

Simultaneous Sources and Sparse Update for Static Electromagnetic Geophysics

Thibaut Astic

EOSC 513 2017: Project
<https://github.com/thast/EOSC513>

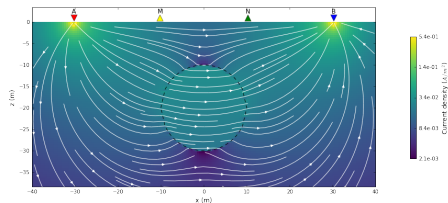
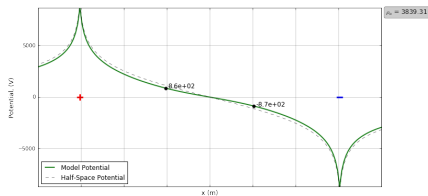
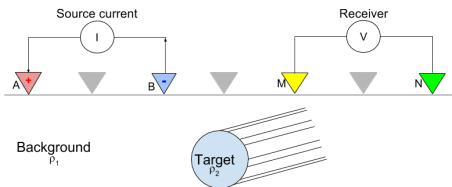
Outline

- 1 Motivation
- 2 Simultaneous Sources
- 3 Sparse Gauss Newton Update
- 4 Discussion and Conclusion

Outline

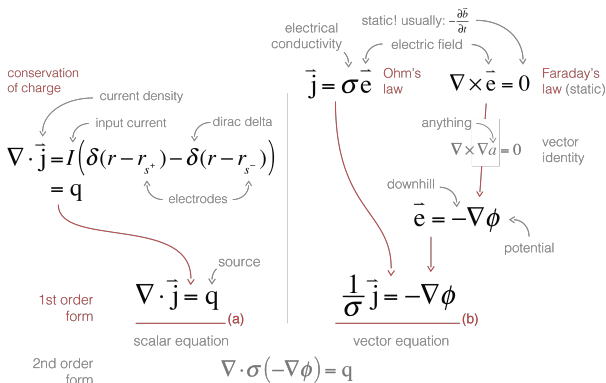
- 1 Motivation
- 2 Simultaneous Sources
- 3 Sparse Gauss Newton Update
- 4 Discussion and Conclusion

DC Resistivity Survey



Currents are injected to the earth. Their flow will be distorted depending upon the conductivity contrast in the earth. These changes can be measurable on the surface electrodes. [Interactive notebooks](#)

DCR Physics: Maxwell's equations



Rowan Cockett, Lindsey J. Heagy, and Douglas W. Oldenburg (2016). "Pixels and their neighbors: Finite volume." *The Leading Edge*, 35(8), 703–706. doi: 10.1190/tle35080703.1

DCR: inverse Problems

- SimPEG: open-source python package for EM Geophysics
- discretize using a Finite Volume Formulation

$$\text{diag}(\mathbf{v})\mathbf{D}\mathbf{M}_f(\sigma^{-1})^{-1}\mathbf{D}^\top\text{diag}(\mathbf{v})\boldsymbol{\phi} = \mathbf{q}. \quad (1)$$

- Solve for $m = \ln(\sigma)$ to ensure positivity of conductivity
- Use a beta-cooling strategy for the regularizer's weight

$$f(m) = \frac{1}{2} \sum_{i=1}^{N_s} \|P_i A(m)^{-1} q_i - d_i\|_2^2 + \beta R(m) \quad (2)$$

$$f(m) = \frac{1}{2} \|PA(m)^{-1}Q - D\|_F^2 + \beta R(m) \quad (3)$$

Usually:

$$R(m) = \frac{1}{2} \|W(m - m_{ref})\|_2^2 \quad (4)$$

DCR: Solving the inverse problem

- solve a linearized problem
- Inexact Gauss-Newton steps

$$f(m) = \frac{1}{2} \|r(m)\|_2^2 + \beta \|W(m - m_{ref})\|_2^2 \quad (5)$$

$$f(m + \delta m) = \frac{1}{2} \|r(m) + J(m)\delta m\|_2^2 + \beta \|W(m + \delta m - m_{ref})\|_2^2 \quad (6)$$

$$(J^T J + \beta W^T W) \delta m = -J^T r(m) - \beta W^T W(m - m_{ref}) \quad (7)$$

