



Inventory decisions for emergency supplies based on hurricane count predictions

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ABSTRACT

This paper addresses a stochastic inventory control problem for manufacturing and retail firms who face challenging procurement and production decisions associated with hurricane seasons. Specifically, the paper presents a control policy in which stocking decisions are based on a hurricane forecast model that predicts the number of landfall hurricanes for an ensuing hurricane season. The multi-period inventory control problem is formulated as a stochastic programming model with recourse where demand during each pre-hurricane season period is represented as a convolution of the current period's demand and an updated estimate of demand for the ensuing hurricane season. Due to the computational challenges associated with solving stochastic programming problems, recent scenario reduction techniques are discussed and illustrated through an example problem. The proposed model specifies cost minimizing inventory strategies for simultaneously meeting stochastic demands that occur prior to the hurricane season while proactively preparing for potential demand surge during the season.

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1. Introduction

Planning inventories of supplies for the hurricane season can be challenging. For instance, in 2004, manufacturing and retail firms experienced stock-outs because they were not prepared for responding to the demand caused by several hurricanes that swept through southeastern United States. In 2005, these firms again experienced shortages due to the extreme demand surge caused by Hurricane Katrina. These experiences motivated firms to be pro-active and more aggressive in their approach to stocking hurricane supplies in 2006. However, large amounts of excess inventory was commonplace because of an inactive season.

This paper introduces stochastic programming methodologies to investigate proactive inventory planning for the hurricane season based on an expert hurricane count prediction model. Demand predictions for the hurricane season during the pre-hurricane season planning horizon are assumed to evolve according to a discrete-time Markov chain. The approach allows the inventory manager to adjust inventory decisions as new information regarding the hurricane season and realizations of pre-hurricane season demands are acquired.

The stochastic inventory model is characterized by multiple periods before the hurricane season in which the inventory manager has the option to adjust the target inventory level of emergency supplies that should be available at the beginning of the ensuing hurricane season. During these pre-hurricane season months, manufacturing and retail organizations determine inventory levels that account for stochastic demands that occur during each period prior to the hurricane season as well as stochastic demand that will occur at the beginning of the season. The hurricane season demand predictions are revised at the beginning of each pre-hurricane season planning period, and these demand predictions are correlated to landfall hurricane count rate predictions. This multi-period stochastic inventory problem is formulated as a stochastic programming model. The solution specifies cost minimizing order/production quantities in which the decision-maker (DM) has flexibility to adjust the inventory policy based on updated hurricane season demand information and as pre-season demand realizations occur.

The paper is organized as follows: In Section 2, related work from the academic literature is reviewed. In Section 3, the stochastic programming model is presented followed by a discussion of how demand scenarios are constructed, which also includes a description of the selected hurricane count prediction model. In Section 4, optimal and heuristic scenario reduction strategies are discussed and illustrated through numerical examples. Finally, in Section 5, the paper is summarized, and ideas for further research are presented.

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2. Literature review

Papers from the supply chain and inventory control literature that address the following topics will be discussed in this section: information updating and sharing, humanitarian relief, and Markovian demand.

Information updating has consistently been an active area of research in inventory control theory. Therefore, only the recent research from the relevant literature is reviewed. For instance Tang et al. (2004) explore the benefits of “Advance Booking Discount” program that enables retailers to update demand forecasts and to respond demand fluctuations by replenishing stocks during the season. The optimal discount price that maximizes the retailer's expected profit is also discussed. Cheng and Wu (2005) examine the impact of information sharing on inventory and expected cost in a two-level supply chain with multiple retailers. Ray et al. (2005) evaluate a make-to-stock firm's different pricing policies to determine the profit-maximizing values for the decision variables. Initially, the price is considered as an independent decision variable, then a mark-up pricing policy is explored. Teng et al. (2005) develop a pricing/lot-sizing model for a retailer to whom the supplier offers a permissible delay in payments. Choi (2007) investigates a dynamic optimization problem that considers pre-season inventory and pricing decisions for fashion retailers to determine the optimal stocking policy. Wenkateswaran and Son (2007) examine the stability conditions for a production and inventory control system using z-transformation techniques considering the frequency of information updates. The stability of the system operating under sufficient and insufficient inventory is examined and stability boundaries are established. In another paper, Choi and Chow (2008) study an inventory management strategy called “Quick Response Program” that requires quick response to market changes by implementing a mean-variance approach. They examine various policies under which the supply chain as a whole will be better-off in terms of expected profit and risk. Finally, Wu and Cheng (2008) derive the optimal ordering policy for a three-echelon supply chain by considering information sharing on inventory and expected cost.

Qualitative descriptions of supply chain management for humanitarian relief have appropriately been the emphasis of this emerging area of research, which is more commonly referred to as *humanitarian logistics*. For instance, Beamon (2004) compares and contrasts the commercial supply chain and the humanitarian relief chain and identifies the unique challenges of relief logistics planning. Similarly, Kovács and Spens (2007) and Oloruntoba and Gray (2006) also describe the unique characteristics of humanitarian logistics while emphasizing that it would be beneficial to leverage best practices and lessons learned from commercial supply chains. Other researchers focus specifically on disaster relief. Kapucu (2007) examines the role of non-profit organizations with respect to responding to a catastrophic disaster via a case study while Smirnov et al. (2007) addresses the similarities of industrial environment and disaster relief operations in decision-making. A comprehensive synthesis of performance measurement in humanitarian logistics is presented in Beamon and Balcik (2008). They compare performance measurement in the humanitarian relief chain with in the commercial supply chain and develop new performance metrics for the humanitarian relief chain.

Only a few papers directly address inventory control problems that are related to humanitarian relief. Beamon and Kotleba (2006) consider a multiple supplier inventory model that determines optimal order quantities and reorder points for long-term emergency relief response. Lodree and Taskin (2008) introduce variations of the newsboy problem to assess the risks

and benefits associated with inventory decisions related to preparing for supply chain disruptions or disaster relief efforts. Lodree and Taskin (2009) and Taskin and Lodree (2009) address inventory planning specific to hurricane events. The difference is that this paper focuses on inventory planning *before* the hurricane season, whereas the latter two papers investigate inventory decisions based on forecasts associated with an observed storm.

