

Evidence for the Location of Quasars in Superclusters

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Summary

Twelve pairs of neighbouring quasars have been found whose redshifts are so nearly equal that they have a high probability of being physically associated. If their redshifts are "cosmological", as we assume throughout, and H_0 is assumed to be $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ the majority have separations between 5 and 30 Mpc. Therefore the "superstructures" providing the association must have dimensions of at least this order, which is that of superclusters. It is plausible therefore to identify them with superclusters. Two of the quasar pairs have redshifts of 2.84 and 3.15 respectively, showing that large superclusters exist at $z \approx 3$. The data are compatible with the assumption that all quasars lie in superclusters. In order to exclude spurious pairs a special effort has been made to determine the probability of accidental co-incidences.

On the average there is about one quasar pair down to $V \sim 20.0$ per square degree, and therefore one supercluster can be identified by a deep survey over such an area; but redshifts accurate to one or two thousandths are generally required to exclude spurious pairs, except at $z > 2.5$, where an accuracy of 0.01 is sufficient. We find that roughly 20% of the powerful superclusters contain one quasar brighter than 20^m ; only 3% have two quasars; but the statistics are still very poor. Extensive quasar surveys can provide important data on superclusters and their evolution from the earliest time when quasars existed.

KEY WORDS: Superclusters; Quasars.

In previous articles one of us has drawn attention to apparent concentrations of quasars around bright galaxies or their companions, and has expressed the belief that these quasars are expelled from, or in some other way related to, these galaxies (Arp, 1980 a, b, c en d; Arp *et al.* 1979). During these investigations it was noted that some of the quasars have surprisingly similar redshifts; in several cases they differ so little that it is difficult to escape the conclusion that the quasars belong together.

In the present paper we concentrate on the problem of these and other cases of quasar pairs with near-equal redshifts, and, as an alternative to the hypothesis that they are related to a neighbour galaxy, here consider the redshifts to be indicative of distances according to Hubble's law. The "belonging together" of these quasars then implies that they lie in the same structure. As these structures are found to have dimensions typical of superclusters we identify them as such.

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There has been growing evidence that many galaxy clusters as well as groups and single galaxies are situated in superclusters; there are even indications that practically all galaxies may belong to such superstructures. It therefore appears reasonable to inquire whether there is any evidence that quasars are likewise concentrated in superclusters.

At quasar distances the quasars themselves are generally the only objects that can be seen. Therefore evidence of superclusters at these distances can, for the present, only come from the quasars themselves. Since, if a supercluster contains more than one observable quasar it may reveal itself by the equality of their redshifts, the way to approach our question must be to search for pairs (or multiples) of quasars with very similar redshifts.

What is the maximum redshift difference allowable to infer common supercluster membership?

Apart from the regions of dense clusters the velocity dispersion in known superclusters is roughly between 300 and 600 km/s. The lower value has been inferred from Gregory and Thompson's publication on the Coma supercluster (1978; cf. their Fig. 2). In the Hercules supercluster (Tarenghi *et al.* 1979, and Chincarini, private communication) the dispersion may be estimated at 500 km/s. As the relation between velocity difference Δv in the rest frame of the source and redshift difference Δz is

$$\Delta z = (1+z)\Delta v/c \quad (1)$$

The corresponding dispersion in z lies between 0.002 and 0.004 for $z = 1$, and between 0.003 and 0.006 for $z = 2$.

But we must also take account of the probable expansion of superclusters. Suppose they expand at the same rate as the universe.

As an example consider a supercluster at $z = 2$ in which there are two quasars at a mutual distance of 20 Mpc, of which 10 Mpc is in the direction of the line of sight. The relative radial velocity due to the expansion would then be roughly 3000 km/s, corresponding with $\Delta z = 0.030$. For superclusters at other z 's Δz would scale roughly as $(1+z)^2$. Since a supercluster is a region of excess density its actual expansion will be less than the Hubble expansion, and the expected values of Δz will therefore be smaller, but we don't know how much; we have no information on the real rate of expansion, except perhaps in the case of the Local Supercluster, for which there are some indications that it has passed its maximum size, and is collapsing. At the time of writing we received a preprint by Ford, Harms, Ciardullo and Bartko with radial-velocity data on two superclusters at $z = 0.116$ and 0.135 , respectively; the data indicate that the first is not expanding, and the second only slowly, if at all.

