Homework13

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November 12, 2018

1. Use integration by substitution to solve the integral below

$\int 4e^{-7x}\,dx$

Answer:

- let $u = -7x$
- $du/\,dx = -7$
- $du = -7\,dx$
- $dx = -\,du/7$
- Rewrite the integral as

$$\int (-4/7)e^{u}\,du$$
$$= (-4/7)e^{u} + C$$
$$= (-4/7)e^{-7x} + C$$

2. Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $dN/\,dt = -3150/t^4 - 220$ bacteria per cubic centimer per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

Answer:

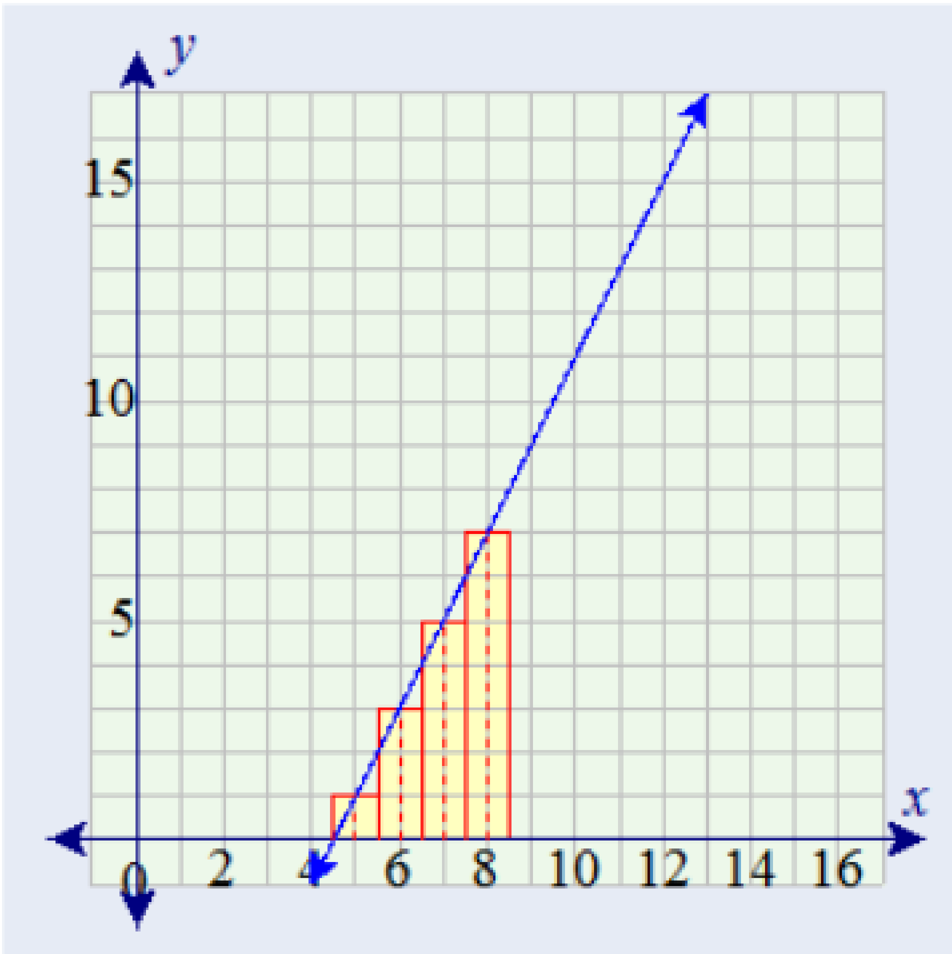
- We are given the initial condition $N(1) = 6530$
- To find $N(t)$ in general, we have to integrate the equation and solve for the arbitrary constant C to get the general closed form solution $N(t)$

$$N(t) = \int dN/\,dt\,dt$$
$$= \int (-3150/t^4 - 220)\,dt$$
$$= 1050/t^3 - 200t + C$$

- To find C, we can use the given initial condition $N(t = 1)$ and solve for C

$$N(1) = 6530 = 1050 - 200 + C$$
$$C = 5680$$
$$thus$$
$$N(t) = 1050/t^3 - 200t + 5680$$

3. Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$



Area =

Answer:

- By looking at the width of each triangle using the grid lines as a guide, we see that the width of all 4 triangles is 1 unit and the length of each rectangle from left to right respectively is 1,3,5 and, 7 units.

