

IDS - Homework 3

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$$\begin{aligned}
 1. \ a) \ F(A, B, C, D) &= D + A(CD + B) + \bar{B}C\bar{D} \\
 &= \underline{D} + \underline{ACD} + AB + \bar{B}C\bar{D} \quad \text{associative law} \\
 &= \underline{D} + AB + \underline{\bar{B}C\bar{D}} \quad \text{absorption law} \\
 &= D + \bar{B}C + AB \quad \text{absorption (variant) law}
 \end{aligned}$$

before	after
L = 8	L = 5
G = 12	G = 7
GN = 14	GN = 8

$$\begin{aligned}
 b) \ F(A, B, C, D) &= (\overline{DD} + AB) \cdot C + DC \\
 &= (\bar{D} + \bar{D} + AB) C + DC \quad \text{de morgan's law} \\
 &= (\bar{D} + AB) C + DC \quad \text{idempotency law} \\
 &= C(\bar{D} + AB + D) \quad \text{distributive law} \\
 &= C(1 + AB) \quad \text{inverse law} \\
 &= C \quad \text{one element law}
 \end{aligned}$$

before	after
L = 7	L = 1
G = 12	G = 1
GN = 13	GN = 1

$$\begin{aligned}
 c) F(A, B, C, D) &= D(AC + C(\overline{B}B + DA)) + AC \\
 &= D(AC + C(B + DA)) + AC && \text{idempotency law} \\
 &= D(AC + BC + ACD) + AC && \text{distributive law} \\
 &= D(C(A + B + AD)) + AC && \text{distributive law} \\
 &= D(C(B + A)) + AC && \text{absorption law} \\
 &= D(BC + AC) + AC && \text{distributive law} \\
 &= BCD + \underline{ACD} + \underline{AC} && \text{distributive law} \\
 &= BCD + AC && \text{absorption law} \\
 &= C(BD + A) && \text{distributive law}
 \end{aligned}$$

before	after
L = 10	L = 1
G = 18	G = 6
GN = 18	GN = 6

2. a)  $F(A, B, C, D) = \sum m(1, 2, 3, 5, 6, 7, 10, 11, 14, 15)$   
 $\prod M(0, 4, 8, 9, 12, 13)$

SOP

AB \ CD	00	01	11	10
$\overline{A}$ 00	0	1	1	1
$\overline{A}$ 01	1	1	1	1
A 11	1	1	1	1
A 10	1	1	1	1
	$\overline{D}$	D	$\overline{D}$	D

POS

AB \ CD	00	01	11	10
$\overline{A}$ 00	0	1	1	1
$\overline{A}$ 01	0	1	1	1
A 11	0	1	1	1
A 10	0	1	1	1
	D	$\overline{D}$	D	$\overline{D}$

Optimized SOP =  $C + \overline{A}D$

Optimized POS =  $(C + D)(\overline{A} + C)$

b)  $F(A, B, C, D) = \prod M(1, 3, 5, 7, 9, 11)$   
 $\sum m(0, 2, 4, 6, 8, 10, 12, 13, 14, 15)$

SOP

AB \ CD	00	01	11	10
$\bar{A}$	0	1	3	2
A	4	5	7	6
$\bar{A}$	8	9	11	10
A	12	13	15	14
$\bar{D}$				
D				
$\bar{B}$				
B				
$\bar{B}$				
B				

POS

AB \ CD	00	01	11	10
$\bar{A}$	0	1	3	2
A	4	5	7	6
$\bar{A}$	8	9	11	10
A	12	13	15	14
$\bar{D}$				
D				
$\bar{B}$				
B				
$\bar{B}$				
B				

Optimized SOP =  $\bar{D} + AB$

Optimized POS =  $(A + \bar{D}) \cdot (B + \bar{D})$

c)  $F(A, B, C, D) = (\bar{A} \cdot A + A \cdot C) \cdot B + \bar{D} A$   
 $= \boxed{ABC} + \boxed{\bar{D} A}$

inverse law

SOP

AB \ CD	00	01	11	10
$\bar{A}$	0	1	3	2
A	4	5	7	6
$\bar{A}$	8	9	11	10
A	12	13	15	14
$\bar{D}$				
D				
$\bar{B}$				
B				
$\bar{B}$				
B				

POS

AB \ CD	00	01	11	10
$\bar{A}$	0	1	3	2
A	4	5	7	6
$\bar{A}$	8	9	11	10
A	12	13	15	14
$\bar{D}$				
D				
$\bar{B}$				
B				
$\bar{B}$				
B				

Optimized SOP =  $ABC + A\bar{D}$

Optimized POS =  $A \cdot (C + \bar{D}) \cdot (B + \bar{D})$

$$\begin{aligned}
 3. a) F(A, B, C, D) &= CD + (AB + \underline{C})C \\
 &= CD + (\underline{AB + C})C \quad \text{idempotency law} \\
 &= \underline{CD} + \underline{C} \quad \text{absorption law} \\
 &= C \quad \text{absorption law}
 \end{aligned}$$

