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Stochastic Optimization for Natural Disaster Asset Prepositioning

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A key strategic issue in pre-disaster planning for humanitarian logistics is the pre-establishment of adequate capacity and resources that enable efficient relief operations. This paper develops a two-stage stochastic optimization model to guide the allocation of budget to acquire and position relief assets, decisions that typically need to be made well in advance before a disaster strikes. The optimization focuses on minimizing the expected number of casualties, so our model includes first-stage decisions to represent the expansion of resources such as warehouses, medical facilities with personnel, ramp spaces, and shelters. Second-stage decisions concern the logistics of the problem, where allocated resources and contracted transportation assets are deployed to rescue *critical population* (in need of emergency evacuation), deliver required commodities to *stay-back population*, and transport the *transfer population* displaced by the disaster. Because of the uncertainty of the event's location and severity, these and other parameters are represented as scenarios. Computational results on notional test cases provide guidance on budget allocation and prove the potential benefit of using stochastic optimization.

Key words: humanitarian logistics; relief operations; stochastic optimization
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1. Introduction

Humanitarian logistics is defined as "the process of planning, implementing, and controlling the efficient, cost-effective flow of and storage of goods and materials as well as related information from point of consumption for the purpose of meeting the end beneficiary's requirements" (Thomas and Mizushima 2005). Until recently, the major thrust of the emergency planners was on operational details. However, this cannot be accomplished without long-term commitments, the most important of which is the pre-establishment of adequate capacity and resources that enable an efficient response. For example, after learning from Hurricane Katrina, U.S. Federal Emergency Management Agency (FEMA) has supplies warehoused, routes planned, and temporary shelters designated in the hurricane-prone region. This deals with questions such as: What assets need to be in place in anticipation of a disaster? And, where should they be located? We refer to this task as "prepositioning" of assets such as warehouses, medical facilities, ramp space, and temporary shelter space. The term prepositioning is also commonly employed to refer to the storage of supplies near a potential area in anticipation of an imminent disaster, but we use it in its long-term, strategic sense.

The need for strategic planning is recognized, among others, by FEMA's Response and Recovery Division Director for Region IX, who points out that the prescript tasks, before disaster happens, such as medical evacuation, facility support, commodity distribution, and temporary housing have to be planned (Fenton 2008b); by US Homeland Security Director of Operation Analysis, "A good strategic model for planning is necessary in the future" (Kapos 2007); by US National Oceanic and Atmospheric Administration, which emphasizes the importance of response and recovery (Reynolds 2008); by US California National Guard's recommendation that a system needs to be in place for "any emergency" (Nelan, 2008); and, by the Asia-Pacific Task Force for Emergency Preparedness (2009) which supports a strategic plan for more effective emergency preparedness, risk reduction, and disaster response.

This paper deals with the strategic planning and resource allocation for humanitarian aid in future cyclic, natural disasters. We introduce a stochastic optimization model to guide strategic resource allocation for cyclic disasters in geographical areas, where history tells us natural disasters occur frequently, for example, hurricanes in the Southeastern United States or typhoons in Southeast Asia. Its stochastic component enables the modeling of multiple areas, which could be affected by disasters of different severity about which we may only have limited, probabilistic information. Our approach also models some operational details in order to assess the consequences of long-term strategies.

2. Literature Survey

Some of the initial work in humanitarian aid logistics was done in locating emergency service facilities for fire stations or ambulances under the paradigm of set covering problems: a demand is covered if there exists at least one available unit within a specified distance from its location (see, e.g., Cabot et al. 1970, Church and ReVelle 1974, Shmoys et al. 1997, Tonegas et al. 1971). These models have evolved over time to account for more detailed information, such as locations of storage areas for critical emergency equipments and supplies (Hale and Moberg 2005). Alsalloum and Rand (2006) extend the models for the maximal covering location problem by replacing the 0–1 deterministic coverage with a probability of covering a demand within the target time and then using sufficient vehicles at each location to satisfy a required performance level.

Barbarosoglu et al. (2002) develop a mathematical model for disaster relief helicopter mission. Baker et al. (2002) model cargo and passenger routes during a military airlift.

Ozdamar et al. (2004) integrate time into a planning model for a natural disaster. They solve a dynamic, time-dependent transportation problem at given time intervals during ongoing delivery of humanitarian aid. The model is deterministic at the core because it assumes the demand, allowed to be unknown, is fixed initially and then forecasted successively. More recently, Yi and Ozdamar (2007) extend that model as a mixed-integer, multicommodity network flow problem focused on vehicle traffic to coordinate the evacuation and support after the disaster.

Hoffman (2006) discusses the challenges in coordinating humanitarian logistics when aid is also offered by non-government organizations and the private sector, and the cross-learning potential. Van Wassenhove (2006) offers this perspective with illustrations of numerous case studies, discussing the importance of managing uncertainty and risk.

The literature reviewed for this study suggests that there is an unfulfilled need for research in humanitarian logistics for managing the supply chain under uncertainty. Certain assumptions such as that the existence of at least one supply center within distance from a demand site is sufficient to cover that site's demand is an oversimplification of the actual problem. For example, in a relief operation demands often require response from multiple units and locations.

Our stochastic optimization model is driven by the expected number of survivors rescued from (possibly) affected areas (AAs). While the abovementioned work by Yi and Ozdamar (2007), Hale and Moberg (2005), Ozdamar et al. (2004), Barbarosoglu et al. (2002), and Baker et al. (2002) address goals similar to ours, all of them do so with a deterministic model and most of them from an operational point of view, such as, how pre-established relief units and assets will respond to disaster relief operations. Thus, the level of detail is catered more toward operational logistics and does not determine the strategic positioning and sizing of relief units and assets. A preliminary version of the model developed in this paper was introduced by Ee Shen (2006) and tested by Heidtke (2007). In this paper, we have enhanced Ee Shen's mathematical model by adding a survival rate, a third type of population, and nested a second objective function. We have also created different scenarios and analyzed the model's results to prove its robustness to changes in critical parameters, among others. Fenton (2008a) points out that planners have initiated efforts in a similar direction as our model, but without the formal use of stochastic optimization.

3. The Problem

3.1. Overview

Our problem posits possible AAs that "may be" hit by a disaster (since no disaster has occurred yet), and candidate relief locations (RLs), where resources already exist or can be prepositioned. RLs are not necessarily located away from the AAs (if collocated, we allow certain relief assets to become unavailable when a disaster hits that specific area).

Our discussions with US emergency planners (Eisner 2007, Fenton 2008a, Nelan 2008, and others in the humanitarian community), based on lessons learned from events like Hurricane Katrina, California wild fires, and Hurricane Gustav, make us confident of the representative partition of the affected population into three categories: *critical population*, which refers to those in need of emergency medical evacuation to RLs; *stay-back population*, which includes those who may stay at the AAs but require delivery of certain commodities from RLs for survival; and *transfer population*, needing only evacuation due to short-term displacement to RLs.

When a person from the critical population is not evacuated, we assume his health deteriorates and results in death. Likewise, we estimate that a certain percentage of stay-back population who are not supplied with commodities will perish. This reflects the need for aggregate essential supplies, which could include potable water in flooded areas, or self-administered vaccines in case of a disease, to be delivered to the AAs. Our first objective function, due to life and death considerations, consists of minimizing total casualties from these two populations. For simplicity, we assume that everybody in the critical population has the same priority, though in principle, they could have varying degrees of emergency. Evacuating the transfer population to temporary shelters, though not as critical as the above, is also important, and becomes a secondary objective in our test cases. (We do not model the subsequent problem of long-term displacement.) Our approach allows this hierarchy to be flexible by letting the planner decide the threshold of achievement for the first objective while optimizing the second objective. Combining these objectives is pertinent because the organizations involved in the relief effort share the resources, and the transportation activity shares the time line (Fenton 2008b, Nelan 2008).

