Homework6

Jonathan Hernandez

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1. A box contains 54 red marbles, 9 white marbles, and 75 blue marbles. If a marble is randomly selected from the box,

what is the probability that it is red or blue? Express your answer as a fraction or a decimal number rounded to four

decimal places.

Answer

There are a total of 138 marbles so the probability of picking a colored marble

is the total number of that colored marble / total marbles

$P(red \cup blue) = P(red) + P(blue) - P(red \cap blue)$

$P(red \cap blue) = 0$ as there is no way to pick a marble that has both colors.

So $P(red \cup blue) = P(red) + P(blue) = 54/139 + 75/139 = 0.9281$

2. You are going to play mini golf. A ball machine that contains 19 green golf balls, 20 red golf balls, 24 blue golf

balls, and

17 yellow golf balls, randomly gives you your ball. What is the probability that you end up with a red golf ball?

Express your answer as a simplified fraction or a decimal rounded to four decimal places.

Answer:

There are a total of 80 golf balls and each one equally likley to be selected.

$P(red) = 20/80 = 0.25$

3. A pizza delivery company classifies its customers by gender and location of residence. The research department has

gathered data from a random sample of 1399 customers. The data is summarized in the table below.

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Gender and Residence of Customers		
	Males	Females
Apartment	81	228
Dorm	116	79
With Parent(s)	215	252
Sorority/Fraternity House	130	97
Other	129	72

What is the probability that a customer is not male or does not live with parents? Write your answer as a fraction or a

decimal number rounded to four decimal places.

Answer:

Let's write out the equation using the asked probabilities:

$$P(\overline{male} \cup \overline{parents}) = P(\overline{male}) + P(\overline{parents}) - P(\overline{male} \cap \overline{parents})$$

(Note that the bar on top of the event is the complenent of that event)

Looking at the table, we see $P(\overline{male}) = 729/1399$ (sum up the females column)

and $P(\overline{parents}) = 932/1399$ (add up all the entries excluding the with parents row)

We also see from the table $P(\overline{male} \cap \overline{parents}) = 477/1399$

(This can be found by looking at the females column and add it up excluding the

number of females who live with their parents)

Finally pluggin in the numbers gives

$P(\overline{male} \cup \overline{parents}) = (732 + 932 - 477)/1339 = 1184/1399 = 0.8463$

4. Determine if the following events are independent.

Going to the gym. Losing weight.

Answer: A) Dependent B) Independent

Let G = event of going to the gym and W = event of losing weight

if these two events are independent then $P(G \cap W) = P(G)P(W)$

These are dependent events as the probability of going to the gym and losing weight

is not equivalent to going to the gym only and losing weight only. Usually they go

hand-in-hand together.

5. A veggie wrap at City Subs is composed of 3 different vegetables and 3 different condiments wrapped up in a

tortilla. If there are 8 vegetables, 7 condiments, and 3 types of tortilla available, how many different veggie wraps

can be made?

Answer: using the answer for combinations that is ${}_nC_r = n!/r!(n-r)!$

$n! = n * (n-1) * (n-2) * ... * 1$

We can apply this equation for each of the foods

for picking 3 vegetables from 8 vegetables there are ${}_8C_3 = 8!/3!5! = 56$

for picking 3 condiments from 7 condiments there are ${}_7C_3 = 7!/3!4! = 35$

for picking 3 types of tortilla for making a single one there are ${}_3C_1 = 3!/1!2! = 3$

we take the product of these numbers $56 * 3 * 35 = 5880$ possible ways to make different veggie wraps.

6. Determine if the following events are independent.

Jeff runs out of gas on the way to work. Liz watches the evening news.

Answer: A) Dependent B) Independent

these would be independent events as the outcomes of these events does not affect the outcome of the other or gives no information about the other.

7. The newly elected president needs to decide the remaining 8 spots available in the cabinet he/she is appointing. If there

are 14 eligible candidates for these positions (where rank matters), how many different ways can the members of the

cabinet be appointed?

Answer: using permutations (where order matters) we can find how many possible ways to vacate 8 spots from selecting 14 people.

that is n = 14 and r = 8 and ${}_nP_r = n!/(n-r)!$

so ${}_{14}P_8 = 14!/6! = 121080960$ so over a 121 million different ways can the members of the cabinet be appointed.

8. A bag contains 9 red, 4 orange, and 9 green jellybeans. What is the probability of reaching into the bag and randomly

withdrawing 4 jellybeans such that the number of red ones is 0, the number of orange ones is 1, and the number of green

ones is 3? Write your answer as a fraction or a decimal number rounded to four decimal places.

Answer: We can use the binomial expansion or probability mass function for each type ot jellybean

For the red jellybeans we can label a success as picking a red jellybean and failure otherwise

we can apply this for the other colored jellybeans using the formula below and

R's builtin dbinom() function.

Binomial expansion (or probability mass function):

$P(k;n,p) = \binom{n}{k} p^k (1-p)^{(n-k)}$

For red, n = 9 and k = 0 and p = 1/22 so $P(k; n,p) = P(0; 9, 1/22) =

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red <- dbinom(0, 9, prob = 1/22)
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For orange, n = 4, k = 1, p = 1/22

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orange <- dbinom(1, 4, prob = 1/22)
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For green, n = 9, k = 3, prob = 1/22

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green <- dbinom(3, 9, prob = 1/22)
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compute the product of the 3 binomial variables:

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red*green*orange
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## [1] 0.0006208513
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9. Evaluate the following expression: 11! / 7!

