Homework15

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1. Find the equation of the regression line from the given points. Round any final value to the nearest hundreth, if necessary.

- (5.6,8.8), (6.3,12.4), (7,14.8), (7.7,18.2), (8.4, 20.8)
- A regression line of the form $y = a + bx$ can be computed as follows for a and b:

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

```r
# set x and y
x <- c(5.6,6.3,7,7.7,8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)
n <- 5 # number of points

sum_y <- sum(y)
sum_x <- sum(x)
sum_xy <- sum(x*y)
sum_x2 <- sum(x^2) # sum of squares
sum_s_squared <- sum(x)^2 # total sum squared

a <- ((sum_y * sum_x2) - (sum_x * sum_xy)) / ((n*sum_x2) - sum_s_squared)
b <- ((n * sum_xy) - (sum_x * sum_y)) / ((n*sum_x2) - sum_s_squared)

# round to nearest hundreth
a <- round(a,2)
b <- round(b,2)
print(paste('slope:', b))
```

```
## [1] "slope: 4.26"
```

```r
print(paste('y-intercept:', a))
```

```
## [1] "y-intercept: -14.8"
```

- Equation of the line is

$$\hat{y} = 4.26 - 14.8x$$

2. Find all local maxima, local minima, and saddle points for the function given below. Write your answers in the form (x,y,z). Separate multiple points with a comma.

- $f(x, y) = 24x - 6xy^2 - 8y^3$
- Let's first take the partial derivaties
  - $f_x(x, y)$
  - $f_y(x, y)$
  - $f_{xx}(x, y)$
  - $f_{yy}(x, y)$
  - $f_{xy}(x, y)$
- Then set $f_x(x, y) = 0$ and $f_y(x, y) = 0$ and solve for x and y. Doing so gives

$$f_x(x, y) = 0 = 24 - 6y^2$$
$$f_y(x, y) = 0 = -12xy - 24y^2$$

- Solving this system of equations gives $y = \pm 2$ and $x = \pm 4$ so we have two critical points
- $(4, -2), (-4, 2)$
- plug these critical points back to $f(x, y)$ gives
  - $f(4, -2) = 64$
  - $f(-4, 2) = -64$
- Let's use the Second derivative test to find the saddle points and relative maximum and minimum for each critical point. let $D = f_{xx}(x, y)f_{yy}(x, y) - f_{xy}^2(x, y)$
  - $f_{xx} = 0$
  - $f_{yy} = -12x - 48y$
  - $f_{xy}^2(x, y) = (-12y)^2 = 144y^2$
  - $D = 0 * (-12x - 48y) - 144y^2 = -144y^2$
- D is negative regardless of the critical points for $y = \pm 2$. Thus the critical points are saddle points of f. That is the saddle points are
- (4,-2,64)
- (-4,2,-64)

3. A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x - 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1. Find the revenue function R(x,y)

- Let's compute the revenue of each brand and then add them up Revenue = price * number of units sold
- For the "house" brand, the revenue is $R(x) = x(81 - 21x + 17y) = 81x - 21x^2 + 17xy$
- For the "name" brand the revenue is $R(y) = y(40 + 11x - 23y) = 40y + 11xy - 23y^2$
- Thus

$$
\begin{aligned}
R(x,y) = R(x) + R(y) = \\
81x - 21x^2 + 17xy + 40y + 11xy - 23y^2 \\
= 81x - 21x^2 + 28xy + 40y - 23y^2
\end{aligned}
$$

Step 2. What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10

- To solve this, we compute $R(2.30, 4.10)$

```
revenuexy <- function(x,y){
  return(81*(x) - 21*(x^2) + 28*(x*y) + 40*(y) - 23*(y^2))
}
revenuexy(2.30,4.10)
```

```
## [1] 116.62
```

- This means that selling the house and name brand for the prices above the revenue she will make is \$116.62.

4. A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x,y) = (1/6)x^2 + (1/6)y^2 + 7x + 25y + 700$ where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

- We are given the cost function and we are also given a constraint/extra equation: $x + y = 96$ This comes from the given information that the company wants to sell a total of 96 units of x and y while minimizing costs.
- We can solve for say y $y = 96 - x$ and plug it into $C(x,y)$ to get a function using only x that is $C(x, 96-x)$, compute $C_x(x, 96-x)$, set it to 0 and find the critical point that is value of x that minimizes the total weekly cost. We find y by solving for $y - 96 - x$

$$
\begin{aligned}
C(x, 96-x) = (1/6)x^2 + (1/6)(96-x)^2 + 7x + 25(96-x) + 700 \\
C_x(x, 96-x) = (1/3)x - (1/3)(96-x) + 7 - 25 \\
= (x/3) - 32 + (x/3) - 18 = (2x/3) - 50 \\
0 = (2x/3) - 50 \\
x = 75
\end{aligned}
$$

- With x found, we can compute y that is $y = 96 - 75 = 21$
- So to minimize total weekly costs and wanting to produce 96 total units, the company should produce 75 units in Los Angeles and 21 units in Denver.

5. Evaluate the double integral on he given region

$$
\iint_R (e^{8x+3y})\, dA; R : 2 \leq (x,y) \leq 4
$$

Write your answer in exact form without decimals.

- We evaluate the integral as follows

$$
\begin{aligned}
\iint_R (e^{8x+3y})\,dx\,dy &= \int_2^4 \int_2^4 (e^{8x+3y})\,dx\,dy \\
&= \int_2^4 \left[\frac{e^{8x+3y}}{8}\right]_2^4 dy \\
&= \int_2^4 \frac{e^{3y}(e^{32}-e^{16})}{8}\,dy \\
&= \frac{(e^{32}-e^{16})}{8}\int_2^4 e^{3y}\,dy \\
&= \frac{(e^{32}-e^{16})}{8}\left[\frac{e^{3y}}{3}\right]_2^4 \\
&= \frac{(e^{32}-e^{16})}{8}\frac{(e^{12}-e^{6})}{3} \\
&= \frac{e^{44}-e^{38}-e^{28}+e^{22}}{24} \\
&= \frac{e^{22}(1-e^{6}-e^{16}+e^{22})}{24}
\end{aligned}
$$