9. Paczynski, B. *Nature* **321,** 419-420 (1986).
10. *Nature* **321,** 377 (1986).
11. Shaver, P. A. & Christiani, S. *Nature* **321,** 585-586 (1986).
12. Bahcall, N. A. *Astrophys. J.* **232,** 689-698 (1979).
13. Bahcall, N. A. *Astrophys. J.* **247,** 787-791 (1985).
14. Rivolo, A. R. *Nature* **317,** 788-789 (1985).
15. Piccinotti, G. *et al. Astrophys. J.* **253,** 485-503 (1982).
16. Quintana, H. & Melnick, J. *Astrophys. J.* **87,** 972-978 (1982).
17. Jones, C. & Forman, W. *Astrophys. J.* **276,** 38-55 (1984).
18. McGlynn, T. A. & Fabian, A. C. *Mon. Not. R. astr. Soc.* **208,** 709-718 (1984).
19. Sunyaev, R. A. & Zel'dovich, Ya. B. *Commts Astr. Space Sci.* **4,** 173-179 (1972).

Multiple quasars for multiple images

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Three quasar pairs that have been identified as the products of gravitational lenses have no obvious visible lenses[1–3]. They are: 1146+111 (redshift $z=1.01$; pair separation $\Delta\theta=157$ arc s)[3], 1635+267 ($z=1.96$; $\Delta\theta=4$ arc s)[1] and 2345+007 ($z=2.15$; $\Delta\theta=7$ arc s)[2]. An even number (two) of quasar images is observed in each case, although an odd number of images is produced by gravitational lenses with non-singular potentials[4]. The absence of a visible lens creates, in the lens hypothesis, a severe 'missing matter' problem. The missing lenses range in estimated mass[3,5] from $>10^{12}\,M_\odot$ for 1635+267 (where $M_\odot$ is the mass of the Sun) to $\sim10^{15}\,M_\odot$ for 1146+111. We investigate here the possibility that these quasar images are pairs of physically distinct quasars in galaxy associations. We show that, at the same redshifts as the quasars, only galaxy associations of exceptional richness would be identifiable in the existing data. Less prominent groups of galaxies, such as those known to be associated with nearby quasars[6], would not be visible. We calculate the probability of pairs of quasars appearing in galaxy associations and find that physical pairs could appear with the observed frequency if, as expected, quasars are more common relative to galaxy associations at earlier epochs. We also discuss observations that can distinguish between the hypothesis of a gravitational lens and that of physical pairs.

Observations of nearby quasars support the suggestion that quasars may come in groups. Stockton[6–8] has shown that at least one third of all nearby quasars are surrounded by small groups of galaxies at the same redshift. For later reference, we note that the observed quasar–galaxy separations range from ~10 to 400 kpc (for Hubble constant $H_0=50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$)[6–8]; the upper limit is fixed by the size of the survey fields. The mean velocity difference between the quasar and the associated galaxies is consistent with zero and has a standard deviation of $\sim400\,\mathrm{km\,s^{-1}}$.

Figure 1*a* illustrates the difficulty of detecting associated galaxies around the high redshift quasar pairs if the galaxies are at the same redshift as the quasar. One has the best chance of observing associated galaxies for 1146+111 (the 'wide pair') as it has the lowest redshift. Figure 1*a* shows a deep image of the field obtained at the Palomar 5-m telescope with the 4-Shooter[9]. The limiting magnitude for galaxies on this *i*-band (8,000 Å) exposure is ~23.5 mag. No prominent association of galaxies is apparent in Fig. 1*a*.

