Data Science

Definition

An interdisciplinary field focused on extracting knowledge and insights from data.

principles.

- Utilizes statistics, data analysis and machine learning
- Involves data collection, cleaning, analysis, and visualization.

M.L

- Def^n. A subset of AI enables systems to learn from data and improve over time without being explicitly programmed.

principles : ∘ Alogrithms identify patterns within data

∘ Includes supervised, unsupervised and reinforcement learning.

A.I

Defn: The simulation of human intelligence process by machines particularly computer systems.

Principles:

- Encompasses a broad range of capabilities including reasoning learning and self correction.
- language processing, robotics and computer vision

Dr. Ignas Semmelweis → Data Science
(Hungarian scientist.)

1kb = 1024 bytes.

Alan turing - father of theoretical C.S.
(Mathematician)
Turing set (Data science)

Types of M.L

Supervised learning
un " "
Semi " "
Reinforcement learning.

John Mccarthy - father of A.I
(mathematician)

Syntax.

Python Notebook. (↑ collab.)

print [illegible]

1- # heading
subheading
" to sub heading

Run time

Run before : ~~before Bo~~
run before focused cell.
(not run after focused cell)

Run cell and below : run focused cell and
after the focused cell.

Green tick → Cell run [illegible].

Share
↓
- copy link
- Done.

Right clique → new folder.

Introduction to Python programming

What is python ?

High level, object Oriented, Interpreted, dynamic semantics.

* opensource → development is public
* Hardware platforms
* Very powerful
* Easy to learn & read
* commercial Data base (embedded devices)
* Testing & debugging
* Standard library
* Ubiquitous.

print (233)
print (4.9)

Fundamental
Python data types

age = 28
↓
variable name

① * Numeric
- Integers
- Float
- complex.

② * Text
↓
String.

③ * Sequence
→ list — mutable, different data
→ Tuple — immutable (delete, add
→ range. (start, stop, step)

list (2, 5, 2)
start, stop, step.

one variable
↓
(stop)

2 variable
(start, stop)

3.

④ * Mapping type
Dictionary.
(phone no, name
details of ~~a~~ person)

(key / pair)
Values.

* Set data type.

* Boolean < true / false

* None.

→ (length = len.)

* ({ })

0 1 2 3 4 5

p y t h o n

-6 -5 -4 -3 -2 -1

s = python.

print (s[1]) = y

Hello" [0:3]

'Hel'

Hello' [0:5]

'Hello'

'Hello' [-3:]

'llo'

'Hello' [0:5:2]

'H lo'

'Hello' [0:5:3]

'Hl'

'Hello' [0:5:-1:-1]

'olle'

'Hello' [::-1]

olleH

'Malayalam' [::-2]

'malayalam'

'Malayalam' [::-1]

'malayalaM'

'ICT Acadamy of Kerala' [3:11]

'Acadamy' s

'ICT Acadamy of kerala'.split()

['ICT', 'Acadamy', 'of', 'kerala']

write code to print a given string is Palyndrom (true/False)

a = 'malayalam'

b = 'malayalam' [::-1]

True

~~x = input~~ ~~print~~ (print in words)

~~print(x)~~

~~x 2136~~

digit_words =

0	1	2	3
1	2	3	4

num = "1234"

num[0] , num[1]

digits { 1; 'one', 2 : 'two' }

digit [int (num [0])

* digit_words = { 1 : 'one' , 2 : 'two' }

otp = input ("Enter otp : ")

otp_int = int (otp [0])

print (digit_words[otp_int])

[1]

⇒ one

* digits = ["zero", "one", "two", "three"
"four", "five", "six", "seven",
Eight", "nine"]

otp = input ("Enter otp . ")

a = int (otp [0])

b = int (otp [1])

c = int (otp [2])

d = int (otp [3])

print (digits [a], digits [b], digits [c],
digit [d])

$\boxed{1\ 2\ 9\ 4\ 8}$

⇒ Two Nine four Eight.

