Homework7

Jonathan Hernandez

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1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of

which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

Answer:

$Y = min(X_1, X_2, ..., X_n)$ that is the minimum of x1, minimum of x2, etc

For this, majority of the $min(X_i)$ will be near 1 and a few may drift off above 1 so it would be a right-skewed distribution

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors).

This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a. What is the probability that the machine will fail after 8 years?. Provide also the

expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

Answer: As a geometric distribution

Let X be the # of years

p = 0.1 in this case

$P(X > 8) = 1 - P(X \leq 8) = 1 - \left[P(X = 1) + P(X = 2) + ... + P(X = 8)\right] = 1 - 0.5695 = 0.4304$

Expected value: 1/p = 1/0.1 = 10 years(agrees with exepcted value in the problem description)

Standard deviation: $\sqrt{1-p}/p = \sqrt{0.9}/0.1 = 9.486$ years

R code

```
p <- 0.1
years <- 8
# compute the sum from 0 to 7 successes
pgeom(years-1, prob = p, lower.tail = FALSE)
```

```
## [1] 0.4304672
```

b. What is the probability that the machine will fail after 8 years?. Provide also the

expected value and standard deviation. Model as an exponential.

The Average for a exponential distribution is $1/\lambda$. The average is given to us

as $\mu = 10$ that is $\lambda = 0.1$

Let X be the number of years, x in this case $= 8$

$P(X > 8) = 1 - P(X \leq 8) = 1 - \left[1 - e^{-\lambda x}\right] = e^{-0.1*8} = 0.4493$

Expected value: $\mu = 10$ years

Variance: $1/\lambda^2 = 1/0.1 = 10 = \mu$ years

R code

```
lambda <- 0.1
years <- 8
pexp(years, rate = lambda, lower.tail = FALSE) # or exp(years*lambda)
```

```
## [1] 0.449329
```

c. What is the probability that the machine will fail after 8 years?. Provide also the

expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8

years)

p = 0.1 x = 0 successes n = 8 trials (years)

$P(X > 8) = 1 - P(X \leq 8) = 1 - \binom{8}{0} 0.1^0 (0.9^8) = 0.5695$

Expected Value: $np = 8 * 0.1 = 0.8$ years

Standard Deviation: $\sqrt{np(1-p)} = \sqrt{8 * 0.1 * 0.9} = 0.848$ years

R code

```
p <- 0.1
n <- 8
x <- 0

pbinom(0,8,prob = 0.1,lower.tail = FALSE)
```

```
## [1] 0.5695328
```

d. What is the probability that the machine will fail after 8 years?. Provide also the

expected value and standard deviation. Model as a Poisson.

rate $= 0.1 = \lambda$

$P(X > 8) = 1 - P(X \leq 8) = 1 - \left[P(X = 0) + P(X = 1) + ... + P(X = 8)\right] = 0.6671$

Expected Value $= \lambda = 10 =$ variance

Standard deviation $= \sqrt{\lambda} = \sqrt{10} = 3.162$ years

R code

```r
x <- 8
lambda <- 10
ppois(x, lambda = lambda,lower.tail = FALSE)
```

```
## [1] 0.6671803
```