

## RECOILING BLACK HOLES IN QUASARS

E. W. BONNING,<sup>1</sup> G. A. SHIELDS,<sup>2</sup> AND S. SALVIANDER<sup>2</sup>

Received 2007 May 23; accepted 2007 July 18; published 2007 August 14

### ABSTRACT

Recent simulations of merging black holes with spin give recoil velocities from gravitational radiation up to several thousand kilometers per second. A recoiling supermassive black hole can retain the inner part of its accretion disk, providing fuel for a continuing QSO phase lasting millions of years as the hole moves away from the galactic nucleus. One possible observational manifestation of a recoiling accretion disk is in QSO emission lines shifted in velocity from the host galaxy. We have examined broad-line QSOs with measurable H $\beta$  and [O III] from the Sloan Digital Sky Survey that have broad emission lines substantially shifted relative to the narrow lines. We find no convincing evidence for recoiling black holes carrying accretion disks. We place upper limits on the incidence of recoiling black holes in QSOs of 0.2% for kicks greater than 800 km s<sup>-1</sup>, 0.08% for kicks greater than 2000 km s<sup>-1</sup>, and 0.04% for kicks greater than 2500 km s<sup>-1</sup> line-of-sight velocity.

*Subject headings:* black hole physics — galaxies: active — quasars: general

### 1. INTRODUCTION

Recent breakthroughs in numerical relativity have allowed the possibility of simulating in complete general relativity the final orbits of a binary black hole up to and past the merger (Pretorius 2006; Campanelli et al. 2006; Baker et al. 2006a). Analytical approximations have for some time indicated that the merger of unequal mass black holes will, through anisotropic emission of gravitational radiation, impart to the final black hole a “kick” of up to more than 1000 km s<sup>-1</sup> (Fitchett 1983; Damour & Gopakumar 2006; Sopuerta et al. 2006; Lousto & Price 2004 and references therein). Numerical simulations have only begun to explore the parameter space of black hole mass ratio, spin magnitude, and spin-orbit orientations in such mergers (Herrmann et al. 2007a, 2007b; Baker et al. 2006b, 2007; González et al. 2007a, 2007b; Koppitz et al. 2007; Campanelli et al. 2007a; Tichy & Marronetti 2007). The largest kick measured in numerical simulations to date is  $\sim 2500$  km s<sup>-1</sup> for spins with spin parameter  $a_* = 0.9$ , antialigned and perpendicular to the orbital angular momentum (González et al. 2007a; Tichy & Marronetti 2007). Extrapolating from numerical results, Campanelli et al. (2007b) predict a maximum recoil kick of  $\sim 4000$  km s<sup>-1</sup> for maximally rotating holes. Such large recoil velocities have significant astrophysical implications for galactic mergers, since even velocities of order 1000 km s<sup>-1</sup> can be greater than the escape velocity of moderate-sized elliptical galaxies and spiral bulges, and much greater than the  $\lesssim 300$  km s<sup>-1</sup> escape velocity for dwarf galaxies (Merritt et al. 2004 and references therein).

Binary supermassive black holes ( $\sim 10^8 M_\odot$ ) will be formed during the merger of galaxies (Begelman et al. 1980). The binary orbit will decay quickly as a result of dynamical friction due to the stellar background. The orbit may stall at a radius  $\sim 1$  pc, but this may be overcome by the presence of a nuclear gas disk (Escala et al. 2004; Berczik et al. 2006). For a black hole merger taking place in an active galactic nucleus (AGN), the accretion disk will remain bound to the recoiling black hole inside the radius  $R = 1.33 \times 10^{18} M_8 / v_{1000}^2$  cm, where the orbital velocity is equal to the recoil velocity. Here  $M_8 = M/10^8 M_\odot$  and  $v_{1000} = v/1000$  km s<sup>-1</sup>. The retained disk mass

in such a case, assuming an  $\alpha$  disk (Shakura & Syunyaev 1973; Frank et al. 2002), will be

$$M(R) = (10^{6.60} M_\odot) \alpha_{-1}^{-4/5} M_8^{1/4} \dot{M}_0^{7/10} R_{17}^{5/4} f^{14/5} \quad (1)$$

or

$$M(v) = (10^{8.02} M_\odot) \alpha_{-1}^{-4/5} M_8^{3/2} \dot{M}_0^{7/10} v_{1000}^{-5/2}, \quad (2)$$

where  $\dot{M}_0$  is the accretion rate in solar masses per year. Stability requires  $M_{\text{disk}} < M_{\text{BH}}$  (see Loeb 2007).