The inventory models where the demand distribution is defined via a Markovian process is also relevant to this research. It seems that Karlin and Fabens (1960) introduced the idea of Markovian demand to the inventory control literature. They claim that if each demand state is defined by different numbers, a base-stock type inventory policy can be obtained. Iglehart and Karlin (1962) prove that a base-stock policy is optimal for a demand process modeled by a discrete-time Markov chain. Song and Zipkin (1993) examine an inventory model in which fluctuations in the demand rate are represented by a continuous-time Markov chain. Beyer et al. (1998) show the existence of an optimal Markov policy for the discounted and average-cost problems where demand is unbounded and costs have polynomial growth. Cheng and Sethi (1999) examine an inventory-promotion decision problem in which the demand state is represented both by environmental factors and the promotion decisions. Hari and Graves (2001) consider a Markov-modulated Poisson demand process and determine closed-form approximations for both inventory and service levels. Finally, Chen and Song (2001) examine a serial multistage inventory problem with Markov-modulated demand.

This paper can also be described as an inventory model with more than one period to prepare for the selling season. The reader is referred to Silver et al. (1998) for an extensive list of references related to this problem. Some earlier papers are Murray and Silver (1966), Hausman and Peterson (1972), Bitran et al. (1986), Matsuo (1990), and Kodama (1995). More recent papers include Choi et al. (2004, 2006).

This paper addresses a stochastic inventory control problem faced by manufacturing and retail firms who expose to challenging procurement and production decisions triggered by hurricane events. The hurricane stocking decisions made in advance of the season are affected by the general predictions regarding the ensuing hurricane season. Hurricane season demand predictions, which are updated and observed at the beginning of each pre-season period, are considered in planning for emergency supply inventory levels as well as pre-hurricane season demand. The contribution of this paper to inventory theory in general is that the proposed model accounts for the possibility of reserving stock in a multi-period setting based on information updates about a demand surge that might occur in a future (and terminal) period, while also accounting for demand uncertainty associated with the current period. Another contribution is that unlike previous inventory models that explicitly incorporate hurricane predictions into demand forecasts, this model allows inventory decisions to be modified during the planning horizon. Finally, our model and its solution methodology contribute to empirical research by analyzing historical hurricane counts and observed index values. These data are used to predict landfall count rate probabilities associated with the upcoming season. The hurricane season demand distribution is ultimately described over these predicted values.

3. Stochastic programming model

The objective of this study is to determine an optimal ordering policy such that (i) demand in each pre-hurricane season period is met and (ii) reserve supplies are stored for the ensuing hurricane

season in a cost effective way. The pre-hurricane season planning problem is introduced as a stochastic programming model in which the procurement/production decisions are given to minimize the expected total cost. The assumptions of the stochastic inventory model are given as follows.

Assumption 1. The annual hurricane landfall count \tilde{n}_h is assumed to follow a Poisson distribution with rate λ .

The Poisson distribution is used to express the probability of hurricane counts occurring in a fixed period of time. From a statistical standpoint, it is appropriate to describe the distribution of the hurricane counts as a Poisson distribution because they occur independently of the time since the last event, and with a known average rate.

Assumption 2. Hurricane season demand is a linear function of predicted hurricane landfall count rates λ_t during month t .

This assumption enables us to define the hurricane season demand distribution in terms of the hurricane count rate probabilities. In real applications, the demand is influenced by various attributes such as hurricane wind speeds, radius of the storm, and the population of the locations hit by hurricanes. For illustrative purposes, the hurricane season demand is introduced as a linear function of λ_t .

Assumption 3. Pre-hurricane season demand and hurricane season demand are independent random variables.

As consistent with intuition, demand tends to be higher during the hurricane season compared to the demand observed during the pre-hurricane season months. More specifically, they are not correlated, and should be described as independent variables.

The nature of the stochastic problem requires making multi-period ordering decisions by considering the uncertainty associated with demand realizations. Inventory control theory leverages dynamic programming, optimal control and stochastic programming as the main approaches to solve multi-period inventory problems. In this study, the inventory control problem is formulated as a multi-stage stochastic programming model with recourse, which can be reduced to a discrete-equivalent linear program. Dupačová et al. (2003) formulate the two-stage and multi-stage stochastic programs with recourse as follows:

$$\min_{\mathbf{d} \in \mathbb{D}} \mathbb{E}_{\mathbf{P}} f(\mathbf{x}, \mathbf{d}) = \int_X f(\mathbf{x}, \mathbf{d}) P(d\mathbf{x}), \quad (1)$$

where \mathbb{D} is the set of feasible first-stage decisions, and $X \subset \mathbb{D}$. $f(\cdot, \mathbf{d})$, $\mathbf{d} \in \mathbb{D}$ is the objective function of the stochastic model. P is the probability measure on the Borel σ -field and the subset X .

Now the stochastic programming inventory problem is described as follows: A demand realization occurs at the end of each period t . Let Q_{kt} denote the order quantity at the beginning of period t under (demand) scenario k , and let c_t be the associated unit ordering cost. Let x_{kt} represent the total demand for a single product during period t under scenario k , and q_{kt} the corresponding scenario probability. Let v_{kt} denote the excess inventory observed at the end of period t under scenario k , and let h_t be the associated unit holding cost. Similarly, u_{kt} is the observed number of shortages at the end of period t under scenario k with corresponding unit cost s_t . Then the multi-stage stochastic programming problem can be expressed as the following linear program. The details about the model can be found in (El Agizy, 1969)

$$\min \sum_{t=1}^T \sum_{k=1}^K q_{kt} \cdot (c_t \cdot Q_{kt} + h_t \cdot v_{kt} + s_t \cdot u_{kt}),$$

$$Q_{kt} + v_{k(t-1)} + u_{kt} - v_{kt} = x_{kt},$$

$$v_{k0} = 0, \quad k = 1, \dots, K,$$

$$Q_{kt}, v_{kt}, u_{kt} \geq 0, \quad t = 1, \dots, T, \quad k = 1, \dots, K. \quad (2)$$

This problem can be shortened by adding the *nonanticipativity* constraints:

$$\begin{aligned} Q_{kt} &= Q_{k't}, u_{kt} = u_{k't} \quad \text{and} \quad v_{kt} = v_{k't} \quad \text{for all } k, k' \text{ for which } x_{k,[1,t]} \\ &= x_{k',[1,t]}, \quad t = 1, \dots, T \end{aligned}$$

These constraints ensure that the decisions taken at period t do not depend on the future observations of the stochastic process, but on the information available up to period t , $x_{[1,t]}$.

