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Two-stage stochastic optimal islanding operations under severe multiple contingencies in power grids

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ABSTRACT

Due to the catastrophic consequences of rolling blackouts, there is an increasing concern with the security and stability of modern power grids. Power grid islanding, as an emergency control operation method, can divide power grids into several self-sufficient islands and can then avoid wide-area blackouts. In this paper, we present a two-stage stochastic programming model to divide the power grids into self-sufficient islands before any multiple failures happen, and optimize islanding operations plan under severe multiple contingencies that lead to extreme situations where rolling blackouts may occur. Line switching and controlled load shedding are main tools for islanding, and the expected penalty for load shedding cost is minimized in consideration of contingencies with certain probabilities to happen. The presented model can give the system operator an efficient and optimal way to properly respond to outages. Several numerical experiments are performed on IEEE test cases to show the effectiveness of the proposed islanding operations method.

1. Introduction

Recent reports of large-scale blackouts in North America, Europe, South Asia, and other areas show that power systems are at times operating close to stability limits (see [1,2]). The power systems are thus very vulnerable to some unexpected events, such as hurricanes, earthquakes, extremely hot weather, system failures and outages, human errors, etc. As a result catastrophic failures or large-scale blackouts may happen. These blackouts have huge influences on the society. For example, the July 2012 Indian blackout affected over 620 million people (see [3]).

A power grid island is a self-sufficient subnetwork in a large-scale power system. To avoid wide-area blackout and minimize the losses, in case of multiple component failures in a power system (called contingencies), defensive islanding intentionally splits an interconnected power system into islands to prevent the further spreading of wide area blackouts. To respond to particular severe contingencies leading to extreme situations where rolling blackouts may occur, it may be desirable to isolate the impacted or failed area from the other parts, which remain operational with limited load shedding to help avoid catastrophic losses; Otherwise the traditional protection schemes will be triggered and used when normal contingencies occur. In other words, controlled islanding is the last line of defense in preventing large-scale blackouts in case of multiple severe contingencies. Compared with uncontrolled islanding, which may happen in a blackout, controlled islanding can help minimize the disturbance of power supply to a system as early as possible and facilitate the recovery process afterwards. Control islanding strategies have been extensively discussed in [4–9].

Power grid islanding has been studied in many papers. Some of them have studied the islanding problem without considering contingencies and in the others contingencies have been taken into account. As examples of the former category, in [10], a review of main aspects of power grid islanding has been presented outlining the islanding schemes according to graph partitioning, minimal cutset enumeration, and generator grouping. Graph partitioning is directly used to partition the vertex sets into several subsets in [11–15] by modeling the topology of the power grid by a graph, and also minimal cutsets are used for partitioning a network in [13,16]. Recently, Senroy, et al. [8] used the decision tree method for controlled islanding, and Pareto optimization has been used in [17] by Vittal and Heydt. Zhao et al. [18] used an ordered binary decision diagram based algorithm for network splitting and extended their work considering corrective actions to make the network more stable in [19]. Some other partitioning methods based on matrices were used to partition the power grid, for example, the spectral

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methods on power flow Jacobian matrix [11], and the spectral and k-means methods on real power flow matrix [20].

Approaches for simultaneously obtaining more than two islands are proposed in [12,14,15] by graph partitioning. Some optimization approaches [11,13,16,17] to power grid islanding relied on heuristic and approximation methods. In the recent work [15], an exact optimization approach for power grid islanding considering load shedding and connectivity constraints, which are crucial for the reliability of islands, was used for identifying multiple islands simultaneously. However, this model is for intentional islanding without considering any failures that lead to cascading blackout. Intentional islanding has also been addressed in [21–24] by several approaches including optimization approaches considering static stability and minimizing load shedding in power grids. Islanding has been proposed in [25] as a protection method in power networks with renewable distributed generators. In [26], optimization approaches based on a constrained programming formulation and heuristic methods have been used for minimizing the load shedding in the region where the failures start and in the topological complement of the region. In [27,28], given an area of uncertainty in the network, a mathematical formulation for the islanding based on mixed integer linear programming has been used to isolate unhealthy components of the network. However, [27,28] considered islanding of power grids under contingencies by bus splitting rather than line switching. On the other hand, line switching has been proved as an effective method for power grid operations to reduce the operational costs as well as to improve the reliability (see [29,30]). In order to trigger line switching some protection devices such as R-Rdot out-of-step relays discussed in [5,31] can be used. Most recently, [32] gave an overview of controlled islanding approaches, notably those using mixed-integer programming and suggested a two-stage stochastic program to tackle the problem of finding robust islanding that hedges against possible uncertainties. Their model is computationally intractable as the number of scenarios in the second stage grows exponentially in the number of uncertain pieces of the grid, and also it decides load shedding and line switching before instead of after any failures happen. Additionally, [33] proposed and thoroughly investigated three load-shedding strategies to prevent cascading failures in the power grids without considering the possibility for islanding.