$\boxed{2}$ 9 4 8

```
digits = ["zero" ... "nine"]

otp = input ("enter otp:")

a,b,c,d = int (otp[0]), int (otp[1]),
          int (otp[2]), int (otp[3])

print ( digits[a], digits [b], digits[c],
        digits [d])
```

2 4 3 9

=> Two four three nine

"one two three" split (" ")

0	1	2	3
"one"	"Two"	"three"	"four"
↓	↓	↓	↓
1	2	3	4

$\boxed{E(x^2) - E(x)^2}$

μ – population mean

$\bar{X}$ – sample mean

σ – standard deviation $= \sqrt{\text{variance}}$

how dispersed it is ← σ^2 – variance → how close it is to the mean

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

Quartiles

Q_1 — 1st quartile ($\frac{1}{4}$th)

Q_2 — 2nd " ($\frac{2}{4}$th)

Q_3 — 3rd " ($\frac{3}{4}$th)

Q_4 — 4th "

mode → most repeated value.

n percentiles → n^{th} percentage of observations ~~[illegible]~~ value.

Eg 25% → 25% of value.

Q_1 — 25 %

Q_2 — 50 % → median

Q_3 — 75 %

mean — avg.

IQR → Inter quartile range $= Q_3 - Q_1$

Scatter plot

Outlayer → Extreme values

meaning point representation

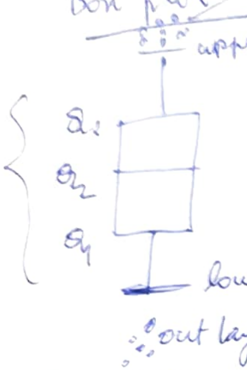
$20 - \frac{3}{2} \times 12$

$20 - 18 = 2$

$\frac{3}{2} \times 12$

$8 - 18$

Box plot → outlayers.



lower limit $= Q_1 - 1.5\,IQR$

upper limit $= Q_3 + 1.5\,IQR$

Statistical point information

Range = max. value − min value.

co. variance = cov(x,y) ⟶ direction of relation.

$\Sigma(x, y)$

$\Sigma x \cdot y$

+ve direction → direct proportion

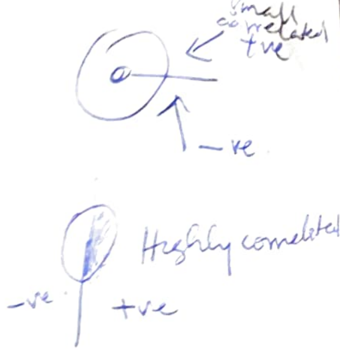
−ve directional → opp. direction

$cov(x,y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n}$

correlation $= \frac{cov(x,y)}{\sigma_x \sigma_y}$

(how strong the relation is)

range is $(-1$ to $+1)$



* (feature selection used on correlation.)

Cosation :

$Z\ \text{score} = \frac{(x - \bar{x})}{\sigma}$

* population → Probability distribution
* Sample.
* Sampling bias

Normal distribution :—

$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\mu - \sigma}$

variance = 1

mean ≠ μ

CLT :-

samp avg. distribution → Normal curve.

Hypothesis test :

Null hypothesis

Alternate hypothesis

Significance level (α)

Typically used values are 0.05 (eg: commerce)) and 0.01 (field like medicine)

p - value.

Z - test (z score).

$(x - \bar{x})$

$z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$) sample mean
) population mean
population S.D.
n - sample size.

Steps :

1 Null hypothesis
2 alternate hypothesis
3 significance level (α)
4 Z-test statistic.
5 find p-value using z-~~test~~ stastic
6 p value < α - we fail to reject null hypothesis
7 There are one tailed & two tailed tests.

avg weight - 50 g.

pop. mean

$\mu = 50.15$

50.3, 49.5, 50.2, 51, 49.7, 50.3, 49.8, 50.5, 49.6, 50.1

s.d = 1.5 g.

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\bar{x} < \mu$ / $>$ — one tail

$x \neq \mu$ differ — Two tail

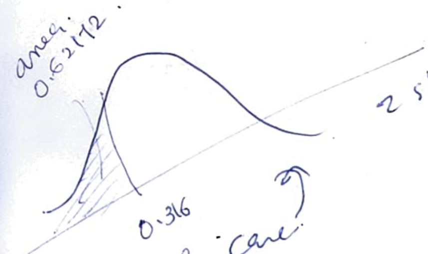
$= \dfrac{50.15 - 50}{1.5/\sqrt{10}} = \dfrac{0.15}{1.5/\sqrt{10}}$

$= 0.316$

null hypothesis → $\bar{x}/\mu$ difference illa

alternate " → difference und.

$\alpha \to 0.05$



one tail case

$p > \alpha$.

accept null hypothesis

Two tail case.