For a black hole binary contained in an accretion disk, the two holes will empty out a “gap” with a radius of approximately twice the binary semimajor axis (Macfadyen & Milosavljevic 2006). This will refill quickly after the kick (Loeb 2007), and QSO activity will resume. The disk mass will be sufficient to fuel QSO activity over a disk consumption time  $t_d \approx M(v)/\dot{M}_0 \approx (10^8 \text{ yr}) \alpha_{-1}^{-4/5} M_8^{3/2} \dot{M}_0^{-3/10} v_{1000}^{-5/2}$ . This “wandering QSO” phase could last for a time comparable to the premerger phase and would result in either a QSO displaced a number of kiloparsecs from the galactic nucleus or QSO emission lines shifted relative to galactic systemic velocity.

Observations of nearby AGNs do not show displaced nuclei (Libeskind et al. 2006). This may be one indication that large kicks rarely occur during an active AGN phase.<sup>3</sup> Alternatively, these kicks may be observed in the velocity of the AGN emission lines. The broad-line region (BLR) of QSOs corresponds to radii within which the disk will remain bound to the post-merger black hole. The BLR dynamical timescale is  $\sim 10^2$  yr, so the BLR should be regenerated quickly from the disk following the recoil and resumption of AGN activity. The narrow emission lines, in contrast, arise from gas predominantly orbiting in the potential of the host galaxy that will not follow the recoiling black hole (Merritt et al. 2006). The displaced QSO will still ionize the interstellar gas, producing narrow emission lines, albeit different in detail from a normal narrow-line region (NLR). Therefore, the broad emission lines associated with a recoiling disk will appear shifted with respect to

<sup>1</sup> LUTH, Observatoire de Paris, CNRS, Université Paris Diderot, 92190 Meudon, France; erin.bonning@obspm.fr.

<sup>2</sup> Department of Astronomy, University of Texas, Austin, TX 78712; shields@astro.as.utexas.edu, triples@astro.as.utexas.edu.

<sup>3</sup> The quasar HE 0450–2958 (Magain et al. 2005) has been suggested as such a candidate, but questions remain about whether an ejected black hole is indicated (Merritt et al. 2006; Kim et al. 2007; Zhou et al. 2007; Feain et al. 2007).

the galaxy systemic redshift as expressed by the narrow emission lines.

We have carried out a search for candidate kicked QSOs using spectra from the Sloan Digital Sky Survey (SDSS)<sup>4</sup> Data Release 5 (DR5). We have focused attention on the broad H $\beta$  and narrow [O III] lines but also consider broad Mg II where available. Note that velocity shifts of broad emission lines are a well-studied phenomenon often attributed to BLR physics or even orbiting binary black holes (e.g., Gaskell 1996; Richards et al. 2002 and references therein). This complicates the task of identifying true examples of recoil.

## 2. OBSERVATIONS

To construct our data set, we downloaded all objects spectroscopically classified “quasar” by the SDSS DR5 pipeline (Schneider et al. 2002; this includes objects with a broad emission line of at least 1000 km s<sup>-1</sup> FWHM). We selected objects with redshift in the range  $0.1 < z < 0.81$  such that the H $\beta$  and [O III] emission lines were both accessible. Emission lines were measured by means of a least-squares fit of a Gauss-Hermite function (Pinkney et al. 2003) to the line profile, together with a linear fit to the continuum in the vicinity of the line. For details on the measurement procedure, including Fe II subtraction, the reader is referred to Salviander et al. (2007). The broad H $\beta$  line was fit after removing an assumed narrow H $\beta$  line with the profile and 10% of the flux of the [O II]  $\lambda$ 5007 line (Baldwin et al. 1981). The broad Mg II line and the narrow [O II] and [S II] lines were measured when accessible. We assumed doublet ratios of unity for Mg II and 1.2 red-to-blue for [O II]. Spectral fits were accepted if the line widths and equivalent widths had an accuracy better than 15% and visual inspections showed a good fit and no artifacts. After the numerical quality cuts, we eliminated objects with FWHM (H $\beta$ ) less than 1500 km s<sup>-1</sup>, which removes the narrow-line Seyfert 1 and any Seyfert type 2 that passed the SDSS quasar criterion (about 7% of the objects). In addition, we removed objects with  $h_3$  for H $\beta$  greater than 0.1 in absolute magnitude, since the broad-line velocity may be affected by the strong asymmetry of these line profiles. After visual inspection, our data set consists of 2598 objects; this is approximately 20% of the original download. Approximately 40% of these objects are brighter than  $M_B = -22.3$ , the Seyfert/QSO boundary (Peterson 1997), and 75% are brighter than  $M_B = -21.3$ .