  Area of the first rectangle = 1*1 = 1

  Area of the second rectangle = 1*3 = 3

  Area of the third rectangle = 1*5 = 5

  Area of the fourth rectangle = 1*7 = 7

- Add them all up and the total area of the rectangles is 16.

4. Find the area of the region bounded by the graphs of the given equations.

$y = x^2 - 2x - 2, y = x + 2$

Answer:

- To compute the area we need to first know what values of x do they both meet at; we can do this by setting them equal to each and solve for x.
- Solving for x gives

$$\begin{aligned} x + 2 &= x^2 - 2x - 2 \\ x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \\ x &= 4, x = -1 \end{aligned}$$

- Now that we have the endpoints, we can compute and plot or graph the two functions and see that $x + 2 \geq x^2 - 2x - 2$ for all x $\epsilon[-1, 4]$.
- Both functions are continous everywhere in the region and we can find the area between curves as
- $\int_{-1}^{4}(x + 2 - (x^2 - 2x - 2))\, dx$ Solving the integral gives

$$\begin{aligned} &\int_{-1}^{4}(x + 2 - (x^2 - 2x - 2))\, dx \\ &= \int_{-1}^{4} -x^2 + 3x + 4\, dx \\ &= \big[-x^3/3 + 3x^2/2 + 4x\big]_{-1}^{4} \\ &= -64/3 + 24 + 16 - (1/3 + 3/2 - 4) \approx 20.833 \end{aligned}$$

5. A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and number of orders per year that will minimize inventory costs.

Answer:

- This is a optimization problem where we need to find the number of orders and number of irons to minimize costs.
- let r be the number of orders and let f be the number of flat irons.
- We are given the cost function $Cost = c(r, f) = 3.75f + 8.25r$
- We are also given the expected value of flat irons to sell: 110 which can be re-written as $rf = 110$
- This is because we can get 110 flat irons by not just doing one order in bulk but in several orders.
- We solve for $f = 110/r$
- We now can write the cost function as a single variable c(r) and compute c‘(r) and set it to zero to find the critical value r that minimizes c(r) that is:

$$\begin{aligned} c(r) &= 3.75(110/r) + 8.25r \\ &= 412.5/r + 8.25r \\ c^{‘}(r) &= -412.5/r^2 + 8.25 \\ 0 &= -412.5/r^2 + 8.25 \\ r^2 &= 412.5/8.25 = 50 \\ r &= \sqrt{50} \end{aligned}$$

- With r found, we can go back to the equation $f = 110/r$ and solve for f. $f = 110/\sqrt{50}$
- Thus the number of flat irons to store is about ceil($\sqrt{50}$) ≈ 8 and the number of orders to put in is about ceil($110\sqrt{50}$) ≈ 16

6. Use integration by parts to solve the integral below $\int ln(9x)x^6\,dx$

Answer:

- Integration by parts is as follows, given two functions $f(x), g(x)$, $\int f(x)g^{`}(x)\,dx = f(x)g(x) - \int f^{`}(x)g(x)\,dx$
- commonly the formula is written down as $\int u\,dv = uv - \int v\,du$ where $u = f(x), v = g(x), du = f^{`}(x)\,dx, dv = g^{`}(x)\,dx$
- let $u = ln(9x)$ and $dv = x^6$, then $du = 1/9x\,dx$ and $v = x^7/7\,dx$ so

$$\begin{aligned}
\int u\,dv &= uv - \int v\,du \\
&= (x^7\ln(9x))/7 - \int (x^7/7)(1/9x)\,dx \\
&= (x^7\ln(9x))/7 - \int x^6/63\,dx \\
&= (x^7\ln(9x))/7 - x^7/441 + C \\
&= x^7(63\ln(9x) - 1)/441 + C
\end{aligned}$$

7. Determine whether f9x) is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$f(x) = 1/6x$

Answer:

- Compute $\int_1^{e^6} 1/6x\,dx$

$$\begin{aligned}
\int_1^{e^6} 1/6x\,dx &= \big[\ln(6x)\big]_1^{e^6} \\
&= \ln(6e^6) - \ln(6) \\
\ln(6) + \ln(e^6) - \ln(6) &= 6 \neq 1
\end{aligned}$$

- This shows the given limits don't compute to 1 which is what is needed to make this a valid probability density function.