Consistent with our strategic approach, decision variables are divided into two stages. The first one deals with strategic decisions that must be implemented well before a disaster strikes. These involve the location and capacity expansion of assets like: health care providers at hospitals, for the critical population; warehouses for storing commodities and ramp space to deliver them by aircraft, for the stay-back population; and temporary shelters at RLs, for the transfer population.

The consequences of strategic decisions are of stochastic nature because they can be realized after only a disaster has occurred. This is reflected by our second, operational stage, which spans the first 3 days after the event. Though nowhere stated formally, humanitarians agree on the fact that the first 72 hours after a disaster are critical to provide relief since communities are not expected to stand on their own for much more than that time (Balcik et al. 2008, Fenton 2008a, Weitz 2006). Decisions for this stage include the engagement and use of Means of Transportation (MoT) in order to evacuate critical and transfer population to the relief hospitals and shelters, as well as delivering of commodities to the stay-back population in the AAs.

3.2. Deterministic and Probabilistic Inputs

Deterministic inputs include: a network of AAs and RLs, with nominal travel times between them (which, of course, may vary by MoT); ramp space for commodity delivery by aircraft, in each AA; health care providers and hospital facilities, and warehouses at each RL; MoT with associated capacity for people (including critical and transfer populations as well as relief workers) and/or commodities; penalty (number of casualties in the stay-back population) per unit of unmet commodities; and pre-positioning cost of additional health care providers, warehouses, ramp space, and post-disaster engagement of MoT.

We model uncertainty about the location and magnitude of the disaster by assuming that the following information is scenario-dependent, i.e., known only in terms of a probability distribution: critical, stay-back, and transfer populations by AA; required relief workers needed to handle the commodities; increased transportation times, such as those due to roads washed away after major flooding; and survival rate for rescued critical population. We assume the probability distribution can be devised by subject matter experts or emergency planners. The creation of these scenarios and the agreement on their characteristics and probabilities is probably the most delicate aspect of the input data, along with the choice of penalties for unmet delivery of commodities.

3.3. Transportation

The logistics problem is conceived as follows: critical population is picked up only by *special mission* MoT. These MoT include a small medical team and are configured with a special layout that allows the transportation of severely injured victims, but not commodities. On the other hand, a *general mission* MoT is configured to transport cargo and relief workers from RLs to the AAs, as well as transfer population from AAs to RLs.

We disallow select MoT from using certain RLs. This could be the case, e.g., of helicopters in the absence of an adequate helipad. Air-based MoT require ramp space at any AA in order to deliver commodities, whereas land-based MoT are assumed to unload and deliver commodities directly to the people. To reduce the complexity of the model, we assume that there are no internal flights or ground transportation between AAs or between RLs. That is, trips for both special mission and general mission MoT start at an RL, travel to an AA, and continue to a (possibly different) RL. For the purpose of strategic planning, we deem it reasonable to assume each MoT will be capable of performing multiple trips, as long as these are restricted by its available operating hours, and no attempt is made to employ a fine-grained model for the detailed schedule of each individual truck or aircraft. While this can be viewed as a model limitation, we believe it is reasonable because of the long-term focus of the approach. However, we acknowledge that, in an actual relief operation, the number of working hours may be hindered by poor planning, long decision cycles from policy makers, other logistic delays, and social unrest.

Commodities, which are aggregated as volume, require relief worker's intervention, especially for unloading and distribution in AAs. Therefore, each ton (or equivalent volume unit) of commodities arriving at a certain RL is associated with a prescribed number of relief workers.

4. The Prepositioning Optimization (PO) Model

In this section we introduce our PO model, a multi-objective, two-stage, stochastic, mixed-integer program. Units are stated within square brackets [.] at the end of each of the definitions. We use the unit "vehicles" to refer to land-vehicles, aircraft, or vessels.

Indices and index sets

A	Set of affected areas; $a \in A$
L	Set of starting and drop off relief locations; $l \in L$
T	Set of MoT (e.g., CH-53 aircraft, HMVV land-vehicle); $t \in T$
T_l	Subset of MoT that can depart from (and drop off at) relief location l
T^R	Subset of MoT that require ramp space for delivery of commodities (aircraft assets)
Ω	Set of disaster scenarios; $\omega \in \Omega$

Deterministic parameters (units)

$h_l^0, h_l^{\max}, c_l^H$	Initial capacity for health personnel at relief location l (persons), maximum capacity expansion (persons), and variable expansion cost (\$/person)
s^H	Critical population that one health care provider can handle (persons)
$s_l^0, s_l^{\max}, c_l^S$	Initial capacity for critical population at relief location l (persons), maximum capacity expansion (persons), and variable expansion cost (\$/person). (These are based on the initial health personnel, maximum health personnel expansion, variable health personnel cost, and s^H)
$r_a^0, r_a^{\max}, c_a^R$	Initial ramp space capacity at affected area a (ft$^3\times$1000), maximum capacity expansion (ft$^3\times$1000), and variable expansion cost (\$/ft$^3\times$1000), respectively
$m_l^0, m_l^{\max}, c_l^M$	Initial capacity for commodities at relief location l (ft$^3\times$1000), maximum capacity expansion (ft$^3\times$1000), and variable expansion cost (\$/ft$^3\times$1000), respectively
$u_t^0, u_t^{\max}, c_t^U$	Initial number of units of MoT t (vehicles), maximum capacity expansion (vehicles), and variable expansion cost (\$/vehicle), respectively
$d_l^0, d_l^{\max}, c_l^D$	Initial shelter capacity for transfer population at relief location l (persons), maximum capacity expansion (persons), and variable expansion cost (\$/person)
$\bar{s}_t$	Capacity for critical population of special MoT t (persons/vehicle$\times$trip)
$\bar{m}_t, \bar{w}_t$	Capacities for commodities (ft$^3\times$1000/vehicle$\times$trip) and relief workers (workers/vehicle$\times$trip), respectively, of general MoT t
$\bar{d}_t$	Capacity for transfer population of general MoT t (persons/vehicle$\times$trip)
h_t	Available hours during the planning time for each unit of MoT t (hours/vehicle)
b	Total budget allocated (\$)
q	Penalty for unmet commodities (i.e., q of the *stay-backs* are assumed to perish per unit of unmet commodities) (persons/ft$^3\times$1000)
α	relaxation level for the first objective when the second objective is optimized (fraction)

Scenario-dependent parameters (units), all under scenario ω

m_a^ω	Demand for commodities in affected area a (ft$^3\times$1000)
s_a^ω	Critical population in affected area a (persons)
λ_a^ω	Survival rate for critical population rescued in affected area a (fraction)
d_a^ω	Number of transfer population in affected area a (persons)
h_{tla}^ω	Trip time (hours) for MoT t to travel from relief location l to affected area a (hours/trip) (The same time is assumed from a to l, so only h_{tla}^ω is defined.)
w_a^ω	Relief workers required to handle commodities at affected area a (workers/ft$^3\times$1000)
p^ω	Probability of scenario ω occurring

Derived sets

L^S, L^M, L^D, A^R	Subset of relief locations, supply locations, shelter locations and affected areas with ramp space, respectively. For example, $L^S = \{l \in L \mid s_l^0 > 0 \text{ or } s_l^{\max} > 0\}$
T^G, T^S	Subsets of general mission MoT (i.e., $\bar{s}_t = 0, \bar{m}_t \geq 0, \bar{w}_t \geq 0, \bar{d} \geq 0$) and special mission MoT (i.e., $\bar{s}_t > 0, \bar{m}_t = \bar{w}_t = \bar{d}_t = 0$), respectively.
K	Subset of four-tuples (t, l, a, l') where MoT t can travel from l to a and then to l': $\{(t,l,a,l') \in T \times L \times A \times L \mid h_{tla}^\omega + h_{tl'a}^\omega \leq \tau_t, t \in T_l \cap T_{l'}\}$, where τ_t is the operating range of t.
K^G, K^S	Subsets of four-tuples (t, l, a, l') where general mission MoT t and special mission MoT t, respectively, can travel from l to a, and then to l':

$K^G = \{(t,l,a,l') \in K | t \in T^G;\ l,l' \in L^M \cup L^D\}$;
$K^S = \{(t,l,a,l') \in K | t \in T^S,\ l' \in L^S\}$

First-stage decision variables (units)

Δs_l	Expansion for health capacity for critical population at drop off relief location l (persons)
Δm_l	Expansion for commodities at relief location l (ft$^3\times$1000)
Δr_a	Expansion for ramp space at affected area a (ft$^3\times$1000)
Δd_l	Expansion for transfer population at relief location l (persons)

Second-stage decision variables (units), all under scenario ω

Δu_t^ω	Additional units of MoT t needed (vehicles)
$S_{tlal'}^\omega$	Critical population rescued by MoT t traveling from l to a and then l' (persons)
S_{ta}^ω	Total critical population rescued by MoT t at affected area a (persons)
US_a^ω	Unmet critical population at affected area a (including rescued but not surviving) (persons)
$M_{tlal'}^\omega$	Commodities delivered by MoT t traveling from l to a and then l' (ft$^3\times$1000)
M_{ta}^ω	Total commodities delivered by MoT t to affected area a (ft$^3\times$1000)
UM_a^ω	Unmet commodities at affected area a (ft$^3\times$1000)
$D_{tlal'}^\omega$	Transfer population transported by MoT t traveling from l to a and then l' (persons)
D_{ta}^ω	Total transfer population transported by MoT t from affected area a (persons)
UD_a^ω	Unmet transfer population at affected area a (persons)
$N_{tlal'}^\omega$	Number of trips from l to a and then to l' by MoT t (trips)
W_{ta}^ω	Number of relief workers carried by MoT t to affected area a (workers)
z_1, z_2	Objective value for the first goal (persons) and second goal (persons), respectively

Formulation:

Objective 1 (minimize): expected casualties from critical and stay-back populations

$$z_1 = \sum_\omega p^\omega \sum_a \left(US_a^\omega + qUM_a^\omega\right), \tag{1}$$

Objective 2 (minimize): expected unmet transfer population

$$z_2 = \sum_\omega p^\omega \sum_a UD_a^\omega, \tag{2}$$

Budget

$$\sum_{l\in L^S} c_l^S\,\Delta s_l + \sum_{l\in L^M} c_l^M\,\Delta m_l + \sum_{l\in L^D} c_l^D\,\Delta d_l + \sum_{a\in A^R} c_a^R\,\Delta r_a + \sum_t c_t^U\,\Delta u_t^\omega \le b, \forall\omega, \tag{3}$$

MoT available and trips

$$\Delta u_t^\omega \ \le\ u_t^{\max},\ \forall t,\omega \tag{4}$$

$$\sum_{(l,a,l')|(t,l,a,l')\in K} (h_{tla}^\omega + h_{tl'a}^\omega) N_{tlal'}^\omega \le\ h_t(u_t^o + \Delta u_t^\omega),\ \forall t,\omega, \tag{5}$$

$$\sum_{(l',a)|(t,l',a,l)\in K} N_{tl'al}^\omega = \sum_{(a,l')|(t,l,a,l')\in K} N_{tlal'}^\omega,\ \forall l, t\in T_l, \omega, \tag{6}$$

Critical population and its transportation

$$\Delta s_l \le s_l^{\max},\ \forall l\in L^s \tag{7}$$

$$\sum_{(t,a)|(t,l,a,l')\in K^S} S_{tlal'}^\omega \ \le\ s_l^o + \Delta s_l,\ \forall l,l'\in L^s, \forall\omega, \tag{8}$$

$$S_{tlal'}^\omega \le \bar{s}_t N_{tlal'}^\omega,\ \forall (t,l,a,l')\in K^S,\ \forall\omega, \tag{9}$$

$$S_{ta}^\omega = \sum_{(l,l')|(t,l,a,l')\in K^S} S_{tlal'}^\omega,\ \forall a\in A, t\in T^S,\ \forall\omega, \tag{10}$$

$$\sum_{t\in T^S} \lambda_a^\omega S_{ta}^\omega + US_a^\omega = s_a^\omega,\ \forall a,\omega, \tag{11}$$

$$\sum_{t\in T^S} S_{ta}^\omega \le s_a^\omega,\ \forall a,\omega. \tag{12}$$

Delivery of commodities for stay-back population

$$\Delta m_l \le m_l^{\max},\ \forall l\in L^M, \tag{13}$$

$$\sum_{(t,a,l')|(t,l,a,l')\in K^G} M_{tlal'}^\omega \le m_l^o + \Delta m_l,\ \forall l\in L^M,\ \forall\omega, \tag{14}$$

$$M_{tlal'}^\omega \le \bar{m}_t N_{tlal'}^\omega,\ \forall (t,l,a,l')\in K^G,\ \forall\omega, \tag{15}$$

$$M_{ta}^\omega = \sum_{(l,l')|(t,l,a,l')\in K^G} M_{tlal'}^\omega,\ \forall t\in T^G, \forall a,\omega, \tag{16}$$

$$\sum_{t\in T^G} M_{ta}^\omega + UM_a^\omega = m_a^\omega,\ \forall a,\omega. \tag{17}$$

Sheltering transfer population

$$\Delta d_l \le d_l^{\max},\ \forall l\in L^D, \tag{18}$$

$$\sum_{(t,l,a)|(t,l,a,l')\in K^G} D^{\omega}_{tlal'} \le d^o_{l'} + \Delta d_{l'},\ \forall l' \in L^D,\ \forall\omega, \tag{19}$$

$$D^{\omega}_{tlal'} \le \bar{d}_t N^{\omega}_{tlal'},\ \forall (t,l,a,l') \in K^G,\ \forall\omega, \tag{20}$$

$$D^{\omega}_{ta} = \sum_{(l,l')|(t,l,a,l')\in K^G} D^{\omega}_{tlal'},\ \forall t \in T^G, \forall a,\omega, \tag{21}$$

$$\sum_{t\in T^G} D^{\omega}_{ta} + UD^{\omega}_a = d^{\omega}_a,\ \forall a,\omega. \tag{22}$$

Ramp space

$$\Delta r_a \le r^{\max}_a,\ \forall a \in A^R, \tag{23}$$

$$\sum_{t\in T^R} M^{\omega}_{ta} \le r^o_a + \Delta r_a, \forall a \in A^R,\ \forall\omega. \tag{24}$$

Relief workers versus commodities

$$\sum_{t\in T^G} W^{\omega}_{ta} \ge w^{\omega}_a \sum_{t\in T^G} M^{\omega}_{ta}, \forall a,\omega, \tag{25}$$

$$\bar{w}_t M^{\omega}_{ta} + \bar{m}_t W^{\omega}_{ta} \le \bar{w}_t \bar{m}_t \sum_{(l,l')|(t,l,a,l')\in K^G} N^{\omega}_{tlal'},\quad \forall t \in T^G, \forall a,\omega. \tag{26}$$

Domain for decision variables

$$\Delta m_l,\ \Delta s_l,\ \Delta r_a,\ \Delta d_l,\ M^{\omega}_{tlal'},\ M^{\omega}_{ta},\ UM^{\omega}_a,\ S^{\omega}_{tlal'},\ S^{\omega}_{ta},\ US^{\omega}_a,\ D^{\omega}_{tlal'},\ D^{\omega}_{ta},\ DM^{\omega}_a \ge 0,\ \forall t,l,l',a,\omega, \tag{27}$$

$$\Delta u^{\omega}_t,\ N^{\omega}_{tlal'},\ W^{\omega}_{ta} \ge 0 \text{ and integer},\ \forall t,l,l',a,\omega. \tag{28}$$

