EFFECTS OF WAVE PACKET PROFILES ON NEUTRINO OSCILLATIONS

Evan Gale[a], and Magdalena Zych[a, b]

[a]ARC Centre of Excellence for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia

[b]Department of Physics, Stockholm University, SE 106 91 Stockholm, Sweden

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

Neutrino Oscillations

Neutrino Oscillations

- Theoretically proposed by Pontecorvo in late 1950s

Neutrino Oscillations

- Theoretically proposed by Pontecorvo in late 1950s
- Experimental tests of solar neutrino flux in mid-1960s

Neutrino Oscillations

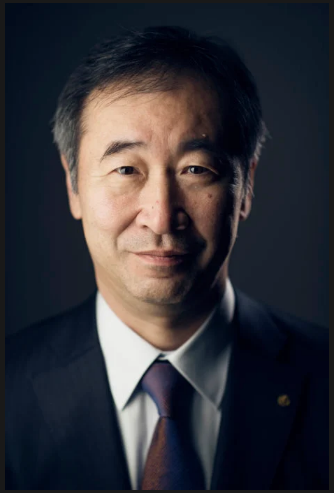
- Theoretically proposed by Pontecorvo in late 1950s
- Experimental tests of solar neutrino flux in mid-1960s
- Discrepancy with predictions, the solar neutrino problem

Neutrino Oscillations

- Theoretically proposed by Pontecorvo in late 1950s
- Experimental tests of solar neutrino flux in mid-1960s
- Discrepancy with predictions, the solar neutrino problem
- Today, discrepancy best explained by neutrino oscillations

Neutrino Oscillations

- Theoretically proposed by Pontecorvo in late 1950s
- Experimental tests of solar neutrino flux in mid-1960s
- Discrepancy with predictions, the solar neutrino problem
- Today, discrepancy best explained by neutrino oscillations



Takaaki Kajita & Arthur B. McDonald, Nobel Prize in Physics 2015

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

How are neutrinos described in quantum mechanics?

Often see oscillations treated with plane waves

How are neutrinos described in quantum mechanics?

Often see oscillations treated with plane waves

Flavour basis ν_α can be related to mass basis ν_i by

How are neutrinos described in quantum mechanics?

Often see oscillations treated with plane waves

Flavour basis ν_α can be related to mass basis ν_i by

$$|\nu\rangle = \sum_i U^*_{\alpha i} \left|\nu_i\right\rangle,$$

How are neutrinos described in quantum mechanics?

Often see oscillations treated with plane waves

Flavour basis ν_α can be related to mass basis ν_i by

$$|\nu\rangle = \sum_i U^*_{\alpha i}\left|\nu_i\right\rangle,$$

where $U_{\alpha i}$ is the mixing matrix,

How are neutrinos described in quantum mechanics?

Often see oscillations treated with plane waves

Flavour basis ν_α can be related to mass basis ν_i by

$$|\nu\rangle = \sum_i U^*_{\alpha i}\left|\nu_i\right\rangle,$$

where $U_{\alpha i}$ is the mixing matrix, and its spacetime evolution is

How are neutrinos described in quantum mechanics?

Often see oscillations treated with plane waves

Flavour basis v_α can be related to mass basis v_i by

$$|v\rangle = \sum_i U^*_{\alpha i} \left| v_i \right\rangle ,$$

where $U_{\alpha i}$ is the mixing matrix, and its spacetime evolution is

$$\left| v_\alpha(t, x) \right\rangle = \sum_i U^*_{\alpha i} e^{-i(E_i t - p_i x)} \left| v_i \right\rangle .$$

How are neutrinos described in quantum mechanics?

How are neutrinos described in quantum mechanics?

Amplitude at detection is given by

How are neutrinos described in quantum mechanics?

Amplitude at detection is given by

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) = \left\langle \nu_\beta(x_d, t_d) \,|\, \nu_\alpha(x_p, t_p) \right\rangle$$

$$= \sum_i U^*_{\alpha i} e^{-i(E_i T - p_i L)} U_{\beta i},$$

How are neutrinos described in quantum mechanics?