3.1. Generating demand scenario probabilities

In order to generate demand scenarios, a Markov chain associated with hurricane count rates is assumed. The stationary transition probability p_{ij} of the Markov chain corresponds to the probability of predicting hurricane landfall count rate of $j = 0, \dots, 5$ during the current pre-hurricane season month given that the previous month's prediction is $i = 0, \dots, 5$. These probabilities are predicted based on the hurricane prediction model developed by Elsner and Jagger (2004). Elsner et al. (2006) further extend Elsner and Jagger (2004) to provide a six-month forecast horizon for annual hurricane counts along the U.S. coastline.

Elsner et al. (2006) determine that the North Atlantic Oscillation (NAO) and the Atlantic Sea-Surface Temperature (SST) variations, referred to as the Atlantic Multidecadal Oscillation (AMO), are the most significant predictors of the annual hurricane landfall. They develop a regression model to predict hurricane count rate using these predictors. A Bayesian approach to regression analysis is conducted to generate samples of posterior coefficients β_{t+1} associated with the NAO and AMO indices, given the prior estimates of these coefficients β_t at period t . The prior distribution for β_t is specified by a multivariate normal distribution, $MVN(\mu_t, \psi_t^{-1})$. In order to determine bootstrap prior estimates for μ_t and ψ_t , the regression equation is fit to the set of hurricane counts using only the pre-hurricane season period (month) t index for NAO and AMO, and a large number of bootstrap samples are generated with replacement. The predicted hurricane count rate λ_t is then updated during period t as shown in Eq. (3) using these bootstrap samples and the NAO_t and AMO_t data observed in period t :

$$\log(\lambda_t) = \beta_{0_t} + \beta_{1_t} \cdot AMO_t + \beta_{2_t} \cdot NAO_t + \beta_{3_t} \cdot (AMO_t \cdot NAO_t). \quad (3)$$

The regression equation given in Eq. (3) is run with the corresponding $t+1$ index data to form the likelihood function $g_t(\lambda_t | \beta_t)$. The posterior density function is then conditioned on these observed hurricane landfall count predictions $f_{t+1}(\beta_{t+1} | \lambda_t)$. In order to analyze $f_{t+1}(\beta_{t+1} | \lambda_t)$, the *Gibbs Sampler* algorithm is implemented. Based on this algorithm, the posterior coefficient values $\beta_{t+1} = [\beta_{0_{t+1}}, \beta_{1_{t+1}}, \beta_{2_{t+1}}, \beta_{3_{t+1}}]$ are generated from $f_{t+1}(\beta_{t+1} | \lambda_t)$. These posterior samples and the observed NAO and AMO index values during pre-hurricane season month $t+1$ are used to predict landfall count rate λ_{t+1} as shown in

$$\begin{aligned} \log(\lambda_{t+1}) &= \beta_{0_{t+1}} + \beta_{1_{t+1}} \cdot AMO_{t+1} + \beta_{2_{t+1}} \cdot NAO_{t+1} + \beta_{3_{t+1}} \\ &\quad \cdot (AMO_{t+1} \cdot NAO_{t+1}). \end{aligned} \quad (4)$$

Eq. (5) gives the predictive density function of hurricane landfall count rate $h_{t+1}(\lambda_{t+1} | \lambda_t)$:

$$h_{t+1}(\lambda_{t+1} | \lambda_t) = \int g_{t+1}(\lambda_{t+1} | \beta_{t+1}) f_{t+1}(\beta_{t+1} | \lambda_t) d\beta, \quad (5)$$

where λ_t corresponds to an observed prediction and λ_{t+1} is the anticipated prediction for period $t+1$. Additionally, $g_{t+1}(\lambda_{t+1}|\beta_{t+1})$ is the density function of λ_{t+1} given the posterior coefficients β_{t+1} .

The Gibbs algorithm is implemented once more to generate predictive samples of λ_{t+1} . The corresponding predictive probabilities are used to determine the transition probabilities of the Markov chain, which are used to estimate demand scenario probabilities.

3.2. Hurricane data analysis and illustrative example

For the numerical example, the pre-hurricane season months of April and May are considered as the inventory planning periods. The hurricane season (June 1–November 30) as a whole is considered to be one period. Table 1 gives the historical NAO and AMO index values associated with the pre-hurricane season months of April and May along with the observed landfall counts for each hurricane season. These data are used to predict the hurricane landfall count rate for the forecasted hurricane season. The NAO values are obtained from the Climatic Research Unit and the AMO values are obtained from the Climatic Diagnostics Center. The predictive distribution of hurricane count rates is analyzed with WinBUGS (Windows version of Bayesian inference using Gibbs Sampling). Initially, bootstrap priors are specified for the regression coefficients. These bootstrap priors and the NAO and AMO April index are used to estimate λ_t , which corresponds to the predicted hurricane landfall count rate during April. Then the Gibbs sampler is run to update the priors until the convergence of the posterior coefficients is achieved. Table 2 is continuation of Table 1, and demonstrates the 1980–2007 hurricane related data.

Table 1

Hurricane landfall count and April–May NAO and AMO index derived by the 1950–1979 data.

Year	n_h	April		May	
		NAO	AMO	NAO	AMO
1950	3	1.61	-0.16	-1.73	-0.34
1951	3	-0.45	0.43	-2.11	0.29
1952	0	2.79	0.06	-0.94	0.28
1953	1	-1.6	0.33	-0.75	0.14
1954	3	-0.26	-0.15	-0.91	0.14
1955	3	2.4	-0.18	0.33	-0.11
1956	3	-1.9	0.11	4.54	-0.14
1957	1	-0.62	-0.46	-0.84	-0.23
1958	1	1.79	1.02	1.1	0.78
1959	0	1.51	-0.08	-2.22	-0.36
1960	3	1.93	0.13	0.07	0.1
1961	2	0.71	-0.08	-0.94	-0.11
1962	1	0.74	0.33	-0.1	0.43
1963	0	-0.46	0.44	1.91	0.23
1964	1	0.95	-0.05	2.51	0
1965	4	2.14	-0.16	-0.08	-0.27
1966	1	1.18	0.47	1.51	0.2
1967	2	-0.76	0.18	-0.46	-0.12
1968	1	-0.71	0.18	-1.5	-0.14
1969	1	1.11	0.91	-0.23	0.78
1970	2	2.52	0.39	1.87	0.47
1971	1	-3.15	-0.23	-0.62	-0.36
1972	3	0.22	-0.08	1.24	-0.34
1973	1	-2.61	0.02	0.37	-0.11
1974	0	-2.3	-0.9	-0.01	-0.99
1975	1	-0.84	-0.47	-2.42	-0.72
1976	1	-1.53	-0.44	1.2	-0.68
1977	1	1.07	-0.19	-1.62	-0.14
1978	1	-3.12	0.53	0.37	0.14
1979	0	-0.79	0.4	1	0.46

Table 2

Hurricane landfall count and April–May NAO and AMO index derived by the 1980–2007 data.