PO is a multi-objective model comprising two optimization problems hierarchically arranged. In the first one, PO-1, we minimize expected casualties resulting from non-rescued (and rescued but not surviving) critical population and the stay-back casualties due to unmet commodities, as given by Equation (1). The second model, PO-2, minimizes unmet demand for transfer population (2):

$$\text{PO-1}: z^*_1 = \min z_1 \quad \text{s.t.}\begin{cases}(1.1)\\(2)\text{-}(9.2)\end{cases} \qquad \text{PO-2}: z^*_2 = \min z_2 \quad \text{s.t.}\begin{cases}(1.2)\\(2)\text{-}(9.2)\\z_1 \le (1+\alpha)z^*_1.\end{cases} \tag{29}$$

Notice that PO-1 might be seen as a bi-objective problem itself, since it seeks to meet the demand of two different groups of people. Our assumption is that both groups are equally important in the sense that failing to meet either demand results in persons to perish. Specifically, (1) accounts for casualties from the critical population, along with a fraction of those who do not receive commodities (q casualties per ft$^3\times$1000). PO-2 minimizes unmet demand for transfer population, but with the additional constraint (29) as an aspiration level based on PO-1's optimal solution. (In our test cases we set the aspiration level to $\alpha = 1\%$.)

All of the remaining constraints are shared by both models. Equation (3) is the budget constraint. Most of the budget allocation is expected to occur during the first stage (expansion of medical facilities, warehouses, shelters, and ramp space). The remaining budget can be allocated to the engagement of additional MoT from the available fleet, usually commercial transportation, arranged beforehand to become available during a disaster, with contractual cost based on the level of utilization (thus, scenario-dependent). It is precisely these constraints that link decision variables involving critical population and commodities. Here, we note a possible enhancement would be to capture the influx of additional funding after the disaster has occurred. While part of this funding may be provided by private donors at the onset of a disaster for different purposes (such as financial help to individuals, reconstruction, etc.), we note that it is not complicated to accommodate an anticipated extra budget, b^{ω}, particular to each scenario, by simply adding b^{ω} to the right-hand side of Equation (3). (This extension has not been explored in our experiments, i.e., we assume $b^{\omega} = 0$ for each ω.)

Constraints (4) bound the maximum capacity expansion for MoT, whereas (5) ensure travel time per MoT does not exceed their available operating hours. Constraints (6) are flow-balance constraints in and out of each of RL. This is a global balance equation by MoT type, understanding that the actual schedule details of each individual vehicle, aircraft, or vessel cannot be anticipated and would become an unnecessary complication for long-term planning purposes.

Constraints (7) limit the allowable increase in health care providers located in the respective RLs. Constraints (8) limit the amount of critical population that can be treated by available health providers. Constraints (9) ensure these people are carried by an MoT configured for special mission, traveling on a given route, but not exceeding the capacity. Constraints (10)–(12) account for "met" and "unmet" demand of critical population at each AA. Specifically, the survival rate in (11) reflects that part of the critical population rescued will perish.

Constraints (13) limit warehouse expansion. Constraints (14) limit delivery from eligible warehouses. Constraints (15) ensure the commodities are carried by existing MoT configured for general mission on each route. Constraints (16) and (17) account for met and unmet demand of commodities for stay-back population at each AA. Likewise, (18)–(22) are constraints for sheltering transfer population.

Constraints (23) and (24) restrict ramp space expansion, which in turn limits commodities delivered by aircraft. Constraints (25) ensure that relief workers arrive at the AAs at a given rate based on the amount of commodities supplied to each AA. Constraints (26) depict total capacity of an MoT on a general mission as a linear function of relief workers and commodities. For example, on every trip a CH-53 could carry up to $W = \bar{w} = 55$ relief workers, or $M = \bar{m} = 1.53$ ft$^3 \times 1000$ commodities, or any linear mix of these such as $W = 50$, $M = 0.139$.

Finally, (27) and (28) define the appropriate domains for the decision variables.

An important observation is that the model is not intended to plan for a single-occurrence event. Since the formulation is in terms of expected value, the answers it provides are valid independent of the number of times the different events being modeled occur, as long as: (a) the given probability distribution for the scenarios is representative of the frequency with which the actual events occur, (b) no two events are ever simultaneous (if they could be, we should define a new scenario with the information of the compound event and its associated probability), and (c) there is no loss of resources, original or expanded, after an event. That is, the optimal prepositioning of assets before a (hypothetical) first disaster should also be optimal after that disaster. This would not be the case if such disaster significantly changes the demographics of the AAs and/or makes us reassess the probability distribution of the events.

5. Test Case Description

In this section, we describe our test case and the assumptions under which it is developed. The test case posits a hurricane striking six possible areas with different severities. For the specific data we use public sources cited in Ee Shen (2006) and Heidtke (2007), and direct consultation with humanitarians (Eisner 2007, Fenton 2008b), among others.

The underlying network is depicted in Figure 1. The same travel time is assumed for all AAs within the same region. Data used for the RLs are given in Table 1. For each category, we provide the initial capacity, the maximum capacity expansion, and its unit cost. For health facilities, capacity reflects the number of personnel at the RL who may provide health care to critical population. In this case, expansion refers to physical capacity and/or contractual obligations for additional health care providers to help in the relief effort should they be needed. In our examples, we assume $s^H = 5$ patients can be treated by each health care provider, existing or added.

Table 1 Relief Location

Asset attributes	Large urban l_1, l_2	Medium urban l_3, l_4	Small rural l_5
Warehouses:			
Initial capacity (ft$^3\times$1000)	150	100	0
Optional expansion (ft$^3\times$1000)	500	2000	4000
Expansion cost (\$/ft$^3\times$1000)	200,000	150,000	100,000
Health facilities:			
Initial capacity (health care providers)	1000	600	600
Optional expansion (health c. p.)	1000	2000	2000
Expansion cost ($/health c. p.)	2000	1500	1500
Shelters for the displaced:			
Initial capacity (persons)	1000	500	0
Optional expansion (persons)	N/A	2000	5000
Expansion cost ($/person)	N/A	1000	1000

All cities have little initial warehouse capacity. Large urban cities have limited possibility for expansion, and at a much higher cost, than the medium urban and small rural cities. Likewise, our test case hypothesizes that additional capacity of health care providers and shelter space is less expensive to establish in rural areas, where there exists substantial room for expansion and lower costs of living.

Data for AAs include ramp space and its possible expansion, as shown in Table 2. Ramp space is required by all aerial MoT in order to land and deliver commodities. Other data depend on the scenario location and severity, as shown in Tables 3 and 4. We postulate five plausible scenarios, with probabilities given in parenthesis.

Figure 1 Network of six possible affected areas ($a_1, \ldots, a_6$) and five relief locations ($l_1, \ldots, l_5$). Numbers on the arcs are travel times (hours) for a CH-53 helicopter. RL l_1 and l_2 are large and urban; l_3 and l_4 are medium-sized and urban, and l_5 is small and rural.

