Homework3

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Problem set 1 (1): What is the rank of the matrix A?

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{pmatrix}$$

Answer: the rank of a matrix is the number of lineraly independent vectors/variables.

To know how many independent variables there are, take the matrix A and put in in row-echelon form using the operations below (did these by hand which when done will give you the row-echelon form)

- R3 = R1 + R3
- R4 = 5R1 - R4
- R4 = 3R3- R4
- swap R4 and R3
- R4 = -2R2 + R4
- R4 = 8R3 + 29R4
- R4 = R4/129

after these operations the matrix is in row-echelon form of the below:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -29 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

you can also take the matrix to reduced-row-echelon form and see there are 4 linarly independent variables

hence the rank(A) = 4

Using R's Matrix library

```
library(Matrix)
A = matrix(c(1,-1,0,5,2,0,1,4,3,1,-2,-2,4,3,1,-3), nrow = 4, ncol = 4)
rankMatrix(A)[1]
```

```
## [1] 4
```

Problem set 1 (2): Given an mxn matrix where m > n, what can be the maximum rank? The minimum

rank, assuming that the matrix is non-zero?

Answer:

For a m x n matrix where m > n the maximum rank a m x n matrix can have is n.

the rank of a matrix is also denoted by min(m,n) where min is the minimum of m and n.

Problem set 1 (3): What is the rank of matrix B?

$B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{pmatrix}$

Answer: Reducing the matrix B to row-echelon form we get using row operations

- R2 = 3R1 - R2
- R3 = 2R1 - R3

to get

$B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

as there is only 1 independent variable the rank of this matrix B is 1.

Check using R

```
B = matrix(c(1, 3, 2, 2, 6, 4, 1, 3, 2), nrow = 3, ncol = 3)
rankMatrix(B)[1]
```

```
## [1] 1
```

Problem set 2: Compute the eigenvalues and eigenvectors of the matrix A.

You'll need to show your

work. You'll need to write out the characteristic polynomial and show your solution.

$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$

Answer: let's first write out the characteristic polynomial using the below equation

$\det(A - \lambda I_n) = 0$ (n in this case is 3)

that is

that is $\left| \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$ (| |) bars means the determinant of the matrix.

$\left| \begin{pmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{pmatrix} \right| = 0$

determinant of the matrix is

$det(A) = (1-\lambda)\Big[(4-\lambda)(6-\lambda) - 0\Big] - 2\Big[(6-\lambda)0 - 5*0\Big] + 3\Big[0 - (4-\lambda)0\Big]$
$= (4-\lambda)(1-\lambda)(6-\lambda) = 0$

solving for λ gives eigenvalues 4, 1 and, 6.

To find the eigenvectors we solve for the vector x in $(A - \lambda I) = 0$ for each eigenvalue λ

for $\lambda = 1$

we solve $\begin{pmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

solving for x $= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

gives $x = x_3 \begin{pmatrix} 0 \\ -5/3 \\ 1 \end{pmatrix}$

for $\lambda = 4$

we solve $\begin{pmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

solving for x $= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

gives $x = x_3 \begin{pmatrix} 0 \\ -5/3 \\ 1 \end{pmatrix}$

for $\lambda = 6$

we solve $\begin{pmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

solving for x $= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

gives $x = x_2 \begin{pmatrix} -2/3 \\ 1 \\ 0 \end{pmatrix}$