Year	n_h	April		May	
		NAO	AMO	NAO	AMO
1980	3	0.03	0.42	-2.26	0.73
1981	1	-3.04	0.48	0.05	0.56
1982	0	-0.99	-0.12	1.1	0.1
1983	0	-1.01	0.74	-0.57	0.57
1984	1	0.33	-0.33	-2.34	-0.33
1985	5	0.34	-0.55	-2.13	-0.57
1986	2	-0.93	-0.45	2.16	-0.45
1987	1	2.59	0.39	-0.81	0.36
1988	1	-2.39	0.33	-1.24	0.15
1989	3	-0.48	-0.7	1.16	-0.69
1990	0	1.77	0.2	-1.19	0.34
1991	1	1.48	-0.29	-0.04	-0.26
1992	1	1.32	-0.16	0.8	0
1993	1	0.83	-0.08	-2.59	0.05
1994	0	1.38	-0.45	-1.43	-0.53
1995	2	-1.81	0.41	-0.36	0.55
1996	2	-0.31	0.46	-1.5	0.38
1997	1	-0.97	0.2	-1.35	0.23
1998	3	-0.39	0.69	-1.26	0.92
1999	3	0.43	0.01	1.03	0.05
2000	0	-3.34	-0.19	0.31	-0.17
2001	0	1.24	-0.14	-0.09	-0.09
2002	1	0.91	0.17	-0.05	-0.08
2003	2	-1.74	-0.03	1.17	-0.09
2004	5	1.08	0.53	-0.67	0.27
2005	5	0.71	1	-0.13	1.18
2006	0	0.57	0.44	-0.22	0.5
2007	2	-0.1	0.47	0.62	0.19

Fig. 1 shows the obtained prior and posterior density function of the regression coefficient β_1 . Similar density functions are obtained for the other coefficients. Visual inspections reveal that the convergence of the chain is observed after 15,000 simulations. The predictive inference for λ_{t+1} is made by setting the hurricane landfall count rate to NA (not available) for the forecasted hurricane season. More specifically, the Gibbs sampler is run once again to generate λ_{t+1} conditional on the posterior coefficients β_{t+1} and the observed NAO and AMO index data at period $t+1$.

Fig. 2 demonstrates the WinBugs output associated with the landfall count rate predictions, and Fig. 3 shows the corresponding hurricane landfall count predictions.

The predicted hurricane count rate probabilities are used to determine the stationary transition probabilities of the six-state Markov chain. For instance, p_{11} is obtained by evaluating $P\{\lambda_{t+1} = 1 | \lambda_t = 1\} = 0.29$ empirically. Remaining entries are obtained in a similar manner. Eq. (6) shows the resulting transition matrix. Recall that the states of the Markov chain correspond to predicting exactly $\lambda = 0, \dots, 5$, respectively,

$$\mathbf{P} = \begin{pmatrix} 0.23 & 0.29 & 0.22 & 0.15 & 0.09 & 0.02 \\ 0.22 & 0.29 & 0.22 & 0.15 & 0.09 & 0.03 \\ 0.23 & 0.29 & 0.22 & 0.13 & 0.1 & 0.03 \\ 0.21 & 0.28 & 0.22 & 0.16 & 0.1 & 0.03 \\ 0.2 & 0.3 & 0.25 & 0.18 & 0.05 & 0.02 \\ 0.21 & 0.31 & 0.18 & 0.18 & 0.1 & 0.02 \end{pmatrix}. \quad (6)$$

By substituting p_{ij} values into the steady-state equations, the following set of equations are obtained:

$$\pi_0 = 0.23\pi_0 + 0.22\pi_1 + 0.23\pi_2 + 0.21\pi_3 + 0.2\pi_4 + 0.21\pi_5,$$

$$\pi_1 = 0.29\pi_0 + 0.29\pi_1 + 0.29\pi_2 + 0.28\pi_3 + 0.3\pi_4 + 0.31\pi_5,$$

$$\pi_2 = 0.22\pi_0 + 0.22\pi_1 + 0.22\pi_2 + 0.22\pi_3 + 0.25\pi_4 + 0.18\pi_5,$$

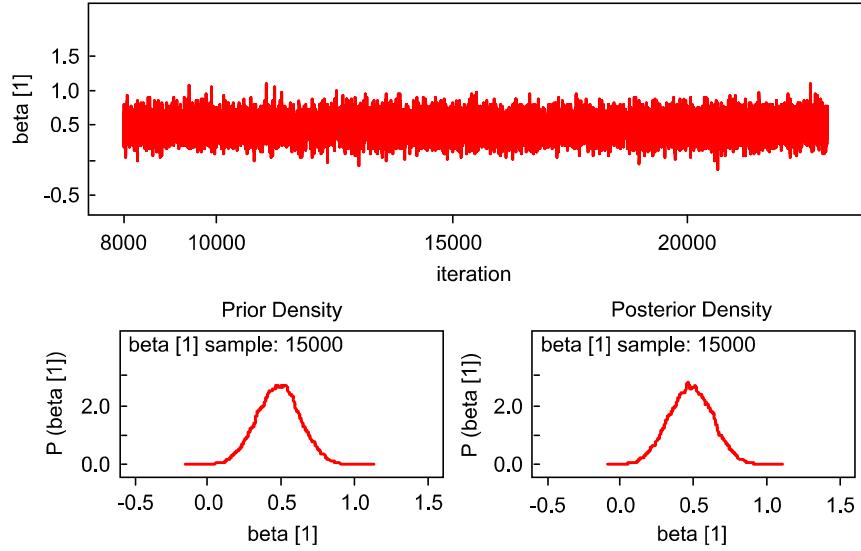


Fig. 1. WinBugs posterior regression coefficients.

