Application of compressive sensing ideas to Static Electromagnetic Geophysics

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EOSC 513 2017: Project
https://github.com/thast/EOSC513

Outline

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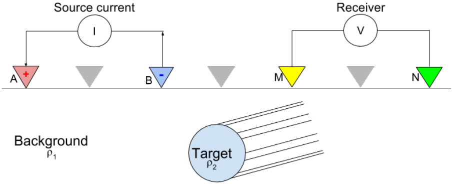
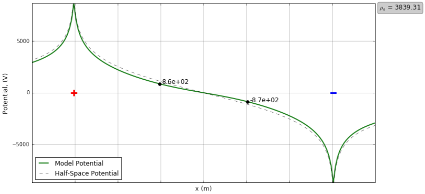
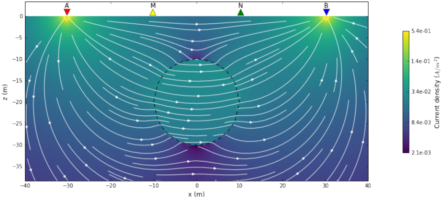
DC Resistivity Survey







Currents are injected to the earth. Theirs flow will be distorted depending upon the conductivity contrast in the earth These changes can be measurable on the surface electrodes. Interactive notebooks

DCR Physics: Maxwell's equations

conservation of charge

current density

input current

dirac delta

$$\nabla \cdot \vec{j} = I\left(\delta(r - r_{s^+}) - \delta(r - r_{s^-})\right) = q$$

electrodes

electrical conductivity

static! usually: $-\frac{\partial \vec{b}}{\partial t}$

electric field

$$\vec{j} = \sigma \vec{e}$$ Ohm's law

$$\nabla \times \vec{e} = 0$$ Faraday's law (static)

anything

$$\nabla \times \nabla a = 0$$ vector identity

downhill

$$\vec{e} = -\nabla \phi$$

potential

source

1st order form

$$\nabla \cdot \vec{j} = q \quad \text{(a)}$$

scalar equation

$$\frac{1}{\sigma}\vec{j} = -\nabla \phi \quad \text{(b)}$$

vector equation

2nd order form

$$\nabla \cdot \sigma(-\nabla \phi) = q$$

Rowan Cockett, Lindsey J. Heagy, and Douglas W. Oldenburg (2016). "Pixels and their neighbors: Finite volume." The Leading Edge, 35(8), 703–706. doi: 10.1190/tle35080703.1

DCR: inverse Problems

- SimPEG: open-source python package for EM Geophysics
- discretize using a Finite Volume Formulation

$$\mathrm{diag}(\mathbf{v})\mathbf{D}\mathbf{M}_f(\sigma^{-1})^{-1}\mathbf{D}^\top\mathrm{diag}(\mathbf{v})\boldsymbol{\phi} = \mathbf{q}. \tag{1}$$

- Solve for $m = \ln(\sigma)$ to ensure positivity of conductivity
- Use a beta-cooling strategy for the regularizer's weight

$$f(m) = \frac{1}{2}\sum_{i=1}^{N_s} ||P_i A(m)^{-1}q_i - d_i||_2^2 + \beta R(m) \tag{2}$$

$$f(m) = \frac{1}{2}||PA(m)^{-1}Q - D||_F^2 + \beta R(m) \tag{3}$$

Usually:

$$R(m) = \frac{1}{2}||W(m - m_{ref})||_2^2 \tag{4}$$

DCR: Solving the inverse problem

- solve a linearized problem
- Inexact Gauss-Newton steps

$$f(m) = \frac{1}{2}||r(m)||_2^2 + \beta||W(m - m_{ref})||_2^2 \tag{5}$$

$$f(m + \delta m) = \frac{1}{2}||r(m) + J(m)\delta m||_2^2 + \beta||W(m - m_{ref})||_2^2 \tag{6}$$