Update m : $m_{k+1} = m_k + \alpha \delta m$ s.t. $f(m_{k+1}) < \gamma f(m_k)$

DCR: inverse Problems Results

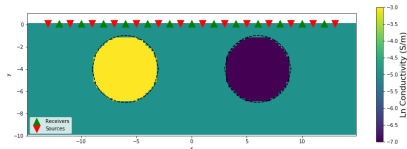


Figure: Initial model and survey

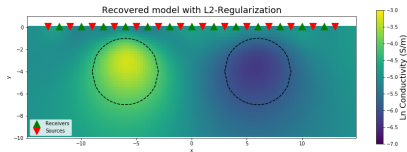


Figure: Recovered model with L2 Regularization

Problematic

- The cost of inverting DC data is mainly driven by the number of sources:
 - Can we decrease this cost? What is the price to pay?
- The diffusive nature of the physics behind DCR and the L2-Regularization are promoting smooth update
 - Can we, in some space, promote sparsity for such a problem?

Outline

- 1 Motivation
- 2 Simultaneous Sources**
- 3 Sparse Gauss Newton Update
- 4 Discussion and Conclusion

Reducing the number of Sources

We start to try one of the basic idea of Compressive Sensing by using simultaneous sources.

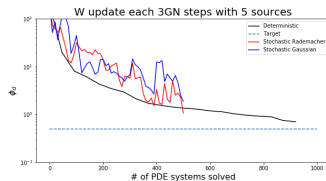
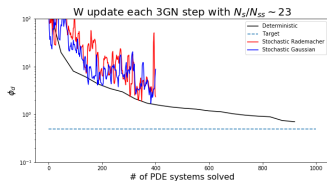
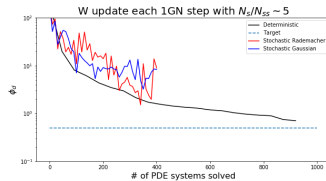
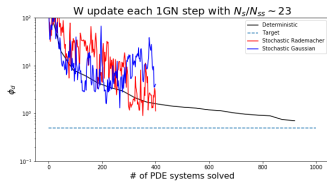
E. Haber, M. Chung, and F. Herrmann, “An Effective Method for Parameter Estimation with PDE Constraints with Multiple Right-Hand Sides,” SIAM Journal on Optimization, vol. 22, no. 3, pp. 739–757, jul 2012.

$$f(m) = \frac{1}{2} \|PA(m)^{-1}QW - DW\|_F^2 + \beta R(m) \quad (8)$$

W a random gaussian matrix of size $N_s \times N_{ss}$ with $N_{ss} \ll N_s$
 The Gauss-Newton step subproblem's size is then reduced by a factor $\frac{N_s}{N_{ss}}$

We have then experiment several strategies for updating W.

Updating W



Outline

- 1 Motivation
- 2 Simultaneous Sources
- 3 Sparse Gauss Newton Update**
- 4 Discussion and Conclusion

Formulating the GN subproblem

Original Data fitting subproblem:

$$\min_{\delta m} \frac{1}{2} \|J(m)\delta m + r(m)\|_2^2 + \beta \|W(m + \delta m - m_{ref})\|_2^2 \quad (9)$$

Now without the L2 penalty but a L1 constrain in some space:

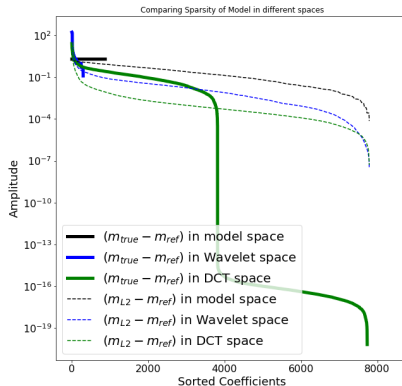
$$\min \|\delta x\|_1 \text{ s.t. } \|J(m)S^H\delta x + r(m)\|_2^2 < tol \quad (10)$$