The redshifts of the peaks of the lines were calculated from the fits to the line profiles. The relative displacement of the broad H $\beta$  line with respect to the peak of [O III] was calculated as  $\Delta v_{\text{H}\beta} = c(z_{\text{H}\beta} - z_{[\text{O III}]})/(1 + z_{[\text{O III}]})$ . The displacement of Mg II from [O III],  $\Delta v_{\text{Mg II}}$ , is defined analogously. A histogram of  $\Delta v_{\text{H}\beta}$  and  $\Delta v_{\text{Mg II}}$  is shown in Figure 1. A Gaussian fit to the distribution of  $\Delta v_{\text{H}\beta}$  gives a mean displacement of +100 km s<sup>-1</sup>, and the FWHM of the distribution is 500 km s<sup>-1</sup>. The fact that there is an overall redshift may largely result from physical processes in the BLR. We find an average [O III] blueshift of 30 km s<sup>-1</sup> relative to [S II] and 40 km s<sup>-1</sup> relative to [O II], similar to the results of Boroson (2005). The largest blueshifts of [O III] relative to [O II] were found for objects with small  $\Delta v_{\text{H}\beta}$ . Interestingly, the 36 objects with measurable [O II] and  $\Delta v_{\text{H}\beta}$  more than 600 km s<sup>-1</sup> from the mean showed good agreement between [O III] and [O II]. Therefore, use of [O III] as a reference velocity does not significantly affect our calculation of the high H $\beta$  line displacements.

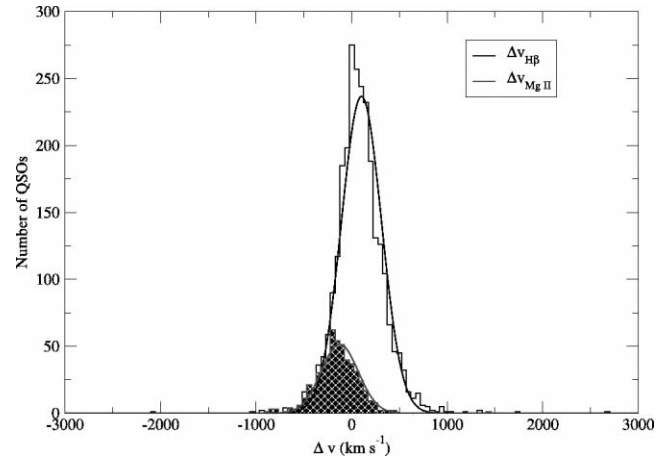


FIG. 1.—Histogram of  $\Delta v_{\text{H}\beta}$  and  $\Delta v_{\text{Mg II}}$  along with Gaussian fits to the data. The distribution is somewhat broader than a Gaussian, with small but significant numbers of outliers with  $|\Delta v| > 1000$  km s<sup>-1</sup>.

In our data set of 2598 objects, the fraction  $f_v$  of  $\Delta v_{\text{H}\beta}$  greater than a given velocity are  $f_{500} = 0.04$ ,  $f_{800} = 0.007$ ,  $f_{1000} = 0.0035$ ,  $f_{1500} = 0.0012$ ,  $f_{2000} = 0.0008$ , and  $f_{2500} = 0.0004$ . Note that these fractions assume that every shift measured is due to a kick. Of the 501 objects with measured  $\Delta v_{\text{Mg II}}$ , only one has  $\Delta v_{\text{Mg II}}$  over 800 km s<sup>-1</sup>, and this object has inconsistent values of  $\Delta v_{\text{Mg II}}$  and  $\Delta v_{\text{H}\beta}$  (see discussion below). If we interpret zero out of 501 objects as an upper limit of 1/501, this implies  $f_{800} < 0.002$ , a tighter limit than from H $\beta$  alone.