The separation between objects belonging to the same supercluster must of course be smaller than its

diameter. Judging from the known large superclusters this may be of the order of 40 Mpc at the present epoch. At the epoch around $z = 2$, where most of the quasars with known redshifts are situated, they will have been smaller. For a rough estimate we assume that at $z = 2$ the diameters would be around 20 Mpc.

The relation between angular diameter θ in degrees and linear diameter L in Mpc is

$$\theta = 57.3 (1+z)L/R \quad (2)$$

where $R = cH^{-1}q_0^{-2} (1+z)^{-1} \{1 - q_0 + q_0 z + (q_0 - 1)(2q_0 z + 1)^{\frac{1}{2}}\}$.

In a universe with $q_0 = 0.05$ ($\Omega = 0.1$) the angular diameters corresponding to the above linear dimensions are of the order of 1° for $z = 2$, and 1.6° for $z = 1$. The above numbers indicate the general ranges in z and separation to look for, but it should be stressed that superclusters exist over a very wide range of linear dimensions.

What should we expect to observe if quasars would be generally located in superclusters? In a recent article one of us (Oort, 1981) has estimated that the average present distance, S_0 , between major superclusters along a line of sight is about 280 Mpc. If the superclusters have not changed their diameters, then at a redshift z the average distance between them would have been $S_z = S_0(1+z)^{-3}$. Actually, at $z \sim 1$ or 2 , the superclusters were probably smaller, and the distances between them correspondingly larger. Had they expanded at the same rate as the universe, S_z would have been equal to $S_0(1+z)^{-1}$. As we have no way of determining their size variation, we adopt as a compromise $S_z = S_0(1+z)^{-2}$. At the average redshift of about 1.0 with which we shall be concerned the average separation between two superclusters along a line of sight would then be $S_{z=1} = 70$ Mpc. With a Hubble constant of $H_0(1+z)$ this corresponds to an interval of 0.05 in z . The range in z to be considered is about 1.5 . On this range we should then expect to encounter 30 superclusters. Due to our very incomplete knowledge of superclusters and our lack of insight into their expansion characteristics, this estimate is evidently quite uncertain; probably by a factor of about 2.

As discussed in Oort (1981) there may be a relation between superclusters and the strong Lyman α absorption systems observed in distant quasars. For the $\text{Ly}\alpha$ lines the average line-of-sight spacing reduced to $z = 0$ was found to be 150 Mpc, i.e. about half the value found directly from the major superclusters; the difference was ascribed to the incompleteness of our knowledge of superclusters.

Now consider a small region in which a reasonably complete search has been made, and sufficiently accurate redshifts have been measured to enable one to spot quasars lying in the same supercluster. Denote the number of quasars in this region by n , and the number of superclusters traversed by a line of sight in the direction of the region by N .

Suppose that all quasars lie in superclusters and that they are randomly distributed over the N superclusters, then the probability of finding i quasars in one supercluster is given by the binomial

$$P_i = \frac{n!}{i!(n-i)!} \frac{1}{N^i} \left(1 - \frac{1}{N}\right)^{n-i}$$

The average number of superclusters that contain two is thus

$$P_2 = \frac{n(n-1)}{2N} \left(1 - \frac{1}{N}\right)^{n-2}, \quad (3)$$

and this is consequently the expected number of "pairs".

The model is inadequate for a number of reasons. In the first place it does not take into account the

steep increase in the quasar density with z ; in the second place it neglects the serious selection effects by which quasar surveys and catalogues are affected; while thirdly it neglects the great dispersion in the sizes of superclusters. The density increase is largely offset by the fact that at greater distances only the more powerful are observed: in most surveys and in the general catalogue the numbers of quasars do not increase very strongly with z . That notwithstanding its evident inadequacy the model may provide a usable approximation for the general number of large superclusters is indicated by the discussion below, where it is shown that the observed number of pairs agrees with that computed from formula (3) if N is estimated from the known major superclusters.

We begin by considering the quasars which were investigated because of their proximity to bright galaxies or their companions. There are twelve cases where two or more quasars close to galaxies were studied (mostly by Arp and collaborators), but only for two a complete and systematic search was made in a substantial area around the galaxy concerned, viz. NGC 2639 (Arp, 1980b) and NGC 3379/84 (Arp, Sulentic and di Tullio, 1979). There is one other case with five quasars (near NGC 4395), the nine remaining cases have only two or three. None of these have been completely surveyed. We therefore begin our statistics with the two areas mentioned.