- $\delta x = S\delta m$
- S a sparsifying matrix
- S^H the inverse transform.
- solved using SPGL1

E. van den Berg and M. P. Friedlander, "Probing the Pareto frontier for basis pursuit solutions", SIAM J. on Scientific Computing, 31(2):890-912, November 2008

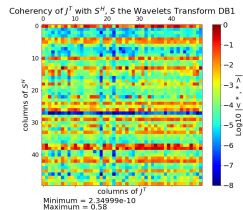
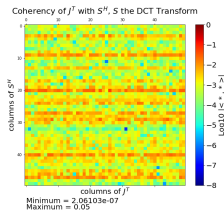
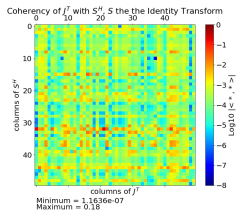
Choosing a sparsifying transform

- The model to recover should be sparse in some space



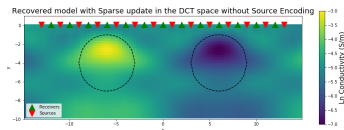
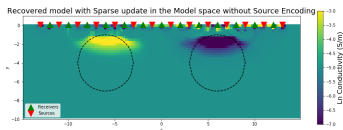
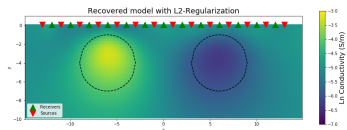
Choosing a sparsifying transform

- the columns of J^T and S^H should be incoherent
- The DC problem is a weighted Laplacian
- The eigenfunctions of the Laplacian are cosine functions



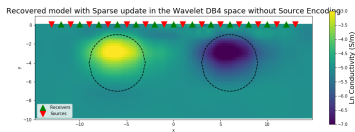
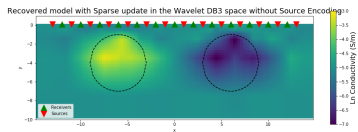
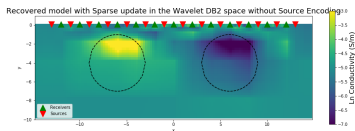
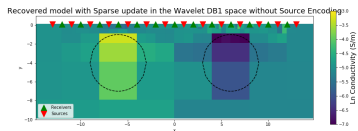
Recovered Models using sparse GN update: $S = \text{Identity}$ or DCT

- Without simultaneous sources

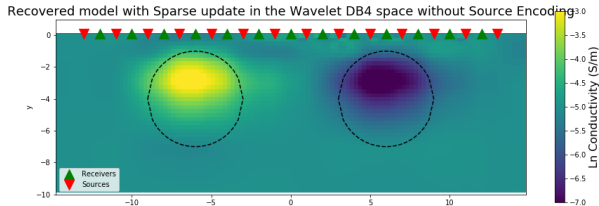
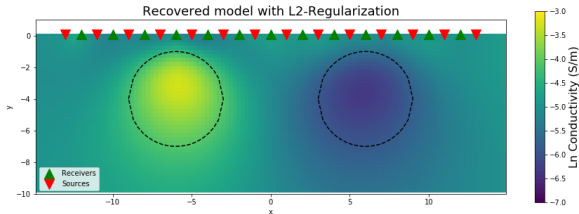


Recovered Models using sparse GN update in Wavelets space

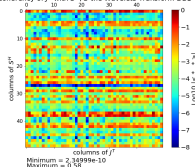
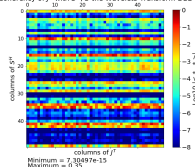
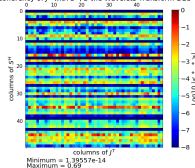
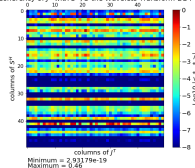
- Without simultaneous sources



Recovered Models using sparse GN update in Wavelets space



Coherency of J and Wavelet transform

Coherency of J^T with S^H , S the Wavelets Transform DB1Coherency of J^T with S^H , S the Wavelets Transform DB2Coherency of J^T with S^H , S the Wavelets Transform DB3Coherency of J^T with S^H , S the Wavelets Transform DB4