## 3. DISCUSSION

Schnittman & Buonanno (2007) compute the black hole recoil velocities for a range of mass ratios, spin magnitudes, and directions from post-Newtonian equations of motion calibrated to the results of the numerical simulations. For mergers of two black holes with equal spin parameter  $a_* = 0.9$  and a restricted set of mass distributions such that  $m_1 m_2 / (m_1 + m_2)^2 \geq 0.16$ , Schnittman & Buonanno (2007) find a fraction  $f_{500} = 0.31$  of recoils greater than 500 km s<sup>-1</sup> and a fraction  $f_{1000} = 0.079$  of recoils greater than 1000 km s<sup>-1</sup>. Convolving the probabilities of Schnittman & Buonanno (2007) with random kick inclinations to the line of sight, we find the predicted observational kick fractions to be  $f_{500} = 0.18$  and  $f_{1000} = 0.054$ .

The observed incidence of H $\beta$  shifts over 1000 km s<sup>-1</sup> is several times less than theoretically expected for rapidly spinning holes with random orientations and similar masses. However, it is difficult to compare our results directly to the kick fractions predicted by relativistic calculations. Without knowing the spin distribution of supermassive black hole binaries, and without full exploration of the parameter space by numerical calculations, computed kick probabilities will be uncertain. The likelihood of black hole mergers early in the luminous phase of an AGN is unknown. One must also account for forces acting on the black hole by the host galaxy. The kick velocity may be substantially reduced by dynamical effects including the inertia of the disk and stars that remain bound to the recoiling hole, the slowing of the black hole by the gravitational potential of the galaxy, and dynamical friction over the luminous life of the wandering AGN. These effects can be significant. For large elliptical galaxies, Merritt et al. (2004) find escape velocities of 1000 km s<sup>-1</sup> or more. For kick velocities near or below this value, the recoiling black hole will slow down substantially or turn around in a fraction of

<sup>4</sup> The SDSS Web site is <http://www.sdss.org>.

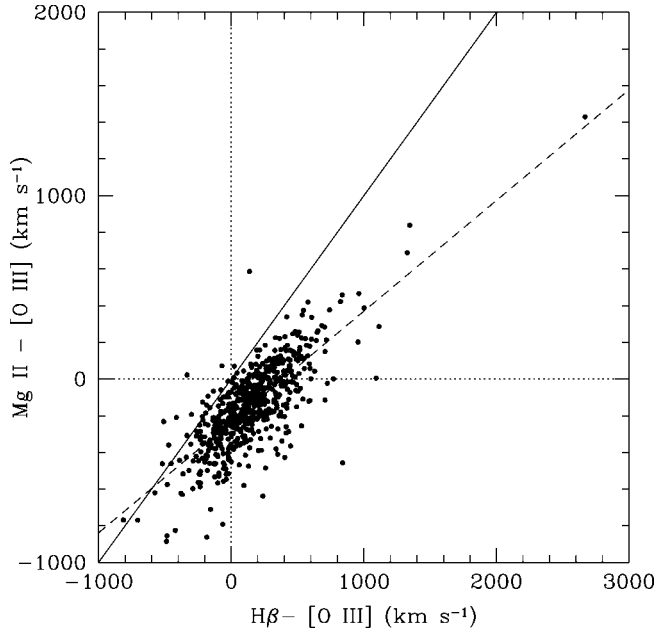


FIG. 2.—Plot of Mg II - and Hβ - [O III] displacements. The dotted line shows a fit to the data of  $\Delta v_{\text{Mg}} = 0.6\Delta v_{\text{H}\beta}$ , and the solid line shows the relation of equality. Removal of the high-redshift outlier does not change the fit.

the luminous phase. However, in the case of kicks over  $\sim 1500 \text{ km s}^{-1}$ , these dynamical effects are relatively less important, and such cases may still be detectable by means of shifted broad lines.

Closer inspection of the objects with large  $\Delta v_{\text{H}\beta}$  suggests that these displacements most likely result from BLR physics rather than recoils.