$$(J^T J + \beta W^T W)\delta m = -J^T r(m) - \beta W^T W(m - m_{ref}) \tag{7}$$

Update m: $m_{k+1} = m_k + \alpha\delta m$ s.t $f(m_{k+1}) < \gamma f(m_k)$

DCR: inverse Problems Results

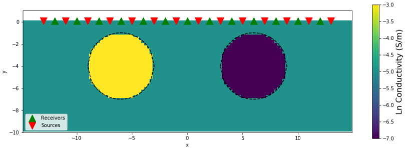


Figure: Initial model and survey

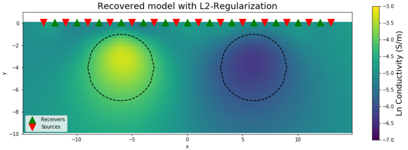


Figure: Recovered model with L2 Regularization

Problematic

- The cost of inverting DC data is mainly driven by the number of sources:
    - Can we decrease this cost? What is the price to pay?
- The diffusive nature of the physics behind DCR and the L2-Regularization are promoting smooth update
    - Can we, in some space, promote sparsity for such a problem?

Outline

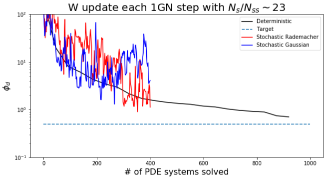
Reducing the number of Sources

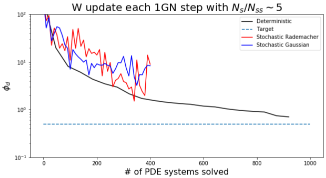
We start to try one of the basic idea of Compressive Sensing by using simultaneous sources.

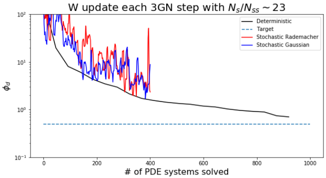
$$f(m) = \frac{1}{2}||PA(m)^{-1}QW - DW||_F^2 + \beta R(m) \tag{8}$$

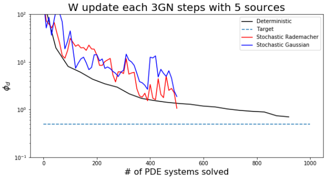
W a random gaussian matrix of size $N_s \times N_{ss}$ with $N_{ss} << N_s$
The Gauss-Newton step subproblem's size is then reduced by a factor $\frac{N_s}{N_{ss}}$
We have then experiment several strategies for updating W.

Updating W


W update each 1GN step with $N_s/N_{ss} \sim 23$
Deterministic
Target
Stochastic Rademacher
Stochastic Gaussian
ϕ_d
# of PDE systems solved


W update each 1GN step with $N_s/N_{ss} \sim 5$
Deterministic
Target
Stochastic Rademacher
Stochastic Gaussian
ϕ_d
# of PDE systems solved


W update each 3GN step with $N_s/N_{ss} \sim 23$
Deterministic
Target
Stochastic Rademacher
Stochastic Gaussian
ϕ_d
# of PDE systems solved


W update each 3GN steps with 5 sources
Deterministic
Target
Stochastic Rademacher
Stochastic Gaussian
ϕ_d
# of PDE systems solved

Outline

Formulating the GN subproblem

Original Gauss-Newton subproblem:

$$(J^T J + \beta W^T W)\delta m = -J^T r(m) - \beta W^T W(m - m_{ref}) \qquad (9)$$

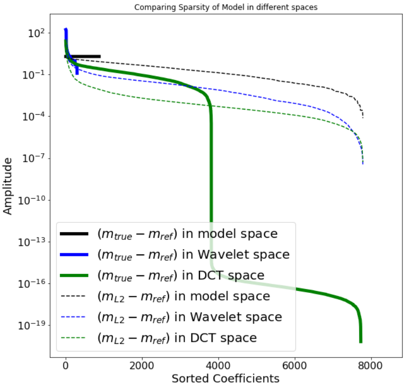
Now without the L2 penalty but a L1 constrain in some space:

$$\min||\delta x||_1 \text{ s.t } ||J(m)S^H \delta x - r(m)||_2^2 < tol \qquad (10)$$

- $\delta x = S\delta m$
- S a sparsifying matrix
- S^H the inverse transform.
- solved using SPGL1
- tolerance is chosen such that it performs at least as the L2-norm regularized problem for each GN step

