hypothesis. The observed redshifts were allowed a large, non-cosmological, component and groups of QSOs could now have discordant redshifts.

Our observation of a close pair of QSOs with discordant redshifts is unlikely under the cosmological hypothesis. Sandage and Luyten[1] estimate that there are five QSOs brighter than magnitude 19.4 per square degree. If this is true and if QSOs are randomly distributed in the sky the chance of finding a second QSO within 5 arc s of a given object is approximately 4×10^{-5}. The probability of finding such a pair among the 250 QSOs with known redshifts is about 1 in 100 if there were no observation selection effects. Present observational techniques certainly discriminate against finding close doubles since normally the search is stopped when one candidate from a group of stars is found to be a QSO.

A local hypothesis which puts 10% of the QSOs in pairs of galactic cluster dimensions, say one minute of arc, is at a statistical advantage by a factor of thirty or so. The radio alignment, when accurately measured, may provide additional evidence. The spectral peculiarities are fascinating but no pre-existing theory suggested them and these observations only raise them for future consideration.

We believe that these observations add support to non-cosmological theories of redshifts. Such theories will receive additional support if an interaction between the two components of a close pair can be shown. Alternatively, additional close pairs may be found. Since it is now known that some large redshift QSOs have neutral colours[8], a search for more QSO pairs should include all stars found near suspected or known QSOs.

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Discrepant Redshifts of QSOs in Clusters of Galaxies and a Close QSO Pair

ALTHOUGH it is the majority view that QSO redshifts are cosmological in origin and related to distance by Hubble's law, several workers[1–3] have proposed that QSOs may be more local objects. The strongest direct evidence for the conventional view is provided by observations of a number of QSOs which have been shown to have redshifts (z) which agree to within about 1% with the redshifts of apparently associated clusters of galaxies[4–8]. These QSO-cluster associations have all been found in searches restricted to QSOs with $z<0.36$ since on the cosmological hypothesis it is only for such low values of z that the associated galaxies would be readily visible. But, as Burbidge and O'Dell[9] have pointed out, by assuming at the outset that the redshifts are cosmological these investigations fail to test the non-cosmological hypothesis. For the past several years we have been attempting to carry out a study of QSOs in clusters or groups which would be free from this objection. Although our results are by no means complete, we present here some of our preliminary findings as they seem to be of relevance to the present controversy.

To avoid any assumptions as to the nature of the redshifts, two of us (D. J. and C. H.) started, not with a list of QSOs, but with a list of 280 radio sources selected from the 4C catalogue and for which we had measured improved positions. The fields around each of these sources were examined on the Palomar Sky Survey Prints for possible QSO identifications, an identification being suggested whenever a blue stellar object (BSO) brighter than 19 mag was found within the error rectangle of the radio position[10]. In searching for possible associations with clusters we adopted the view that a QSO is typically brighter than the brightest cluster galaxies by up to 2 or 3 mag. We therefore noted all cases in which three or more galaxies lay within 1 (arc min)^2 of the suggested QSO identification and which on the Palomar Sky Survey E print were approximately equal to it in brightness. We noted QSOs with less than three nearby galaxies only if these were brighter than 19 mag and lay within 10 arc s. This approach not only differs from that adopted in other QSO-cluster investigations but also differs significantly from that of Burbidge *et al.*[11] who looked for an association between QSOs and much brighter galaxies. It assumes that the QSOs are at cosmologically significant distances rather than very local ones but not necessarily at distances given by Hubble's law.

A search was also made for situations in which more than one BSO lay close to the radio position, a few examples of this type having already been noted[12]. It seemed possible that these might represent physical associations of QSOs the significance of which would be easier to establish than QSO-galaxy associations. They would therefore provide a more powerful method of investigating any possible non-cosmological contribution to the QSO redshifts, particularly if this non-cosmological component were present only in high redshift objects, say $z>1$.

A BSO or possible BSO was found within the search area of fifty-three of the 280 sources examined. Of these fifty-three, two (4C04.54 and 4C28.40) were found to be within 1 arc min of a second BSO, three (4C24.23, 4C11.45 and 4C26.48) lay close to one or more galaxies and the sixth (4C11.50) showed both a close pair of BSOs and a galaxy within the error rectangle. The latter four cases were considered to represent the most probable associations and to be worthy of further study, the remaining forty-nine have been published elsewhere[10]. The adopted radio positions of the four selected sources and their position relative to the nearby BSOs are given in Table 1 and enlargements of the fields around each of the sources are shown in Fig. 1.

The most convincing case for a QSO-cluster association is provided by 4C24.23; the BSO is seen clearly near the edge of a small compact group of five galaxies which may represent the brightest members of a cluster extending over a diameter of about 2 arc min. The BSO is the brightest member of the group, consistent with it being a QSO at the same distance. The BSO identified with 4C11.45 is also the brightest member of a small compact group but both the galaxies and the BSO are some 2 mag fainter than in the case of 4C24.23. A line of three brighter galaxies within 2 arc min may be members of the same cluster. 4C26.48 and 4C11.50, each with only a single galaxy within 10 arc s, are less convincing. 4C11.50 was, however, also of par-

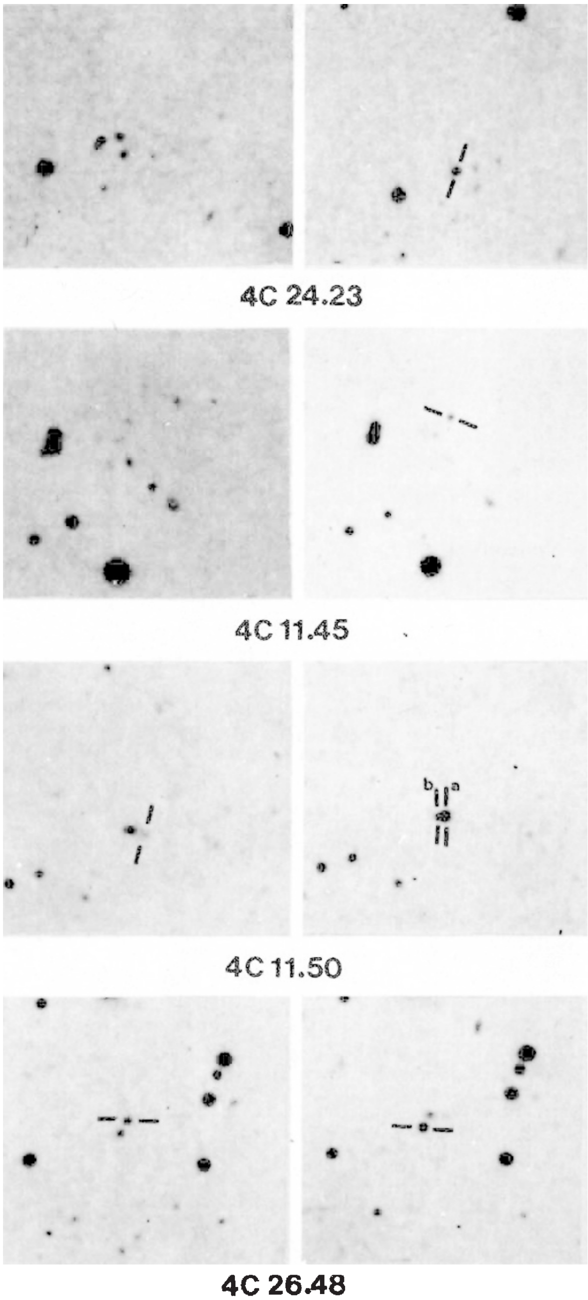

Fig. 1 Enlargements from the Palomar Sky Survey Prints of the region around the four sources, the E enlargement being on the left and the O enlargement on the right in each case. The quasistellar objects are marked on the O enlargements, and for 4C11.50 and 4C26.48 the nearby galaxies are marked on the E enlargement. Each picture is about 3 (arc min)^2 and north-east is at the top left hand corner.

ticular interest because of the presence of the two BSOs separated by only a few seconds of arc. The pair of BSOs can be clearly seen on the Palomar O plate although the fainter of the two is barely visible on the E plate which is of rather low quality. The galaxy which is visible only on the E plate is of comparable magnitude to this fainter BSO.

At this stage, because of the low accuracy of the radio positions, it was not certain that the BSOs we had noted were really QSOs rather than chance associations with blue stars. Nor could the possibility be excluded that one or more of them was a radio-quiet QSO with the radio emission originating in one of the nearby galaxies. A search of the literature showed, however, that 4C11.45 and 4C26.48 had already been suggested and confirmed as QSO identifications[13–15]. Since then we have taken optical spectra of

Table 1 Positions of the Four Radio Sources and the Relative Positions of the Optical Identification

Source	Adopted radio position (1950) Right ascension h	min	s	Declination °	′	″	S_{430} f.u.	Relative position of optical identification optical-radio ″	″
4C24.23	10	48	46.8	24	04	18	(2.9)	0	18S
4C11.45	13	18	50.0	11	22	30	4.8	4P	5N
4C11.50	15	48	22.0	11	30	00	1.8	6P	12S
4C26.48	16	23	12.8	26	57	30	2.8	12P	12S

4C24.23 using the 200-inch telescope at Palomar and the two BSOs near 4C11.50 have been studied by Baldwin *et al.*[16] at Lick Observatory. Both 4C24.23 and the fainter member of the 4C11.50 pair (4C11.50b) have been confirmed as QSOs and the brighter member of the pair (4C11.50a) was noted as having a continuum spectrum. In addition Ekers (private communication) has determined structures and accurate positions (± 0.5 arc s) for all four radio sources using the Westerbork Aperture Synthesis array, and Fanaroff (private communication) has measured the structure of 4C11.50 using the Cambridge aperture synthesis instrument. Optical positions accurate to about ± 0.3 arc s have been measured by Argue and by Murdoch (private communications).

A full account of these observations will be published elsewhere; at present we note that the accurate optical and radio positions confirm that for 4C24.23, 4C11.45 and 4C26.48 the radio emission does indeed originate in the QSO. The observations of 4C11.50 are particularly interesting since they show that the source is double with a separation of ~15 arc s in position angle 152° but with one component coincident with 4C11.50a rather than the confirmed QSO, 4C11.50b. This indicates that the source is a 3C273-type double and that 4C11.50a is also a QSO. The separation of the two QSOs is only 4.5 arc s so the possibility that they are physically associated inevitably arises. The nature of 4C11.50a has also been confirmed independently of these radio observations by Wampler *et al.*[17] in a parallel optical investigation and redshifts have been measured for both it and 4C11.50b. A summary of the relevant optical information is given in Table 2.

Although the redshifts of none of the galaxies close to any of the four QSOs have been measured, we estimate from their magnitudes that they cannot in any case exceed a value of approximately 0.3. It can be seen from Table 2 that, apart from 4C11.50b, the QSO redshifts exceed this value of *z* to such an extent that they cannot be associated with the galaxies if the redshifts of QSOs and galaxies are both distance indicators. The possibility remains, however, that the

Table 2 Optical Data in the Suggested QSO Galaxy Association

Source	QSO magnitude	Redshift	Notes
4C24.23	18.5	1.27	QSO in small group of galaxies
4C11.45	19	2.17 (ref. 15)	QSO in faint cluster of galaxies *
4C11.50 (a)	17 †	0.43 (ref. 17)	Close pair of QSOs. 19 mag galaxy 10″ preceding
(b)	18.5	1.90 (ref. 17)	
4C26.48	17.5	0.78 ‡	18 mag galaxy 10″ north preceding

* Clarke, Bolton and Shimmins[13] have previously noted this cluster.

† The identification of 4C11.50a as a QSO has already been made by Wills and Bolton[18], but neither the second QSO nor the galaxy was noted. The group of three objects has also recently been noted by Murdoch and Hazard (to be published) in an identification programme based on Molonglo radio positions.

‡ Provisional value communicated by M. Schmidt.

Table 3 Results of Counts

No. of galaxies	0	1	2	3	4	5	6	7	8	9	10
No. of (a) regions in which observed	178	90	28	1	2	1	1	0	0	0	0
No. of (b) regions in which observed	54	79	69	49	24	13	5	1	3	2	1
No. of (c) regions in which observed	22	38	40	22	6	1	0	1	0	0	0

associations are genuine but that the redshifts are not distance indicators. To support this view it would be necessary to obtain very strong evidence to rule out chance associations. In the absence of evidence showing radio or optical links to the nearby galaxies the only evidence which can be provided must be based on counts of galaxies and clusters of galaxies.

As it has been shown that the radio emission in all four examples arises in the QSO and not in the galaxies or clusters of galaxies, we need to know only the probability that out of the chosen sample of 280 radio sources four quasistellar radio sources will be found in the observed configurations; the probability of a radio-quiet QSO lying close to a galaxy need not be considered. Out of the 280 radio sources, fifty-three were suggested as possibly associated with QSOs with probably 30% representing chance coincidences with blue stars. This indicates that we are dealing with a list containing about forty QSOs which is consistent with the percentage of QSOs (~15%) usually found in source lists at the flux level of the 4C survey. The probability of two cases of a single galaxy being observed within 10 arc s of a QSO is then easily estimated from the published galaxy counts. Counts by Hoskins *et al.*[19] give the probability of a galaxy above +19 mag occurring by chance within 10 arc s of a randomly chosen position as about 0.02 and hence the probability of observing two such cases in a sample of forty QSOs is about 0.3. This ignores the restriction placed on the initial survey of noting only galaxies close to QSOs when they were of comparable magnitude on the Palomar E plate. Even if this were taken into account, however, the probability could not be reduced to a significant level.

The probability that the remaining two QSOs would be observed by chance in small groups of galaxies is more difficult to assess as no figures are available for the surface density of such groups, and in any case the definition of a group is to a large extent subjective. To provide some estimate we have counted the number of galaxies seen above the plate limits in 300 randomly selected regions of sky (a) for 1 (arc min)^2 centred on each of the chosen points; (b) for 4 (arc min)^2 centred again on the chosen point; and (c) for 130 such regions we have counted the maximum of galaxies which were observed in any 1 (arc min)^2 not centred on the chosen point but still containing it. The results of these counts are shown in Table 3. The corresponding number of galaxies observed around 4C24.23 and 4C11.45 is shown in Table 4.

Similar numbers of galaxies would be observed by chance in about one trial in a hundred. In our random counts in most cases where the number of galaxies counted exceeded five per area, they were all at the plate limits, and with our selection criteria would not have been considered as clusters at the same distance as at least the brighter BSOs. A probability of 0.01 is therefore a conservative estimate of the chance of finding a QSO in small groups of the type we have noted. Adopting this estimate we then find that the probability of finding two such cases in our sample is about 0.08. If the redshifts of the QSOs associated with 4C24.23 and 4C11.45 lay in the region of 0.1 to 0.3, even this probability would have led us to believe that we were dealing with physical associations and that the QSO redshifts are indeed distance indicators.

Table 4 Number of Galaxies around 4C24.23 and 4C11.45

No. of galaxies in region	(a)	(b)	(c)
4C24.23	4	10	6
4C11.45	5	8	6

The probability is, however, too high by some orders of magnitude for our results to lend much support to the non-cosmological hypothesis. Therefore, although the possibility that the associations are genuine cannot be ruled out, the realistic interpretation is that we have simply chance associations of QSOs with small clusters of galaxies. We point out, however, that in their appearance on the Sky Survey Plates, 4C24.23 and 4C11.45 appear at least as convincing QSO-galaxy associations as those which have been used to support the cosmological hypothesis. That the only QSO-galaxy associations suggested prior to a redshift measurement should show discrepant redshifts is at least of some interest, if only to demonstrate that all possible associations do not necessarily support the conventional view.

An examination of the figures of galaxy counts presented here shows that it will be extremelv difficult to establish a non-cosmological contribution to the QSO redshifts on the basis of an association with clusters of galaxies unless additional information can be obtained to rule out the possibility of chance associations. The detection near 4C11.50 of a possibly physically associated pair of QSOs seems, however, to offer a more promising approach. Assuming that these are 10^5 radio-quiet QSOs in the sky brighter than +19 mag (ref. 20), the probability of detecting one within 5 arc s of a quasistellar radio source in a sample of forty such objects is only of the order of 6×10^{-4}. In spite of their different redshifts the probability that we are dealing with a real association is therefore high, particularly as Stockton[21] has already reported two radio-quiet QSOs separated by only 35 arc s, and which he estimates have a probability of occurring by chance of only 6×10^{-3}. To establish beyond doubt the existence of associations of QSOs will require many more examples, preferably with more than two members, and it is to be hoped that the examples of two or more blue objects close to the position of a radio source which have already been reported[12,13] will now be investigated optically. It may be that a significant test of the nature of QSO redshifts is at last in sight. A detailed account of the optical observations of 4C11.50 and their significance is given elsewhere by Wampler *et al.*[17].

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Force on a Wire in a Magnetic Field

McCaig[1] has suggested that a shape factor might be involved in determining the force on a current-carrying wire in a magnetic field when there is a permeability contrast between the wire and its surroundings.

Certainly, the **B** seen by the conduction electrons in the wire would then be different from that present before the wire was inserted, apparently leading to a force which depended on the wire shape and permeability as well as the current, but this then gives difficulties when considering the mutual force between two different wires carrying the same current.

The problem of the cylindrical wire with uniform current density has in fact been solved by Stratton (ref. 2, section 4.21); a simple solution of the more general two-dimensional problem is now presented and discussed. I shall show that there is no shape factor for the force in the general two-dimensional case or for an arbitrary complete circuit. There is, however, a shape factor for the torque.

For simplicity let us consider a two-dimensional equilibrium situation with all quantities independent of z, that is, a long straight wire in a transverse field. We wish to calculate the force per unit length required to hold the wire stationary.

The external medium is assumed to be an incompressible fluid; it is homogeneous, isotropic and linear, and has relative permeability μ_e.

Before the wire is inserted there exists a uniform field

$$\mathbf{B}_0 = \mu_e\mu_0 \, \mathbf{H}_0 = \mu_e\mu_0 H_0\hat{\mathbf{x}}$$

The medium is of sufficient extent, and the field sources sufficiently distant, that the insertion of the wire does not significantly alter the reluctance seen by the sources. Nor are the sources affected by the field of the current.

The long straight wire carries a current $\mathbf{I}_0 = \mathbf{I}_0\hat{\mathbf{z}}$. The wire can have arbitrary mechanical and electrical properties, arbitrary cross-section, and arbitrary current distribution across it, provided all these are independent of z.

The field produced by the current will in general not be circular, and inserting the wire into the field will also distort $\mathbf{H}_0$. Outside the wire the total field $\mathbf{H}$ (strictly $\mathbf{B}/\mu_e\mu_0$) can be expressed in terms of a vector potential

$$\mathbf{H} = \text{curl}[T(\rho,\lambda)\hat{\mathbf{z}}]$$

with

$$\nabla^2 T = 0.$$

The appropriate solution for T is

$$T = H_0 y + a_0 \ln(1/\rho) + \sum_{n=1}^{\infty} \rho^{-n}(a_n \cos n\lambda + b_n \sin n\lambda)$$

corresponding to $H_0\hat{\mathbf{x}}$, a circular field $a_0\hat{\boldsymbol{\lambda}}/\rho$, and line dipole, quadrupole (and so on) fields proportional to a_n/ρ^{n+1}. Only the circular field contributes to $\oint \mathbf{H}.\mathrm{d}\mathbf{l}$ taken round a circle, so $a_0 = I_0/2\pi$, independent of the choice of origin.

The force of electromagnetic origin acting on the wire will come from both body forces (magnetic forces on the current and on the magnetisation) and surface forces (non-uniform pressure of the surrounding fluid). Fortunately it is the total force which is given by integrating the Maxwell stress over any surface surrounding the wire and entirely in the fluid (ref. 2, section 2.29):

$$\mathbf{F} = \mu_e\mu_0 \int [(\mathbf{H}.\hat{\mathbf{n}})\mathbf{H} - (1/2)H^2\hat{\mathbf{n}}]\mathrm{d}S$$

where $\mathbf{H}$ is the total field.

Let us make the integration surface the cylinder $\rho = \alpha$. (It is easily shown that the ends do not contribute.) In equilibrium the force cannot depend on the radius of this surface, so the only non-zero contribution to the integral must come from the interaction of the uniform field $\mathbf{H}_0$ with the circular field $\mathbf{H}_I = I_0\hat{\theta}/2\pi\rho$. Keeping only these terms, and remembering $\mathbf{H}_I.\hat{\mathbf{n}} = 0$, it follows that

$$\mathbf{F} = \mu_e\mu_0 \int [(\mathbf{H}_0.\hat{\mathbf{n}})\mathbf{H}_I - (\mathbf{H}_0.\mathbf{H}_I)\hat{\mathbf{n}}]\mathrm{d}S$$

Performing the integration we find the force per unit length to be

$$\mathbf{f} = I_0\mu_e\mu_0 H_0\hat{\mathbf{y}} = I_0 B_0\hat{\mathbf{y}} = \mathbf{I}_0 \times \mathbf{B}_0$$

Thus the force on the wire depends only on the total current it carries, and on the value of the uniform $\mathbf{B}_0$ present before the wire was inserted; it is independent of the shape and material of the wire, and the distribution of current in it.

Therefore the total electromagnetic force per unit length of the long straight wire is simply $\mathbf{I}_0 \times \mathbf{B}_0$. Unfortunately it does not seem possible to give a simple interpretation of this overall result in terms of the various forces acting on the wire.

To indicate the complexity of the problem, consider the much simpler situation when there is no $\mathbf{B}_0$, but only the current $\mathbf{I}_0$. One can see physically that provided the fluid has uniform permeability, and (effectively) extends to infinity, there is no reason for there to be any net force on the wire. That the net force is zero is confirmed very quickly by the Maxwell stress argument used above. However, let us consider the individual forces.

There will be induced magnetisation $\mathbf{M}(\rho,\lambda)$ in the fluid and in the material of the wire. The total field $\mathbf{B}(\rho,\lambda)$ can be expressed as $\mathbf{B} = \mathbf{B}_I + \mathbf{B}_i + \mathbf{B}_e$, where $\mathbf{B}_I$ is that field which would be given by the current I_0 in free space, $\mathbf{B}_i$ that from the (internal) wire magnetisation, and $\mathbf{B}_e$ that from the (external) fluid magnetisation.

The resultant body force on the wire will come not only from the integral of the Lorenz force $\mathbf{j}_0 \times \mathbf{B}$, but also from the integral of the force $(\mathbf{M}.\nabla)\mathbf{B}$ exerted on the wire magnetisation by the gradient of $\mathbf{B}$. Many of the contributions must cancel on integration, because they correspond to the 'action' and 'reaction' of forces between different parts and properties of the wire, but two terms are left. These are the volume integrals of $\mathbf{j}_0 \times \mathbf{B}_e$ and $(\mathbf{M}.\nabla)\mathbf{B}_e$, both involving the field whose source is the (non-uniform) magnetisation of the surrounding fluid.

In addition to these body forces there will also be a surface force given by the integration of the (non-uniform, non-