Would galaxies associated with the quasars have been identified in this exposure? We answered this question by using data on low-redshift clusters of galaxies transformed to take account of *k*-corrections[10] (no evolution) and of metric differences (for deceleration parameter $q_0=1/2$). We selected *i*-band exposures of 1146+111 because this filter is most likely to reveal any associated galaxies at $z\simeq1$. Figure 1*b* shows the simulated data when the cluster CL0024+1654, which is one of the richest

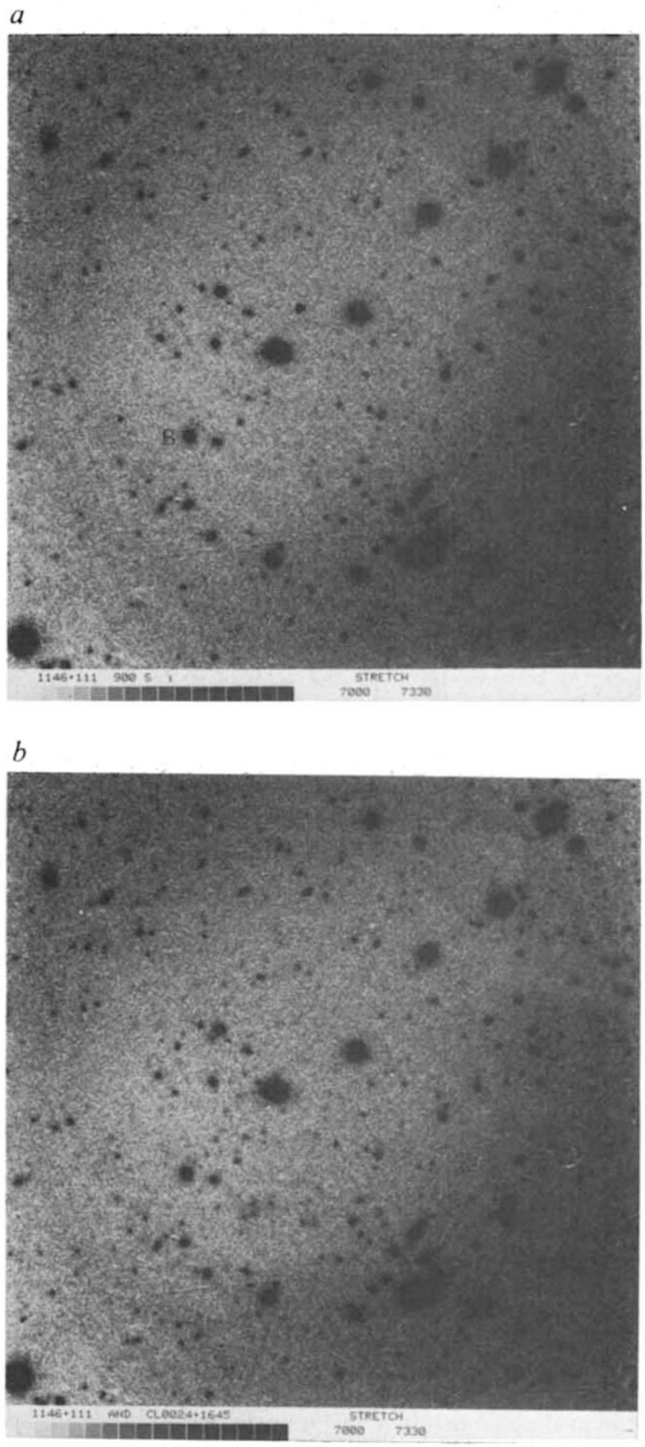

Fig. 1 *a*, An *i*-band CCD (charge-coupled device) frame of the field containing the quasar-pair 1146+111 B, C. North is up, east is to the left, and the picture is 250 arc s on a side. The seeing is 1.4 arc s (FWHM). *b*, As *a*, except that a frame of the very rich cluster of galaxies CL0024+1654 has been added to the quasar picture. The image of the cluster ($z=0.39$) has been processed so that it appears as if it were at the redshift of the pair of quasars ($z=1.01$). The centre of the cluster is ~30 arc s south-west of quasar B.

known galaxy clusters (Abell richness ⩾4 (ref. 11)), is transformed to the redshift of the quasar pair and superimposed on the field. Only the brightest cluster galaxies are visible on the simulated image. We conclude that a cluster of Abell richness 4 would have been detected around 1146+111, but only barely. A similar simulation made with a typical richness-3 cluster shows that such a system would at best be only marginally detected.