NGC 2639: A complete search was made in an area of 193×190 , in which ten quasars were found, with z -values ranging from 0.3 to 2.1, two of which are uncertain. They are concentrated around a companion galaxy $27'$ SSE of the $12^m 4$ spiral NGC 2639. Among these there are two pairs with practically identical redshifts, viz. $1.522 \pm .002$ and $1.525 \pm .002$; and $0.303 \pm .002$ and $0.305 \pm .002$, respectively (cf. Table 1). In order to determine the probability of these near co-incidences being accidental we proceed as follows.

Consider a set of n quasars and number these from 1 to n ; suppose their redshifts to be evenly distributed over a range a_z . If the z -distribution is random the probability k for an arbitrary member of the set to have a redshift differing less than Δz from that of number 1 is evidently

$$k = 2\Delta z/a_z. \quad (4)$$

The expected number of accidental pairs with such a Δz containing number 1 will then be $k(n-1)$. Similarly, the expected number of such pairs containing number 2 and not number 1 (which we have already counted) is $k(n-2)$. For number 3 the number of new pairs is $k(n-3)$; and so on for the other members of the set. In total the expected number of spurious pairs will therefore be

$$k(n-1)\frac{n}{2}. \quad (5)$$

In the case of the quasars near NGC 2639 we have $\Delta z = .003$, $a_z = 1.8$, $n = 8$ (we leave out the two with uncertain redshifts). The expected number of accidental pairs is therefore 0.08. The two pairs that are actually present are thus probably pairs that are really related. The separations between the components are 0.66 and 0.68 respectively, corresponding to 13 and 7 Mpc (cf. Table 1). There is another quasar in the field (U5, $z = 1.494$), $V = (18.3)$ whose redshift differs 0.028 (corresponding to 3300 km/s) from that of the pair (U3, U15) at $z = 1.52$. It might belong to the same supercluster; if this were expanding at the Hubble rate it would lie 13 Mpc in front of the pair; however, as the pair itself shows no indication of any expansion this interpretation is quite uncertain, the more so since the measured redshift is itself uncertain.

For NGC 3379/84 a complete search was made in an area of $30'$ radius, in which eight quasars were found, which were subsequently observed spectroscopically.

Initially two pairs were found with nearly identical z , of which, however, one redshift turned out to be wrong according to a more complete observation (E.M. Burbidge, private communication). The remaining pair (U15 and U8 in Arp et al 1979) has $z=1.131\pm.002$ and $z=1.134\pm.002$. Assuming a true range Δz of 1.0 we find $k=0.006$, so that by formula (5) the expected number of spurious pairs is 0.09. Again the observed pair is thus likely to be real. This is consistent with the conclusion by Arp, Sulentic and di Tullio (1979) that the pairs mentioned must be truly related. The field contains two other faint quasars whose redshifts are relatively near that of the U15/U8 pair, viz. U4 with $z = 1.107$, and U5 with a less certain redshift of 1.192. In the pair's frame of reference the former has a velocity of 3700 km/s and lies at a lateral distance of 10 Mpc. Though the pair itself shows no sign of any expansion the supercluster might expand in a direction perpendicular to the line joining the pair. If it did, and if the expansion velocity would be equal to the Hubble expansion U4 might be a member lying approximately 25 Mpc in front of the pair. Its membership is clearly doubtful. For U5 it is still more so.

The case of a pair near M82 belongs in the same category as the three pairs discussed above. Three faint quasars were found by Hoag within 10' from the galaxy (cf. Arp, 1980a). Accurate redshifts were measured by E.M. Burbidge et al (1980). Two have $z = 2.049$ and 2.0440 respectively; the redshift of the third is given as (2.1). The separation is $0^{\circ}59$, corresponding to 1.3 Mpc. With $\Delta v = 490 \text{ km s}^{-1}$ the first two belong undoubtedly together, while the third is very likely a member of the same supercluster; in view of their relatively small separation they might even be members of a large galaxy cluster.