Amplitude at detection is given by

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) = \left\langle \nu_\beta(x_d, t_d) \,|\, \nu_\alpha(x_p, t_p) \right\rangle$$

$$= \sum_i U^*_{\alpha i} e^{-i(E_i T - p_i L)} U_{\beta i},$$

where $T = t_d - t_p$ and $L = x_d - x_p$.

How are neutrinos described in quantum mechanics?

Amplitude at detection is given by

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) = \langle \nu_\beta(x_d, t_d) | \nu_\alpha(x_p, t_p) \rangle$$
$$= \sum_i U^*_{\alpha i} e^{-i(E_i T - p_i L)} U_{\beta i} \,,$$

where $T = t_d - t_p$ and $L = x_d - x_p$.

Transition probability found by

$$P_{\nu_\alpha \to \nu_\beta}(L) = \int dT \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) \right|^2 .$$

Problems with plane wave treatment

Problems with plane wave treatment

Giunti [1] gives several reasons to consider wave packets

[1] C. Giunti, Found. Phys. Lett. **17**, 103 (2004).

Problems with plane wave treatment

Giunti [1] gives several reasons to consider wave packets

- Finite lifetime of atomic transitions, so *“no source of waves vibrates indefinitely”*

[1] C. Giunti, Found. Phys. Lett. **17**, 103 (2004).

Problems with plane wave treatment

Giunti [1] gives several reasons to consider wave packets

- Finite lifetime of atomic transitions, so *“no source of waves vibrates indefinitely”*
- Localisation of interactions implies that, *“the particle cannot be described by an unlocalized plane wave”*

[1] C. Giunti, Found. Phys. Lett. **17**, 103 (2004).

Problems with plane wave treatment

Giunti [1] gives several reasons to consider wave packets

- Finite lifetime of atomic transitions, so *“no source of waves vibrates indefinitely”*
- Localisation of interactions implies that, *“the particle cannot be described by an unlocalized plane wave”*

Beuthe [2] adds that,

[1] C. Giunti, Found. Phys. Lett. **17**, 103 (2004).
[2] M. Beuthe, Phys. Rep. **375**, 105 (2003).

Problems with plane wave treatment

Giunti [1] gives several reasons to consider wave packets

- Finite lifetime of atomic transitions, so *“no source of waves vibrates indefinitely”*
- Localisation of interactions implies that, *“the particle cannot be described by an unlocalized plane wave”*

Beuthe [2] adds that,

“this approach implies a perfectly well-known energy–momentum and an infinite uncertainty on the space–time localization of the oscillating particle.

[1] C. Giunti, Found. Phys. Lett. **17**, 103 (2004).
[2] M. Beuthe, Phys. Rep. **375**, 105 (2003).

Problems with plane wave treatment

Giunti [1] gives several reasons to consider wave packets

- Finite lifetime of atomic transitions, so *“no source of waves vibrates indefinitely”*
- Localisation of interactions implies that, *“the particle cannot be described by an unlocalized plane wave”*

Beuthe [2] adds that,

“this approach implies a perfectly well-known energy–momentum and an infinite uncertainty on the space–time localization of the oscillating particle. Oscillations are destroyed under these assumptions.”

[1] C. Giunti, Found. Phys. Lett. **17**, 103 (2004).
[2] M. Beuthe, Phys. Rep. **375**, 105 (2003).

How should we formalise the wave packet treatment?

How should we formalise the wave packet treatment?

- Quantum mechanics

How should we formalise the wave packet treatment?

- Quantum mechanics
- Quantum field theory

How should we formalise the wave packet treatment?

- Quantum mechanics (intermediate wave packet approach)
- Quantum field theory

How should we formalise the wave packet treatment?

- Quantum mechanics (intermediate wave packet approach)
- Quantum field theory (external wave packet approach)

How should we formalise the wave packet treatment?

- Quantum mechanics (intermediate wave packet approach)
- Quantum field theory (external wave packet approach)

How should we formalise the wave packet treatment?

- Quantum mechanics (intermediate wave packet approach)
- Quantum field theory (external wave packet approach)

$$|\nu_\alpha(t,x)\rangle = \sum_i U^*_{\alpha i}\psi_i(t,x)\,|\nu_i\rangle \ ,$$

where

$$\psi_i(t,x) = \frac{1}{\sqrt{2\pi}}\int dp_i\,\widetilde{\psi}_i(p_i)e^{-i(E_i t - p_i x)} \ .$$

How should we formalise the wave packet treatment?