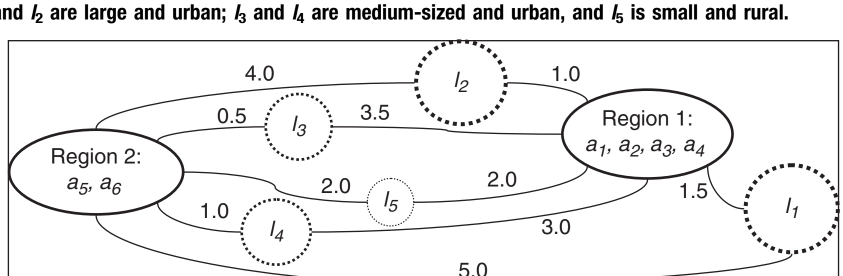


Table 2 Ramp Space Data by Affected Area

Ramp space attributes	a_1	a_2	a_3	a_4	a_5	a_6
Initial capacity (ft^3×1000)	30	10	10	30	20	10
Optional expansion (ft^3×1000)			100			
Expansion cost (\$/ft^3×1000)			12,000			

For example, Table 3 describes scenario ω_1 as a mass flooding in Region 1, which may occur with a probability of 20%. This scenario would make air transportation of commodities to AA a_2 impossible, and would increase ground transportation times to AAs a_3 and a_4 by 50% with respect to the nominal values. Similarly, Table 4 shows larger critical, stay-back and transfer populations in AAs $a_1, \ldots, a_4$ under scenario ω_1, which contrasts with AAs a_5 and a_6, where no damage is envisaged.

In our baseline model, we set to $q = 37$ casualties per ft^3×1000 of unmet commodities. This figure is, for example, equivalent to 1.0 casualties per metric ton if we assume an average commodity density of 1.3 g/cm^3. For simplicity, we employ a unique survival rate $\lambda = \lambda_a^\omega = 1\, \forall a, \omega$. Clearly, more detailed information could further refine our results by combining different rates by area and/or scenario. We also set $w_a^\omega = 10$ relief workers required to handle each ft^3×1000 of commodities, regardless of the scenario and area. Finally, our baseline case budget is \$30 million. Later, we perform sensitivity analysis for a range of budget levels and values of and λ and q.

Table 5 summarizes the data used for MoT. Nominal travel times are calculated based on the speed of MoT and the distance between RLs and AAs (see Figure 1). However, as stated above, certain scenarios contemplate delayed or even impeded routes from an RL to certain AAs, making travel times different. We also assume that MoT are ready-to-use and no maintenance issues or breakdowns occur. We limit the operation for all MoT to 20 or 21 per day. The remaining time is assumed for gas refill and change of shift for the flight and land operators. For example, during a 3-day horizon, a CH-53 could make 30 round trips between RL l_2 and AA a_1. The rescue CH-53S and MV-22S capacities are based on the maximum number of patients (from the critical population) the aircraft can carry. Worker capacity refers to the number of relief workers that can be carried. For example, the tractor trailer and the box van can carry a small number of relief workers in the main cab, but the passenger van can carry a full complement.

Table 3 Scenarios, Probabilities, and Type of Impact

Scenario	Description
$\omega_1(p^{\omega_1} = 0.2)$	Region 1 (a_1, a_2, a_3, a_4) affected severely
(Mass flooding, Region 1)	Airport in a_2 flooded, 50% road delays into a_3 and a_4
$\omega_2(p^{\omega_2} = 0.3)$	Region 1 (a_1, a_2, a_3, a_4) affected moderately
(Moderate flooding, Region 1)	
$\omega_3(p^{\omega_3} = 0.1)$	Region 2 (a_5, a_6) affected severely
(Mass flooding, Region 2)	Relief location l_3 not available
$\omega_4(p^{\omega_4} = 0.2)$	Region 2 (a_5, a_6) affected moderately
(Moderate flooding, Region 2)	
$\omega_5(p^{\omega_5} = 0.2)$	No area is affected
(No flooding)	

Table 4 Data for Affected Areas and Scenarios

Characteristics of affected areas	$\omega_1(p^{\omega_1} = 0.2)$	$\omega_2(p^{\omega_2} = 0.3)$	$\omega_3(p^{\omega_3} = 0.1)$	$\omega_4(p^{\omega_4} = 0.2)$	$\omega_5(p^{\omega_5} = 0.2)$
Critical population (persons)					
a_1, a_4	8000	2000	0	0	0
a_2, a_3	2000	0	0	0	0
a_5, a_6	0	0	8000	2000	0
Commodities (ft^3×1000)					
a_1, a_4	300	50	0	0	0
a_2, a_3	100	25	0	0	0
a_5, a_6	0	0	400	100	0
Transfer population (persons)					
a_1, a_4	20,000	5000	0	0	0
a_2, a_3	5000	0	0	0	0
a_5	0	0	10,000	2000	0
a_6	0	0	15,000	3000	0

6. Computational Results

We have implemented the PO model using the general algebraic modeling language (GAMS, Brooke et al. 1998), using Cplex 11.0 (ILOG 2008) as the solver engine, on a 2 GHz laptop computer with 2 GB of RAM. The objectives we report below are the values of z_1 and z_2 after having solved PO-2 using an aspiration level $\alpha = 1\%$.

6.1. Solution Interpretation and Analysis

6.1.1. Baseline Test Case and Related Results. Table 6 shows the achieved objective values. The expected value of perished critical population is 698. Given the survival rate ($\lambda = 1$), this figure is due exclusively to non-rescued critical population. Unmet demand for commodities causes the remaining 678 casualties. The combined objective is $z_1 = 1376$ perished. For the second objective, the expected number of transfer

Table 5 Summary of Means of Transportation Data (Heidtke 2007)

Name	Type	Initial units available vehicles	Maximum expansion vehicles	Expansion cost (variable) ($ per ft^3 $\times$1000)	Capacity commodity (ft$^3\times$1000)	Capacity critical population (persons)	Capacity relief worker (persons)	Capacity transfer population (persons)	Available hours	Operating range (hours)
CH-53S	Rescue helicopter	15	20	2,600,000	0	24	0	0	60	8
CH-53G	General purpose helicopter	5	20	2,600,000	1.530	0	55	55	60	8
MV-22S	Rescue VSTOL aircraft	15	20	4,160,000	0	12	0	0	60	10
MV-22G	General purpose VSTOL aircraft	5	20	4,160,000	0.858	0	24	24	60	10
C-130J	Cargo aircraft	5	36	70,314	4.551	0	92	92	60	5
C-17	Cargo aircraft	5	36	174,908	8.736	0	102	102	60	5.33
B747	Cargo aircraft	1	4	254,620	6.190	0	366	366	60	14.23
DC-10	Cargo aircraft	3	8	277,176	4.618	0	0	0	60	6.14
A300	Cargo aircraft	4	11	19,896	13.822	0	0	0	60	5.47
MD-11	Cargo aircraft	4	10	66,350	21.100	0	0	0	60	8.21
Tractor trailer	Cargo vehicle	25	300	27,968	5.256	0	3	3	63	16.5
Box van	Passenger vehicle	25	500	75,384	1.300	0	3	3	63	10
Passenger bus	Passenger vehicle	25	250	6000	0	0	56	56	63	12

population that cannot be transported to shelters is $z_2 = 15,400$.

We notice that almost all the critical population can be rescued except in the most severe scenario, ω_1, which affects all the areas in Region 1. There is a small amount of unmet demand for commodities in the severe scenarios, ω_1 and ω_3, and none in the other scenarios. Overall, the expected value of non-rescued critical population and perished stay-backs due to unmet commodities are about 9% and 5% of the total, respectively. Thus, PO-1 has balanced efforts for these two populations, but additional resources are still necessary to cover the most severe scenarios (especially ω_1) and the transfer population.

At the budget level of \$30 million, warehouse expansion occurs at 237,000 ft^3 at l_5. Approximately 1500 health care providers are added at l_5, 5000 at l_2, and 5300 at l_3. The ramp space expansion is 1000 ft^3 at a_3, 3000 ft^3 at a_4, 80,000 ft^3 at a_5, and 40,000 ft^3 at a_6. Expansion of shelters, if any, is very small and six tractor trailers are also added under scenario ω_1.

6.1.2. Exploration of Budget Levels. We vary the budget from \$10 million to \$100 million in increments of \$10 million holding all other input values constant. (We also enforce expenditure persistence, i.e., a minimum expenditure in each category given by the solution for the previous budget level.)