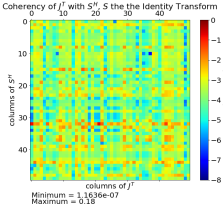
Choosing a sparsifying transform

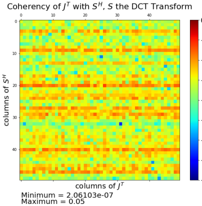
- The model to recover should be sparse in some space

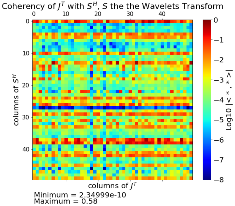


Choosing a sparsifying transform

- the columns of J^T and S^H should be incoherent
- The DC problem is a weighted Laplacian
- The eigenfunctions of the Laplacian are cosine functions

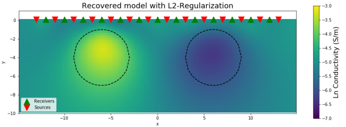


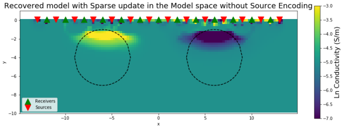


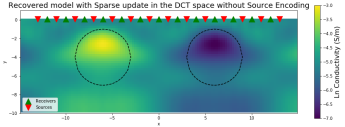


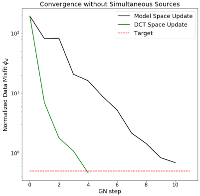
Recovered Models using sparse GN update

- Without Source Encoding









Recovered Models using sparse GN update

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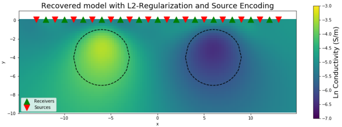
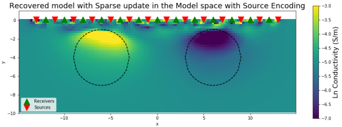
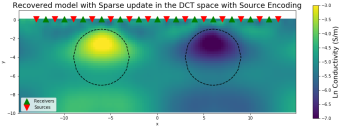
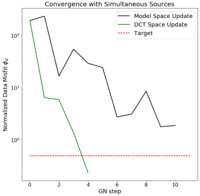


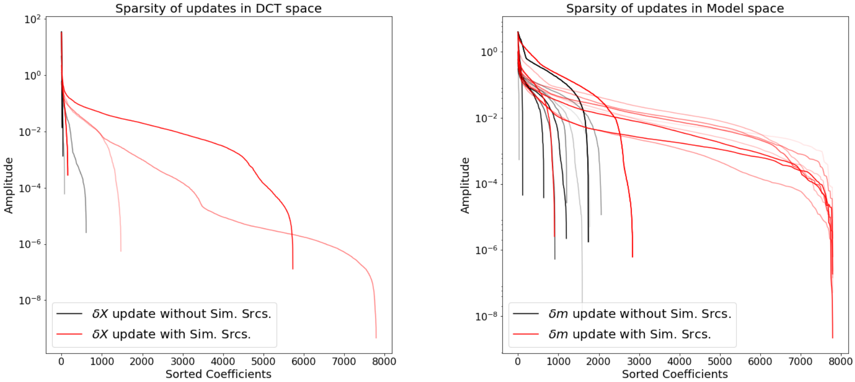






A curious behaviour

- Simulateneous Sources is killing sparsity



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Discussion and Conclusion

- Simultaneous Sources
  - In terms of number of PDE solved, the overall convergence rate is similar or worse
  - each step is a lot less memory-expansive, Useful if we have too many sources for our memory
- Sparse GN Update
  - Antagonization between sparsity in model space and the diffusive behavior of the Jacobian
  - Sparse update in model space are expansive to solve, sparse in DCT space is easier but recovered model is still smooth and display periodic patterns
  - Simultaneous Sources has been a killer for sparsity during our experiment. Several hypothesis can be formulated to explain this behavior but no definitive answer.

Thank you!

All scripts and more are available on Github/thast/

Bonus: Another way to recover a compact model through prior information and vector quantization: