LETTERS TO NATURE

PHYSICAL SCIENCES

Statistical analysis of close pairs of QSOs

THE observation of close pairs of QSOs with very different redshifts has been suggested to be evidence in support of the noncosmological redshift hypothesis[1,2]. The statistical arguments used by Stockton[1] and by Wampler *et al.*[2] have recently been attacked by Bahcall and Woltjer[3] on the ground that the method used is one of *a posteriori* statistics. Statistics aside, they have ignored the point that in one case[2], the evidence is not only based on close proximity of the two QSOs but also on two coincidences in wavelengths in the spectra of the two objects. Although the argument against *a posteriori* statistics is correct in the absolute sense, this method is widely used in astronomy and has generally been accepted, except in cases in which the hypothesis under consideration has not reached the point of general acceptability. The redshift controversy is a good example of this. Evidence as to the existence of non-cosmological redshifts must, because we have few ways of measuring distance, depend upon finding conflicts between distances derived from the redshifts (assumed cosmological) and distances derived in other ways[4] —either through statistical arguments or by finding luminous connections between pairs of objects. If, as seems to be the case, most workers use the redshift as the prime criterion of distance, and subordinate other methods of measurement to it, no solution to the problem is possible.

One or two examples of the prejudicial approach used in this problem will suffice:

(1) Several cases of QSOs with low redshift close to small groups of galaxies, and with redshifts similar to those of one or more members of the respective groups, have been found. Although some of these associations may be real, the statistical evidence is not convincing[5]; nevertheless, these juxtapositions are cited by many as proof of the cosmological origin of QSO redshifts. It is instructive to examine some of the analyses which attached statistical significance to such associations. For example, Gunn[6] found a small group of galaxies near the QSO PKS2251+11 ($z = 0.33$) agreeing in redshift within the observational uncertainty ($\Delta z \simeq \pm 0.01$). He finds that the brightest member of the group, with a calculated absolute magnitude -21.2, lies 28″ from the QSO; then he asks: "What is the probability of finding a galaxy of absolute magnitude -21 or brighter within 30″ of a random position and within 3,000 km s^{-1} of a given redshift near 0.3?" The answer, 0.001 before multiplying by the number of tries, is then purported to be "a realistic assessment of the probability of a chance coincidence". While the choice of 3,000 km s^{-1} can be justified since it follows from the observational uncertainty, the choices of the magnitude limit, -21, and the separation limit 30″ are obviously predicated upon the observed values -21.2 and 28″, respectively, and thus constitute *a posteriori* information.

(2) Another good example is found in a paper by Wolf and Bahcall[7]. They wish to argue that a galaxy in a cluster is physically associated with the N-system Ton256. We quote them as follows: "There appears to be a galaxy about 8″ from Ton256; the redshifts of the galaxy and Ton256 are identical, to within the observational error of 300 km s^{-1}. The density of galaxies in the central region of the cluster is ~0.08 galaxies (arc min)$^{-2}$; so the probability of having a galaxy appear within 8″ from the QSO, assuming a random distribution of the galaxies near the center of the cluster, is only about 0.4%. We thus assume that the galaxy is really associated with the QSO and is in a bound orbit around the QSO." Again, this is a classic piece of statistical analysis argued *a posteriori*, presumably considered acceptable because the redshifts of the two objects are the same.

In what follows we reconsider the pairs already discovered and then make an *a priori* calculation based on realistic parameters such that a test involving the discovery of further close pairs of QSOs may enable a significant statement to be made.

Let $\Gamma(<m)$ be the sky density of QSOs brighter than m. Then the expected number lying by chance within an angular separation θ of N arbitrary coordinates is

$$\langle n \rangle = 2.4 \times 10^{-7} N\Gamma(<m)\theta^2, \quad (1)$$

with Γ expressed in (arc deg)$^{-2}$ and θ, in arc s. A plot of $\langle n \rangle$ against θ for various $N\Gamma$ is shown in Fig. 1. For N corresponding to the number of QSOs with known redshifts (~ 300), and Γ ($m_B < 19.4$) taken from the Sandage–Luyten survey [~ 5 QSO (arc deg)$^{-2}$], the value for the product $N\Gamma$ is ~ 1,500 tries per QSO (arc deg)$^{-2}$. This number could be increased to ~ 3,000 by extending the survey to $m_B <$ 20. Of course, it is always possible to consider a more restricted sample, yielding a smaller value for $N\Gamma$, but to obtain a value much larger than 3,000 would require deep plates

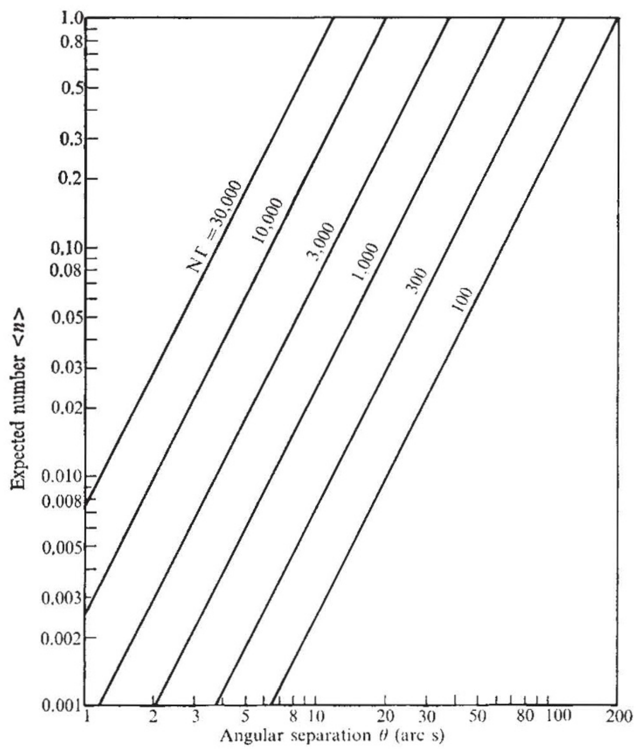

FIG. 1 Dependence of the expected number of apparent pairs upon pre-selected class intervals defined by the maximum angular separation θ to be included and the product of the number of tries N and the sky density $\Gamma(<m)$ of QSOs brighter than the adopted completeness limit m of the sample to be considered.

or deep scans around each of the known QSOs. From a practical point of view it would be difficult to go to $m_B = 21$ on the Palomar Sky Survey because objects at the very limit of the Survey, lying say 5″ from a brighter object, will tend to be missed and in any case would be very difficult to examine spectroscopically.

The QSO 1548+115a lies about 5″ from the previously identified QSO 1548+115b (ref. 2). Using the observed separation, the expected number of such pairs, for $N\Gamma$ $(m > 19.4) \approx 1{,}500$, is only about ~ 0.01; however, since this choice of separation θ is based on *a posteriori* information, the quoted probability is only an approximate lower limit to a realistic *a priori* probability. In order to estimate an upper limit to the *a priori* probability, we can consider the class interval determined by the next nearest separation known—namely, the 35″ separation of Ton155 and Ton156 (ref. 1). Adopting $\theta = 35''$, the number of pairs with smaller separations expected by chance is ~ 0.4 (see ref. 3).

The statistical significance of the pair 1548+115a,b is thus not well defined; it ranges from $\sim 99\%$ confidence to about 60%. Consequently, the *a priori* statistical significance of this pair cannot be demonstrated. Nevertheless, the proximity of the association is curious, particularly in view of the spectroscopic coincidences already mentioned. Unless additional evidence is uncovered, however, the proximity of 1548+115a,b will remain just a curiosity cited by some as evidence for non-cosmological redshifts, but disregarded by others as a mere coincidence. We therefore propose the statistical test described below.