$p \to 1 - 0.62172$

$= 0.3759$

$\Rightarrow 2 \times 0.3759 = 0.7518 = p$ value.

One sampled test

Null hypothesis (H_0) : sample weight is not higher than population

Alternate hypothesis (H_1) : " " higher than population.

p value : 0.6172 α : 0.05.

here $p > \alpha$ $\therefore$ accept H_0.

Type T test

One sampled t-test. — the sample mean ≠ reference sample. $df = n-1$

independent sample t-test. $df = \cancel{n(n+1)}\ n_1+n_2-2$

Paired samples t test $df = n-1$

One sample test

→ null hypo

→ alternate hypo

→ α.

→ calculate T-test Statistic

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$ s → sample S.D.

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{N}(x_i-\bar{x})^2}$$ n - sample size

t < critical we fail to reject null hypothesis

$$t = \frac{50.15-50}{\sqrt{\frac{1}{9}(}} = \frac{0.15}{0.51049} = \frac{0.2938 \times \sqrt{10}}{\sqrt{10}}$$

$= 0.9291$

$\alpha = 0.05$

T test

Alpha (α), df, tailed

↓

T. test static $\xrightarrow{\text{tables}}$ critical t-value.

$0.9 < 1.833$.

Independent sample test

$df = n_1 + n_2 - 2$

$t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ for one tail

sample mean gp1

" gp2.

$t = \dfrac{\bar{x}_1 - \bar{x}_2}{S_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ for 2 tailed

$S_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$ pooled S.D

S.D of gp1 S.D of gp2.

sample size 1

t < critical value — we fail to reject null hypothesis.

50.1, 49.8, 50.3, 50

49.9, 50.6, 50.2, 50.1

$\bar{x}_1 = 49.94$

$\bar{x}_2 = 50.24$

C.V 2.3065

$S_1 = \sqrt{\frac{1}{n-1} \Sigma (x_1 - \bar{x})^2}$

→ 0.30495

$t = \cancel{5.70125}$ −1.6464

$S_2 = 0.2702$

$S_p = 0.0832$

$0.0832\sqrt{2/5} = 0.05262$

df = 8

t → 1.6464

t < C.V. accept H_0.

B.	G
50	48.
81	50.
61	39.
63	53.
83	60.
64	47.
75	60.
43	85.
	52.
8	52.
	63.
	49.

$n_1 = 8$

$n_2 = 12$ 1410

$\bar{x}_1 = 65$ (4)

$\bar{x}_2 = 54.833.$

$S_1 = 14.1925$

$S_2 = 11.5431$

$$Sp = \sqrt{\frac{7 \cdot S_1^2 + 11 \cdot S_2^2}{20 - 2 = 18}}$$

$$= \sqrt{159.75} \quad \checkmark 12.6392$$

$$t = \frac{65 - 54.833}{Sp \cdot \sqrt{\frac{1}{8} + \frac{1}{12}}}$$

$C.V = 2.101$

$= \cancel{0.02904}\ \cancel{0.1394}\ 1.7623$

$t < C.V$ accept H_0

Import numpy as np from scipy import stats.

b_1 = np.array([ ])

b_2 = np.array([ ])

Stat.ttest

Matrices.

variables

10x15

row comes 1st then columns.

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} 4 \times 3 \quad (m \times n)$$

rectangular matrix.

square matrix $\rightarrow n \times n$

diagonal matrix $\rightarrow \begin{bmatrix} 12 & 0 \\ 0 & 13 \end{bmatrix}$

Symmetric matrix $\rightarrow \begin{bmatrix} 12 & 13 \\ 13 & 12 \end{bmatrix} \Rightarrow [M = M^T]$

$a_{ij} = a_{ji}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ square & symmetric.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ identity matrix.

Operation :- Addition & subtraction
same order matrix can be addition operation

$$A+B = \begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}5\\7\\9\end{bmatrix}$$

$$D = \begin{bmatrix}6 & 3\\9 & 4\\0 & 7\end{bmatrix}_{3\times2} \quad C = \begin{bmatrix}1 & 2\\3 & 4\\11 & 31\end{bmatrix}_{3\times2} \quad D+C = \begin{bmatrix}7 & 5\\12 & 8\\11 & 38\end{bmatrix}_{3\times2}$$

Multiplication.

