1 Introduction

2 Roadmap

- Exploratory Inventory Location Problem for Newsvendor pre-allocation
- contrived grid/contrived geography no-routing power grid repair scheduling with 1 crew
- contrived grid/contrived geography perfect information road repair with 1 crew
- inclusion of routing into power grid
- formulation of real geography and transition to GIS data
- transition from pipeflow power to DC grid model
- consult electrical engineers RE: power grid analysis
- generate real hurricane scenarios
- 1-2 response style interactions
- 1-2-1 revision style interactions
- joint solution
- incomplete information in roads

3 Exploratary ILP

4 Power Grid Models

4.1 NonInteracting/NonRouting

4.1.1 Variable Glossary

- C_{FlowIJ} is steady state flow on the grid before the hurricane
- C_{lineIJ} is the capacity limit for the power line going from IJ
- $C_{RepairTimeI}$ is the time to repair node I
- $C_{demandI}$ is the power demand at location I in the pre-disaster steady state
- $C_{GeneratorCapacityK}$ is the maximum power generation for generator K

4.1.2 Variables

- X_{ij}^t is the flow from i to j at time t
- G_k^t is the production from generator k at time t
- Y_i^t is 1 if node i is functioning at time t
- W_{ij}^t is 1 if line ij is functioning at time t
- F_i^t is 1 is node i is serviced at time t
- E_{ij}^t is 1 is line ij is serviced at time t

4.1.3 Sets

- L is the set of nodes
- P is the set of power lines
- T is the planning horizon
- J is the set of Generators

4.1.4 Model

$$Minimize \sum_{i,j \in L} \sum_{t \in T} |C_{FlowIJ} - X_{ij}^t|$$

Subject to:

(1) $\sum_{i \in L} X_{ik} + G_k^t = \sum_{j \in L} X_{kj} + C_{demand\_k} Y_i^t \ \forall t \in T \ \forall k \in L$

(2) $\sum_{i \in L} C_{Demand\_i} Y_i^t = \sum_{k \in J} G_k^t \ \forall t \in T$

(3) $G_k^t \leq C_{GeneratorCapacity\_k} \ \forall t \in T \ \forall k \in J$

(4) $X_{ij}^t \leq C_{lineIJ} W_{ij}^t \ \forall t \in T \ \forall i, j \in P$

(5) $\sum_{i \in L} C_{RepairTime\_i} F_i^t + \sum_{i,j \in P} C_{RepairTime\_ij} S_{ij}^t + \sum_{i \in L} \leq 8 \ \forall t \in T$

(6) $Y_i^t \leq \sum_{0}^{t} F_i^t + initial \ \forall i \in L \forall t \in T$

(7) $W_{ij}^t \leq \sum_{0}^{t} S_{ij}^t + initial \ \forall i, j \in L \forall t \in T$

(8) $\sum_{t \in T} F_i^t \leq 1 \ \forall i \in L$

(9) $\sum_{t \in T} S_{ij}^t \leq 1 \ \forall i, j \in L$

4.1.5 Explanation of Constraint Systems

- Constraint (1) defines flow balance equations for each node
- Constraint (2) defines input/output network balance. This is assuming Generator ramp time can be ignored, but that's fine since excess power can always be dropped to ground.
- Constraint (3) constrains power generation to be in the realm of feasible production
- Constraint (4) constrains line flow to be inside line capacity
- Constraint (5) constrains/decides what gets done during a shift
- Constraints (6) and (7) handle defining operations

4.1.6 Comments

- I'm assuming that we're staying in the region of safe production for generators, a later thing to think about is "pushed" generators where they can be run in overdrive for short periods of time

4.2 NonInteracting/Routing

4.2.1 Variable Glossary

- C_{FlowIJ} is steady state flow on the grid before the hurricane
- C_{lineIJ} is the capacity limit for the power line going from IJ
- $C_{RepairTimeI}$ is the time to repair node I
- $C_{RepairTimeIJ}$ is the time to repair line IJ
- $C_{TravelIJ}$ is the travel time between nodes I and J
- C_{broken} is a coefficient of "broken-ness" representing the average slowdown from debris on the road and minor flooding
- $C_{demandI}$ is the power demand at location I in the pre-disaster steady state
- $C_{GeneratorCapacityK}$ is the maximum power generation for generator K