Possible explanation
(hypothesis): asymptotic
Incoherence

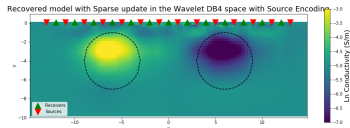
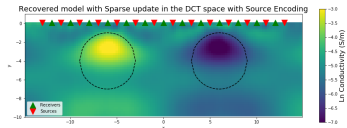
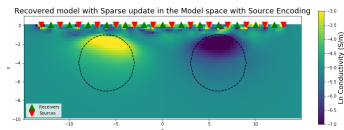
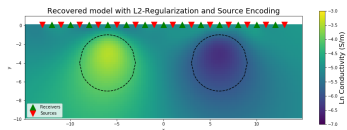
Adcock, Ben, et al.

”Breaking the coherence
barrier: A new theory for
compressed sensing.”

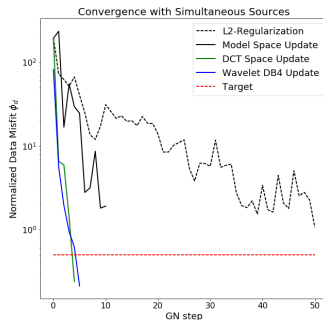
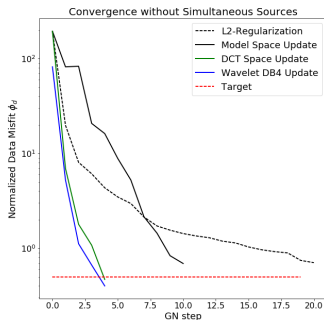
Forum of Mathematics,
Sigma. Vol. 5. Cambridge
University Press, 2017.

Recovered Models using sparse GN update and Simultaneous Sources

- With simultaneous sources



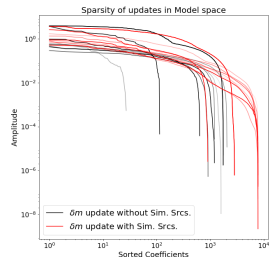
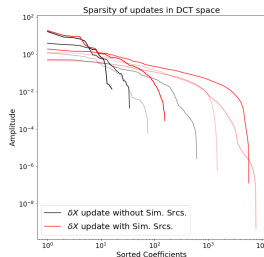
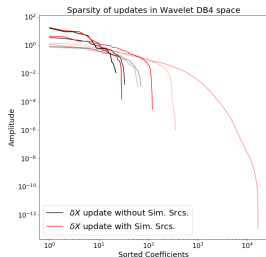
Convergence Comparison



- L2 is still faster as each SPGL1 GN update is much more expensive to compute
- SPGL1 GN update seems even harder with Simultaneous Sources!

A curious behavior

- Simultaneous Sources is killing the sparsity



Outline

- 1 Motivation
- 2 Simultaneous Sources
- 3 Sparse Gauss Newton Update
- 4 Discussion and Conclusion**

Discussion and Conclusion

- Simultaneous Sources
 - In terms of number of PDE solved for L2, the overall convergence rate is similar or worse
 - each step is a lot less memory-expansive, Useful if we have too many sources for our memory
 - for SPGL1, it has appeared to make things even worse, with longer time necessary to compute each GN update

Discussion and Conclusion

- Sparse GN Update
 - At first, antagonization between sparsity in model space and the diffusive behavior of the Jacobian
 - Sparse update in model space are expensive to solve, sparse in DCT space is easier but recovered model is still smooth and display periodic patterns
 - Surprisingly, Wavelets have been an efficient transform despite a somehow high maximum for coherency
 - Simultaneous Sources has been a killer both for sparsity and computation times during our experiment
 - We did not study much the influence of the tolerance for the data fitting term in SPGL1 or the strategy to set it.
Considering the time required for each iteration and the few iterations required to reach the target, we have probably been quite strict.

Thank you!

All scripts and more are available on [Github/thast/](https://github.com/thast/)

Bonus: Another way to recover a compact model through prior information and vector quantization:

