

# A mathematical model for post-disaster road restoration: Enabling accessibility and evacuation



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## ABSTRACT

This paper focuses on the planning of road restoration efforts during disaster response and recovery. The primary objective is to maximize network accessibility for all locations in the area during the restoration process so that survivors are evacuated and road side debris is removed as soon as possible. We propose a dynamic path based mathematical model that identifies criticality of blockages and clears them with limited resources. This model is more efficient than link based models and can solve restoration problems for realistic size networks within reasonable time. Algorithm performance is demonstrated using two instances based on districts in Istanbul.

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## 1. Introduction and problem description

Post-disaster road restoration constitutes the first step in disaster response and recovery (FEMA, 2007). In any kind of disaster, whether hurricane or earthquake, the goal is to maximize survival rates. It is essential to be able to reach survivors and offer them relief and a possibility to evacuate the affected region during the first few days after the disaster strikes. Road network disruptions impede timely access to help and delay evacuation to shelters.

This paper addresses the issue of restoring blocked links in a road network with the goal of opening access paths for all locations as early as possible (during the first three days of response). A dynamic path based mathematical model is proposed to solve this problem. The assumptions are as follows: The operation is carried out by a limited number of equipments (denoted by  $D$ ); each equipment can handle one blocked road (link) at a time; each blocked link,  $e_i$ , has an estimated restoration time,  $c_i$ . These assumptions imply that at any point in time, at most  $D$  links can be processed simultaneously by a team of  $D$  work groups.

A path  $p$  consists of an ordered sequence of links between an origin–destination (O–D) pair and it is assumed to be open when all of its links  $e_i$  are clear. Each location (node) in the network needs access to shelters or local relief distribution centers and to temporary debris dump sites (so that any debris cleared from that node can be sent out). A set of access paths are defined for each node in the network, and the union of these paths compose the total set of pre-defined access paths,  $P$ , to be cleared. Note that a link can be an element of more than one path and that some paths may already be open due to lack of damage/debris.

Given a pre-defined set of access paths,  $P$ , to be cleared, the model optimally identifies the order of blocked links,  $e_i$ , to be restored during a three-day workshift ( $T_{max}$ ). The output of the model is a restoration schedule for clearing blocked links. The

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goal is to maximize the total weighted earliness of all paths' restoration completion times. A path's restoration completion time is denoted as  $t_p$ . The earliness of a path's restoration completion time is defined as the difference between the operation due date,  $T_{max}$ , and  $t_p$ . Here, a priority related weight  $Q_p$  is assigned to each path and it represents the urgency of restoring path  $p$  based on criteria such as the expected traffic flow on the path. In the objective function, each path's earliness value is multiplied by  $Q_p$  and summed over all paths, resulting in the total weighted earliness.

An earliness based goal maximizes cumulative network accessibility throughout the restoration operation. Paths that are not cleared until period  $T_{max}$  do not contribute to the objective function value. Hence, the objective function implicitly tries to maximize the number of open paths as well.

The proposed dynamic path based model is an integer program because it contains binary variables that represent the open/closed status of every blocked link and the status of every path over the time span of  $T_{max}$ . Though path based models are very compact and efficient with regard to the number of binary variables, they still involve a high complexity. In order to reduce the computational burden, a 'divide-and-conquer' strategy is adopted, i.e., we decompose the affected area into smaller zones (districts) and solve the scheduling problem for each zone under several equipment allocation scenarios. Then, we use a resource allocation model to distribute the specialized equipment among these districts equitably.

The proposed solution approach is novel in the sense that it is the first path based approach that considers maximizing dynamic network accessibility throughout the time span of the restoration process. Path based models found in the literature (e.g., see [Matisziw and Murray, 2009](#)) try to identify a given number of critical links whose failure will block a maximum number of paths. In these models, equipment limitations are not considered, there is no scheduling involved, and there is a bound on the number of links considered. Although this information is very valuable in mitigation studies, it provides little help in the response phase where the set of failed links does not overlap with the set of identified critical links. In the literature, there exist non-path based scheduling approaches involving formulations based on the multi-tour Traveling Salesman Problem ([Feng and Wang, 2003](#)), the multi-machine scheduling problem with sequence dependent travel times between restoration jobs ([Chen and Tzeng, 2000](#)) and network flow formulations ([Yan and Shih, 2009](#)). These are link based models whose complexities are so high that they can at best be solved by heuristics. On the other hand, the reduced complexity of the proposed path based model allows us to solve scheduling problems of realistic size without resorting to heuristics ([Yan and Shih, 2012](#)). However, it does not consider operational decisions such as equipment routing.

In subsequent sections, we summarize the literature related to this problem, then we describe the two mathematical models, finally we discuss the implementation of these models on two exemplary districts in the city of Istanbul (Turkey).

## 2. Literature survey

Phase 1 activities of disaster recovery and response involve clearing roadside debris and restoring the road network in order to open up evacuation routes and other important lifeline paths so that traffic flow is enabled in affected areas. The restoration operation can be conducted efficiently by identifying the optimal order in which critical blocked links in the road network are cleared. The goal is to maximize the overall earliness of path restoration times, which leads to maximizing cumulative network accessibility throughout the operation. The concept of network accessibility is closely linked to network vulnerability which is defined as a susceptibility to incidents that can result in considerable reductions in road network serviceability (availability for usage) ([Berdica, 2002](#)). The goal of minimizing road network vulnerability is used in both mitigation studies (reinforcing critical infrastructure) and response studies (enabling speedy network accessibility for relief). Although many models and measures are proposed to measure network vulnerability (e.g. [Chen et al., 2007](#); [Erath et al., 2010](#)), the increase in travel time and travel distance seem to be the most common measures. [Chang and Nojima \(2001\)](#) define an accessibility measure based on the extension of post-disaster shortest paths between all pairs of locations in the road network. [Sohn \(2006\)](#) enhances this measure by weighing locations according to their populations and traffic densities. [Giovinazzi and Nicholson \(2010\)](#) discuss measures of post-disaster network reliability such as connectivity, travel time, capacity and accessibility. Some researchers such as [Sohn \(2006\)](#) and [Jenelius and Mattsson \(2012\)](#) identify the importance of network links/areas by evaluating accessibility measures by closing one link/area at a time and observing its impact on the whole network. However, this sequential approach does not take the combined effects of blocked links on major paths, hence, if the restoration scheduling plans are made according to such link criticality indices, the resulting plans might be suboptimal. Other studies develop commodity flow models to assess economic loss under link damages ([Cho et al., 2001](#); [Kim et al., 2002](#); [Ham et al., 2005](#)). These methods consider only commodity destinations.

In the area of disaster response and recovery, repair schedules for blocked links are obtained by optimization methods. For instance, [Chen and Tzeng \(2000\)](#) propose a two-level mathematical model for sequencing road repair tasks over time, imposing a due date. Travel times between tasks are considered but repair resources are not limited. The goal is to minimize travel weighted traffic flow. The model is quite complex, therefore, a genetic algorithm is proposed. In another study, the authors use a multiobjective GA to solve the same problem on a realistic network with 24 nodes ([Chen and Tzeng, 1999](#)).

[Feng and Wang \(2003\)](#) consider the following constraints in emergency road rehabilitation: limitations on dozers, manpower and a finite debris clearing capacity of 250 m<sup>3</sup>/h for dozers. The authors consider goals that include maximizing the total length of accessible roads, the total number of cleared roads and minimizing the risk of work teams. In their model each

road is visited only once on a route of a work team, all work should be completed in 3 days, and each road is assigned to one work team. The formulation is based on a complex multi-tour traveling salesman problem. The model is tested on a small 50 node network with 10 damaged roads.

Yan and Shih (2009) propose an integrated road repair and relief distribution model with the goal of minimizing operation completion time. A time augmented network flow model with work team trips and relief material flows (over repaired roads) is proposed with an equity constraint on demand satisfaction. A three-step heuristic is proposed: blocked links are prioritized; worker schedules and commodity flows are optimized. A 46 node network with 25 repair points is solved for a 3-day span with a time bucket of 15 min. Due to model size, this small network is solved in 900 CPU seconds.

Matisziw and Murray (2009) identify vital node and arc blockages that would prevent traffic flow the most with both mitigation and response intentions (to protect most critical linkages in pre-disaster phase and recover them first in post-disaster phase). The authors maximize flows over broken source-sink paths while identifying the most critical  $k$  links. The introduction of path aggregation constraints dispose of the necessity to enumerate all source-sink paths. Results of the model are illustrated on a network of 23 nodes and 34 arcs. In an earlier study, Murray et al. (2007) propose a similar model that identifies the most critical links that make a set of  $k$  facilities inaccessible. This model is tested on a fiber optic communications network. Maya Duque and Sörensen (2011) address the repair problem under budget constraints using a fixed cost network flow formulation for minimizing the cost of flows from each rural center to the nearest regional center.

Nolz et al. (2011) define different measures to ensure the safe delivery of critical goods such as maximizing the number of alternative paths, minimizing unreachability of all alternative paths and maximizing the number of paths whose risk value is below a threshold. The authors propose a memetic algorithm that sequences repair tasks to develop a Pareto frontier for these objectives.