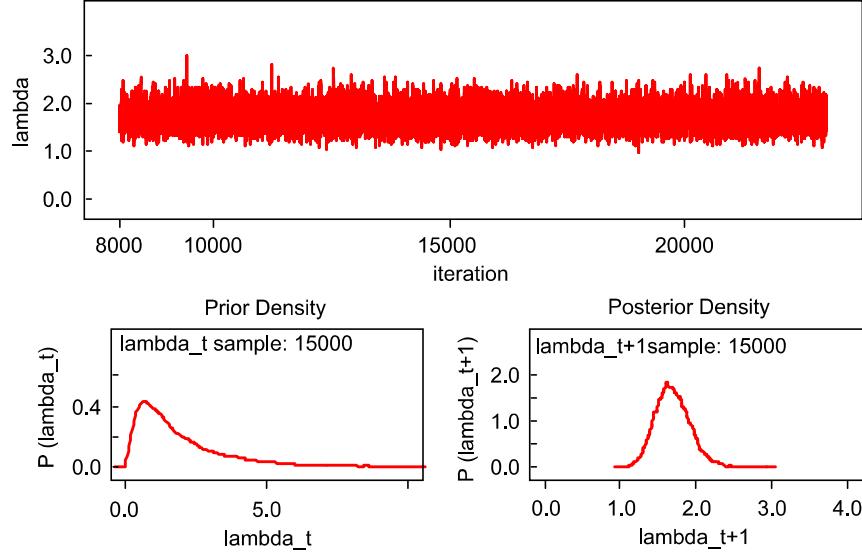


Fig. 2. WinBugs predictive hurricane count rates.

$$\pi_3 = 0.15\pi_0 + 0.15\pi_1 + 0.13\pi_2 + 0.16\pi_3 + 0.18\pi_4 + 0.18\pi_5,$$

$$\pi_4 = 0.09\pi_0 + 0.09\pi_1 + 0.1\pi_2 + 0.1\pi_3 + 0.05\pi_4 + 0.1\pi_5,$$

$$\pi_5 = 0.02\pi_0 + 0.03\pi_1 + 0.03\pi_2 + 0.03\pi_3 + 0.02\pi_4 + 0.02\pi_5,$$

$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5. \quad (7)$$

The simultaneous solutions to the last six equations provide the unique solution as

$$\pi_0 = 0.22, \quad \pi_1 = 0.29, \quad \pi_2 = 0.22,$$

$$\pi_3 = 0.15, \quad \pi_4 = 0.09, \quad \pi_5 = 0.03. \quad (8)$$

The steady-state probabilities given in Eq. (8) are used to define the hurricane season demand distribution. Suppose the likely outcomes of the hurricane season demand are 200, 250, 300, 350, 400, and 450 corresponding to $\lambda = 0, 1, \dots, 5$, respectively. Assuming that each period's pre-hurricane season demand is equally likely to be 100 or 150, the underlying stochastic demand distribution can be described as shown in Table 3.

Using the data in Table 3, the stochastic programming model (2) becomes

$$\min \sum_{t=1}^2 \sum_{k=1}^{36} q_{kt} \cdot (c_t \cdot Q_{kt} + h_t \cdot v_{kt} + s_t \cdot u_{kt}),$$

$$Q_{kt} + v_{k(t-1)} + u_{kt} - v_{kt} = x_{kt}, \quad t = 1, \dots, 2, \quad k = 1, \dots, 36,$$

$$v_{k0} = 0, \quad k = 1, \dots, 36,$$

$$Q_{kt}, v_{kt}, u_{kt} \geq 0, \quad t = 1, \dots, 2, \quad k = 1, \dots, 36 \quad (9)$$

plus the nonanticipativity constraints:

$$Q_{k1} = Q_1, \quad k = 1, \dots, 36,$$

$$Q_{k2} = Q_{21}, \quad u_{k2} = u_{21}, \quad v_{k2} = v_{21}, \quad k = 1, \dots, 6,$$

$$Q_{k2} = Q_{22}, \quad u_{k2} = u_{22}, \quad v_{k2} = v_{22}, \quad k = 7, \dots, 12,$$

$$Q_{k2} = Q_{23}, \quad u_{k2} = u_{23}, \quad v_{k2} = v_{23}, \quad k = 13, \dots, 18,$$

$$Q_{k2} = Q_{24}, \quad u_{k2} = u_{24}, \quad v_{k2} = v_{24}, \quad k = 19, \dots, 24,$$

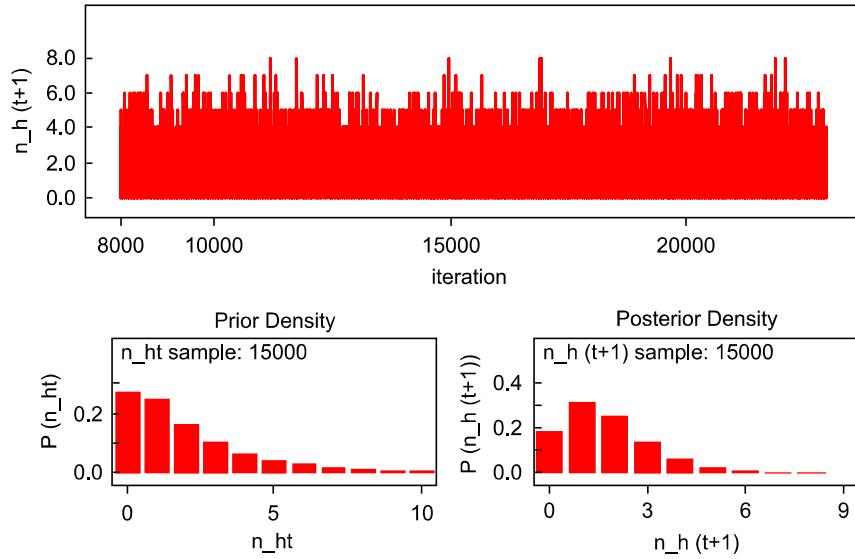


Fig. 3. WinBugs predictive hurricane counts.

Table 3
Demand distribution.

Demand (pre-hurricane season)	Probability	Demand (hurricane season)	Probability	Demand	Probability (weighted)
100	0.5	200	0.22	650	0.17
150	0.5	250	0.29	750	0.27
		300	0.22	850	0.23
		350	0.15	950	0.17
		400	0.09	1050	0.12
		450	0.03	1150	0.04

Table 4
Results of the original model.