There is one other case in which a considerable number of accurate redshifts are available in a restricted area, and where quasar pairs might therefore be observed. This is a field observed by Arp and Hazard (1980); they were struck by two peculiar configurations of quasars in two adjacent regions on a plate taken with the U.K. Schmidt telescope as part

of a programme aimed at investigating quasar clustering. In one of the concentrations ten redshifts were measured with the 5-metre reflector at Palomar in an area of $51' \times 52'$. In the second concentration, about 3 to 4 degrees from the first, redshifts were measured for six quasars in a similar area. The peculiarity of the concentrations has been discussed by Arp and Hazard. Here we limit ourselves to the observation that the first concentration contains a pair with identical redshifts, of 1.010 ± 0.001 and 1.011 ± 0.001 respectively; the separation is $0^{\circ}05$, corresponding to 0.9 Mpc. The second concentration has no definite pairs. The expected number of spurious pairs with a separation less than $0^{\circ}5$ is 0.07 for each of the two areas. The pair indicated is therefore very probably real, as stressed by the authors.

There are two more series of observations where a sizeable number of quasars have been observed in small areas: with a transmission grating on the Kitt Peak 4-m telescope Sramek and Weedman (1978) found 91 quasars in 29 small areas, of $34'$ side. Spectrophotometry of 8 good-quality plates yielded reasonably reliable redshifts and magnitudes for 46 (Barbara Vaucher and Weedman, 1980). Among the latter there is one narrow pair, with $z = 2.82$ and $z = 2.88$, respectively at $12^{\text{h}}59^{\text{m}}44^{\text{s}}+34^{\circ}26'$ and $13^{\text{h}}00^{\text{m}}08^{\text{s}}+34^{\circ}33'$ (cf. the table in the Appendix). The chance of its being real is about 1/2. With a similar outfit on the 4-m telescope on Cerro Tololo Hoag and Smith (1977; cf. also Osmer 1980) observed a number of regions with a total area of about 5 square degrees around $0^{\text{h}}-43^{\circ}$. There are three narrow pairs in their list which might be real (cf. the table in the Appendix, Osmer 1980).

From the preceding discussions we conclude that the existence of pairs of quasars which may show the presence of large superstructures is well established. The probability that the majority of the pairs would be spurious is extremely small.

The results for the deep fields are summarized in Table 1. Leaving out the field of M82, for which we have no statistical information, we have five regions

TABLE 1.
Quasar pairs in deep-survey areas with accurate redshifts

Pair	Field	Area	n	α 1950				δ	z	Quasar pairs				Expected spurious pairs
				h	m	s	o	'		V	separation		Mpc	
1	NGC 2639 ¹⁾	1.3x1.0	10	8	40	4	+50	23	0.303	19.3	0.67	7.0	0.08	
2									0.305	18.1				
									1.522	(19.1)				0.64
3	NGC 3379/84 ²⁾	radius 0.6	8	10	45	38	+12	54	1.131	19.7	0.26	5.2	0.09	
									1.134	18.7				
									NGC 4395	5				12
4	M 82	radius 10'	3	9	53	7	+69	52	2.049	20.	0.05	1.3	small	
									2.044	21.				
									(2.1)	21.				
5	Arp & Hazard 11/46 + 11/12	51'x52'	10	11	46	14	+11	12	1.010	(19.5)	0.05	0.9	0.14	
									1.011	(18.9)				
									Arp & Hazard 11/30 + 10/40	40'x60'				6

1) There is a fifth quasar in this field whose redshift of 1.494 differs 0.028 from that of the second pair (see text).

2) There are two other 19^{m} quasars in this field with redshifts resembling those of the tabulated pair, viz., U4 with $z = 1.107$ and U5 with $z = 1.192$; see text.

TABLE 2.
Other quasar pairs

Pair	α		δ		z	V	separation		reality	
	h	m	s	'			o	Mpc		
6	1	43	19	- 1	36	3.14	18.8	0.54	11.8	almost certain
		43	49	- 1	1	3.16	19.0			
7	4	3	14	-13	16	0.571	17.17	1.07	16.3	very probable
		5	27	-12	20	0.574	17.07			
8	16	18	7	+17	44	0.555	16.41	1.28	19.2	probable
		23	14	+17	22	0.552	19.0			
9	22	4	32	-19	12	1.067	17.80	1.26	24.3	uncertain
		6	41	-18	3	1.071	19.15			
10 ¹⁾	23	19	33	-38	20	0.37	17.3	0.90	10.9	almost certain
		21	26	-37	31	0.37	18.9			
11	23	55	5	-36	25	2.073 ²⁾	18.0	1.60	34.8	uncertain
		57	6	-34	52	2.07	17.0			
12 ³⁾	23	55	10	-38	58	2.85	18.4	1.18	26.2	probable
	0	0	31	-39	49	2.83	18.8			

1) There are two other quasars in the CTIO Curtis Schmidt survey which are almost certainly situated in the same supercluster, viz.