Eg :-

* Scalar multiplication → $M\times2 = 2M$, $M \times \frac{1}{2} = \frac{M}{2}$
* matrix multiplication ⇒

$(m\times p) \times (p\times n) \Rightarrow m\times n$.

$$\begin{bmatrix}15 & 6 & 4\\1 & 2 & 3\end{bmatrix}_{2\times3}\begin{bmatrix}-5 & -1\\-6 & 0\\11 & 24\end{bmatrix}_{3\times2} = \begin{bmatrix}-75+-36+44 & -15+0+96\\-5-12+33 & -1+0+72\end{bmatrix} = \begin{bmatrix}-67 & 81\\16 & 71\end{bmatrix}_{2\times2}$$

Transpose matrix $(A) = A^T$.

- Transpose
- symmetric
- Identity
- Determinant

Square matrix

a_{22}

→ Determinant :- square matrix, $|M| = \begin{vmatrix}a & b\\c & d\end{vmatrix} = (ad-bc)$

$$M = \begin{bmatrix}10 & 12\\9 & 7\end{bmatrix}$$

$$\begin{matrix}a & b & c\\d & e & f\\g & h & i\end{matrix}$$

$|M| = 70 - 108 = -38$

$MM^{-1} = I$

M^{-1} does not exist when $|M|=0$

$$A = \begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}_{2\times2} \quad v = \begin{bmatrix}1\\0\end{bmatrix}_{2\times1} \quad u = \begin{bmatrix}0\\1\end{bmatrix} \quad w = \begin{bmatrix}1\\1\end{bmatrix}$$

$$Av = \begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}_{2\times2}\begin{bmatrix}1\\0\end{bmatrix}_{2\times1} = \begin{bmatrix}2\\0\end{bmatrix}_{2\times1}$$

$$Au = \begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

$$Aw = \begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$$

* v, u are the eigen vectors of A.
* 2, 3 are eigen values of A.

Eigen values and E vectors

$Av = \lambda v$ eigen value condition

vector, E value

$(A - \lambda I)v = 0 \Rightarrow (A - \lambda I) = 0$

$$B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$2 - 3\lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$(\lambda - 5)(\lambda + 2) = 0$$

$$\lambda = 5, \ \lambda = -2$$

$$B - \lambda I = \begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix}$$

$$|B - \lambda I| = 0 \Rightarrow (1-\lambda)(2-\lambda) - 12 = 0$$

$$\begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$-4x_1 + 4x_2 = 0$$

$$3x_1 + -3x_2 = 0 \quad x + 4y = 0$$

$$3x_1 + 4x_2 = 0$$

$$3x_1 = 4x_2$$

$$x_1 = \frac{4}{3}x_2$$

$$x = -\frac{4}{3}y$$

$$x\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-4x + 4y =$$

$$x = y$$

$$x_1\begin{bmatrix} -\frac{4}{3} \\ 1 \end{bmatrix}$$

$$x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$O = \begin{bmatrix} 0 & 1 \\ +1 & 0 \end{bmatrix}$$

$$AM = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ +1 & 0 \end{bmatrix}}\underset{2\times 1}{\begin{bmatrix} 2 \\ 0 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

emin

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1, \ \lambda = 1, \ \lambda = -1$$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$-x_1 + x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$$x_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$x_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ +1 \end{pmatrix}$$

$$x_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$(2-\lambda)^2 = 0$.

$\lambda = 2, 2$.

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$x_2 = 0$.

$$\begin{pmatrix} x_1 \\ 0 \end{pmatrix} \Rightarrow x_1 * \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Probability.

event = get an outcome from sample space.

probability = chances of happening. $\{H, T\}$

$$= \frac{(\text{event})\ \text{count}}{(\text{Sample})\ \text{count}}.$$

~~T, T, T, H, T, H, T, T, H, T~~

Law of large nos → sample large no: assignment expected probability aryo.

Events

independent dependent.

Coin

$P(H) = 1/2$

die

$P(\text{odd}) = 3/6$

$P(\text{odd}, H) = P(\text{odd}) \cdot P(H) = 1/2 \cdot 1/2 = 1/4$

$P(A \cap B) = P(A) \cdot P(B/A)$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Joint probability.