In this paper, we propose a power grid islanding scheme, through line switching and controlled load shedding, by considering a set of severe contingencies with a probability distribution. Each contingency state, with a probability to happen, includes failures of multiple grid components (buses, transmission lines and/or generators), which can lead to rolling large-scale blackouts. Additionally, based on the fact that most blackouts are starting from failures in a small "geographic area" (see [32]), we assume failed components of a severe contingency can be isolated into a single island in a grid. First, we partition a grid into several self-sufficient subnetworks, each of which can run separately with an acceptable load shedding amount. It should be noted that before any failures in the system, no islanding operations are employed at a normal state. The subnetworks comprising the entire grid are just defined artificially so that once failures happen, islanding operations by line switching can be used to isolate the failed part, and the other operational parts can operate with controlled load shedding.

We present a two-stage programming model to divide the power grid into self-sufficient islands before any failures happen, and optimize islanding operations plan under multiple severe contingency scenarios. The expected penalty for load shedding cost is minimized in consideration of contingency states with certain probabilities to happen. The whole model will give the system operator an efficient and optimal way to properly respond to failures happening in a grid.

Table 1
Sets and indices.

Symbol	Description
V	Set of buses (indexed by i, j)
E	Set of transmission lines (indexed by e)
G	Set of generators (indexed by g)
$\mathcal{C}$	Set of contingencies (indexed by c)
i_e, j_e	From/to buses (bus number) of transmission line $e = (i_e, j_e)$

The rest of this paper is organized as follows. In Section 2, a two-stage stochastic programming model is formulated for the islanding operations for a set of contingency scenarios. Several aspects, including special cases and solution approaches are also discussed. Section 3 performs comprehensive numerical experiments of our proposed model. Finally, Section 4 concludes this paper.

2. Models for islanding operations

2.1. Nomenclature

The sets, indices, parameters, and decision variables used in the model are defined respectively in Tables 1–3.

2.2. Stochastic programming model for islanding operations under contingencies

For a power grid consisting of buses in V, transmission lines in E and generators in G, the islanding operations follow these steps: (1) K islands in this power grid are predetermined before any contingency happens. (2) Once a contingency happens with one or more failed components, an islanding operation is used to

Table 2
Parameters.

Symbol	Description
D_i	Load demand at bus $i \in V$
$\overline{P_g}$	Generation capacity of generator $g \in G$
F_e	Transmission capacity of line $e \in E$
B_e	Susceptance of line $e \in E$
C_i	Penalty cost for load shedding at $i \in V$
$Prob(c)$	Probability of contingency $c \in \mathcal{C}$ such that $\sum_{c \in \mathcal{C}} Prob(c) = 1$
$d^{(c)} \in \{0,1\}^{\lvert V\rvert+\lvert E\rvert+\lvert G\rvert}$	A vector used to define the contingency c, 1 if the element is failed and 0 otherwise
K	Number of islands proposed in a grid (indexed by k)
ε	Threshold for load shedding amount in operational islands

Table 3
Decision variables.

Symbol	Description
$x_{ik} \in \{0,1\}$	$x_{ik} = 1$ if bus i is in island k and 0 otherwise
x	Vector formed by x_{ik}'s for all i and k, which decides all islands of the grid
$y_e \in \{0,1\}$	$y_e = 1$ if the end buses i_e and j_e are in the same islands and 0 if the end nodes are in different islands
$Q(x, c)$	Load shedding cost under islands formed by x in contingency c
$p_g^{(c)}$	Power generation level by generator g in contingency c
$f_e^{(c)}$	Power flow on line $(i, j) \in E$ in contingency c
$s_i^{(c)}$	Load shedding amount at bus $i \in V$ in contingency c
$\theta_i^{(c)}$	Phase angle of bus $i \in V$ in contingency c
$z_e^{(c)} \in \{0,1\}$	$z_e^{(c)} = 1$ if line e is switched on in contingency c, and 0 otherwise
$u_k^{(c)} \in \{0,1\}$	$u_k^{(c)} = 0$ if island k is operational in contingency c, and 1 otherwise

isolate one island with failed components and the other parts of the grid are still operational with an acceptable load shedding amount. (3) When considering all contingency scenarios with a probability distribution, the expected load shedding cost should be minimized.