23^h 3^m 50^s -39° 6', $z=0.36$, $V=17.7$ at a distance of 3^o 44', or 41.6 Mpc, from the pair,
and 23 22 19 -41 30, $z=0.37$, $V=17.6$ at a distance of 3^o 60', or 43.5 Mpc, from the pair.

The separation between the two outermost members is 51.5 Mpc.

2) Uncertainty $\pm .009$.

3) Two other high-redshift quasars have been observed in the same vicinity:

0^h 2^m 16^s -42° 14', $z=2.76$, $V=17.4$ and 0^h 18^m 24^s -42° 12', $z=2.86$, $V=18.6$,

but it is doubtful whether they lie in the same supercluster, the former differing too much in z , the latter being 4^o 8', or 106 Mpc away in the lateral direction, which would imply an improbably large diameter for this early epoch.

TABLE 2a
A triple of nearby objects

	α		δ		z	distance from centre		V	M
	h	m	s	'		of group	Mpc		
12	27	31	+12	6	0.061	11.51	32.0	19.0	-17.3
		29	48	+20	25	0.064	3.31	15.3	-21.0
13	0	43	+36	8	0.060	14.04	40.3	17.5	-18.8

with an average of 8 quasars per region, among which there are four well established pairs; in a given direction the average number of superclusters containing two quasars is therefore $p_2 = 0.8$. Inserting this number in formula (3) and taking $n = 8$ we derive

$$N = 28.$$

This is the number of superclusters in a given direction which are sufficiently powerful to contain quasars. It is interesting that this number agrees with the number of major superclusters expected on the basis of our, admittedly scant, knowledge of superclusters in our vicinity. The number is about one third of the number of strong $\text{Ly}\alpha$ absorptions, and indicates that if these are due to superclusters only one third of these

superclusters are of sufficient importance to contain quasars. It should again be stressed that all the numbers quoted are uncertain by a factor of at least 2.

From the above values for p_2 and N the number of superclusters containing various numbers of quasars, i , may be found from the expression for P_i preceding formula (3) and are shown in Table 3. We see that 75% of the superclusters capable of containing quasars do not contain any, and 21% contain only a single quasar, and cannot therefore be observed.

We observe from Table 1 that the separations are generally of the order of 5 to 10 Mpc, considerably smaller than the diameters of known superclusters. The difference may be partly an effect of selection due to the small sizes of the fields.

TABLE 3

Estimated numbers of superclusters seen in a given direction which contain i observable quasars (sample discussed in the text)

i	nr	fraction
0	21	0.75
1	6	0.21
2	0.8	0.03
3	0.06	0.002

A search for additional quasar pairs has been made by us in the new general catalogue of quasars by Adelaide Hewitt and G. Burbidge (1980) as well as in the major special surveys. Seven probable pairs were found; they are listed in Table 2. The search was essentially confined to pairs with separations less than $1^{\circ}5$, because for larger separations there is generally no possibility to distinguish real pairs from accidental co-incidences. For the same reason we considered principally those quasars for which z has been observed with 3-decimal accuracy, except for quasars with $z > 2.5$, or < 0.05 , where 2-decimal accuracy suffices to recognize the genuine pairs. During the general search a number of additional pairs were noted which are given in a table of possible pairs in the Appendix. For these it is estimated that there is approximately an even chance that they are accidental co-incidences. We did not aim at any sort of completeness, and the criteria for inclusion in the table have been somewhat arbitrary. For instance, there must exist real pairs with separations larger than the 2° limit which was generally adhered to in picking out the pairs considered in this article, but it is as yet impossible to distinguish these from accidental co-incidences.

For Table 2 the chance of being spurious was estimated in a similar manner as before except that k is now the probability that a quasar will be found within a given distance r (usually $1^{\circ}5$, in one case $1^{\circ}0$) as well as in a given interval of $2\Delta z$. For pairs 6 and 12 in Table 2 all known quasars with $z > 2.5$ were taken into consideration. There are about 90. All but 36 of these lie either in the area of about 520 square degrees of the CTIO Curtis Schmidt Survey around -40° declination (Osmer and Smith 1980) or in the area of 392 square degrees around 0° declination of the Michigan-Tololo Survey (Lewis et al 1979), or in two small areas around $2^{\text{h}}50^{\text{m}}$ and $16^{\text{h}}18^{\text{m}}$.