4.2.2 Variables

- X_{ij}^t is the flow from i to j at time t
- G_k^t is the production from generator k at time t
- Y_i^t is 1 if node i is functioning at time t
- W_{ij}^t is 1 if line ij is functioning at time t
- S_{ij}^t is 1 if line ij is serviced at time t
- K_{ij}^t is 1 if node j follows node i in the tour at time t
- F_i^t is 1 is node i is serviced at time t
- E_{ij}^t is 1 is line ij is serviced at time t

4.2.3 Sets

- L is the set of nodes
- P is the set of power lines
- R is the set of roads
- T is the planning horizon
- J is the set of Generators

4.2.4 Model

$$Minimize \sum_{i \in L} \sum_{t \in T} C_{FlowIJ} - X_{ij}^t$$

Subject to:

(1) $\sum_{i \in L} X_{ik} + G_k^t = \sum_{j \in L} X_{kj} + C_{demandK} \ \forall t \in T \ \forall k \in L$

(2) $\sum_{i \in L} C_{DemandI} Y_I^t = \sum_{k \in J} G_k^t \ \forall t \in T$

(3) $G_k^t \leq C_{GeneratorCapacityK} \ \forall t \in T \ \forall k \in J$

(4) $X_{ij}^t \leq C_{lineIJ} W_{ij}^t \ \forall t \in T \ \forall i, j \in P$

(5) $\sum_{i \in L} C_{RepairTimeI} F_i^t + \sum_{i,j \in P} C_{RepairTimeIJ} S_{ij}^t + \sum_{i \in L} \sum_{j < i \in L} K_{ij}^t C_{TravelIJ} C_{broken} \leq 8 \ \forall t \in T$

(6) $Y_i^t \leq \sum_{0}^{t} F_i^t + initial \ \forall i \in L$

(7) $W_{ij}^t \leq \sum_{0}^{t} S_{ij}^t + initial \ \forall i, j \in L$

(8) $\sum_{j \in L} K_{0j}^t \geq 1$

(9) $\sum_{j \in L} K_{ij}^t - \sum_{j \in L} K_{ji}^t = 0 \ \forall t \in T \ \forall i \in L$

(10) A subtour elimination constraint

4.2.5 Explanation of Constraint Systems

- Constraint (1) defines flow balance equations for each node
- Constraint (2) defines input/output network balance. This is assuming Generator ramp time can be ignored, but that's fine since excess power can always be dropped to ground.
- Constraint (3) constrains power generation to be in the realm of feasible production
- Constraint (4) constrains line flow to be inside line capacity
- Constraint (5) constrains/decides what gets done during a shift
- Constraints (6) and (7) handle defining operations
- Constraints (8)-(10) handle the routing side of the problem

4.2.6 Comments

- I'm assuming that we're staying in the region of safe production for generators, a later thing to think about is "pushed" generators where they can be run in overdrive for short periods of time

4.3 DC Power Flow/Shortest Path "Routing"

4.3.1 Glossary

- C_{lineE} is the capacity limit for the power line E
- $C_{RepairTimeI}$ is the time to repair node I
- $C_{RepairTimeE}$ is the time to repair line E
- $C_{SP(i)}$ is the shortest path to node i from the central depot
- C_{broken} is a coefficient of "broken-ness" representing the average slowdown from debris on the road and minor flooding

- $C_{demandI}$ is the power demand at location I in the pre-disaster steady state
- $C_{GeneratorCapacityK}$ is the maximum power generation for generator K
- B_e is the line susceptance (imaginary part of admittance, also inverse of resistance) for power line e

4.3.2 Variables

- X_e^t is the flow on line e at time t
- G_k^t is the production from generator k at time t
- V_i^t is 1 if node i is functioning at time t
- W_e^t is 1 if line e is functioning at time t
- S_e^t is 1 if line e is serviced at time t
- F_i^t is 1 is node i is serviced at time t
- θ_i^t is the phase angle for the power flow at i in time t

4.3.3 Sets

- N is the set of nodes
- E is the set of power lines
- R is the set of roads
- T is the planning horizon
- o(i) is the set of lines with origin i
- d(i) is the set of lines with destination i
- o(e) is the origin node of line e
- d(e) is the destination node of line e

4.3.4 Model

$$Minimize \sum_{i \in N} \sum_{t \in T} (1 - W_i^t) * C_{demand\_i}$$

Subject to:

(1) $X_e^t = B_e * (\theta_{origin}^t - \theta_{destination}^t) \forall t \in T \ \forall e \in E$