Liberatore et al. (2012) propose a network flow model for relief distribution over a network with broken linkages where a budget constraint is imposed on fixing roads. Several goals are proposed, such as delivery time, satisfied demand and reliability of arcs used for distributing relief.

As mentioned previously, the proposed mathematical model falls under the category of path based models that are more compact than the link based scheduling formulations discussed above. However, unlike other path based models, the proposed approach is dynamic and aims at maximizing cumulative network accessibility during restoration. This is crucial for post disaster survivors during the first 3 days of response. Additionally, the temporal nature of the proposed model allows us to handle the repair scheduling problem, hence, repair resource limitations can be considered. To summarize, the proposed model is both compact and realistic and its objective function is directed to maximize the effects of humanitarian efforts on beneficiaries.

### 3. The mathematical model

In this section, we formally define the Debris Clearance Scheduling Model (DCSM) that schedules the road restoration work in a region with the goal of maximizing the total weighted earliness of all cleared paths. As mentioned in Section 1, the DCSM is an integer programming model solved to optimality for each zone or district.

Before we can define the DCSM, we should provide the notation used in the model.

*Sets and parameters:*

$P$	Set of predefined paths, indexed by $p \in P$ ,
$A$	Set of links, indexed by $i \in A$ ,
$A_p$	Set of links on path $p \in P$ ,
$B_i$	Set of adjacent links, one of which should be cleared before link $i$ can be accessed,
$T$	Set of time periods, indexed by $t, t' = 1, \dots, T_{max}$ ,
$c_i$	Number of periods required to restore link $i$ ,
$Q_p$	Priority weight of path $p \in P$ , $Q_p \in (0, 1]$ ,
$D$	Number of equipment available for restoration.

*Decision variables:*

$x_{i,t}$	{1 if link $i$ becomes completely restored in period $t$ , 0 otherwise}
$z_{p,t}$	{1 if path $p$ becomes completely restored in period $t$ , 0 otherwise}

Then DCSM can be formulated as in Eqs. (1)–(9) given below:

#### Model DCSM

$$\text{Max } w = \sum_{p \in P} \sum_{t \in T} Q_p (T_{\max} - t + 1) z_{p,t} \quad (1)$$

$$\text{s.t. } \frac{1}{|A_p|} \sum_{i \in A_p} \sum_{t' \in T: t' \leq t} x_{i,t'} \geq z_{p,t} \quad \forall p \in P, t \in T \quad (2)$$

$$\sum_{i \in A: c_i > 0} \sum_{t' = t+1}^{t+c_i} x_{i,t'} \leq D \quad \forall t \in T \quad (3)$$

$$\sum_{t \in T: t \leq c_i} x_{i,t} = 0 \quad \forall i \in A : c_i > 0 \quad (4)$$

$$x_{i,1} = 1 \quad \forall i \in A : c_i = 0 \quad (5)$$

$$\sum_{t \in T} x_{i,t} \leq 1 \quad \forall i \in A \quad (6)$$

$$z_{p,t} \leq 1 \quad \forall p \in P, t \in T \quad (7)$$

$$\sum_{j \in B_i} \sum_{t' \leq t - c_i} x_{j,t'} \geq x_{i,t} \quad \forall i \in A, t \in T \quad (8)$$

$$x_{i,t} \in \{0, 1\} \quad \forall i \in A, t \in T \quad (9)$$

$$z_{p,t} \in \{0, 1\} \quad \forall p \in P, t \in T \quad (10)$$

In the above formulation, Eq. (1) gives the objective function, which is the sum of the weighted earliness values of all paths restored between time periods 1 to  $T_{\max}$ . Eq. (2) ensures that a path  $p$  assumes restored status only if all links on path  $p$  are restored. Eq. (3) limits the number of equipment that can be used at any period to  $D$ . Eq. (4) sets the earliest completion time for the restoration work of each blocked link, while the next equation sets the completion time for links with no blockage to period 1. Eqs. (6) and (7) respectively ensure that a link and a path assumes restored status at most once. Eq. (8) ensures that a blocked link that is sandwiched between two or more blocked links can be restored after at least one of these links are restored. The last two equations give the integrality constraints on variables  $x_{i,t}$  and  $z_{p,t}$ .

In this model travel times of equipment and work teams between consecutive restoration tasks are ignored, because restoration times are much longer than travel times. Thereby, the complexity of vehicle routing constraints is avoided. Without the vehicle routing constraints the resulting path based model can be solved to optimality within reasonable computation time. Hence, we can say that the DCSM is tactical rather than operational.