In this paper, we assume that all transmission lines can be switched off in case of a contingency. As discussed in Section 1, most blackouts start from failed component(s) in a small part of the grid, and we assume also that the failed components can be isolated in one island. Additionally, we assume that the number K of islands, which is bounded by the number of buses with generators, is given, following the same assumption in [15,26].

First, we predetermine K islands before any severe contingency happens. Again, the subnetworks comprising the entire grid are just defined artificially with no control measures taken. This first step is just to identify the set of islands that can be formed when a contingency occurs. Then, during a severe contingency, the islanding operations includes these steps: switching lines on or off to isolate the failed components, adjusting the generation level by ramping up and/or down generators, and using controlled load shedding for the operational part(s).

To optimally determine the islands and the islanding operations during different contingencies in $\mathcal{C}$, we use a two-stage stochastic programming model, where the first stage model is (in the following, unless explained, "$\forall i, \forall e, \forall g, \forall c$" denotes for all $i \in V$, $e \in E$, $g \in G$, $c \in \mathcal{C}$, respectively; "$\forall k$" means for all $k = 1, 2, \ldots, K$)

$$\min \sum_{c \in \mathcal{C}} Prob(c) \cdot Q(x, c) \tag{1a}$$

$$s.t. \sum_{k=1}^{K} x_{ik} = 1, \forall i \tag{1b}$$

$$\sum_{i \in V} \mathbf{1}_i^g x_{ik} \geq 1, \forall k \tag{1c}$$

$$y_e = \sum_{k=1}^{K} x_{i_e k} x_{j_e k}, \forall e \tag{1d}$$

connectivity *constraints* for each island (1e)

$$x_{ik}, y_e \in \{0, 1\}, \forall i, e, k \tag{1f}$$

Constraints (1b) ensure that each bus belongs to exactly one island. Constraints (1c) ensure that each island has at least one generator, where $\mathbf{1}_i^g \in \{0, 1\}$ is a parameter to indicate whether bus i has a generator connected to itself if $\mathbf{1}_i^g = 1$ or not if $\mathbf{1}_i^g = 0$. Constraints (1d) denote that for line $e = (i_e, j_e)$, if both ends in the same island, $y_e = 1$, and if two ends are in different islands, $y_e = 0$, which means that line e can be switched off under a contingency state for islanding operations. For each island, constraints (1e) ensure all buses are connected to the generating bus inside that island, and the details will be discussed in the following section. The last constraints (1f) are binary requirements for x_{ik}'s and y_e's. Therefore, the kth predefined island includes buses $\{i \in V : x_{ik} = 1\}$, which includes at least one generating bus.

The objective function $\sum_{c \in \mathcal{C}} Prob(c) \cdot Q(x, c)$ denotes the expected load shedding cost over all contingency scenarios, with contingency c having a load shedding cost $Q(x, c)$ under the islands formed by x, and is decided in the second stage. The second stage model is used for islanding operations in case of a contingency $c \in \mathcal{C}$, and can be formulated as follows under contingency c decided by $d^{(c)}$:

$$Q(x, c) = \min \sum_{i \in V} C_i s_i^{(c)} \tag{2a}$$

$$s.t. f_e^{(c)} = B_e(\theta_{i_e}^{(c)} - \theta_{j_e}^{(c)}) z_e^{(c)} (1 - d_e^{(c)})(1 - d_{i_e}^{(c)})(1 - d_{j_e}^{(c)}), \forall e \tag{2b}$$

$$-F_e z_e^{(c)} (1 - d_e^{(c)})(1 - d_{i_e}^{(c)})(1 - d_{j_e}^{(c)}) \leq f_e^{(c)} \leq F_e z_e^{(c)} (1 - d_e^{(c)})(1 - d_{i_e}^{(c)})(1 - d_{j_e}^{(c)}), \forall e \tag{2c}$$

$$\sum_{g: i_g = i} p_g^{(c)} + \sum_{e: j_e = i} f_e^{(c)} = (D_i - s_i^{(c)}) + \sum_{e: i_e = i} f_e^{(c)}, \forall i \tag{2d}$$

$$0 \leq p_g^{(c)} \leq \overline{P_g}(1 - d_{i_g}^{(c)})(1 - d_g^{(c)}), \forall g \tag{2e}$$