(2) $G_i^t - \sum_{l \| o(l)=i} X_l^t + \sum_{l \| d(l)=i} X_l^t = C_{demand\_i} \ \forall t \in T \ \forall i \in N$

(3) $G_k^t \leq C_{GeneratorCapacityK} V_k^t \ \forall t \in T \ \forall k \in N$

(4) $-C_{line\_e} W_e^t \leq X_e^t \leq C_{line\_e} W_e^t \ \forall t \in T \ \forall e \in E$

(5) $-C_{line\_e} V_{o(e)}^t \leq X_e^t \leq C_{line\_e} V_{o(e)}^t \ \forall t \in T \ \forall e \in E$

(6) $-C_{line\_e} V_{d(e)}^t \leq X_e^t \leq C_{line\_e} V_{d(e)}^t \ \forall t \in T \ \forall e \in E$

(7) $\sum_{i \in L} C_{RepairTimeI} F_i^t + \sum_{e \in E} C_{RepairTime\_e} S_e^t + \sum_{i \in L} F_i^t C_{SP(i)} + \sum_{e \in E} S_e^t * min(C_{SP(o(e))}, C_{SP(d(e))}) \leq 8 \ \forall t \in T$

(8) $V_i^t \leq \sum_{0}^{t-1} F_i^t + initial \ \forall i \in L$

(9) $W_e^t \leq \sum_{0}^{t-1} S_e^t + initial \ \forall e \in E$

4.3.5 Explanation of Constraint Systems

- Constraint (1) defines flow based on line limits and line susceptance as per Salmeron, Ross, and Baldick 2004
- Constraint (2) defines node power balance so that inflow has to match outflow at each node.
- Constraint (3) constrains power generation to be in the realm of feasible production conditional on the relevant node being operational
- Constraints (4)-(6) constrains line flow to be inside line capacity conditional on the relevant elements being operational
- Constraint (7) constrains/decides what gets done during a shift and handles shortest path travel time.
- Constraints (8) and (9) handle defining operations

4.3.6 Comments

- The assumption in this version of the model is that a vehicle can only do one operation per trip, so routing reduces to shortest path
- This also assumes DC power flow, which is a much more through version of power flow than pipeflow
- note that from constraint 9, once W is working, we can chose whether or not it's engaged

5 Road Models

5.1 Basic Routing Repair

5.1.1 Glossary

- C_{ij} is a measure of the value of the road to relief supply delivery efforts
- L_{ij} is the length of the road between nodes i and j when everything is working as normal
- R_{ij} is the time to repair the road between i and j

5.1.2 Variables

- X_{ij}^t is 1 if the road between nodes i and j is working at time t
- S_{ij}^t is the length of the road between i and j at time t.
- K_{ij}^t is the decision variable that is 1 if j follows i in the tour at time t and 0 else.

5.1.3 Sets

- T is the set of time over the time horizon
- N is the set of nodes on the graph

5.1.4 Model

$$Minimize \sum_{t \in T} t \sum_{i,j \in N} C_{ij} * (1 - X_{ij}^t)$$

Subject to:

(1) $\sum_{i,j \in N} S_{ij}^t K_{ij}^t \leq 8 \;\; \forall t \in T$

(2) $S_{ij}^t = max(L_{ij}, (1 - X_{ij}^t) R_{ij}) \;\; \forall t \in T \; \forall i, j \in N$

(3) $\sum_{j \in N} K_{ij}^t - \sum_{j \in N} K_{ji}^t = 0 \;\; \forall t \in T \; \forall i \in N$

(4) $X_{ij}^t <= \sum_{v=0}^{t-1} K_{ij}^v + starting \;\; \forall t \in T \; \forall i, j \in N$

(5) $\sum_{i,j \in S; i \neq j} X_{ij}^t \leq |S| - 1 \;\; \forall S \subset N; \; S \neq \emptyset$

5.1.5 Explanation of Constraint Systems

- Constraint 1 is a scheduling constraint so that each tour has to be less than 8 hours of stuff
- Constraint 2 defines the length of a road to be either the travel length if it's working or the repair cost if it hasn't yet
- Constraint 3 is path connectivity for the tour
- Constraint 4 defines the functionality of each road. While it doesn't bind to 1, because there's a penalty for not being 1, it will choose 1 if possible
- Constraint 5 eliminates subtours to ensure a valid tour

5.1.6 Comments

- we assume R_ij is 10x the length of the road representing time to drive down the road and clear debris/move trees/drain flooded patches/etc