14 · Coffee 18 · Tea.

more sugar 12 more sugar 3

no sugar 2 no sugar 15

	more sugar	no sugar	
C	12/32	2/32	14/32
T	3/32	15/32	18/32
	15/32	17/32	

$$P(C, m) = \frac{12}{32}$$

$$P(C) \cdot P(m) = \frac{\cancel{14}^{7}}{32} \cdot \frac{15}{\cancel{32}_{16}}$$

$$\frac{14}{32} \cdot \frac{12}{14} = \frac{12}{32}$$

unconditional

$C = \frac{15}{32}$ $T = \frac{17}{32}$ more sugar $= \frac{2}{32}$

no sugar $= \frac{30}{32}$

only for Independent events :—

$P(C)$

$P(C, m) = P(C) \cdot P(m) = \frac{15}{32} \cdot \frac{30}{32}$

$P(C, n) = P(C) \cdot P(n)$

$P(T, m) = P(T) \cdot P(m)$

$P(T, n) = P(T) \cdot P(n)$.

only for independent events.

	m	n	
C	$\frac{15 \cdot 2}{32^2}$	$\frac{30 \cdot 15}{32^2}$	$\rightarrow \frac{480}{32^2}$
T	$\frac{17 \cdot 2}{32^2}$	$\frac{17 \cdot 30}{32^2}$	$\frac{544}{32^2}$
	$\frac{64}{32^2}$	$\frac{960}{32^2}$	

$P(C) = P(C, m) + P(n, m) = \frac{480}{32^2} = \frac{15}{32}$

$P(T) = \frac{17 \cdot 2}{32^2} + \frac{17 \cdot 30}{32^2} = \frac{544}{32^2} = \frac{17}{32}$

$\frac{15 \cdot 2}{32 \cdot 32} + \frac{30 \cdot 15}{32 \cdot 32}$

$\frac{480}{32 \cdot 32}$

Marginal probability

Joint probabilities

Specs — $9/32$ without specs $23/32$

Girls — $15/32$

Boys — $17/32$

	S	nS	
G	$P(G, S)$	$P(G, nS)$	$\rightarrow P(G)$
B	$P(B, S)$	$P(B, nS)$	$\rightarrow P(B)$
	$\downarrow$ $P(S)$	$\downarrow$ $P(nS)$	

marginal

	S	N.S.	
G	$15/32 \cdot 9/32 = 135/32^2$	$15/32 \cdot 23/32$ $345/32^2$	$15/32$
B	$17/32 \cdot 9/32$ 153	$17/32 \cdot 23/32$ $391/32^2$	$17/32$
	$9/32$	$23/32$	1

marginal probability $\Rightarrow 9/32$

$P(G, S) = P(G) \cdot P(S)$

$P(G) = 15/32 \cdot 9/32 + 15/32 \cdot 23/32$

$= 15/32\,(32/32) = 15/32$

* Venn diagram

* conditional probability

$P(A|B)$ given B

$P(B|A)$ given A

$P(R|C) = \frac{P(R \cap C)}{P(C)}$

$P(C|R) = \frac{P(C \cap R)}{P(R)}$

$P(R \cap C) = P(R|C) \cdot P(C)$

$P(R \cap C) = P(C|R) \cdot P(R)$

$P(R|C) \cdot P(C) = P(C|R) \cdot P(R)$

$P(R|C) = \frac{P(C|R) \cdot P(R)}{P(C)}$: Bayes theorem.

$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$.

$P(A)$ — Prior

$P(B|A)$ — likelihood

$P(A|B)$ — posterior

$P(B)$ — evidence.

Raining

$P(R) = 0.75$

$P(R') = 0.25$

$P(C) = 0.6$

$P(C') = 0.4$

$P(C/R)$

not raining

$P(C) = 0.1$

$P(C') = 0.9$

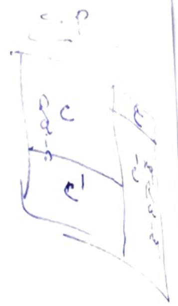
$P(C/R')$

$$P(R|C) = \frac{P(C/R) \cdot P(R)}{P(C)}$$

$$= \frac{0.6 \times 0.75}{0.6 \times 0.75 + 0.1 \times 0.25}$$

$0.6\times$

$$= \underline{\underline{0.9474}}$$



$$P(C) = P(C \cap R) + P(C \cap R')$$
$$= P(C/R) \cdot P(R) + P(C/R')P(R')$$

$$P(R|C) = \frac{P(C/R) \cdot P(R)}{P(C/R)P(R) + P(C/R')P(R')}$$