In the implementation of the model, the set of predefined paths,  $P$ , is built as follows: We assume that several important destinations can be defined for each node to have access. Here, without loss of generality, we assume that each node in the network should gain access to two primary destinations: the first being a predesigned evacuation route and the second, a temporary debris dump site (so that possible debris can be transported). For each node, we calculate the shortest paths between the node and these two primary destinations. Note that if we detect an open path from a node to a destination, we do not include the relevant shortest path in  $P$ . The set of predefined paths  $P$  consists of all shortest path pairs for all nodes in the network. Hence, the magnitude of the set  $P$  is of  $O(N)$ , where  $N$  is the number of nodes in the network. Thus, by opening up all paths for every location at the earliest possible time, maximum network accessibility is achieved with regard to both relief distribution and restoration operations. Note that in the objective function summing up the weighted earliness of all paths leads to maximizing cumulative prioritized network accessibility during  $T_{\max}$ .

We solve the DCSM for each district separately. In the DCSM, the number of equipment that can be used for restoration at any time period is assumed to be limited to  $D$ . However, given a total number of equipment that are available at a large disaster area (e.g. a city), the allocation of this scarce resource is another critical decision. In order to achieve a good allocation of equipment among districts, we solve the DCSM for each district under a set of scenarios, each corresponding to a different number of equipment allocation to that district. Once we obtain the total earliness values for each district under each scenario, we solve the following Equipment Allocation Model (EAM) to determine the optimal allocation of equipment among districts. In this model, let the parameters and decision variables be as follows.

#### Sets and parameters:

$J$	Set of districts, indexed by $j \in J$ ,
$S_j$	Set of equipment availability scenarios for district $j \in J$ , indexed by $s \in S_j$ ,
$d_{j,s}$	Number of equipment available for district $j \in J$ under scenario $s \in S_j$ ,
$w_{j,s}$	Total earliness objective function value of district $j \in J$ under scenario $s \in S_j$ ,
$D'$	Total number of equipment to be allocated among all districts $j \in J$ .

Decision variables:

$y_{j,s}$	{1, if scenario $s$ is selected for district $j$ , 0 otherwise}
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Given the above notation, EAM can be defined as in Eqs. (10)–(16):

Model EAM

$$\text{Max } z_1 = \sum_{j \in J} \sum_{s \in S_j} w_{j,s} y_{j,s} \quad (11)$$

$$\text{s.t. } \sum_{s \in S_j} y_{j,s} = 1 \quad \forall j \in J \quad (12)$$

$$\sum_{j \in J} \sum_{s \in S_j} d_{j,s} y_{j,s} = D' \quad (13)$$

$$y_{j,s} \in \{0, 1\} \quad \forall j \in J, s \in S_j \quad (14)$$

In the above model, the objective function in Eq. (11) maximizes the total earliness over all districts in the disaster area. Here, the values of  $w_{j,s}$  are obtained by solving the model DCSM for all districts  $j$  under given equipment allocation scenarios.  $w_{j,s}$  are the optimal objective function values of these district scenario models. Eq. (12) ensures that exactly one equipment scenario is selected for each district, while Eq. (13) limits the total number of equipment allocated to all districts to the total availability,  $D'$ . Finally, Eq. (14) defines the integrality constraint on variables  $y_{j,s}$ . The model in (11)–(14) maximizes the total earliness over all districts but equity among districts is not considered. Alternatively, we propose the modified model EAM' which minimizes the maximum difference between the earliness values of any two districts:

Model EAM'

$$\text{Min } z_2 = s_{\max} \quad (15)$$

$$\text{s.t. } \sum_{s \in S_{j_1}} w_{j_1,s} y_{j_1,s} - \sum_{s \in S_{j_2}} w_{j_2,s} y_{j_2,s} = s_{j_1 j_2}^+ - s_{j_1 j_2}^- \quad \forall j_1, j_2 \in J : j_1 \neq j_2 \quad (16)$$

$$s_{j_1 j_2}^+ + s_{j_1 j_2}^- \leq s_{\max} \quad \forall j_1, j_2 \in J : j_1 \neq j_2 \quad (17)$$

$$s_{j_1 j_2}^+, s_{j_1 j_2}^- \geq 0 \quad \forall j_1, j_2 \in J : j_1 \neq j_2 \quad (18)$$