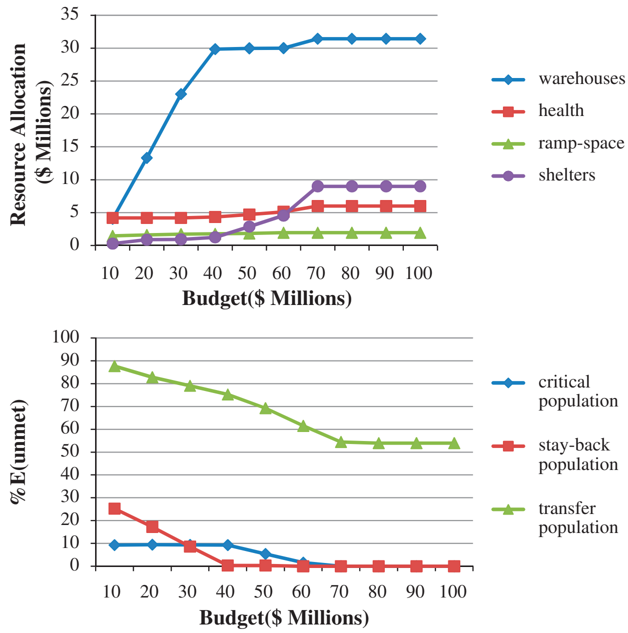
Figure 2(a) shows the budget allocation to each expansion category, and Figure 2(b) depicts the percentage of expected unmet goals. Allocation levels increase progressively in warehouses and shelters, and remain fairly constant for expansion of ramp space and health facilities. The pronounced slope for

Table 6 Objective Function Terms

Terms in the objective functions	$\omega_1(p^{\omega_1} = 0.2)$	$\omega_2(p^{\omega_2} = 0.3)$	$\omega_3(p^{\omega_3} = 0.1)$	$\omega_4(p^{\omega_4} = 0.2)$	$\omega_5(p^{\omega_5} = 0.2)$	Expected value
Critical population	20,000	4000	16,000	4000	0	7600
Rescued	16,520	4000	15,984	4000	0	6902
→ *Perished*	*3480*	*0*	*16*	*0*	*0*	*698*
Stay-back population commodities	800	150	800	200	0	325
Delivered	739	150	739	200	0	307
Unmet	61	0	61	0	0	18
→ *Stay-back perished*	*2260*	*0*	*2260*	*0*	*0*	*678*
Transfer population	50,000	10,000	25,000	12,000	0	17,900
Transported	3125	3125	3125	3125	0	2500
→ *Not transported*	*46,875*	*6875*	*21,875*	*8875*	0	*15,400*

z_1 is the expected value of perished (from the critical and stay-back populations). z_2 is the expected value of non-transported from transfer population.

Figure 2 Baseline case: (a) Budget allocation; (b) Unmet goals



warehouse expansion is a clear indication that initial conditions in warehouse capacity are the most compelling limitation to minimize casualties. For the lowest budget ($10 million), however, the budget allocation in health facility expansion slightly exceeds that of warehouses. The reason is that the initial capacity for special mission MoT is sufficient for most (over 90%) of the critical population, but there are not enough health care providers to take care of them. Thus, PO-1's strategy is to match MoT and health care providers first, because otherwise (without the medical resources) the large initial fleet of special MoT is useless. While not all medical emergencies from critical population have been met, the remaining ones would require the allocation of budget to both additional special mission MoT and health facility expansion, whose combined costs do not offset the penalties for undelivered commodities. Likewise, the expansion of ramp space is justified by an initial fleet of general mission MoT, which requires more ramp space than initially available. Humanitarians can derive valuable lessons from these tradeoffs between increased budgets, their distribution, and how they affect expected casualties.

The high cost to rescue the remaining critical population also justifies the bottleneck shift to warehouse expansion, until the budget is $40 million, in order to decrease the unmet demand for commodities. Given some of the MoT that transport the commodities are also used to provide transportation out of the AAs for transfer population, we also observe a modest but progressive, nearly linear investment in shelter space. Beyond the $40 million budget, additional allocation to expand special MoT capacity and health facilities occurs again. This enables transportation and health care to the remaining 9% of critical population.

Allocation to shelter increases until the expansion capacity is exhausted: Total shelter space in all five RLs after maximum expansion is 12,000. There are two scenarios where there is more transfer population than this number: 50,000 (20% probability) and 25,000 (10% probability), so even after expanding up to maximum shelter space, on the average, there still will be $(50{,}000-12{,}000)\times 0.2+(25{,}000-12{,}000)\times 0.1 = 8900$ transfer population left behind at AAs. Thus, as shown in Figure 2(a), all the allocations stabilize after a budget of $70 million.

6.1.3. Effects of Survival Rate, λ, and the Penalty q. Our excursions on these parameters allow $\lambda = 1$ (baseline case), 0.8, 0.6, 0.4, and 0.2, and $q = 18.5$, 37 (baseline), 55.5, 74, 92.5, and 111. All other data are kept constant. Overall results (see Table 7) show that, as the survival rate decreases, so does the picked-up critical population and the investment in health expansion. In the baseline case, 6902 persons (out of an expected maximum of 7600) are rescued. Since $\lambda = 1$ in this case, all of them are assumed to survive. However, as we decrease the survival rate down to 0.2, pick-ups decrease by up to 15.9% (i.e., 5806 persons, of whom only $0.2\times 5806 = 1161$ will actually survive). The drop in pick-ups could be much more dramatic if there was not an existing health and transportation infrastructure to accommodate this fraction of critical population to be rescued. This is evident from the fact that, at $\lambda = 0.2$, there is no investment in health expansion, but the investment increases in warehouse capacity, ramp space, and/or transportation. In other words, our critical population is using all of the initial capacity of special MoT and health care providers because this capacity "is already there," but expansion in this category has been discouraged by the low survival rate and the need to meet other exigencies. This effect becomes more noticeable as the penalties for unmet commodities increase, where even for larger values of λ we observe the investment in health is, in general, a lesser priority.

Figure 3 shows the investment in all categories for select budget levels of $10 million, $30 million and $50 million, but varying $q = 37$ and 74 (casualties per $ft^3\times 1000$), and survival rates levels, λ, equal to 1.0, 0.8, 0.6, 0.4, and 0.2. The investments in health capacity expansion shown in Figure 2(a) occur again in Figure 3, except when λ is very low (0.2), or medium $(0.4, 0.6)$ combined with a larger q (74) or a low budget. Additional warehouse capacity is always accompanied by a fairly constant investment in ramp-space, regardless of the budget level, q and λ.

Table 7 Solution Sensitivity to Changes in Survival Rate and Penalty for Unmet Commodities

S = Average critical population rescued (maximum is 7600 persons)							M = Average commodities delivered (maximum is 325 ft^3×1000)						
λ	q = 18.5	37	55.5	74	92.5	111	λ	q = 18.5	37	55.5	74	92.5	111
1.0	0.0	6902	0.0	− 1.2	0.0	− 14.7	1.0	0.0	307	0.0	0.0	0.0	3.9
0.8	− 0.2	− 0.2	− 0.2	− 0.2	− 14.7	− 14.7	0.8	0.0	0.0	0.0	0.0	3.9	3.9
0.6	0.1	− 0.1	− 0.1	− 14.7	− 14.7	− 14.7	0.6	0.0	0.0	0.0	3.6	3.9	3.9
0.4	− 1.2	− 6.6	− 14.3	− 15.9	− 15.9	− 15.9	0.4	− 4.6	0.7	2.6	2.9	3.3	3.6
0.2	− 6.5	− 15.9	− 15.9	− 15.9	− 15.9	− 15.9	0.2	0.0	2.3	3.3	3.3	3.3	3.3

H = Investment in health ($ million)							W = Investment in warehouses ($ million)						
λ	q = 18.5	37	55.5	74	92.5	111	λ	q = 18.5	37	55.5	74	92.5	111
1.0	0.0	4.2	0.0	− 14.3	− 14.3	− 95.2	1.0	− 1.3	23.8	0.0	2.9	3.4	17.6
0.8	0.0	0.0	0.0	0.0	− 95.2	− 95.2	0.8	1.3	0.0	0.4	0.4	17.6	17.2
0.6	0.0	0.0	0.0	− 95.2	− 95.2	− 95.2	0.6	− 1.3	− 0.8	− 0.4	16.8	16.8	17.2
0.4	− 14.3	− 50.0	− 88.1	− 100.0	− 100.0	− 100.0	0.4	− 18.5	3.4	12.6	14.3	16.8	17.2
0.2	− 50.0	− 100.0	− 100.0	− 100.0	− 100.0	− 100.0	0.2	0.8	10.9	14.7	15.1	15.5	16.0

The value for the baseline case (highlighted) is shown in the original units. Other values represent the percent variation with respect to that value.