1. The largest shifts occur only for objects with large Hβ FWHM. (This differs from the findings of Sulentic et al. [2007] and Richards et al. [2002] for C IV  $\lambda 1550$ .) The distribution of FWHM for our measured Hβ lines peaks at about  $3000 \text{ km s}^{-1}$ , but for objects with shifts greater than  $600 \text{ km s}^{-1}$  from the mean we find an average FWHM of  $\sim 5000 \text{ km s}^{-1}$ , with only one object (SDSS J141959.21+610143.6) narrower than  $3000 \text{ km s}^{-1}$ . We have no reason to expect recoils to prefer larger FWHM.

2. For recoils, all broad lines should have approximately the same velocity shift. Figure 2 shows that typically  $\Delta v_{\text{Mg}} \approx 0.6\Delta v_{\text{H}\beta}$  for high as well as low shifts. No subset of objects stands out to the eye as noteworthy candidates to be recoils.

3. We have constructed composite spectra (Fig. 3) for objects with  $\Delta v_{\text{H}\beta}$  greater than  $600 \text{ km s}^{-1}$  from the mean, less than  $-600 \text{ km s}^{-1}$  from the mean, and with absolute value of the shift less than  $600 \text{ km s}^{-1}$ . In all three composites, the [Ne v] FWHM is several hundred kilometers per second greater than that of the lower ionization lines (Table 1). This is often, although

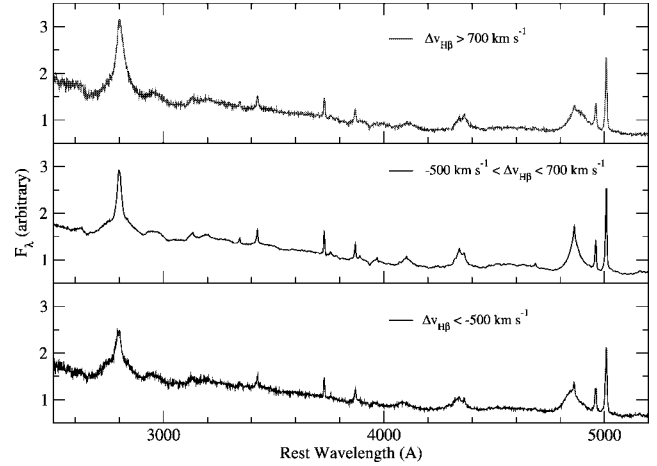


FIG. 3.—Composite spectra for the highest Hβ redshifts relative to [O III] (top panel, 33 objects), highest blueshifts (bottom panel, 24 objects), and shifts centering around the mean (middle panel, 2541 objects).

not always, the case in AGNs (e.g., Whittle 1985). However, in the case of a kicked black hole, one might expect a “left behind” NLR, or interstellar medium ionized by the off-center QSO, to show line widths consistent among the narrow lines. Alternatively, if there were a compact NLR bound to the black hole, it would contribute a [Ne v] component centered on the velocity of the black hole. Neither of these cases appears in the composite spectra, suggesting that the QSOs with high  $\Delta v_{\text{H}\beta}$  have normal NLRs. Table 1 also shows that the shifted QSOs have similar narrow-line intensities as the unshifted QSOs.

4. Finally, we visually inspected the high-shift objects to see if any of them stood out as kick candidates. Of particular interest was SDSS J091833.82+315621.1, shown in Figure 4, which has the largest Hβ shift in our sample ( $\Delta v_{\text{H}\beta} = 2667 \text{ km s}^{-1}$  and  $\Delta v_{\text{Mg}} = 1231 \text{ km s}^{-1}$ ). While the shifted Hβ line is striking in appearance and symmetric in shape, there seems to be little else in the spectrum to distinguish it from a non-kicked BLR. The NLR line ratios and intensities are normal, and its  $\Delta v_{\text{Mg}}$  is consistent with  $0.6\Delta v_{\text{H}\beta}$ , as shown in Figure 2.

Some highly blueshifted objects, such as SDSS J120354.76+371137.2 ( $\Delta v_{\text{H}\beta} = -513 \text{ km s}^{-1}$ ) or SDSS J135800.40+404358.1 ( $\Delta v_{\text{H}\beta} = -813 \text{ km s}^{-1}$ ) have a sharp cutoff on the red side of the line, often accompanied by a ledge or shoulder. In these cases, the asymmetry shifts the peak, but the wings more nearly center on the [O III] redshift. Similarly, Richards et al. (2002) found that large blueshifts (relative to [O III]) in the broad C IV and Mg II lines can be attributed to a diminution of flux in the red wing of the line rather than to a true shift.