Q_1^*	Scenarios	Q_2^*	Expected cost
1700	1	0	\$87,243.6
	2	0	
	3	0	
	4	100	
	5	200	
	6	300	

$$Q_{k2} = Q_{25}, \quad u_{k2} = u_{25}, \quad v_{k2} = v_{25}, \quad k = 25, \dots, 30,$$

$$Q_{k2} = Q_{26}, \quad u_{k2} = u_{26}, \quad v_{k2} = v_{26}, \quad k = 31, \dots, 36, \quad (10)$$

where $t = 1, 2$ corresponds to the pre-hurricane season months of April and May, respectively. The weighted probabilities are determined using the pre-hurricane season and hurricane season demand probabilities. For instance, 0.17 is calculated as follows. Similar calculations are made to develop the demand distribution:

$$\begin{aligned} & 300 \cdot (0.22 \cdot 0.5) + 350 \cdot (0.22 \cdot 0.5) \\ & 300 \cdot 0.11 + 350 \cdot 0.255 + 400 \cdot 0.255 + 450 \cdot 0.185 + 500 \cdot 0.12 + 550 \cdot 0.06 + 600 \cdot 0.015 \\ & \approx 0.17. \end{aligned} \quad (11)$$

Excel Solver™ is used to obtain the optimal ordering policy for the numerical example. The following data are used to solve the linear program, and the results are shown in Table 4: $c_1 = 30$, $c_{t+1} = c_t \cdot 4$, $s = 300$, $h_t = c_t/2$.

Note that the nonanticipativity constraints are added to the problem. In other words, there is only one first period decision, namely Q_1 , and there are 6 second period ordering and recourse decisions, one for each scenario. There are 91 variables and 42

constraints in the linear program. The solution yields a total expected cost of \$87,243.6. The optimal solution given in Table 4 can be interpreted as follows: Order/produce 1700 units at the beginning of the current period (April). If the observed demand associated with the month of April is 950, then order/produce 100 units at the beginning of May. Similarly, order/produce 200 (300) units at the beginning of May if April's demand is 1050 (1150), respectively.

3.3. Comparison of proposed and traditional approaches

In this section, the above-mentioned problem is explored in a different case. For this version, the DM chooses not to prepare for the hurricane demand in advance of the season but only for the period's demand. In other words, the DM is reactive and responds to hurricane season demand only after the hurricane season begins. Initially, the multi-period problem that accounts only for pre-hurricane season demand is solved. Then the optimum ordering quantity given at the beginning of the season Q_1^* is calculated considering the predicted hurricane season demand. Since stock-outs cost more during the season due to the increased demand surge, we assume that the unit shortage cost becomes relatively more expensive $s_3 = 900$. Based on our previous assumption about the cost parameters, the other unit costs become $c_3 = 480$, $h_3 = 240$. Table 5 shows the total expected cost obtained from the solution of the integrated stochastic model and the associated optimum ordering quantities.

The results reveal that the proposed model has lower expected cost \$87,243 < \$177,465. Additionally, when the optimal order quantities obtained from the solution of both modeling approaches are compared, our model yields a significantly larger

order quantity $1700 > 250$. Finally, we determine that better service levels are achieved under various demand scenarios based on the proposed model when compared to the traditional multi-period inventory model.

4. Scenario reduction

In most practical cases, the stochastic process is approximated with many realizations. In order to numerically solve such problems, Heitsch and Römisch (2003) and Dupačová et al. (2003) introduce an optimal scenario reduction methodology based on probability metric minimization. They define the optimal scenario reduction of a given discrete approximation as the determination of a scenario subset of prescribed cardinality that is closest to the original distribution. The optimal reduction approach described in their papers suggests considering the following probability distance function:

$$\hat{\mu}_c(P, Q) = \min \left\{ \sum_{\substack{i,j=1 \\ j \notin J}}^N c(x_i, x_j) \cdot \eta_{ij} : \eta_{ij} \geq 0, \sum_{i=1}^N \eta_{ij} = q_j, \sum_{j=1}^N \eta_{ij} = p_i \right\},$$

$$D(J; q) := \hat{\mu}_c \left(\sum_{i=1}^N p_i \cdot \delta_{x_i}, \sum_{j \notin J} q_j \cdot \delta_{x_j} \right), \quad (12)$$

where $J \subset \{1, \dots, N\}$ is the index of withdrawn scenarios with fixed cardinality and Q is the discrete approximation of P . Based on the optimal reduction concept, the optimal index set J^* , and the optimal weights q^* are determined such that $D(J; q)$ is minimized. Then, the new probabilistic weights q_j , $j \in \{1, \dots, N\} \setminus J$ are assigned to each remaining scenario $x_j, j \notin J$ using the following optimal redistribution rule.

Theorem 1 (Heitsch and Römisch, 2003). Given $J \subset \{1, \dots, N\}$, the probability distance is defined as follows:

$$D_J = \min \left(D(J; q) : q_j \geq 0, \sum_{j \notin J} q_j = 1 \right) = \sum_{i \in J} p_i \cdot \min_{j \notin J} c(x_i, x_j) \quad (13)$$

and the minimum value is attained at

$$q_j^* = p_j + \sum_{i \in J \setminus \{j\}} p_i \quad \text{for each } j \notin J \quad (14)$$

where $j(i) \in \operatorname{argmin}_{j \notin J} c(x_i, x_j)$ for each $i \in J$.

Theorem 1 implies that the new probability of a kept scenario is equal to the sum of its original probability and of all probabilities of the closest withdrawn scenarios determined based on the c metric. Then the optimal index set J^* for scenario reduction with given cardinality is determined by solving the following problem formulated by (Heitsch and Römisch, 2003)

$$\min \left\{ D_J := \sum_{i \in J} p_i \cdot \min_{j \notin J} c(x_i, x_j) : J \subset \{1, \dots, N\}, n(J) = N - n \right\}. \quad (15)$$

Table 5
Results of the original model.

Planning horizon	Q_1^*	Q_2^*	Q_3^*	Expected cost	Total expected cost
Pre-hurricane season	250	0	–	\$13,875	\$13,875 + \$163,590 = \$177,465
Hurricane season	–	–	250	\$163,590	

As discussed by Heitsch and Römisch (2003) when $n(J) = N - 1$, Eq. (15) reduces to

$$\min_{J \in \{1, \dots, N\}} \sum_{i=1}^N p_i \cdot c(x_i, x_j). \quad (16)$$

Eq. (16) yields the best possible deterministic approximation of the initial distribution such that the redistribution rule assigns $q_j^* = 1$ to the preserved scenario.

4.1. Numerical example

The numerical example presented in the previous section is resolved using the optimal scenario reduction concept. The number of deleted scenarios is fixed as $n(J) = 4$. Table 6 gives the selected index sets and their corresponding probability distances, and it reveals that $J^* = \{1, 3, 5, 6\}$ gives the minimum distance with $D_J^* = 60$. Therefore, these scenarios should be removed from the original set of scenarios.