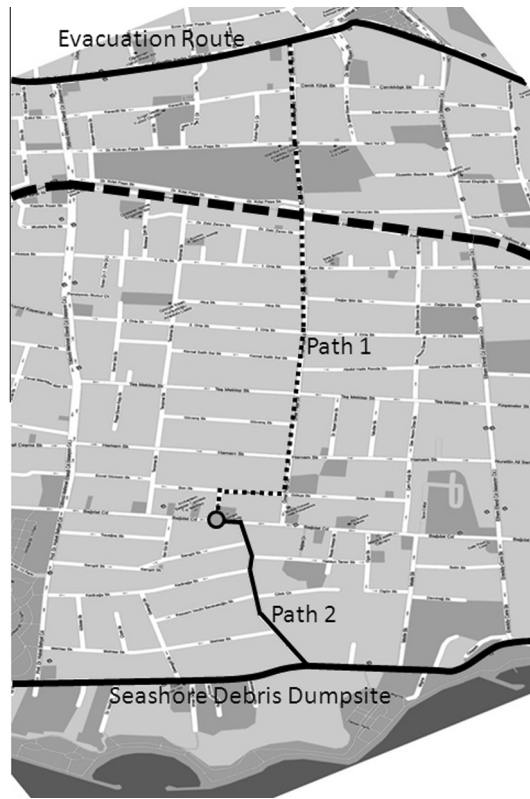
and (11)–(13).

In this model, the left hand side of Eq. (16) is the difference between the earliness values of two districts  $j_1, j_2 \in J$ . Obviously, variables  $s_{j_1 j_2}^+$  and  $s_{j_1 j_2}^-$  cannot be positive simultaneously for the same district pair in any basic feasible solution. Thus, if the left hand side of Eq. (16) is positive,  $s_{j_1 j_2}^+$  is assigned to the value of the left hand side and  $s_{j_1 j_2}^-$  is set to zero. Conversely, in case of a negative left hand side,  $s_{j_1 j_2}^-$  is assigned to the absolute value of left hand side. Therefore, the right hand side of Eq. (16) results in the absolute value of the difference between the earliness values of districts  $j_1, j_2 \in J$ . The value of  $s_{\max}$  is assigned to the maximum difference between all pairs of districts in Eq. (17). Finally, the objective function in (15) minimizes the value of  $s_{\max}$ .

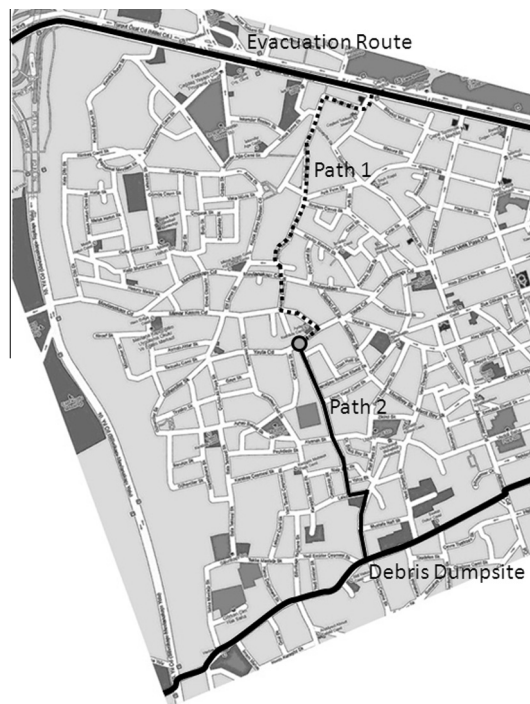
#### 4. Computational results

We demonstrate our approach on two test instances based on the road networks of two districts in Istanbul, Turkey. The first district (Caddebostan) depicted in Fig. 1a contains 212 road segments out of which 49 are blocked, whereas the larger (Fatih) district has 386 segments where 79 are blocked (Fig. 1b). The restoration times for blocked segments vary between 1 and 10 h. For both districts, the set of predefined paths  $P$  has been constructed as explained in Section 3. In Fig. 1a, the seashore in the south of the map is assumed to be a continuous temporary dumpsite (as was the case in the 1999 Izmit earthquake). The evacuation route in the north of the map is a 40 km. long avenue officially declared as an emergency route by AKOM (Disaster Coordination Center of Istanbul). Accordingly, two predefined paths that lead to the evacuation route and the dumpsite are marked for an exemplary node on the map. Path 1 in Fig. 1a leads to the nearest node on the evacuation route while Path 2 leads to the nearest node on the seashore (here, there are no other unblocked paths from this node to either destination). The same approach is implemented to determine the set  $P$  for the Fatih district. The two districts differ in terms of the network characteristics: Caddebostan lies in the relatively newer part of the city and has a network that resembles a Manhattan type block structure whereas Fatih, which is in the heart of old Istanbul contains a relatively more complex network. On the other hand, both districts present a challenge in terms of recovery efforts: both districts lie in the very high risk earthquake zone and they both have narrow streets that usually have one or two lanes. Moreover, most streets have many tall buildings on both sides, which are likely to block access in case of any structural damage due to an earthquake. In both districts, priority weights  $Q_p$  are assumed to be one for all paths  $p \in P$ .