Possibly more promising are objects which, besides having symmetrical broad lines, also have similar  $\Delta v_{\text{H}\beta}$  and  $\Delta v_{\text{Mg}}$  in addition to a [Ne v] line with similar broadening as the

TABLE 1  
COMPOSITE SPECTRA

LINE	EW (Å)			FWHM ( $\text{km s}^{-1}$ )		
	Redshift	Blueshift	$ \Delta v_{\text{H}\beta}  < 600 \text{ km s}^{-1}$	Redshift	Blueshift	$ \Delta v_{\text{H}\beta}  < 600 \text{ km s}^{-1}$
[O II] .....	3.07	2.44	2.76	582	472	506
[O III] .....	17.64	19.28	15.41	541	536	420
[Ne III] .....	2.73	2.10	1.90	748	595	516
[Ne V] .....	1.95	1.58	1.5	775	615	650



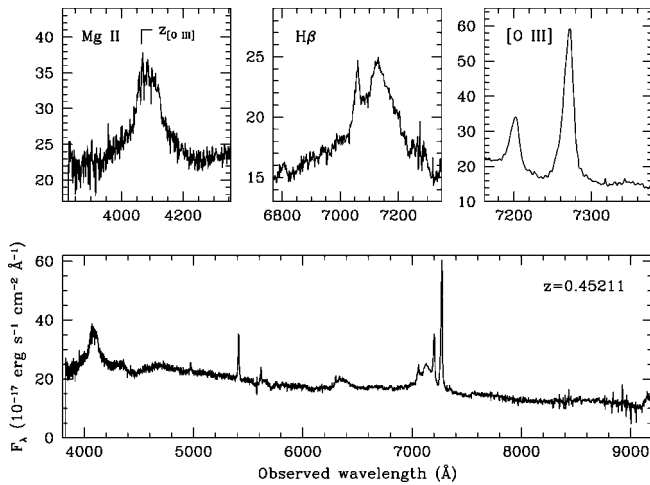


FIG. 4.—SDSS J091833.82+315621.1, the largest shifted object in our sample with  $\Delta v_{H\beta} = 2667 \text{ km s}^{-1}$ .

lower ionization narrow lines. Two such objects in our sample, albeit with fairly low signal-to-noise ratio spectra, are SDSS J134812.36+052402.6, with  $(\Delta v_{H\beta}, \Delta v_{Mg}) = (-706, -769) \text{ km s}^{-1}$  and FWHM  $([\text{Ne v}], [\text{O III}], [\text{O II}]) = (380, 308, 304) \text{ km s}^{-1}$ , and SDSS J103144.53+415420.8, with  $(\Delta v_{H\beta}, \Delta v_{Mg}) = (-518, -462) \text{ km s}^{-1}$  and FWHM  $([\text{Ne v}], [\text{O III}], [\text{O II}]) = (561, 572, 582) \text{ km s}^{-1}$ . Such objects may merit further investigation.

In summary, we find a number of QSOs with displaced broad-line peaks relative to the narrow lines. However, for a variety of reasons, few if any of these are likely candidates for recoiling black holes. The comparison of observed velocity shifts with predictions of numerical relativity is complicated by dynamical effects that may severely reduce the black hole's velocity during the wandering QSO phase. The frequency of the largest velocity shifts may be most useful, but the interpretation of these statistics requires more comprehensive results from numerical relativity and careful modeling of the wandering black hole's orbital velocity. In this context, studies of shifted emission lines in quasars may have the potential to put useful constraints on the processes affecting black hole mergers in AGNs. For example, Bogdanović et al. (2007) propose that in gas-rich mergers, the spin and orbital angular momenta of the black holes become aligned in a way leading to small ( $<200 \text{ km s}^{-1}$ ) kicks. If black holes do not merge until after the QSO phase, there would be no disk to fuel a wandering QSO. Such a phenomenon would have to be detected through other means, such as tidal disruption of stars in the galactic bulge (Gezari et al. 2006) or enlargement of the galactic core (Merritt et al. 2004).

The authors thank the anonymous referee for valuable suggestions and Richard Matzner for enlightening discussions. E. W. B. is supported by Marie Curie Incoming European Fellowship contract MIF1-CT-2005-008762 within the Sixth European Community Framework Programme.

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