$$0 \leq s_i^{(c)} \leq D_i, \forall i \tag{2f}$$

$$z_e^{(c)} = \sum_k ((1 - u_k^{(c)}) x_{i_e k}) \sum_k ((1 - u_k^{(c)}) x_{j_e k}), \forall e \tag{2g}$$

$$x_{i_e k} + x_{j_e k} \geq u_k^{(c)} d_e^{(c)}, \forall e, k \tag{2h}$$

$$x_{ik} \geq u_k^{(c)} d_i^{(c)}, \forall i, k \tag{2i}$$

$$x_{i_g k} \geq u_k^{(c)} d_g^{(c)}, \forall g, k \tag{2j}$$

$$\sum_k (\sum_{i \in V} s_i^{(c)} x_{ik} (1 - u_k^{(c)})) \leq \epsilon \sum_k (\sum_{i \in V} D_i x_{ik} (1 - u_k^{(c)})) \tag{2k}$$

$$\sum_{k=1}^{K} u_k^{(c)} = 1 \tag{2l}$$

$$z_e^{(c)}, u_k^{(c)} \in \{0, 1\}, \forall i, e, k \tag{2m}$$

With probability $Prob(c)$, the contingency $c \in \mathcal{C}$ includes failures of system components, which can be failed buses, transmission lines, and/or generators. In [35], several graph algorithms and interdiction methods are used to select contingency states in a power grid. In this paper, following contingency c found in [35], we use $d^{(c)}$ to indicate failed component in contingency c. If $\sum_{i \in V} d_i^{(c)} + \sum_{e \in E} d_e^{(c)} + \sum_{g \in G} d_g^{(c)} = 1$, c includes only one failed component and is a single contingency state or N-1 contingency, defined by the North American Electric Reliability Corporation (NERC, [34]); If $\sum_{i \in V} d_i^{(c)} + \sum_{e \in E} d_e^{(c)} + \sum_{g \in G} d_g^{(c)} = k \geq 2$, 2 or more failures are included in contingency c, which is referred as a N-k contingency state by NERC.

Under contingency c with one or more failed components, line switching and controlled load shedding are useful techniques to avoid catastrophic losses and therefore can avoid wide-area blackouts. For edges connecting buses in predetermined different islands, denoted by $E' = \{e \in E : y_e = 0\}$, under a contingency state c, edge $e \in E'$ can be properly switched off to isolate an island with failures, while the other parts of the grid can be whole or individual islands, and are still operational with an acceptable load shedding amount.

In formulation (2a)–(2m) for contingency state c, the objective (2a) is to minimize the penalty cost for load shedding under contingency state c. Constraints in (2b)–(2c) includes two cases for each line e: if the line is failed ($d_e^{(c)} = 1$) or one of its two ends ($d_{i_e}^{(c)} = 1$ or $d_{j_e}^{(c)} = 1$), or the line is switched off ($z_e^{(c)} = 0$), the flow on line e is 0 ($f_e^{(c)} = 0$); if the line and its two ends are not failed ($d_e^{(c)} = d_{i_e}^{(c)} = d_{j_e}^{(c)} = 1$), and the line is not switched off ($z_e^{(c)} = 1$), the flow on the line is $f_e^{(c)} = B_e(\theta_{i_e}^{(c)} - \theta_{j_e}^{(c)})$ and within its capacity limits $[-F_e, F_e]$, following the Kirchhoff's law. Constraints in (2d) present the balance constraints for each bus, the in-flow plus the generation levels from its connected generators are equal to out-flow plus the satisfied demand amount $D_i - s_i^{(c)}$. Constraints in (2e) ensure generation limits if both the generator itself and its connected bus are not failed, or the case that the generation is 0 if one of them is failed. Similarly, constraints (2f) ensure that the load shedding

