Week 14: Taylor Series Approximations

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In Exercises 31-32, approximate the value of the given definite integral by
using the first 4 nonzero terms of the integrand's Taylor series.
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- This discussion will cover only #31 that is

$\int_0^{\sqrt{\pi}} sin(x^2)$

- First, let's compute the Taylor series for $sin(x^2)$. We know that the Taylor series of $sin(x)$ is

$$\begin{aligned} sin(x) &= \sum_{n=0}^{\infty} (-1)^n x^{2n+1}/(2n+1)! \\ &= x - x^3/3! + x^5/5! - x^7/10! + ... \end{aligned}$$

- We can compute the Taylor series for $sin(x^2)$ by just simply substituting x for x^2

$$\begin{aligned} sin(x^2) &= \sum_{n=0}^{\infty} (-1)^n x^{2(2n+1)}/(2n+1)! \\ &= \sum_{n=0}^{\infty} (-1)^n x^{4n+2}/(2n+1)! \\ &= x^2 - x^6/3! + x^{10}/5! - x^{14}/7! + ... \end{aligned}$$

- Now lets substitute the above equation into the integral

$$\begin{aligned} \int_0^{\sqrt{\pi}} sin(x^2)\,dx &= \int_0^{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n x^{4n+2}/(2n+1)!\,dx \\ &= \int_0^{\sqrt{\pi}} x^2 - x^6/3! + x^{10}/5! - x^{14}/7! + ...\,dx \\ &= \left[x^3/3 - x^7/(7*3!) + x^{11}/(11*5!) - x^{15}/(15*7!) + ...\right]_0^{\sqrt{\pi}} \\ &= (\sqrt{\pi})^3/3 - (\sqrt{\pi})^7/(7*3!) + (\sqrt{\pi})^{11}/(11*5!) - (\sqrt{\pi})^{15}/(15*7!) + ... \end{aligned}$$

let's try to estimate the numerical approximation of the first 4 terms above to compute $\int_0^{\sqrt{\pi}} sin(x^2)$

```r
terms <- 0:3 # number of nonzero terms to use

# Function to compute the approximate definite integral of sin(x^2) from a non-zero
# upper limit to 0 (lower limit) using Taylor Series
integral_sin_x_squared_func <- function(x, n){
  numerator <- x^(4*n+3)
```

```r
  denominator <- (4*n+3) * factorial(2*n+1)
  return( (-1^n) * (numerator / denominator))
}

sin_x_squared_approx <- integral_sin_x_squared_func(pi^0.5, 0)
sin_x_squared_approx
```

```
## [1] -1.856109
```