Pair no. 6 comes from the Michigan-Tololo Survey. This contains 6 quasars with $z > 2.5$. Taking $r = 1^{\circ}0$, $2\Delta z = 0.05$, and $a_z = 0.7$ for the quasars with $z > 2.5$, we have $k = \frac{0.05}{0.7} \times \frac{3.14}{392} = 5 \times 10^{-4}$. With $n = 6$ we find from (5) that the expected number of spurious pairs with separations less than $1^{\circ}0$ is 0.008. The pair has thus a very high probability of being real. The last pair comes from the CTIO Curtis Schmidt Survey, which contains 19 quasars with $z > 2.5$. In this case

$k = \frac{0.05}{0.7} \times \frac{7.06}{520} = 1.0 \times 10^{-3}$; the expected number of spurious pairs with separations less than $1^{\circ}5$ is 0.17, so that the pair has a fair probability of being real.

Pair No. 10, with $z = 0.37$, comes likewise from the CTIO Curtis Schmidt Survey. In the main part of this survey, between $2^{\text{h}}0^{\text{m}}$ and $5^{\text{h}}0^{\text{m}}$, covering 340 square degrees, there are seven objects with $z < 0.10$. Among these the expected number of accidental pairs with $\Delta z < 0.01$ and separations $< 1^{\circ}5$ is

$\frac{0.01}{0.10} \times \frac{7.07}{340} \times 6 \times \frac{7}{2} = 0.004$. The pair is therefore almost certainly real. There are moreover two other quasars in this survey within 4° of the pair, with

$z = 0.36$ and 0.37 , respectively (cf. Table 2). The probability of a chance co-incidence is roughly 0.003 for each of these, so that the four objects are probably members of one, relatively nearby, supercluster, extending over at least 52 Mpc.

For the other pairs the reality is more difficult to ascertain. From a careful analysis we have estimated that in the entire catalogue one should expect 0.16 accidental pairs like No. 7; this is if no account is taken of the exceptional brightness of the pair 7 ($V=17.17$ and 17.07 respectively). If one does take this into consideration the expectation of a spurious pair would become nearly an order of magnitude smaller. We thus consider Pair No. 7 as being very probably a genuine one.

Pairs 9 and 11 lie similarly in arbitrary regions, where no special surveys were made. The pairs are not exceptionally bright. We estimate that about 0.2 accidental pairs of similar qualifications should be expected in the whole catalogue. It is therefore somewhat uncertain whether these pairs are real. For Pair 11 the evidence might be improved by determining more reliable redshifts.

Pair No. 8 is contained in a survey field observed by Savage et al. (1978). The probability of a spurious pair in this field is 0.12. As this is the only small field not in the vicinity of a bright galaxy where a considerable number of redshifts were measured with sufficient accuracy, we consider the pair as probably real.

Pair No. 7 had been noted before, and has been mentioned by one of us (Arp, 1980b) in a discussion of pairs of quasars aligned across galaxies.

Pair No. 6 was first brought to our attention by T. Heckman.

Pairs 6 and 12 are particularly interesting in that they furnish evidence of the existence of large superclusters at these early epochs. The velocity differences of $\sim 1500 \text{ km s}^{-1}$ are not larger than what should be expected. At this early stage superclusters are presumably still expanding along their large axes. If they would expand at the Hubble rate the expansion coefficients would be 476 and $435 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively. If the dimensions along the line of sight would be of the same order as the observed transverse separations the expected velocity ranges due to expansion would then be 5600 km s^{-1} and 11000 km s^{-1} , respectively. The observed velocity differences are only $1/4$ and $1/7$ of these values, and are therefore perfectly reasonable.

A special case is that of the three nearby objects listed in Table 2a. These were found from the first edition of the quasar catalogue (Burbidge et al 1977). They can hardly be classified as quasars: two have absolute magnitudes of -17 and -19 , respectively, the third -21 . There are eight objects with $z < 0.100$ in this catalogue. The number of chance configurations of three within a region of 60° diameter and having $\Delta z < 0.004$ is roughly 0.001, so that the triple is very probably "physically connected". The distance between the two outer members is 70 Mpc. The supercluster in which they are situated is therefore large, but not exceptionally so.