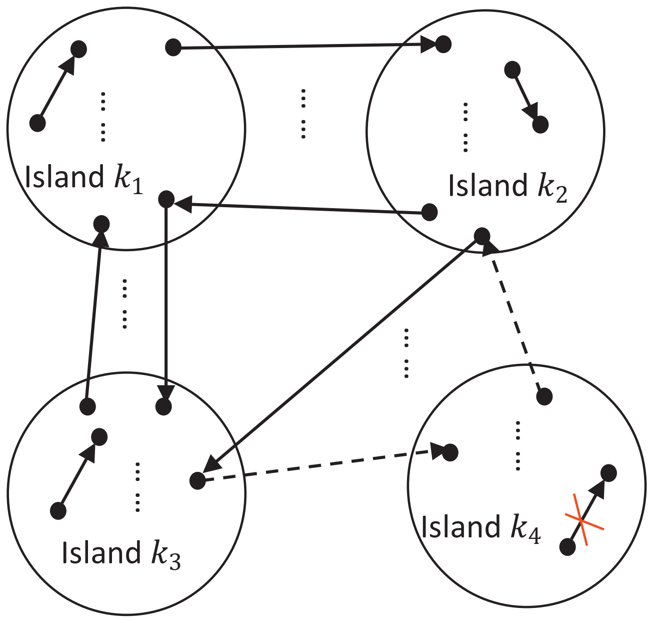

Fig. 1. Islanding operations under a contingency state (Island k_4 has failed components).

amount cannot exceed the demand amount. Fig. 1 is used to explain constraints in (2g)–(2m). In this figure, we assume that all failed components in a contingency are isolated in island k_4, while all other islands are operational.

As defined above, the constraint (2l) with binary requirements in (2m) ensures that there is exactly one failed island under contingency state c and it should be isolated to avoid the cascading blackout.

For $e \in E$, constraint (2g) includes these cases: (i) if two ends are in the same operational island, e.g., k_1, which is operational under c, constraint (2g) becomes $z_e^{(c)} = (1-u_{k_1}^{(c)})(1-u_{k_1}^{(c)}) = 1$; (ii) if two ends of e in different islands, e.g., k_1, k_2, both of which are operational, constraint (2g) becomes $z_e^{(c)} = (1-u_{k_1}^{(c)})(1-u_{k_2}^{(c)}) = 1$; (iii) if one end of e is in an operational island, e.g., k_2, and the other end is in a failed island, e.g., k_4, constraint (2g) becomes $z_e^{(c)} = (1-u_{k_2}^{(c)})(1-u_{k_4}^{(c)}) = 0$, which implies $z_e^{(c)} = 0$, e.g., the lines connected to a failed island are switched off to isolate it; (iv) if both ends are in the same failed island, e.g., k_4, which is failed under c, constraint (2g) becomes $z_e^{(c)} = (1-u_{k_4}^{(c)})(1-u_{k_4}^{(c)}) = 0$, which will imply all 0 flows in the failed island. Therefore, constraints (2g) imply that during a contingency state, the grid is divided into two parts, one part includes a failed island (no demand can be satisfied as lines inside this island have zero flow), and all other islands are operational. If the operational islands are originally connected, the non-failed part will be working as a whole part; if they are originally not connected, the non-failed part will be operated as individual islands.

Next, the constraints in (2h)–(2j) ensure that at least one end of a failed line is in the failed part, the failed buses/generators stay in the failed part. The constrain (2k) presents the controlled load shedding requirements: at most ϵ fraction of the whole demand in all operational parts can be shed.

In summary, the second stage presents an islanding operation under a severe contingency scenario, while the switched off lines between failed and operational parts are determined in the first stage. Once, some multiple failures happen in the power system, the operator will first identify the failed island and then switch off lines connected to this island to isolate it following the predetermined islanding partition of the power grid. To isolate this failed part, the large-scale and cascading blackout can be avoided.

2.3. Discussions on the islanding operations model

2.3.1. Connectivity constraints for each island

As discussed above in the first-stage constraints in Section 2.2, the kth island is formed by buses $\{i \in V : x_{ik} = 1\}$, and the transmission lines induced by them. To ensure each island is connected, following the same method in [15] to ensure the connectivity of islands, the multi-commodify flow constraints for (1e) are given as follows:

$$\begin{cases} x_{i_k k} = 1, \forall k \\ \sum_{e:i_e=i_k} v_{ek} = \sum_{i \in V} x_{ik} - 1, \forall k \\ \sum_{e:i_e=i} v_{ek} + x_{ik} = \sum_{e:j_e=i} v_{ek}, \forall k, i \in V, i \neq i_k \\ 0 \le v_{ek} \le x_{i_e k} \sum_{i \in V} x_{ik}, \forall e, k \\ 0 \le v_{ek} \le x_{j_e k} \sum_{i \in V} x_{ik}, \forall e, k \end{cases} \tag{3}$$

where we identify a root i_k for island k (usually a generating bus can be chosen as the root) and v_{ek} denotes the type k flow on line e. Here, "$\forall e$" indicates that for all $e \in E$ and all reverse edges of edges in E.