How should we formalise the wave packet treatment?

Nussinov [3] argued that wave packets cause decoherence

[3] S. Nussinov, Phys. Lett. B **63**, 201 (1976).

How should we formalise the wave packet treatment?

Nussinov [3] argued that wave packets cause decoherence

- Wave packet spreads over time and decoheres when width larger than oscillation length

[3] S. Nussinov, Phys. Lett. B **63**, 201 (1976).

How should we formalise the wave packet treatment?

Nussinov [3] argued that wave packets cause decoherence

- Wave packet spreads over time and decoheres when width larger than oscillation length
- Separation of wave packets due to different group velocities of mass eigenstates

[3] S. Nussinov, Phys. Lett. B **63**, 201 (1976).

How should we formalise the wave packet treatment?

Nussinov [3] argued that wave packets cause decoherence

- Wave packet spreads over time and decoheres when width larger than oscillation length
- Separation of wave packets due to different group velocities of mass eigenstates

Kayser [4] first to study oscillations with wave packets,

[3] S. Nussinov, Phys. Lett. B **63**, 201 (1976).
[4] B. Kayser, Phys. Rev. D **24**, 110 (1981).

How should we formalise the wave packet treatment?

Nussinov [3] argued that wave packets cause decoherence

- Wave packet spreads over time and decoheres when width larger than oscillation length
- Separation of wave packets due to different group velocities of mass eigenstates

Kayser [4] first to study oscillations with wave packets, and Giunti [5] first to obtain explicit results with Gaussians

[3] S. Nussinov, Phys. Lett. B **63**, 201 (1976).
[4] B. Kayser, Phys. Rev. D **24**, 110 (1981).
[5] C. Giunti, C. W. Kim, and U. W. Lee, Phys. Rev. D **44**, 3635 (1991).

Why a Gaussian?

Why a Gaussian?

Can obtain analytic results!

Why a Gaussian?

Can obtain analytic results!

> *"A gaussian momentum distribution is the most convenient one for the calculation of several integrations …*
>
> *Other distributions which are sharply peaked around an average momentum lead to the same results after their approximation with a gaussian …*
>
> *Therefore, the gaussian momentum distributions can be taken as approximations of the real momentum distributions from the beginning." [6]*

[6] C. Giunti, JHEP 2002, **017** (2002).

Why a Gaussian?

Can obtain analytic results!

> *"A gaussian momentum distribution is the most convenient one for the calculation of several integrations …*
>
> *Other distributions which are sharply peaked around an average momentum lead to the same results after their approximation with a gaussian …*
>
> *Therefore, the gaussian momentum distributions can be taken as approximations of the real momentum distributions from the beginning." [6]*

[6] C. Giunti, JHEP 2002, **017** (2002).

Why a Gaussian?

Can obtain analytic results!

> *"A gaussian momentum distribution is the most convenient one for the calculation of several integrations …*
>
> *Other distributions which are sharply peaked around an average momentum lead to the same results after their approximation with a gaussian …*
>
> *Therefore, the gaussian momentum distributions can be taken as approximations of the real momentum distributions from the beginning." [6]*

[6] C. Giunti, JHEP 2002, **017** (2002).

Really?

Really?

A Gaussian is described by its first two moments (mean and variance).

Really?

A Gaussian is described by its first two moments (mean and variance). We can have non-Gaussian wave packets!

Really?

A Gaussian is described by its first two moments (mean and variance). We can have non-Gaussian wave packets!

Conditions on when a distribution $f(p) = \exp[-g(p)]$ can be approximated by a Gaussian [7].

[7] E. Kh. Akhmedov and J. Kopp, JHEP **2010**, 8 (2010).

Really?

A Gaussian is described by its first two moments (mean and variance). We can have non-Gaussian wave packets!

Conditions on when a distribution $f(p) = \exp[-g(p)]$ can be approximated by a Gaussian [7].

Expanding about minimum $p = P$, we require

$$\frac{1}{4!}\left|g^{(iv)}(P)\right| \ll \frac{1}{2}\left|g''(P)\right|^2$$

[7] E. Kh. Akhmedov and J. Kopp, JHEP **2010**, 8 (2010).

Lorentzian wave packets

Lorentzian wave packets

Neutrinos in the context of the Mössbauer effect described by a Lorentzian wave packet [8, 9]

[8] E. Kh. Akhmedov, J. Kopp, and M. Lindner, JHEP **2008**, 005 (2008).
[9] J. Kopp, JHEP **2009**, 049 (2009).

Lorentzian wave packets

Neutrinos in the context of the Mössbauer effect described by a Lorentzian wave packet [8, 9]

$$\tilde{\psi}(p; \bar{p}, \gamma) = \mathcal{N}\left[\frac{\gamma}{(p-\bar{p})^2+\gamma^2}\right]$$

[8] E. Kh. Akhmedov, J. Kopp, and M. Lindner, JHEP **2008**, 005 (2008).

[9] J. Kopp, JHEP **2009**, 049 (2009).

Lorentzian wave packets

Neutrinos in the context of the Mössbauer effect described by a Lorentzian wave packet [8, 9]

$$\tilde{\psi}(p; \bar{p}, \gamma) = \mathcal{N}\left[\frac{\gamma}{(p-\bar{p})^2 + \gamma^2}\right]$$

Moments undefined. Cannot be approximated by a Gaussian!

[8] E. Kh. Akhmedov, J. Kopp, and M. Lindner, JHEP **2008**, 005 (2008).

[9] J. Kopp, JHEP **2009**, 049 (2009).

Relativistic minimum uncertainty (RMU) wave packets

Relativistic minimum uncertainty (RMU) wave packets

Can be expressed in a Lorentz invariant form

Relativistic minimum uncertainty (RMU) wave packets

Can be expressed in a Lorentz invariant form

$$\tilde{\psi}(p_\mu; a_\mu) = \mathcal{N} \exp\left[- a_\mu p^\mu \right],$$

Relativistic minimum uncertainty (RMU) wave packets

Can be expressed in a Lorentz invariant form

$$\tilde{\psi}(p_\mu; a_\mu) = \mathcal{N} \exp\left[-a_\mu p^\mu \right],$$

where $a_\mu = (\alpha, -\beta) \in \mathbb{C}^2$ and transforms as a vector.

Why RMU wave packets?

Why RMU wave packets?

Let us reconsider Gaussian wave packets

Why RMU wave packets?

Let us reconsider Gaussian wave packets

$$\psi(p;\bar{p},\sigma_p) = \mathcal{N}\exp\left[-\frac{(p-\bar{p})^2}{4\sigma_p^2}\right].$$

Why RMU wave packets?

Let us reconsider Gaussian wave packets

$$\psi(p;\bar{p},\sigma_p) = \mathcal{N}\exp\left[-\frac{(p-\bar{p})^2}{4\sigma_p^2}\right].$$

Gaussian minimises Heisenberg-Robertson uncertainty relation

$$\sigma_x\sigma_p = \left|\frac{1}{2i}\langle[\hat{x},\,\hat{p}]\rangle\right| = \frac{\hbar}{2}.$$

Assuming that position and momentum are independent

Robertson-Schrödinger uncertainty relation

Robertson-Schrödinger uncertainty relation

More generally, can have non-vanishing covariance

Robertson-Schrödinger uncertainty relation

More generally, can have non-vanishing covariance

$$\sigma_x^2 \sigma_p^2 - \sigma_{xp}^2 = \left| \frac{1}{2i} \left\langle [\hat{x},\ \hat{p}] \right\rangle \right|^2$$

Robertson-Schrödinger uncertainty relation

More generally, can have non-vanishing covariance

$$\sigma_x^2 \sigma_p^2 - \sigma_{xp}^2 = \left| \frac{1}{2i} \left\langle [\hat{x},\ \hat{p}] \right\rangle \right|^2$$

One obtains squeezed Gaussian wave packets

Robertson-Schrödinger uncertainty relation

More generally, can have non-vanishing covariance

$$\sigma_x^2 \sigma_p^2 - \sigma_{xp}^2 = \left| \frac{1}{2i} \left\langle [\hat{x},\, \hat{p}] \right\rangle \right|^2$$

One obtains squeezed Gaussian wave packets

$$\psi(p; \bar{x}, \bar{p}, \sigma_p) = \mathcal{N} \exp\left[- \frac{(p - \bar{p})^2}{4\sigma_p^2} + i\bar{x}p \right]$$

What about RMU wave packets?

What about RMU wave packets?

Define relativistic velocity operator

What about RMU wave packets?

Define relativistic velocity operator

$$\hat{v} \equiv i[\hat{H}, \hat{x}] = \frac{\hat{p}}{\hat{H}}$$

What about RMU wave packets?

Define relativistic velocity operator

$$\hat{v} \equiv i[\hat{H}, \hat{x}] = \frac{\hat{p}}{\hat{H}}$$

Minimise uncertainty between position and *velocity*

What about RMU wave packets?

Define relativistic velocity operator

$$\hat{v} \equiv i[\hat{H}, \hat{x}] = \frac{\hat{p}}{\hat{H}}$$

Minimise uncertainty between position and *velocity*

$$\sigma_x^2 \sigma_v^2 - \sigma_{xv}^2 = \left| \frac{1}{2i} \left\langle [\hat{x},\ \hat{v}] \right\rangle \right|^2$$

What about RMU wave packets?

Minimise uncertainty between position and *velocity*

$$\sigma_x^2 \sigma_v^2 - \sigma_{xv}^2 = \left| \frac{1}{2i} \langle [\hat{x},\ \hat{v}] \rangle \right|^2$$

Squeezed RMU wave packets! Generalised from Ref. [10]

$$\tilde{\psi}(E_p, p; \alpha, \beta) = \mathcal{N} \exp[-\alpha E_p + \beta p]\,,$$

where α, β determined by the moments of velocity and space(time)

[10] M. H. Al-Hashimi and U.-J. Wiese, Ann. Phys. **324**, 2599 (2009).

What about RMU wave packets?

Squeezed RMU wave packets! Generalised from Ref. [10]

$$\tilde{\psi}(E_p, p; \alpha, \beta) = \mathcal{N} \exp[-\alpha E_p + \beta p] \, ,$$

where α, β determined by the moments of velocity and space(time)

[10] M. H. Al-Hashimi and U.-J. Wiese, Ann. Phys. **324**, 2599 (2009).

What about RMU wave packets?

Squeezed RMU wave packets! Generalised from Ref. [10]

$$\tilde{\psi}(E_p, p; \alpha, \beta) = \mathcal{N} \exp\Big[-\alpha E_p + \beta p \Big],$$

where α, β determined by the moments of velocity and space(time)

Reduce to Gaussians in non-relativistic limit…

[10] M. H. Al-Hashimi and U.-J. Wiese, Ann. Phys. **324**, 2599 (2009).

What about RMU wave packets?

Squeezed RMU wave packets! Generalised from Ref. [10]

$$\tilde{\psi}(E_p, p; \alpha, \beta) = \mathcal{N} \exp\left[-\alpha E_p + \beta p \right],$$

where α, β determined by the moments of velocity and space(time)

Reduce to Gaussians in non-relativistic limit… and to Lorentzians in the ultra-relativistic limit!

[10] M. H. Al-Hashimi and U.-J. Wiese, Ann. Phys. **324**, 2599 (2009).

What about RMU wave packets?

Squeezed RMU wave packets! Generalised from Ref. [10]

$$\tilde{\psi}(E_p, p; \alpha, \beta) = \mathcal{N} \exp\Big[-\alpha E_p + \beta p \Big],$$

where α, β determined by the moments of velocity and space(time)

Reduce to Gaussians in non-relativistic limit… and to Lorentzians in the ultra-relativistic limit! (In configuration space and neglecting mass)

[10] M. H. Al-Hashimi and U.-J. Wiese, Ann. Phys. **324**, 2599 (2009).

