Homework14

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`This week, we'll work out some Taylor Series expansions of popular functions.`

- $f(x) = 1/(1-x)$
- $f(x) = e^x$
- $f(x) = \ln(1+x)$
  For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as a R-Markdown document.
- Taylor Series definition

Let $f(x)$ have derivatives of all orders at x=c.

- The taylor series of $f(x)$, centered at c is

$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$

- for $f(x) = 1/(1-x)$, let's compute some derivatives of $f(x)$ to get the idea of how the series acts, note that the function's range is all real numbers $\neq 0$
  - $f'(x) = 1/(1-x)^2$
  - $f''(x) = 2/(1-x)^3$
  - $f'''(x) = 6/(1-x)^4$
  - $f^{(4)}(x) = 24/(1-x)^5$
- It looks like the n^{th} derivative is

$f^{(n)}(x) = n!/(1-x)^n$ so we compute the Taylor Series as

$$\sum_{n=0}^{\infty} \frac{n!/(1-x)^n}{n!}(x-c)^n$$
$$= \sum_{n=0}^{\infty} \left[\frac{(x-c)}{(1-x)}\right]^n$$

- This is the Taylor series centered at c. Letting c = 0 and computing the derivatives at c gives $f(x) = \sum_{n=0}^{\infty} x^n$

- For $f(x) = e^x$, it's easy to see how \$f(x) = f{'}(x) = f{''}(x) = f{'''}(x) = $\ldots$ and thus $f^{(n)}(x) = e^x$ and also the range of e^x is from $(0, \infty)$
- The Taylor Series centered at c

$$\sum_{n=0}^{\infty} \frac{e^x}{n!}(x-c)^n$$

- Letting c = 0 and computing the derivatives at c gives

$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

- For $f(x) = \ln(1+x)$, the range of the function is $(-\infty, \infty)$
- First few derivatives of $\ln(1+x)$ are
  - $f'(x) = 1/(1+x)$
  - $f''(x) = -1/(1+x)^2$
  - $f'''(x) = 2/(1+x)^3$
  - $f^{(4)}(x) = -6/(1+x)^4$
  - $f^{(5)}(x) = 24/(1+x)^5$
- The pattern we see for the n^{th} derivative is $f^{(n)}(x) = (-1)^{n+1}(n-1)!/(1+x)^n$ for $n > 0$
- The Taylor series centered at c

$$\begin{aligned}
\ln(1+x) + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n-1)!}{(1+x)^n n!}(x-c)^n \\
= \ln(1+x) + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-c)^n}{n(1+x)^n}
\end{aligned}$$

- Letting c = 0 and computing the derivatives at c gives
- $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$