The first set of constraints in (3) ensures that root $i_k \in V$ is in island k. The second set ensures that type k flow from root is equal to the size of island $k-1$, while the third ensures that each bus in island k consumes 1 unit of type k flow. The last two sets ensure the lower and upper bounds of type k flow on each line. Therefore, by all constraints (3) of multi-commodity flow, the connectivity of all islands is guaranteed.

2.3.2. Lines between/inside operational islands

In constraints (2g), we switch off all lines connected to a failed island and also lines inside the failed island. For lines between two operational islands, we switch on all of them. The following constraints will include the cases that lines connecting operational islands can be switched off during contingency states.

$$z_e^{(c)} \le \left(1 - \frac{1}{2}\sum_k u_k^{(c)}(x_{i_e k} + x_{j_e k})\right) + y_e, \forall e \tag{4}$$

For $e \in E$, constraint (4) indicates that: (i) if two ends are in same operational island, e.g., k_1, which is operational under c, constraint (4) becomes $z_e^{(c)} \le (1-((1/2)(u_{k_1}^{(c)}(1+1)))) + 1 = 2$, which is redundant (in fact, $z_e^{(c)} = 1$ in this case by (4)); (ii) if two ends of e in different islands, e.g., k_1, k_2, both of which are operational, constraints (4) becomes $z_e^{(c)} \le (1-((1/2)(u_{k_1}^{(c)} + u_{k_2}^{(c)}))) + 0 = 1$, which is also redundant (line e can be switched on or off if necessary); (iii) if one end of e is in an operational island, e.g., k_2, and the other end is in a failed island, e.g., k_4, constraints (4) becomes $z_e^{(c)} \le (1-((1/2)(u_{k_2}^{(c)} + u_{k_4}^{(c)}))) + 0 = 1-((1/2)(0+1)) = (1/2)$, which implies $z_e^{(c)} = 0$, e.g., the lines connected to a failed island are switched off to isolate it; (iv) if both ends are in the same failed island, e.g., k_4, which is operational under c, constraint (4) becomes $Z_e^{(c)} \le (1 - \frac{1}{2}(u_{k_4}^{(c)}(1+1))) + 1 = 1$, which is redundant.

Additionally, if we relax the constraints in (2g) from equality to "$\le$" type, lines inside operational islands can also be switched off if necessary.