Propagators for different wave packets

Propagators for different wave packets

Plane wave:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) = \sum_i U_{\alpha i}^* e^{-i(E_i T - p_i L)} U_{\beta i}$$

Propagators for different wave packets

Plane wave:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) = \sum_i U^*_{\alpha i} e^{-i(E_i T - p_i L)} U_{\beta i}$$

Gaussian:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) \sim \sum_i U^*_{\alpha i} \exp\left[-\frac{(L - \bar{v}_i T)^2}{4\sigma_x^2} - i(\bar{E}_i T - \bar{p}_i L) \right] U_{\beta i}$$

Propagators for different wave packets

Plane wave:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) = \sum_i U^*_{\alpha i} e^{-i(E_i T - p_i L)} U_{\beta i}$$

Gaussian:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) \sim \sum_i U^*_{\alpha i} \exp\left[-\frac{(L - \bar{v}_i T)^2}{4\sigma_x^2} - i(\bar{E}_i T - \bar{p}_i L) \right] U_{\beta i}$$

RMU:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) \sim \sum_i U^*_{\alpha i} \frac{(T - i\alpha) m_i K_1\left(-m_i \sqrt{(T - i\alpha)^2 + (L - i\beta)^2} \right)}{\sqrt{(T - i\alpha)^2 + (L - i\beta)^2}} U_{\beta i}$$

Implications for decoherence

1.

Implications for decoherence

1. Gaussian wave packets spread over evolution,

Implications for decoherence

1. Gaussian wave packets spread over evolution, while RMU wave packets maintain their localisation [11]

[11] C. E. Wood and M. Zych, Phys. Rev. Research **3**, 013049 (2021).

Implications for decoherence

1. Gaussian wave packets spread over evolution,
   while RMU wave packets maintain their localisation [11]

   In general, $\sigma_x^{(RMU)}(t) \leq \sigma_x^{(G)}(t)$ for all time

[11] C. E. Wood and M. Zych, Phys. Rev. Research **3**, 013049 (2021).

Implications for decoherence

1. Gaussian wave packets spread over evolution, while RMU wave packets maintain their localisation [11]

   In general, $\sigma_x^{(RMU)}(t) \leq \sigma_x^{(G)}(t)$ for all time

2. RMU wave packets follow semi-classical trajectories, so different mass eigenstates do not separate!

[11] C. E. Wood and M. Zych, Phys. Rev. Research **3**, 013049 (2021).

Can neutrinos propagate with equal velocities?

Can neutrinos propagate with equal velocities?

Controversy in early 2000s whether group velocities can be equal [12-14]

[12] Y. Takeuchi, Y. Tazaki, S. Y. Tsai, and T. Yamazaki, Mod. Phys. Lett. A **14**, 2329 (1999).

[13] S. De Leo, G. Ducat, and P. Rotelli, Mod. Phys. Lett. A **15**, 2057 (2000).

[14] L. B. Okun and I. S. Tsukerman, Mod. Phys. Lett. A **15**, 1481 (2000).

Can neutrinos propagate with equal velocities?

Controversy in early 2000s whether group velocities can be equal [12-14]

Consensus is debate settled as “no” from kinematical arguments

[12] Y. Takeuchi, Y. Tazaki, S. Y. Tsai, and T. Yamazaki, Mod. Phys. Lett. A **14**, 2329 (1999).
[13] S. De Leo, G. Ducat, and P. Rotelli, Mod. Phys. Lett. A **15**, 2057 (2000).
[14] L. B. Okun and I. S. Tsukerman, Mod. Phys. Lett. A **15**, 1481 (2000).

Can neutrinos propagate with equal velocities?

Controversy in early 2000s whether group velocities can be equal [12-14]

Consensus is debate settled as “no” from kinematical arguments

1. If $v_1 = v_2$, then $p_1/E_1 = p_2/E_2$ and $p_1/p_2 = E_1/E_2$

[12] Y. Takeuchi, Y. Tazaki, S. Y. Tsai, and T. Yamazaki, Mod. Phys. Lett. A **14**, 2329 (1999).
[13] S. De Leo, G. Ducat, and P. Rotelli, Mod. Phys. Lett. A **15**, 2057 (2000).
[14] L. B. Okun and I. S. Tsukerman, Mod. Phys. Lett. A **15**, 1481 (2000).

Can neutrinos propagate with equal velocities?

Controversy in early 2000s whether group velocities can be equal [12-14]

Consensus is debate settled as “no” from kinematical arguments

1. If $v_1 = v_2$, then $p_1/E_1 = p_2/E_2$ and $p_1/p_2 = E_1/E_2$
2. Since $E_i = \gamma m_i$, then $p_1/p_2 = E_1/E_2 = m_1/m_2$

[12] Y. Takeuchi, Y. Tazaki, S. Y. Tsai, and T. Yamazaki, Mod. Phys. Lett. A **14**, 2329 (1999).