The D_J values shown in Table 5 are calculated using Euclidean distances. For instance, D_J^* is determined as follows:

$$D_J^* = 100 \cdot 0.17 + 100 \cdot 0.23 + 100 \cdot 0.12 + 200 \cdot 0.04 = 60. \quad (17)$$

Table 7 gives the Euclidean distances $c(x_i, x_j)$ used to determine the optimal weights for the remaining scenarios.

The optimal weights for scenarios 2 and 4 are calculated as shown in

$$q_2^* = \sum_{i=1}^3 = 0.17 + 0.27 + 0.23 = 0.67,$$

$$q_4^* = \sum_{i=4}^6 = 0.17 + 0.12 + 0.04 = 0.33. \quad (18)$$

Stochastic programs can have more than one candidate scenario that has the same probability distance to another scenario. For instance, in this example different values can be assigned to optimal weights by incorporating the probability of deleted

Table 6
Probability distances.

J	D_J	J	D_J	J	D_J
{1,2,3,4}	212	{1,2,5,6}	81	{2,3,4,5}	119
{1,2,3,5}	140	{1,3,4,5}	86	{2,3,4,6}	94
{1,2,3,6}	132	{1,3,4,6}	61	{2,3,5,6}	70
{1,2,4,5}	90	{1,3,5,6}	60	{2,4,5,6}	80
{1,2,4,6}	82	{1,4,5,6}	70	{3,4,5,6}	109

Table 7
Euclidean distances (c metric).

(i,j)	2	4
1	100	300
3	100	100
5	300	100
6	400	200

scenario $i=3$ to the scenario $j=4$ since $i=3$ has the same proximity both to $j=2, 4$. Eq. (19) shows the resulting optimal weight values:

$$q_2^* = \sum_{i=1}^2 = 0.17 + 0.27 = 0.44,$$

$$q_4^* = \sum_{i=3}^6 = 0.23 + 0.17 + 0.12 + 0.04 = 0.56. \quad (19)$$

Then the reduced versions of the stochastic programming models are solved using the previously defined unit costs. The reduced models defined by $n=2$ consist of 15 variables, and 6 constraints obtained by considering the nonanticipativity property of stochastic programs. Table 8 demonstrates the results of these reduced models.

The reduced model 1 recommends ordering 1700 units at the beginning of April, and no order should be given in May. Similar to this model, the reduced model 2 indicates that initially 1700 units should be ordered. However, if the demand realization at the end of April is 950, then this model suggests ordering an additional 200 units. Otherwise no order should be given at the beginning of May. Table 9 presents the solutions for all the potential reduced models given that $n(J)=N-n$.

4.2. Solution quality

In stochastic programming models, it is crucial to evaluate the quality of the reduced trees. In this context, one does not search for the best approximation of the initial distribution but for the quality of the optimal solutions (values). In this study, the accuracy is defined as the ratio of the optimal first-stage decisions obtained from the solutions of the reduced model and the original model. Recall that only the (deterministic) first stage is the appropriate outcome of the stochastic program. The tree serves to model the demand uncertainty. Table 9 reveals that the models carried by the number of scenarios $n=1, 2$ are reduced in an optimal way since the value of first-stage optimal decisions obtained from both of the reduced models are exactly the same as that of the original model. On the other hand, reduced models having $n=3, 4, 5$ scenarios have an accuracy of $\frac{750}{1700} \times 100 \approx 44\%$. These results suggest that the accuracy of the reduced trees tends to increase as they are supported with a small number of scenarios. In other words, while the stochastic process is approximated with less number of scenarios by implementing the scenario reduction approach, the accuracy of the values (solu-

Table 8
Results of reduced models.

Reduced model	Q_1^*	Scenario i	q_i^*	Q_2^*	Expected cost
1	1700	2	0.67	0	\$76,180.8
		4	0.33	0	
2	1700	2	0.44	0	\$82,290
		4	0.56	200	

Table 9
Optimal values (solutions) of reduced models.

n	J^*	D_j^*	Q_1^*	Expected cost	Relative error
1	$J = \{1, 2, 4, 5, 6\}$	114	1700	\$63,750	$114/114 = 100\%$
2	$J = \{1, 3, 5, 6\}$	60	1700	\$76,180.8	$60/114 = 52.63\%$
3	$J = \{1, 5, 6\}$	37	750	\$127,695	$37/114 = 32.46\%$
4	$J = \{5, 6\}$	20	750	\$153,660	$20/114 = 17.54\%$
5	$J = \{6\}$	4	750	\$163,260	$4/114 = 3.50\%$

tions) obtained from reduced models increases. This finding is consistent with intuition such that one would expect to obtain more accurate results as the uncertainty associated with the stochastic process reduces. Similar interpretations can be made for the expected costs associated with the reduced models.

In order to evaluate the performance of the reduced models, the stochastic inventory problem is initially solved on the reduced tree. Then, the values of all the first-stage (root) variables are fixed, and resolved on the original tree. As a result, the out-of-sample performance of the reduced-tree solution is obtained assuming that the original tree is a good-enough approximation of the true distribution. The reduced models carried by $n=1, 2$ give the same optimal expected cost value as the initial optimum value (\$87,243.6). The reduced models having 44% solution accuracy result in an expected cost of (\$174,975). These findings indicate that as the accuracy of the optimal first stage solutions of the reduced model decreases so does the cost efficiency.

4.3. Heuristic algorithm

In most of the stochastic problems where the stochastic process is represented by many scenarios, Eq. (15) cannot be solved optimally. Therefore, Dupačová et al. (2003) and Heitsch and Römisch (2003) develop heuristic algorithms to approximate solutions of Eq. (15). In this study, the *simultaneous backward reduction* algorithm, which includes all the previously deleted scenarios in each backward step, is used. The algorithm determines an index set J to be removed from the original set of scenarios based on the solution of the following equation given in (Heitsch and Römisch, 2003)

$$l_k \in \arg \min_{\substack{l \in J^{[k-1]} \\ i \in J^{[k-1]} \cup \{l\}}} \sum p_i \cdot \min_{j \notin J^{[k-1]} \cup \{l\}} c(x_i, x_j), \quad (20)$$

where $J^{[k-1]} = \{l_1, \dots, l_{k-1}\}$ is defined as the index set of deleted scenarios up to and including step $k-1$.