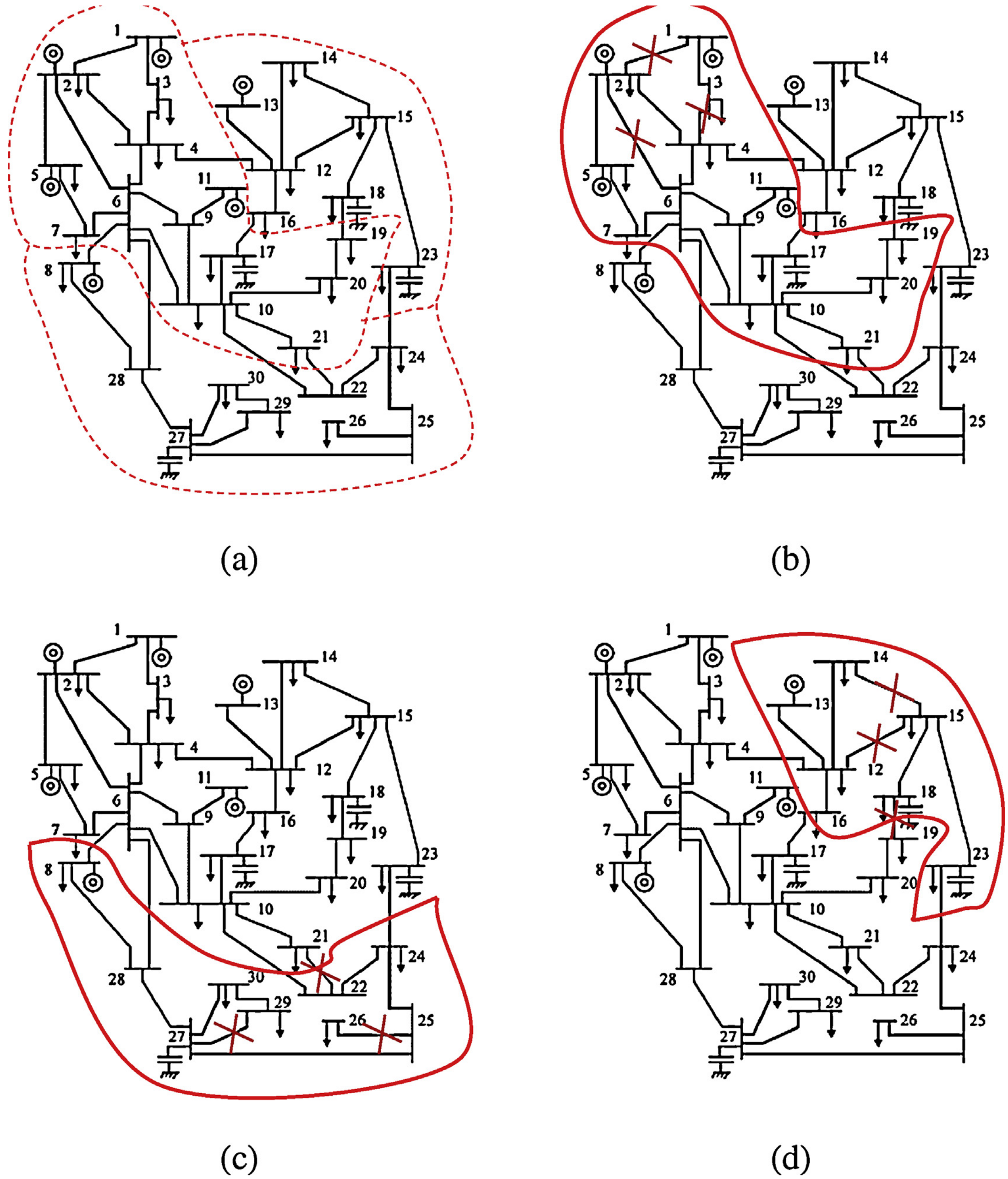

Fig. 2. Islands and islanding operations under contingency states consisting of failures only on transmission lines.

2.3.3. Valid inequalities for the islanding operations model

In this part, some valid inequalities are presented for the two-stage stochastic programing model of the islanding operations.

The inequalities

$$z_e^{(c)} \le 1 - d_e^{(c)}, \forall c, e \tag{5}$$

implying the "off" status of failed lines, are valid for the islanding operations.

If there are no root requirements (in connectivity constraints) for each island, the anti-cycle inequalities

$$x_{11} = 1, x_{21} + x_{22} = 1, \cdots, x_{K1} + x_{K2} + \cdots + x_{KK} = 1, \tag{6}$$

are valid for the islanding operations. Here, to avoid degeneracies of assignment, we assign the first bus to the first island, the second bus to either the first or second island, till the *K*th bus should be in one of the first *K* islands.

2.3.4. Linearization of nonlinear terms

In the formulation (1) and (2) for each contingency state $c \in \mathcal{C}$, there are some nonlinear terms, such as $x_{i_e k} x_{j_e k}, \theta_{i_e} z_e^{(c)}, u_k^{(c)} x_{i_e k}, u_k^{(c)} x_{j_e k}, s_i^{(c)} x_{ik}(1 - u_k^{(c)})$, they all fall into the following two types:

- 4.1) $y = w \cdot x$ for one continuous variable x and a binary w, where x has lower and upper bounds L, U, respectively, i.e., $L \le x \le U$. Then $y = w \cdot x$ can be linearized by

$$\begin{cases} y \ge Lw \\ y \le Uw \\ y \ge x - U(1-w) \\ y \le x - L(1-w) \\ w \in \{0, 1\} \end{cases}$$