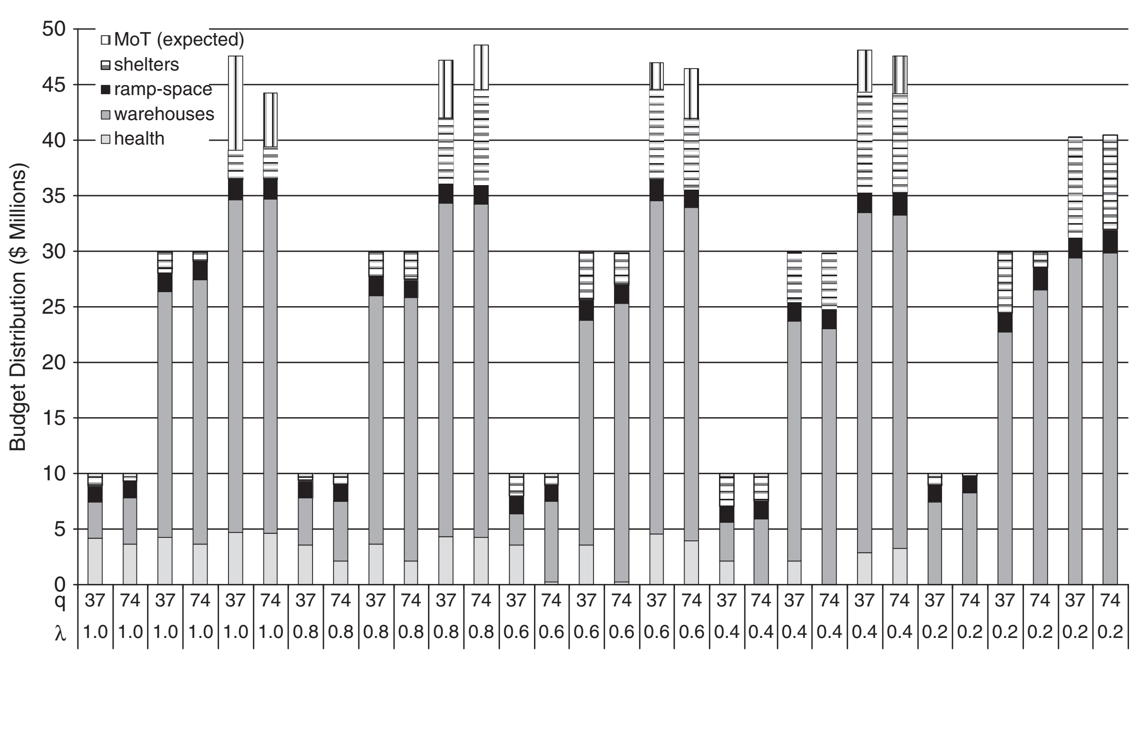
This occurs because no investment in additional air-based MoT is sought, especially at lower budget levels. (The gap to reach the $50 million mark in these cases is due to the fact that, under some scenarios, we do not need to spend all the budget left from the first stage.) Finally, investment in shelter space increases with the budget, tends to be the same or less as q increases (to allow for additional warehouse expansion), and decreases as the survival rate increases.

6.2. Assessing the Stochastic Model

In this section we assess the benefit of using the stochastic PO solution over other solutions obtained from deterministic versions of PO, hereafter POD. POD is defined as a restricted version of PO, where $|\Omega| = 1$, i.e., the scenario to occur is known with certainty.

Let us assume a planner has a validated, POD-like optimization tool. The question is what value the stochastic version of such tool could add to the planning

Figure 3 Allocation of $10 M, $30 M, and $50 M Budgets for Select Values of q and λ.



process. From our baseline case, we carry out two standard comparisons between POD and PO: ''Wait-and-See'' (W&S) analysis and ''Value of Stochastic Solution'' (VSS) analysis (Birge and Louveaux 1997, pp. 137–152), as explained below. In order to do this, we need to define some additional notation:

POD^{ω}, deterministic optimization model for scenario ω.

$z_1^*(\mathrm{POD}^{\omega}), z_2^*(\mathrm{POD}^{\omega})$, optimal objective function values for deterministic model POD^{ω}.

$z_1^*(\mathrm{PO}^{\omega}), z_2^*(\mathrm{PO}^{\omega})$, optimal objective function values for scenario ω in stochastic model PO.

$\mathrm{PO}/\hat{\Delta}$, stochastic PO model where first-stage variables have been fixed as $(\hat{\Delta}m, \hat{\Delta}s, \hat{\Delta}d, \hat{\Delta}r)$.

$z_1^*(\mathrm{PO}^{\omega}/\hat{\Delta}), z_2^*(\mathrm{PO}^{\omega}/\hat{\Delta})$, optimal objective function values for $\mathrm{PO}/\hat{\Delta}$ model under scenario ω.

$E_{\Omega}(.)$, expected value of the argument over the probability space defined by Ω

$z^*(.)$, optimal solution pair $\left(z_1^*(.), z_2^*(.)\right)$ for the problem in the argument.

The W&S analysis compares the best possible solution for each individual scenario, $z^*(\mathrm{POD}^{\omega})$, with the stochastic solution for that scenario, $z^*(\mathrm{PO}^{\omega})$, and the average over all scenarios. From Table 8, the expected value of perfect information for objective z_1 is 24%, and for objective z_2 is 25%. The worst case is for low-probability, severe scenario ω_3, for which z_1 is 47% worse in the stochastic solution than with perfect information. However, as we shall see, the PO solution will still be attractive when all scenarios are considered simultaneously.

Given that we do not expect the POD^{ω} solution to be suitable for other scenarios than ω itself, we now assess the value of the stochastic solution (VSS). Solving $\mathrm{POD}^{\hat{\omega}}$ yields a first-stage solution, $\hat{\Delta}^{\hat{\omega}}$, which permits us to solve $\mathrm{PO}/\hat{\Delta}^{\hat{\omega}}$ and assess differences with respect to PO. For some scenarios, $z^*(\mathrm{PO}^{\omega}/\hat{\Delta}^{\hat{\omega}})$ might be preferred to $z^*(\mathrm{PO}^{\omega})$ (this is more likely to occur if ω is close to $\hat{\omega}$). The difference between the objective values of both solutions is called VSS, which in this case has two components, one for each objective. By design, we know that it must be $z^*(\mathrm{PO}) \succ z^*(\mathrm{PO}/\hat{\Delta}^{\hat{\omega}})$, on the average.

The last three columns in Table 8 show how different $\mathrm{POD}^{\hat{\omega}}$ (for certain $\hat{\omega}$) perform under scenarios $\omega = \omega_1, \ldots \omega_5$, in comparison with $z^*(\mathrm{PO}^{\omega})$ for those same scenarios. We try: (a) $\hat{\omega} = \omega_1$; (b) $\hat{\omega} = \omega_3$; and (c) $\hat{\omega}$ built from an average of $\omega_1, \ldots, \omega_4$ (denoted ω_{1234}), where the fifth ''no-disaster'' scenario has been excluded so it does not reduce the average demand and resource allocation.