Galaxy associations at the same redshifts as the quasar images considered here would have escaped detection on state-of-the-art images if the associations were less rich than Abell class 3. Most (>95%) galaxy associations observed at low redshift satisfy this requirement.

Would we expect the existing observations to have revealed three pairs of quasars in galaxy associations if the quasars are randomly distributed among the associations? The number of pairs of quasars that would be expected is

$$\text{Number of quasar pairs} \simeq f^2\left(\frac{n_{\mathrm{QSO}}}{n_{\mathrm{assoc}}}\right)\frac{N_{\mathrm{QSO}}}{2} \qquad (1)$$

where f is the probability that a given quasar is found in a (galaxy) association. The number density of galaxy associations is n_{assoc}; n_{QSO} is the quasar number density. For definiteness, we assume that the total number of relevant quasars, N_{QSO}, is ~2,000. For number densities in the literature[12–16], the calculated number of pairs is $\sim 800 f_{\mathrm{rc}}^2$ for rich clusters, $50 f_{\mathrm{cg}}^2$ for compact groups, and $5 f_{\mathrm{gg}}^2$ for groups in general. The above numbers are uncertain because the quasars have not been observed in a homogeneous way (n_{QSO} depends on limiting absolute magnitude and redshift, taken here to be $M_{\mathrm{B}} = -25.5$ and $z \simeq 1.5$). For specificity, we assume that the number density of galaxy associations is constant in co-moving coordinates, and the quasars evolve as given by their observed densities at these redshifts[16].

We conclude that the discovery of three physical pairs of quasars in galaxy groups is plausible based on the known densities of quasars and galaxy groups. For example, if $f_{\mathrm{rc}} \simeq 0.05$ (like galaxies) and $f_{\mathrm{groups}} \gtrsim 0.3$ (as for nearby quasars), then several quasar pairs may be expected in rich clusters and in groups.

What do we expect to observe for quasar groups? (1) Quasar pairs would be more likely than higher multiplicities (such as the triples predicted by the gravitational lens hypothesis) because of the rarity of quasars. (2) Quasar pairs in groups similar to the nearby quasar–galaxy groups[6–8] would have projected separations in the range ~1 arc s to ~1 arc min (for $z \gtrsim 1$). The smaller separations would be favoured if both quasars were massive and near the centre of the group. Separations of $\lesssim 1$ arc s are not expected because typical separations among galaxies in groups are $\gtrsim 10$ kpc. By contrast, the gravitational lens hypothesis requires many lensed images with smaller separations. Separations as large as a few arc minutes, as observed for 1146+111, could correspond to quasar pairs in large groups or rich clusters. (3) The velocity difference between the two quasars (probably among the most massive members of the system) may be less than or equal to the typical velocity difference between galaxies in a group, which is a few hundred[6] kilometres per second (~100 km s^{-1} for compact groups[14]). (4) The continuum and emission-line spectra are not expected to be identical for the members of the quasar pair. Some evidence of mutual photoionization might be discernible. However, the spectra should be at least as similar as for unrelated quasars at the same redshift. It would be useful to obtain high signal-to-noise spectra for a large number of quasars, in order to develop quantitative statistical measures of the similarity of spectra of physically distinct quasars. (5) In contrast to what is expected on the basis of the lens hypothesis, there is no requirement that background objects be lensed, nor that there be correlated time variations (on the lens timescale). (6) The Zeldovich–Sunyaev effect would not be expected to be large enough to be measured with current techniques. (7) An association of galaxies at the same redshift as the quasars might be found on extremely deep exposures of the field of 1146+111.