We have run our computational experiment on a 64-bit XEON Quattro CPU computer and each instance was allowed a maximum of 3600 s. CPU time. We have also used an optimality threshold of 0.05%, i.e. the run was terminated before 3600 s.



**Fig. 1a.** Road map for the Caddebostan district.



**Fig. 1b.** Road map for the Fatih district.

**Table 1**

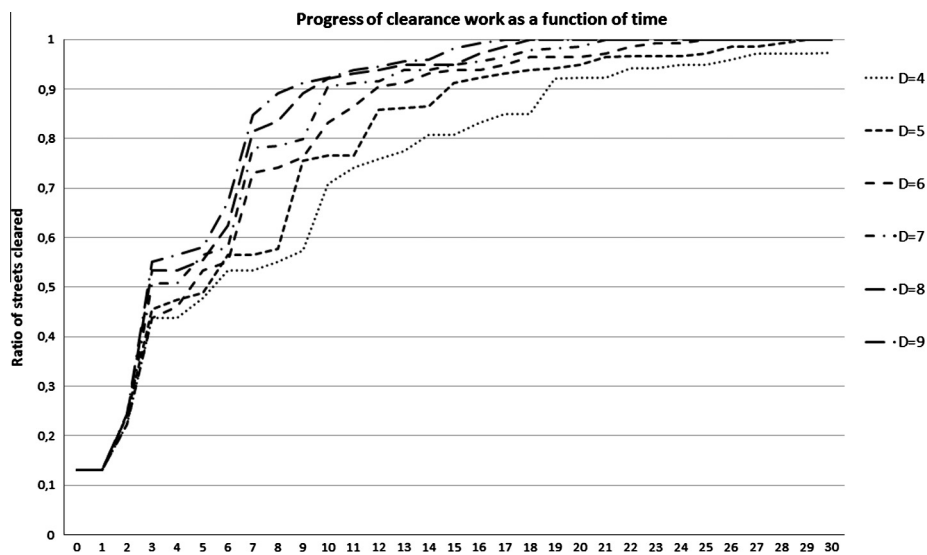
Computational results of DCSM for the Caddebostan district.

Number of equipment ( $D$ )	Makespan (h)	Number of restored paths	Best Feasible (BF)	Upper Bound (UB)	% Gap (UB – BF)/BF	CPU time (s)
4	30	267	6179	6458.663	4.526	3600
5	29	274	6548	6664.079	1.773	3600
6	25	274	6796	6845.174	0.724	3600
7	21	274	6969	6995.670	0.383	3600
8	18	274	7091	7094.500	0.049	3303
9	17	274	7185	7188.591	0.050	779

**Table 2**

Computational results of DCSM for the Fatih district.

Number of equipment ( $D$ )	Makespan (h)	Number of restored paths	Best Feasible (BF)	Upper Bound (UB)	% Gap (UB – BF)/BF	CPU time (s)
11	30	483	10,377	10899.000	5.030	3600
12	28	486	10,661	11023.000	3.396	3600
13	26	486	10,862	11196.000	3.075	3600
14	26	486	11,052	11276.000	2.027	3600
15	25	486	11,241	11375.000	1.192	3600
16	21	486	11,387	11500.000	0.992	3600
17	21	486	11,523	11573.306	0.437	3600
18	21	486	11,634	11674.470	0.348	3600

**Fig. 2.** Progress of restoration work under various equipment scenarios for the Caddebostan district.

if the optimality gap falls below 0.05%. GAMS 23.3.3 (single thread) was used for modeling and CPLEX 12.1 was used to solve the resulting models. For all runs, we have used a unit time period of one hour and set the value of  $T_{max}$  as 30 periods (hours), which corresponds to three 10-h work shifts. Since the first phase of recovery is to be completed within three days, a  $T_{max}$  value of 30 h appears reasonable.

Tables 1 and 2 display the results of the DCSM for Caddebostan and Fatih districts, respectively, under different equipment availability scenarios. Each row of these tables corresponds to a DCSM run where the number of equipment ( $D$ ) is set to the value indicated in the first column. The second column displays the makespan, i.e. the number of periods required to complete the restoration effort. In cases where the restoration cannot be completed within 30 h (which is the case for the first row of each table), a value of 30 h is displayed. The third column gives the number of paths that are cleared at the end of the indicated makespan. Note that all of the 274 paths in Caddebostan and all of the 486 paths in Fatih can be cleared in all equipment availability scenarios except for the scenarios with the minimum number of equipment for each district (4 and 11 equipments, respectively). Columns labeled BF and UB display the best feasible solution found by the DCSM and the