Answer: 11!/7! $= 11 * 10 * 9 * ... * 1/7 * 6 * 5 * ... * 1 = 11 * 10 * 9 * 8 = 7920$

10. Describe the comoplement of the given event:

67% of subscribers to a fitness magazine are over the age of 34.

Answer: Let E be the event that subscribers of a fitness magazine are over the age of 34.

Then the complement that is E' is:

subscribers of a fitness magazine are under the age of 34.

11. If you throw exactly three heads in four tosses of a coin you win $97.

If not, you pay me $30.

Step1. Find the expected value of the proposition. Round your answer to two decimal places.

Answer: The expected value of this game/event can be computed as follows:

Let X be the event of flipping 4 coins.

$E[X] = wp - l(1-p)$ where w = amount gained, l = amount loss and p is the probability of getting three heads exactly out of 4 tosses of a coin.

We can use the Binomial distribution as the coin flips are independent that is $P(k;n,q) = \binom{n}{k} q^k (1-q)^{(n-k)}$ where n = 4, k = 3, and q = 1/2 probability of getting a head.

$p = P(3; 4, 1/2) = \binom{4}{3}.5^3(1-.5)^{(4-3)} = 1/4$

then, (1-p) = 3/4. This makes sense as there are 4 ways out of 16 to get exactly

three heads (1/4) and 12 out of 16 ways otherwise (12/16 = 3/4)

w = 97 and l = 30 and thus $E[X] = 97 * .25 - 30 * .75 = 1.75$ as the expected value.

Step 2. If you played this game 559 times how much would you expect to win or lose? (Losses must be entered as

negative.)

Answer: let n = 559, if we repeat this game n times we can find out how much we could win

by taking E[X] and multiplying it by n that is $nE[X] = 559 * 1.75 = 978.25$ so this is game

that is beneficial to play.

12. Flip a coin 9 times. If you get 4 tails or less, I will pay you $23. Otherwise you pay me $26.

Step 1. Find the expected value of the proposition. Round your answer to two decimal places.

Answer:

Let X be event of flipping a coin 9 times.

$E[X] = wp - l(1-p)$

w = 23 and l = 26

to find p, use the Binomial distribution formula. To find the probability of getting 4

tails or less, we have add up the successes of getting 0 tails, 1 tail, 2 tails, 3 tails and 4 tails

that is

$$\begin{aligned} p = p(X < 4) = P(0; 9, 0.5) + P(1; 9, 0.5) + P(2; 9, 0.5) + P(3; 9, 0.5) + P(4; 9, 0.5) = \\ \binom{9}{0}.5^0(1-.5)^{(9-0)} + \binom{9}{1}.5^1(1-.5)^{(9-1)} + \binom{9}{2}.5^2(1-.5)^{(9-2)} \\ + \binom{9}{3}.5^3(1-.5)^{(9-3)} + \binom{9}{4}.5^4(1-.5)^{(9-4)} = 0.5 \end{aligned}$$

This makes sense as with $2^9 = 512$ possibilities there are 206 ways to get 4 tails or less flipping 9 coins

so the expected value is $E[X] = 23 * 0.5 - 26 * 0.5 = -1.5$

Step 2. If you played this game 994 times how much would you expect to win or lose? (Losses must be entered as

negative.)

Answer: let n = 994 then we would expect to have $nE[X] = 994 * -1.5 = -1491$ so if you play this game many times,

you much more likley to lose alot of money.

13. The sensitivity and specificity of the polygraph has been a subject of study and debate for years. A 2001 study of the

use of polygraph for screening purposes suggested that the probability of detecting a liar was .59 (sensitivity) and that

the probability of detecting a "truth teller" was .90 (specificity). We estimate that about 20% of individuals selected for

the screening polygraph will lie.

a. What is the probability that an individual is actually a liar given that the polygraph detected him/her as such? (Show

me the table or the formulaic solution or both.)

b. What is the probability that an individual is actually a truth-teller given that the polygraph detected him/her as

such? (Show me the table or the formulaic solution or both.)

c. What is the probability that a randomly selected individual is either a liar or was identified as a liar by the

polygraph? Be sure to write the probability statement.

Answer:

Let P(truth) = probablity of person telling truth =

Let P(lie) = probability of person lying

Let P(+) = Polygraph reveals truth

Let P(-) = Polygraph reveals lie

a. Goal is to find $P(lie|-)$

$P(lie|-) = P(lie \cap -)/P(-)$

$P(truth) = 0.8, P(lie) = 0.2$ as per information given

$specificity = P(+|truth) = 0.9$

$sensitivity = P(-|lie) = 0.59$

$P(lie \cap -) = P(lie) * P(-|lie) = 0.2 * 0.59 = 0.118$

We also need to find $P(truth \cap -)$ to compute $P(-)$

$P(truth \cap -) = P(truth) * P(-|truth) = 0.8 * 0.1 = 0.08$

$P(-) = P(lie \cap -) + P(truth \cap -) = 0.118 + 0.08 = 0.198$

Finally, $P(lie|-) = 0.118/0.198 = 0.595$

b. Goal is to find $P(truth|+)$

$P(truth|+) = P(truth \cap +)/P(+)$

$P(truth \cap +) = P(truth) * P(+|truth) = 0.8 * 0.9 = 0.72$

$P(lie \cap +) = P(lie) * P(+|lie) = 0.2 * 0.41 = 0.082$

$P(+) = P(truth \cap +) + P(lie \cap +) = 0.72 + 0.082 = 0.802$

Finally, $P(truth|+) = 0.72/0.802 = 0.897$

c. Goal is to find $P(lie \cup -)$

$P(lie \cup -) = P(lie) + P(-) - P(lie \cap -) = 0.2 + 0.198 - 0.118 = 0.28$