We observe that, by planning for ω_1 only, we improve the first goal by 19% (as anticipated by our above analysis on perfect information). However, this solution neglects ω_3, which suffers 383% more than it would if the PO solution were implemented instead. This can be explained by the fact that scenario ω_1 affects Region 1, whereas ω_3 affects Region 2 (see Tables 3 and 4, and Figure 1). Thus, ω_1 consolidates most of its relief assets at RLs l_1 and l_2, which are far from Region 2 (allowing fewer trips within the 72-hour span). Also, medium urban RL l_3 (which is far from Region 1, but attractive in terms of expansion of warehouse and health facilities) would become unavailable under ω_3. On average, the VSS of the PO plan compared with the $\hat{\Delta}^{\omega_1}$ plan is 47%. Likewise, when we plan for ω_3 only, $\hat{\Delta}^{\omega_3}$ severely neglects the scenario ω_1. In this case, VSS = 61%. Finally, in the customized plan for the average scenario ω_{1234}, all individual comparisons are favorable to the PO plan, and VSS = 256%.

6.3. Model Statistics

Table 9 describes a number of model instances and the challenges they present in terms of tractability. Our baseline instance is called Run #0. This problem has approximately 14,000 constraints and 20,000 variables, of which 7000 are integer, but it solves very rapidly for

Table 8 "Wait and See" and "Value of Stochastic Solution" Assessments

ω	p^{ω}	$z^*(\mathrm{PO}^{\omega})$: stochastic model	$z^*(\mathrm{POD}^{\omega})$: W&S solutions (%)	$z^*(\mathrm{PO}^{\omega}/\hat{\Delta}^{\omega_1})$ (%)	$z^*(\mathrm{PO}^{\omega}/\hat{\Delta}^{\omega_3})$ (%)	$z^*(\mathrm{PO}^{\omega}/\hat{\Delta}^{\omega_{1234}})$ (%)
ω_1	0.2	$z_1 = 5740$	−19	−19	83	167
		$z_2 = 46{,}875$	0	0	0	−19
ω_2	0.3	$z_1 = 0$	0	0	0	0
		$z_2 = 6875$	−100	1	2	−100
ω_3	0.1	$z_1 = 2276$	−47	383	−47	703
		$z_2 = 21{,}875$	0	0	0	−41
ω_4	0.2	$z_1 = 0$	0	0	0	0
		$z_2 = 8875$	−100	0	2	−100
Expected value		$z_1 = 1376$	−24	47	61	256
		$z_2 = 15{,}400$	−25	0	1	−42

Percentages refer to the difference with respect to the PO solution for the given scenario, or in expected value.

Table 9 Model Statistics

Run #	|A|	|L|	|T|	|Ω|	Remarks	Variables (integer) (×1000)	Constraints (×1000)	CPU z_1 (sec)	CPU z_2 (sec)
0	6	5	13	5	Baseline	20 (7)	14	0.8	1.1
1	20	10	13	5		117 (49)	77	4.0	5.7
2	40	20	13	5		1096 (390)	708	89.5	180.7
3	6	5	13	25		106 (36)	72	4.9	8.4
4	20	10	13	25		1021 (359)	668	63.4	151.1
5	40	20	13	25		5603 (1990)	3621	Unable to load	
6	6	5	13	100		427 (145)	291	26.2	57.8
7	20	10	13	100		3289 (1153)	2155	Suboptimal	
8	40	20	13	100			Unable to generate		
9	100	25	13	1	Dense	636 (228)	408	322.0	740.7
10	100	25	13	1	Sparse	636 (228)	408	48.6	104.6

both objectives. Runs #1 through #10 vary the number of AAs, RLs, or scenarios. We maintain the number of MoT types as in our baseline case because we feel it is representative of most problems of interest. In each new run, data for AAs, RLs, and/or scenarios are randomly generated based on the existing data for an analogous entity in the baseline case multiplied by a random factor uniformly distributed on the interval $(0.5, 2)$. For example, if AA a_1 is used to instantiate a new area, a given datum in the new area will follow that of a_1 affected by the abovementioned generated random factor. A similar procedure is used for new, multiple-indexed data (such as trip times).

We observe a rapid, exponential growth in the problem size as we increase the size of AAs, RLs, or scenarios. Cplex preprocessor reduces this size considerably; for example, for Run #2, the post-processed model has 513,000 variables and 143,000 constraints (half and a fifth of the original, respectively). Even with these reductions, we find several issues: In Run #5, the model is generated by GAMS but cannot be loaded into Cplex; in Run #7, Cplex produces a feasible solution, but runs out of memory before optimality; in Run #8, the model cannot be generated by GAMS. Thus, the largest PO instances we have been able to solve by directly employing commercial solvers are for 20 AAs, 10 RLs, and 25 scenarios, or for 6 AAs, 5 RLs, and 100 scenarios.

In all of these runs, we have also observed that little computational time is spent in the mixed-integer programming (branch and bound algorithm) itself: the linear relaxation is normally strong. This is expected given the nature of the integer variables in the problem (such as the aggregate number of trips made by each MoT type between each RL and each AA), which are usually zero or large. If a near-optimal integer solution can be easily found after the continuous solution, the essential difficulty is the course of dimensionality for the continuous PO model. This also suggests the use of decomposition schemes such as Benders partitioning (Benders 1962), suited to tackle two-stage stochastic programs (see, e.g., Birge and Louveaux 1997, p. 157, Escudero et al. 1999, Marin and Salmeron 1998), because it relies on the iterative solution of one scenario at a time (which we can do for a large number of AAs and RLs, as shown in Runs #9 and #10). Other decomposition strategies could be used to split the problem into two separate models involving variables for critical population and commodities, respectively. To do this, the budget linking constraints in (3) must be eliminated, e.g., by Lagrangean relaxation, or by parametric search on the portion of the total budget b that is allocated to each group of variables.

7. Summary, Conclusions, and Future Research

We have introduced a stochastic optimization model, PO, to plan the strategic arrangement of budget-limited supplies and assets in advance of major disasters. Our analysis reflects:

(a) A marked trend in the allocation of our budget to the different categories of expansion based on: (i) the mismatch between the initial MoT capacity and that of health providers, warehouses, and ramp spaces, (ii) the budget level, and (iii) planners' estimates for survival rates and penalties (casualties) for undelivered commodities; and,
(b) The benefits of using stochastic optimization over simpler, deterministic models to deal with uncertainty in the disaster's location and severity.

Since both the modeling and data employed have been based on input and feedback from experts, we have confidence in the strength of these conclusions. Emergency planners can gain insights into their own planning problem by using PO to analyze their own sets of data.

The analysis of the results for our baseline case and a number of excursions suggest, for example, that matching existing transportation capacity and health capacity for critical population is a must-do unless the survival rate is very low or penalties for unmet commodity delivery are high combined with a low budget. As more budget becomes available, expansion of warehouses and delivery of commodities takes priority, because the cost for additional special transportation and health facilities for the last pockets of critical population is too expensive.

All of our analysis is based on stochastic optimization, which allows the planner to enter a variety of scenarios (e.g., by severity or location of the disaster). The solution rendered by our model exhibits substantial benefits by accounting for all of the scenarios simultaneously, but without being subordinated to any of them.

Our model can be further enhanced in a number of ways by: incorporating alternative objectives, such as the total budget itself (for a desired level of performance on the other goals); employing a survivability curve of critical population over time, and assigning different priorities among survivors; expanding the scope of needs to incorporate security and communication; and, considering a time horizon over which budget becomes a decision variable, leading to a multi-stage model, in which capacity is built over the horizon as additional resources become available, and events which may occur over the years influence subsequent budgetary and infrastructure decisions.

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