The vicinity of all ~ 300 QSOs with known redshifts has not been examined closely, and, in fact the method of identification of QSOs from radio positions may have biased the sample against the discovery of close pairs because when a suggested identification close to the radio position is found to be a QSO, no further examination of other nearby objects is usually undertaken. Consequently, more associations may exist (compounding the probabilities). We propose, therefore, that the vicinity around each known QSO be examined for possible conjugates down to $m_B \approx 20$, since it should be feasible to examine any such candidates spectroscopically. For $N\Gamma \simeq 3{,}000$, the expected number with chance separations less than $\theta = 10$ arc s is only $\langle n \rangle \simeq 0.08$ (see equation 1 and Fig. 1). Thus, the discovery of a second pair of QSOs—or two more pairs of QSOs if the known case 1548+115a,b is excluded from the sample—would be statistically significant at 99.5% confidence, based on *a priori* class intervals.

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E. M. BURBIDGE
G. R. BURBIDGE
S. L. O'DELL

*Department of Physics,
University of California, San Diego,
La Jolla, California 92037*

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Absorption of high energy heavy nuclei and γ rays at the surface of hot neutron stars

SINCE the discovery of pulsars and their association with rotating magnetic neutron stars, it has been realised that pulsars can efficiently accelerate particles to high energies and effectively contribute to the observed cosmic-ray density in the Galaxy. Acceleration of particles to very high energies by the low frequency electromagnetic waves beyond the light cylinder is one mechanism proposed[1,2]. Others[3–5] have considered the possibility of accelerating particles by the electric field near the surface of the neutron star arising from non-zero $\mathbf{E}\cdot\mathbf{B}$. High energy photons are also likely to be produced in association with these particles and these have been observed from the Crab pulsar. Presumably, younger pulsars may radiate larger numbers of more energetic γ rays.

Though pulsars are potential sources of very high energy particles and γ rays, it is not clear whether the radiation can escape the pulsar environment and reach interstellar space. Various absorption processes can contribute to the destruction of these radiations. Absorption of high energy γ rays in their interactions with the radiation field of the pulsar may occur[6–8]. Destruction of high energy cosmic rays, especially the photodisintegration of heavy nuclei in the vicinity of pulsars, has also been considered[9,10]. Heavy nuclei may be destroyed by fragmentation in the high density nebular matter surrounding pulsars in their early stages[11]. All these processes become important in the early stages of pulsars because of the simultaneous presence of particles of very high energy and large radiation and matter density.

Here I point out that high energy heavy nuclei and γ rays may also be absorbed because of their interactions with the blackbody photons emitted from the surface of the hot neutron star. Neutron stars have surface temperatures greater than 10^7 K when they form and subsequently the temperature decreases[12]. Tsuruta *et al.*[13] have carried out improved cooling calculations taking into account the influence of superfluidity of the matter of the neutron star, the magnetic field and other factors. Their calculations show that for the first 100 yr the surface temperature is 10^7 K and remains greater than 5×10^6 K for another few hundred years for several models. So there is a high density of photons close to the neutron star during the early stages. It is precisely during this period that pulsars lose most of their energy and accelerate particles to high energies. I have discussed the implication of high surface temperature for the radio emission of pulsars elsewhere[13]; here I concentrate on other effects.

If a is the radius of the neutron star and T its surface temperature, the differential spectrum of photons emitted from the surface is

$$n(\epsilon)\, d\epsilon = \frac{2\epsilon^2}{h^3 c^2}\left(e^{\epsilon/kT} - 1\right)^{-1} \text{ photons } (\text{cm}^2 \text{ s sr erg})^{-1} \quad (1)$$

Considering the interaction between a γ ray at O, at a distance R from the centre, travelling radially, and photons from the surface of the neutron star (see Fig. 1) the optical depth for γ–γ interaction is

$$\frac{d\tau}{dR} = \iint (2\pi a^2 \sin\alpha\, d\alpha \cos\phi)(r^{-2}) (1 - \cos\beta) n(\epsilon)\, d\epsilon\, \frac{\sigma(E_\gamma, \epsilon)}{c} \quad (2)$$

Here $\sigma(E_\gamma, \epsilon)$ is the cross section for pair production in the collision of a γ ray of energy E_γ with a photon of energy ϵ, the term $(2\pi a^2 \sin\alpha\, d\alpha \cos\phi)$ is the element of area normal to OA, the term r^{-2} represents the inverse square reduction and the term $(1 - \cos\beta)$ takes care of the relative velocity between the two photons. The integration with respect to α