Apparently quasars could occur in superclusters. About 20% of the large superclusters could contain one quasar; about 3% contain more than one. Superclusters have been found up to large values of z ; at $z \sim 3$ they can already have dimensions up to 25 Mpc.

At the present moment the data are scarce. They can, however, be augmented almost indefinitely by surveying areas of a few degrees diameter down to a faint limit and measuring accurate redshifts for the quasars discovered. With an estimated surface density of 10 per square degree an area of 2 degrees diameter would contain 30 quasars. On the basis of the very rough data

in Table 3 there would on the average be one genuine pair in such a region, so that one supercluster would be discovered. In addition searches for large-redshift quasars should be made over large areas. These might prove to be the most interesting. For objects with $z > 2.5$ redshifts with an accuracy of ± 0.01 would suffice to pick out genuine pairs. In the long run reliable information could so be obtained concerning the frequency and diameters of superclusters at various epochs in the past. A comparison of the sizes of superclusters at high z with those at small redshifts might, for instance, yield new evidence in the still existing controversy on the cosmological interpretation of some categories of quasars.

Note added to the manuscript

Since the article was written Arp discovered another pair of quasars with identical redshifts, one just south of NGC 2549 comp S, the other just north east

of NGC 2534. Both have $z = 2.40$, with an estimated mean error of ± 0.005 . No other quasars were detected and measured over the 20 sq. degree area of the Schmidt plate on which these quasars were found. The quasars are therefore probably connected. Their separation is $2^{\circ}9'$, corresponding to 47.0 Mpc. The 1950 positions of the quasars are

and $8^{\text{h}}14^{\text{m}}16^{\text{s}} +57^{\circ}52'54''$
8 9 11 +55 49 45

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APPENDIX

The table lists pairs noticed during our search, but for which there is only about an even chance of being real or spurious.

h	α		δ		V	z	Separation		Reference
	m	s	m	s			o	Mpc	
0	1	24	-42	56	18.8	2.03			
	1	57	-42	55	18.4	2.04	0.10	2.2	Osmer 1980
0	3	40	-42	53	19.6	1.85			
	3	47	-43	2	18.4	1.86	0.15	3.2	Osmer 1980
0	18	48	-38	49	19.0	2.29			
	19	34	-39	41	19.8	2.28	0.88	19.3	Osmer & Smith 1980
0	17	44	-39	43	18.4	1.94			
	26	52	-39	16	19.2	1.95	1.82	39.3	Osmer & Smith 1980
1	31	44	+ 1	32	17.8	0.41			
	31	44	+ 0	0	17.8	0.40	1.53	19.4	Lewis <i>et al</i> 1979
2	31	54	-41	4	19.9	2.30			
	39	44	-42	1	18.6	2.30	1.78	38.9	Osmer & Smith 1980
12	58	42	+35	39	18.28	0.323			
13	1	42	+35	49	18.65	(0.330)	0.64	7.1	Deveny <i>et al</i> 1971 Braccisi <i>et al</i> 1970
12	59	44	+34	26	19.1	2.82			
13	0	0	+34	33	19.7	2.88	0.13	2.9	Vaucher & Weedman 1980
13	8	17	+38	13	17.56	2.090			
	11	20	+36	16	18.41	2.084	2.06	44.8	Deveny <i>et al</i> 1971 Braccisi <i>et al</i> 1968
16	34	1	+62	52	20.6	0.988			
	34	20	+58	55	18.0	0.985	3.95	74.9	Baldwin <i>et al</i> 1973 Carswell & Walsh 1980
22	19	55	-39	29	18.5	2.022			
	25	30	-40	25	18.1	2.02	1.42	31.0	Osmer & Smith 1980
22	24	14	-40	52	18.5	2.33			
	24	37	-39	47	19.0	2.34	1.09	23.9	Osmer & Smith 1980
22	24	56	-40	21	19.4	1.91			
	24	26	-40	8	19.5	1.92	0.24	5.2	Osmer 1980
22	47	56	-39	41	19.4	2.61			
23	4	30	-42	20	17.6	2.61	4.13	90.3	Osmer & Smith 1980
23	41	48	+ 1	4	18.6	1.96			
	45	56	+ 0	23	18.0	(1.96)	1.20	25.9	MacAlpine <i>et al</i> 1977
23	53	20	+15	25	18.0	1.801			
	54	45	+14	29	18.2	1.810	0.99	21.2	Wills & Wills 1976 Kinman & Burbidge 1967

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