[13] S. De Leo, G. Ducat, and P. Rotelli, Mod. Phys. Lett. A **15**, 2057 (2000).

[14] L. B. Okun and I. S. Tsukerman, Mod. Phys. Lett. A **15**, 1481 (2000).

Can neutrinos propagate with equal velocities?

Controversy in early 2000s whether group velocities can be equal [12-14]

Consensus is debate settled as “no” from kinematical arguments

1. If $v_1 = v_2$, then $p_1/E_1 = p_2/E_2$ and $p_1/p_2 = E_1/E_2$
2. Since $E_i = \gamma m_i$, then $p_1/p_2 = E_1/E_2 = m_1/m_2$
3. $E_1/E_2 \simeq 1$ in ultra-relativistic regime, not generally true for m_1/m_2

[12] Y. Takeuchi, Y. Tazaki, S. Y. Tsai, and T. Yamazaki, Mod. Phys. Lett. A **14**, 2329 (1999).
[13] S. De Leo, G. Ducat, and P. Rotelli, Mod. Phys. Lett. A **15**, 2057 (2000).
[14] L. B. Okun and I. S. Tsukerman, Mod. Phys. Lett. A **15**, 1481 (2000).

Can neutrinos propagate with equal velocities?

Can neutrinos propagate with equal velocities?

Let's be more careful!

Can neutrinos propagate with equal velocities?

Let's be more careful!

Should one take $\bar{v} = \langle p \rangle / \langle E \rangle$?

Can neutrinos propagate with equal velocities?

Let's be more careful!

Should one take $\bar{v} = \langle p \rangle / \langle E \rangle$?

In general, one has $\bar{v} = \langle \partial_p E \rangle \neq \langle p \rangle / \langle E \rangle$!

Can neutrinos propagate with equal velocities?

Let's be more careful!

Should one take $\bar{v} = \langle p \rangle / \langle E \rangle$?

In general, one has $\bar{v} = \langle \partial_p E \rangle \neq \langle p \rangle / \langle E \rangle$!

RMU wave packets have $\bar{v} = \langle \partial_p E \rangle = \mathrm{Re}(\beta) / \mathrm{Re}(\alpha)$, while

Can neutrinos propagate with equal velocities?

Let's be more careful!

Should one take $\bar{v} = \langle p \rangle / \langle E \rangle$?

In general, one has $\bar{v} = \langle \partial_p E \rangle \neq \langle p \rangle / \langle E \rangle$!

RMU wave packets have $\bar{v} = \langle \partial_p E \rangle = \mathrm{Re}(\beta)/\mathrm{Re}(\alpha)$, while

$$\frac{\langle p \rangle}{\langle E \rangle} = \frac{\mathrm{Re}(\beta)}{\mathrm{Re}(\alpha)} \left(\frac{1}{1 - \chi_{m,\mathrm{Re}(\alpha),\mathrm{Re}(\beta)}} \right).$$

Can neutrinos propagate with equal velocities?

Let's be more careful!

Should one take $\bar{v} = \langle p \rangle / \langle E \rangle$?

In general, one has $\bar{v} = \langle \partial_p E \rangle \neq \langle p \rangle / \langle E \rangle$!

RMU wave packets have $\bar{v} = \langle \partial_p E \rangle = \mathrm{Re}(\beta)/\mathrm{Re}(\alpha)$, while

$$\frac{\langle p \rangle}{\langle E \rangle} = \frac{\mathrm{Re}(\beta)}{\mathrm{Re}(\alpha)} \left(\frac{1}{1 - \chi_{m,\mathrm{Re}(\alpha),\mathrm{Re}(\beta)}} \right).$$

Only agrees in semi-classical regime, for $\sigma_x \gg \lambda_c \equiv 1/m$

Summary

- Neutrino wave packets could have a non-Gaussian profile
- If described by RMU wave packets, then neutrinos are highly localised, and decoherence is heavily suppressed
- Propagation at equal velocities should be taken seriously, and could be experimentally tested

Evan Gale,
e.gale@uq.edu.au