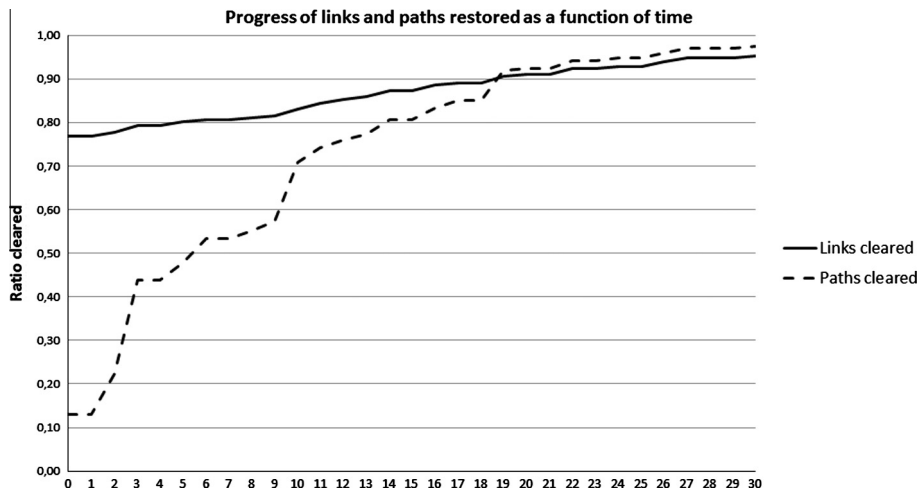


Fig. 3. Progress of link and path clearance over time.

corresponding upper bound. The following column presents the percentage optimality gap for the obtained solution. Finally, the CPU time required for the computation of the solution is reported in the last column.

For both districts, we see that the DCSM can be solved within 5% of the optimal solution and the optimality gap decreases as the number of equipment ( $D$ ) increases. This suggests that the problem gets easier as the value of  $D$  increases. As expected, a higher value of  $D$  also results in a better objective function value (i.e. total earliness) as well as a reduced makespan. In the case of Caddebostan, we also see a reduced CPU time requirement for higher settings of  $D$ . Fig. 2 provides a more detailed look at the model results as a function of time. In this figure, we plot the ratio of paths that are restored over a time span of 30 h under each equipment allocation scenario. The progress of the restoration activity for each scenario indicates that a



Fig. 4a. Progress of clearance work in the first 10 h of the time horizon.





Fig. 4b. Progress of clearance work in the second 10 h of the time horizon.

higher percentage of paths can be opened sooner with more equipment. However, even with an equipment number as low as 4, it is possible to open over 80% of the paths within the first 14 h. Fig. 3 illustrates the progress of links and paths cleared for the scenario with 4 dozers. In this figure, the slope of link clearance exhibits a uniform progress over time, while the slope of path clearance is much steeper at the beginning of the time horizon than it is in the later periods. This shows that the most critical links that are shared by many paths are cleared first. The optimality gap for the 4-dozers scenario is around 4.5%, therefore the plotted solution may not be optimal. However, the sequence of clearance work determined by the model appears to be quite efficient. Overall, the results in Tables 1 and 2 suggest that the DCSM can be used to obtain high quality solutions within reasonable time.

To gain additional insight into the structure of the solutions generated by the model, we provide the progress of work on the 4-dozers Caddebostan scenario in Figs. 4a–4c. In these figures, the horizontal bold dashed line in the north indicates the railway and C1, C2, and C3 mark the only three railroad crossing arteries that reach the evacuation route in the north of the map. Dotted lines indicate blocked roads. The numbers next to blocked roads indicate the specific times that they are cleared. In order to display the progress of clearance work, we mark the links cleared in the first 10 h of the schedule in Fig. 4a, the next 10 h in Fig. 4b and the last 10 h in Fig. 4c. In Fig. 4a, we observe that the clearance work on the three main arteries is completed in the first 10 h. All paths leading to the evacuation route pass through these three arteries. By clearing these main arteries, over 70% of the evacuation paths are opened simultaneously, which is indicated by the steep slope of cleared paths in the first 10 h in Fig. 3. In Fig. 4b, clearance work is continued on clusters of blocked links that are not on the main arteries so as to improve accessibility in those regions. Over 90% of the paths are opened in the next 10 h in this manner. Finally, in Fig. 4c, we observe that peripheral roads such as dead end streets are opened.

Next, we demonstrate how these results can be used to allocate the available equipment between the two districts. Tables 3 and 4 display the results of EAM and EAM' for various settings of  $D'$ . Each column in these tables corresponds to a separate model run for the  $D'$  value indicated in the first row. The second and third rows show how the equipment allocation model distributes the  $D'$  number of equipment between the Caddebostan and Fatih districts. Finally, the last row in each table provides the objective function value of the resulting allocation. All results displayed in these two tables are optimal and the CPU times are negligible due to the small problem size.