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

$$\begin{aligned}
 \text{SOM: } F(A, B, C, D) &= \sum_m (2, 3, 6, 7, 10, 11, 14, 15) \\
 \text{POM: } F(A, B, C, D) &= \prod_m (0, 1, 4, 5, 8, 9, 12, 13)
 \end{aligned}$$

$$\begin{aligned}
 b) F(A, B, C, D) &= \overline{D}A + B(CB + \underline{DD}) \\
 &= \overline{D}A + B(CB + D) \\
 &= \overline{D}A + \underline{CBB} + BD \\
 &= \overline{D}A + CB + BD \\
 &= \overline{D}A + B(C + D)
 \end{aligned}$$

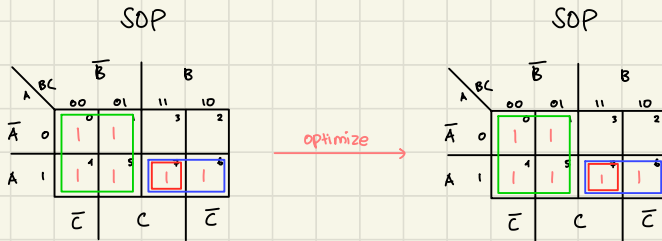
A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$\begin{aligned}
 \text{SOM: } F(A, B, C, D) &= \sum_m (5, 6, 7, 8, 10, 12, 13, 14, 15) \\
 \text{POM: } F(A, B, C, D) &= \prod_m (0, 1, 2, 3, 4, 9, 11)
 \end{aligned}$$

1. a)  $F(A, B, C) = A(BA + BC) + \overline{B}\overline{B}$

$= AAB + ABC + \overline{B}\overline{B}$  distributive law

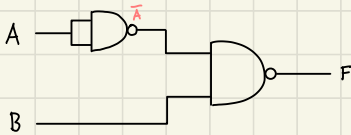
$= AB + ABC + \overline{B}$  idempotency law



Optimized SOP:  $F = A + \overline{B}$

$\overline{\overline{F}} = \overline{A + \overline{B}}$  double complement (to make converting into NAND gates easier)

$F = \overline{(\overline{A} \cdot B)}$  involution & de Morgan's law



$$b) F(A, B, C) = (BC + CC)A + CB$$

$$= (BC + C)A + CB$$

Idempotency law

$$= \boxed{ABC} + \boxed{AC} + \boxed{CB}$$

distributive law

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SOP

	BC		$\bar{B}$		B	
$\bar{A}$	00	01	11	10		
0	0	0	1	1		
1	1	1	1	1		
	$\bar{C}$		C		$\bar{C}$	

Change to POS

POS

	BC		$\bar{B}$		B	
$\bar{A}$	00	01	11	10		
0	0	0	0	0		
1	0	0	0	0		
	C		$\bar{C}$		C	

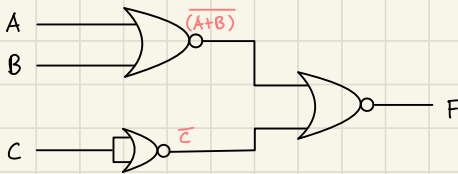
Optimized POS :  $F = C \cdot (A+B)$

$$\bar{\bar{F}} = \overline{C \cdot (A+B)}$$

double complement (to make converting into NOR gates easier)

$$F = \bar{C} + \overline{(A+B)}$$

involution & de morgan's law



5. SOM :  $F(A, B, C, D, E) = \sum_m (0, 1, 2, 3, 5, 6, 12, 13, 14, 15, 17, 18, 24, 25, 26, 27, 29, 30)$

		$\bar{A}$				
		$\bar{D}$		$D$		
BC	DE	00	01	11	10	$\bar{C}$
	00	0	1	1	1	
$\bar{B}$	01	1	1	1	1	$C$
	11	1	1	1	1	
$B$	10	1	1	1	1	$\bar{C}$
	00	1	1	1	1	
		$\bar{E}$	$E$	$\bar{E}$	$E$	

		A				
		$\bar{D}$		D		
BC	DE	00	01	11	10	$\bar{C}$
	00	1	1	1	1	
$\bar{B}$	01	1	1	1	1	C
	11	1	1	1	1	
B	10	1	1	1	1	$\bar{C}$
	00	1	1	1	1	
		$\bar{E}$	E	$\bar{E}$	E	

Prime implicants :  $\bar{A}\bar{B}$ ,  $\bar{B}\bar{C}$ ,  $\bar{A}C$ ,  $BC$ ,  $A\bar{C}$ ,  $AB$

Essential Prime implicants : none