We have examined the published data for 1635+267, 2345+007 and 1146+111. For these objects, the existing information is insufficient to distinguish between a single object with two images formed by a large unseen mass or a pair of quasars belonging to an association of galaxies. However, a pair of images with no visible prominent lens is more simply interpreted as a pair of quasars rather than the lensing of a single object. Of the three systems considered here, 1146+111 is the most likely candidate for a physical pair because its separation is much greater than that for any of the confirmed[5] gravitational lenses.

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1. Djorgovski, S. & Spinrad, H. *Astrophys. J.* **282**, L1-4 (1984).
2. Weedman, D., Weymann, R. J., Green, R. F. & Heckman, T. M. *Astrophys. J.* **255**, L5-9 (1982).
3. Turner, E. L. *et al. Nature* **321**, 142-144 (1986).
4. Burke, W. L. *Astrophys. J.* **244**, L1 (1981).
5. Turner, E. L. *IAU Symp.* No. 117, 209-218 (1986).
6. Stockton, A. *Astrophys. J.* **223**, 747-757 (1978).
7. Stockton, A. *IAU Symp.* No. 92, 89-97 (1979).
8. Stockton, A. *Astrophys. J.* **257**, 33-39 (1982).
9. Gunn, J. E. *et al. Bull. Am. astr. Soc.* **16**, 477 (1984).
10. Schneider, D. P., Gunn, J. E. & Hoessel, J. G. *Astrophys. J.* **264**, 337-355 (1986).
11. Schneider, D. P., Dressler, A. & Gunn, J. E. *Astr. J.* (in the press).
12. Bahcall, N. A. *Astrophys. J.* **232**, 689-698 (1979).
13. Bahcall, N. A. *Astrophys. J.* **270**, 20-38 (1983).
14. Williams, B. A. & Rood, H. R. *Astrophys. J.* (in the press) (1986).
15. Hickson, P. *Astrophys. J.* **255**, 382-391 (1982).
16. Schmidt, M. & Green, R. F. *Astrophys. J.* **269**, 352-374 (1983).

Simulating the sunspot cycle

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Fossil records of presumed solar activity[1,2] in 680-Myr-old varves reveal two periods, ~314 and 350 varve years respectively, which also appear in recent sunspot records. These periodicities, combined with a nonlinear feature of sunspot number and a theory of the asymmetry of the solar cycle, enable the sunspot record to be extrapolated back to 1800 so well that the next magnetic cycle may be predicted. Fossil records offer the hope of going beyond numerical analysis into physics; one result is the demonstration that part of the 8–15 yr spread of the 11-yr cycle is due to shifts in the year of sunspot minimum caused by the 350-yr cycle. The annual sunspot number can be represented by[3]:

$$R_{\mathrm{sim}}(t) = |\mathscr{H}[\mathrm{Re}\{E(t)\exp\{i[\omega_0 t + \phi(t)]\}\} + U(t)]| \qquad (1)$$

where ω_0 is the angular frequency corresponding to the magnetic period (~22 yr), $E(t)$ is an instantaneous envelope amplitude, $\phi(t)$ is the associated instantaneous phase, $U(t)$ is an additive undulation of low amplitude, $\mathscr{H}$ is a nonlinear function, and $R_{\mathrm{sim}}(t)$ is the estimated annual mean sunspot number. Here I consider these parameters, in turn, to derive an evaluation of equation (1) from AD 1800 to 2000.

The magnetic cycle, generally described as having a 22-yr period, consists of two '11-yr' semicycles. Equation (1) indicates that the durations of successive semicycles should diverge oppositely from the mean. Thus in odd-numbered semicycles, such as cycle 19 which began in 1954.2, the negative bias constituted by $U(t)$ advances the start and retards the finish, making the semicycle long, while the same bias has affected cycle 20 in the opposite sense. (For this interpretation, one adopts the notion of an alternating oscillation[4], whose topology harmonizes with an internal 22-yr torsional oscillator[5].) The duration of the magnetic cycle should thus be more stable[6] than that of the semicycles, an implication confirmed by Fig. 1. Part of the spread in semicycle length is therefore due to a magnetic field that adds algebraically, equally in both hemispheres, to the sunspot-