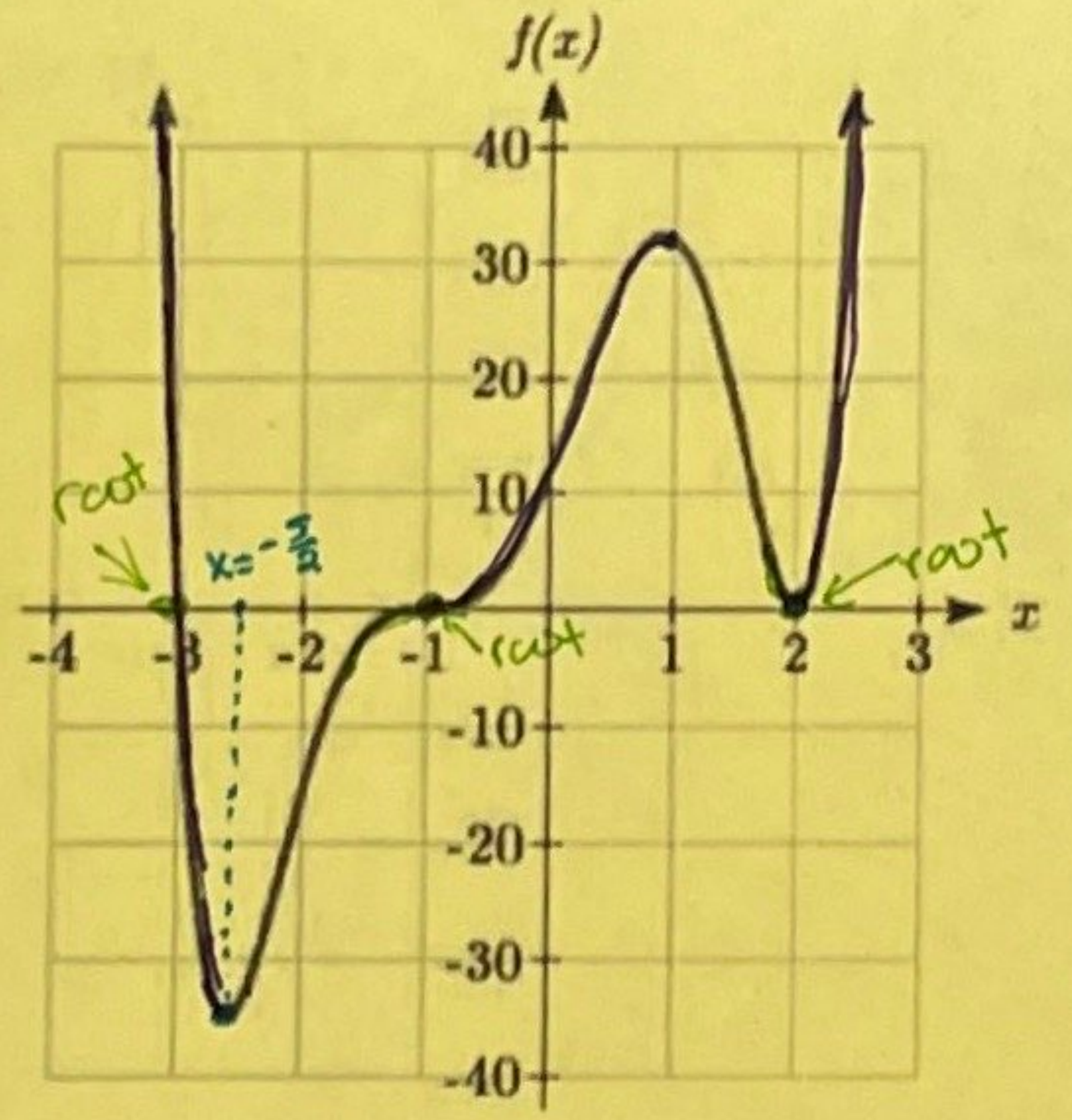
b.) For what values of x does the graph of g have a relative extrema? Be specific and justify your answer.

The rate of change of $g(x)$ changes from pos. to neg. $\therefore$ Rel. Max at $x=2, x=4$

The rate of change of $g(x)$ changes from neg. to pos. $\therefore$ Rel. Min at $x=3$

c.) For what values of x does the graph of g have a point of inflection? Justify your answer.

The rate of change of $g(x)$ changes from inc. to dec. (or dec. to inc.) $\therefore$ P.O.I. at $x=3$



2.) Given the graph of a polynomial function $f(x)$:

a.) For what values of x does the graph of f have a local minimum? Justify your answer.

$f(x)$ changes from dec. to inc. $\therefore$ Rel. Min at $x = -\frac{5}{2}, x = 2$

b.) For what values of x is the graph of f concave up? Justify your answer.

Rate of change of $f(x)$ is increasing $\therefore x \in (-\infty, -\frac{5}{2}) \cup (-1,1) \cup (2,\infty)$

Imagine drawing secant lines in these intervals (they are all above the curve)

But this is not a complete justification (rate of change is inc./dec. is the complete justification)

c.) Determine the equation of $f(x)$ from the graph.

Roots of $f(x)=0$: $x=-3, x=-1, x=2$

At $x=-1$: graph of f looks like a cubic polynomial $\Rightarrow$ triple root

At $x=2$: graph of f looks like a parabola $\Rightarrow$ double root

$\therefore f(x) = (x+3)(x+1)^3(x-2)^2$