The stochastic programming inventory model is solved by implementing the simultaneous backward reduction algorithm to illustrate the application of the heuristic algorithm. The first step requires the deletion of only one scenario. Through the following steps, the index l_k is determined given that the previous index set $\{l_1, \dots, l_{k-1}\}$ is optimal. When the number of withdrawn scenarios becomes $n(J)=4$, the algorithm is terminated. The following steps are implemented to solve the example problem.

Step 1: The Euclidean probability distances are calculated for the scenarios $l=1, 2, 3, 4, 5, 6$ as $D_{J^1} = 17, 27, 23, 17, 12, 4$. The minimum distance is obtained at $D_{J^1} = 4$. Thus, $l_1 = 6$.

Step 2: The Euclidean probability distances are calculated for the remaining scenarios $l=1, 2, 3, 4, 5$ as $D_{J^2} = 21, 31, 27, 21, 20$. The minimum distance is obtained as $D_{J^2}^* = 20$. Thus, $l_2 = 5$.

Step 3: The Euclidean probability distances are calculated for the remaining scenarios $l=1, 2, 3, 4$ as $D_{J^3} = 37, 47, 43, 53$. The minimum distance is obtained as $D_{J^3}^* = 37$. Thus, $l_3 = 1$.

Step 4: The Euclidean probability distances are calculated for the remaining scenarios $l=2, 3, 4$ as $D_{J^4} = 81, 60, 70$. The minimum distance is obtained as $D_{J^4}^* = 60$. Thus, $l_4 = 3$. Since $n(J)=4$ is achieved, the algorithm is terminated.

Table 10 shows the results of this reduced model carried by $j=2, 4$, which shows that the optimal redistribution rule will result in the same optimal weights. Therefore, the resulting stochastic programming model will have the same arguments as the optimally reduced one.

The simultaneous backward reduction algorithm yields optimal solution (value) except for the reduced model supported by $n=1$. This arises from the fact that the best possible scenario $i=3$ has already been deleted in the previous backward step. In other words,

Table 10

Euclidean distance matrix 4.

(i,j)	2	4	$\sum_{i \in \{6,5,1,3\}} p_i \cdot \min_{j \in \{2,4\}} C(x_i, x_j)$
6	400	200	$D_{j4}^* = 0.04 \cdot 200 + 0.12 \cdot 100 + 0.17 \cdot 100 + 0.23 \cdot 100 = 60$
5	300	100	
1	100	300	
3	100	100	

while the optimal reduction method directly solves Eq. (15), the heuristic algorithm implements the scenario reduction process recursively in a stepwise fashion. For this deterministic problem, the algorithm yields an optimal order quantity of $Q_1^* = 1900$ with an expected cost of \$71,250. The index set $i = 6, 5, 1, 3, 2$ is deleted and so the reduced model is defined only by the scenario $j = 4$. These results indicate that the simultaneous backward reduction algorithm works reasonably well. Therefore, it can be used in lieu of optimal reduction approach to reduce the number of scenarios where the initial distribution is represented by many scenarios.

5. Conclusions

This paper explores a stochastic inventory problem in which predictions associated with the ensuing hurricane's season demand distribution evolves according to a Markov chain. The states of the Markov chain represent predicted hurricane count rates for the ensuing season, and hurricane season demand forecasts are assumed to be proportional to these rates. The system is formulated as a multi-stage stochastic programming model with recourse. The underlying demand distribution is developed such that both the pre-hurricane season demand and the hurricane season demand are considered at each pre-hurricane season decision epoch. The demand distributions associated with each pre-hurricane season period are assumed to be known to the inventory manager at the beginning of the planning horizon. However, the hurricane season demand distribution is based on periodic information updating. The hurricane landfall count rate predictive probabilities, which are used to define the hurricane season demand distribution as a Markov chain, are estimated via a widely accepted hurricane prediction model developed by Elsner and Jagger (2004).

From an academic perspective, the modeling approach presented in this paper is novel in that it accounts for information updates regarding the demand surge that will occur at the beginning of the season. It also accounts for demand uncertainties during each period prior to the season. This approach introduces the notion of reserving stock in a multi-period inventory problem with stochastic demands in anticipation of a possible demand surge in a future and terminal period.

From a practitioner's perspective, the proposed model enables inventory managers to determine an appropriate stock level that should be available at the beginning of the hurricane season while simultaneously determining stock levels for pre-hurricane season demand periods. The model also allows production/procurement decisions to be altered as necessary during the planning horizon. A comparison of the results of our model to that of a basic stochastic programming model for multi-period inventory control suggests that cost savings can be realized in the long run. This can be achieved by reserving stock during the pre-season periods in preparation for the seasonal demand surge. This approach seems reasonable when inventory holding costs are inexpensive, procurement/production costs are increasingly expensive as the season approaches, and the demand surge during the season is significant. Under these circumstances, it is more beneficial to have large order/production quantities during the earlier periods

and carry extra inventory than to have large quantities close to the beginning of the season. Further, it is better to keep low inventories during the early stages of the pre-season planning periods.

In real-world applications, the stochastic demand process is likely to consist of many scenarios and/or stages. For these situations, the stochastic programming approach becomes less efficient and requires the implementation of other methodologies. In this paper, the scenario reduction approach introduced by Heitsch and Römis (2003) is implemented to find the optimal set of scenarios to represent the underlying distribution. It is determined that the optimal scenario reduction method results in approximately 44% accuracy when at least half of the scenarios are removed. However, as the problem gets larger, the running time of the algorithm substantially increases. For future study, it would be beneficial to develop approximations of a demand process described by many scenarios through the implementation of the scenario generators. It is also worth exploring the quality of the reduced scenario model where the discrete demand process is represented by many scenarios. Additionally, a case can be developed in which a different demand distribution is introduced for each state. Finally, the existence of an optimal state-dependent base-stock policy can be investigated. These extensions would require additional modeling approaches and solution techniques.

Although this paper is presented from the perspective of the profit driven private sector firm, seasonal prediction of hurricane landfalls is also of interest to government and service organizations. For example, military organizations and electric power companies often pre-position manpower and equipment in anticipation of a potential hurricane event. This pre-positioning decision also inherits the risk of over-preparation and under-preparation with respect to the demand induced by the hurricane event. Not-for-profit service organizations such as the American Red Cross face similar risks and decisions related to stocking and staffing evacuation shelters. The approach to demand forecasting presented in this paper seems to have potential for helping organizations make effective supply chain related decisions within the context of emergency supplies and hurricane seasons. In closing, perhaps the proposed modeling approach could be adapted to accommodate other kinds of predictable disasters such as floods and wildfire.

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