Clearly, the allocations suggested by the two models, namely EAM and EAM' differ significantly. In EAM, where the objective is to maximize the total earliness of the resulting allocation, Caddebostan receives equal or less number of equipment in all scenarios than it does in EAM', which minimizes the maximum difference between the two districts. This is due to the fact that the average earliness value for Fatih scenarios are roughly two-thirds higher than that of Caddebostan. Therefore, total



Fig. 4c. Progress of clearance work in the last 10 h of the time horizon.

Table 3

Computational results for EAM.

$D'$	15	16	17	18	19	20	21	22	23	24	25
Caddebostan	4	5	5	6	6	6	6	7	7	7	8
Fatih	11	11	12	12	13	14	15	15	16	17	17
$z_1$ Value	16,556	16,925	17,209	17,457	17,658	17,848	18,037	18,210	18,356	18,492	18,614

Table 4

Computational results for EAM'.

$D'$	15	16	17	18	19	20	21	22	23	24	25
Caddebostan	4	5	6	7	8	9	9	9	9	9	9
Fatih	11	11	11	11	11	11	12	13	14	15	16
$z_2$ Value	4198	3829	3581	3408	3286	3192	3476	3677	3867	4056	4202

earliness can be achieved by allocating more equipment to Fatih. However, this may not be a preferable solution in terms of equity. A more equitable solution can be targeted by using the EAM' model.

## 5. Conclusion

This study addresses a crucial aspect of disaster response: restoration of road networks. Here, the goal is to make all locations in the affected area accessible for receiving help and evacuation. The restoration process is carried out by a limited number of equipment. It is essential to restore the network as much as possible during the first 3 days of response in order to maximize survival rate.

Several approaches have been proposed in the literature to solve this problem. These can be summarized into three categories. The first category takes the sequential approach in deciding on the criticality of road links: consider the failure of one link at a time and measure network accessibility. These studies do not take the combined effects of link blockages on major

paths, hence, if the restoration scheduling plans are made according to such sequential lists of critical links, the resulting plans might be suboptimal. A second category of models involve traveling salesman type of network models or time augmented network models that consider the movement of equipment and relief simultaneously. These models are often too complex to be solved to optimality and the use of heuristics is inevitable. A third category of path based models involve identifying a given number of critical links that can be restored within a given budget so that traffic flow is maximized throughout a set of paths. These path based models are static, and, in the response phase, if the set of actually failed links does not overlap with the set identified in the static model, then, the information has no value.

The model proposed here is a dynamic path based model (the first one to our knowledge) that accomplishes the restoration scheduling task while remaining compact. Thanks to its dynamic nature, it maximizes network accessibility throughout the restoration process, which is crucial for increasing survival rates. Solving the model on two realistic size networks, we show that the model can produce solutions that are within 5% above the best possible solution within an hour of computation time.

The contributions of this paper to the literature lie in the DCSM model that is developed here. The essential contribution is that as a first path based tactical resource planning model introduced for post-disaster road restoration scheduling, the DCSM is more efficient than link based restoration models proposed in the literature (e.g., Feng and Wang, 2003; Chen and Tzeng, 2000; Yan and Shih, 2009, 2012). Link based models are too complex to be solved optimally. Another contribution lies in the novelty of the proposed objective function of the DCSM. The DCSM aims at maximizing cumulative network accessibility during the restoration operation which is of crucial importance for post disaster survivors during the first 3 days of response. While other path based models have mitigation purposes such as pre-disaster infrastructure re-enforcement under a given budget (e.g., see Matisziw and Murray, 2009), the DCSM is a tactical post-disaster restoration scheduling model that considers limited resources explicitly. To summarize, the DCSM is both compact and realistic with a goal that is directed at enabling relief efforts for beneficiaries.

It is only fair to discuss the limitations of the DCSM. As a tactical model, the DCSM does not include detailed travel routes for the restoration equipments in each district, thereby, avoiding the solution of the multi-vehicle routing problem that needs to be addressed later. Here, this problem is regarded as an operational issue that should be considered after the equipment allocation decisions are made. The fact is that modeling approaches that have dealt with both tactical and operational decisions simultaneously are intractable for realistic size problems. Therefore, our future work in the area will focus on a hierarchical planning system that integrates both the tactical and operational aspects of this problem.

The EAM model proposed here is implemented after the DCSM model is executed for different districts. The EAM model is a subsequent tactical model that allocates restoration equipment equitably among different districts. This model is easy to solve and selects the best equipment allocation scenario for each district given that total equipment availability is limited. In summary, we find that the proposed models can be used by disaster response practitioners in constructing road restoration plans for affected cities.

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