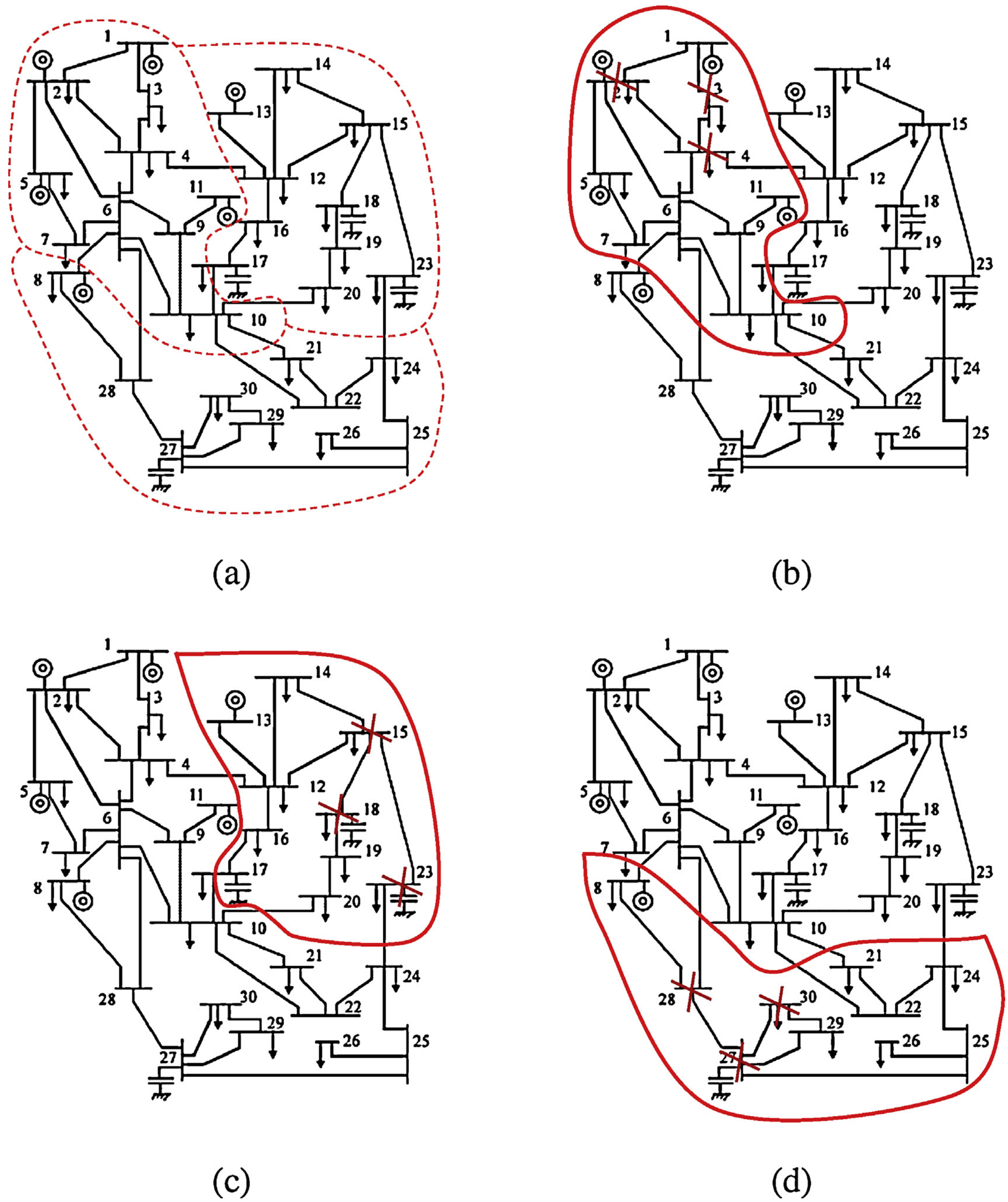

Fig. 3. Islands and islanding operations under contingency states consisting of failures only on buses.

- 4.2) $w = w_1 w_2 \ldots w_n$ for n binary variables. This product can be linearized by

$$\begin{cases} w \le w_i, i = 1, 2, \ldots, n \\ w \ge \sum_{i=1}^{n} w_i - (n-1) \\ w \ge 0 \\ w_i \in \{0, 1\}, i = 1, 2, \ldots, n \end{cases}$$

Therefore, through these linearization techniques, the above two-stage mixed integer stochastic program can be reformulated as a mixed integer linear program (MILP). In the next section, we will use CPLEX to solve MILPs.

3. Numerical experiments

In this section, all MIP formulations are implemented in C++ using CPLEX 12.3 via IBM's Concert Technology library, version 2.9. All experiments were performed on a Linux workstation with 4 Intel(R) Core(TM)2 CPU 2.40GHz processors and 8 GB RAM.

First, we perform our numerical experiments on a modified IEEE 30-Bus system (see [36]). This test system was modified to have a total generation capacity of 330 MW from 6 generators and a total demand 137.5 MW on 30 buses. In Figs. 2–5, we consider different types of severe contingency set $\mathcal{C}$. More precisely, each contingency set has 3 contingency states with equal probabilities and each state has at least three failures to indicate the extreme system operation conditions and in which system security requirements may not be satisfied by conventional $N-1$ security criteria. Additionally, we assume $K = 3$ and $\epsilon = 0.1$. As discussed in Sections 1 and 2, we

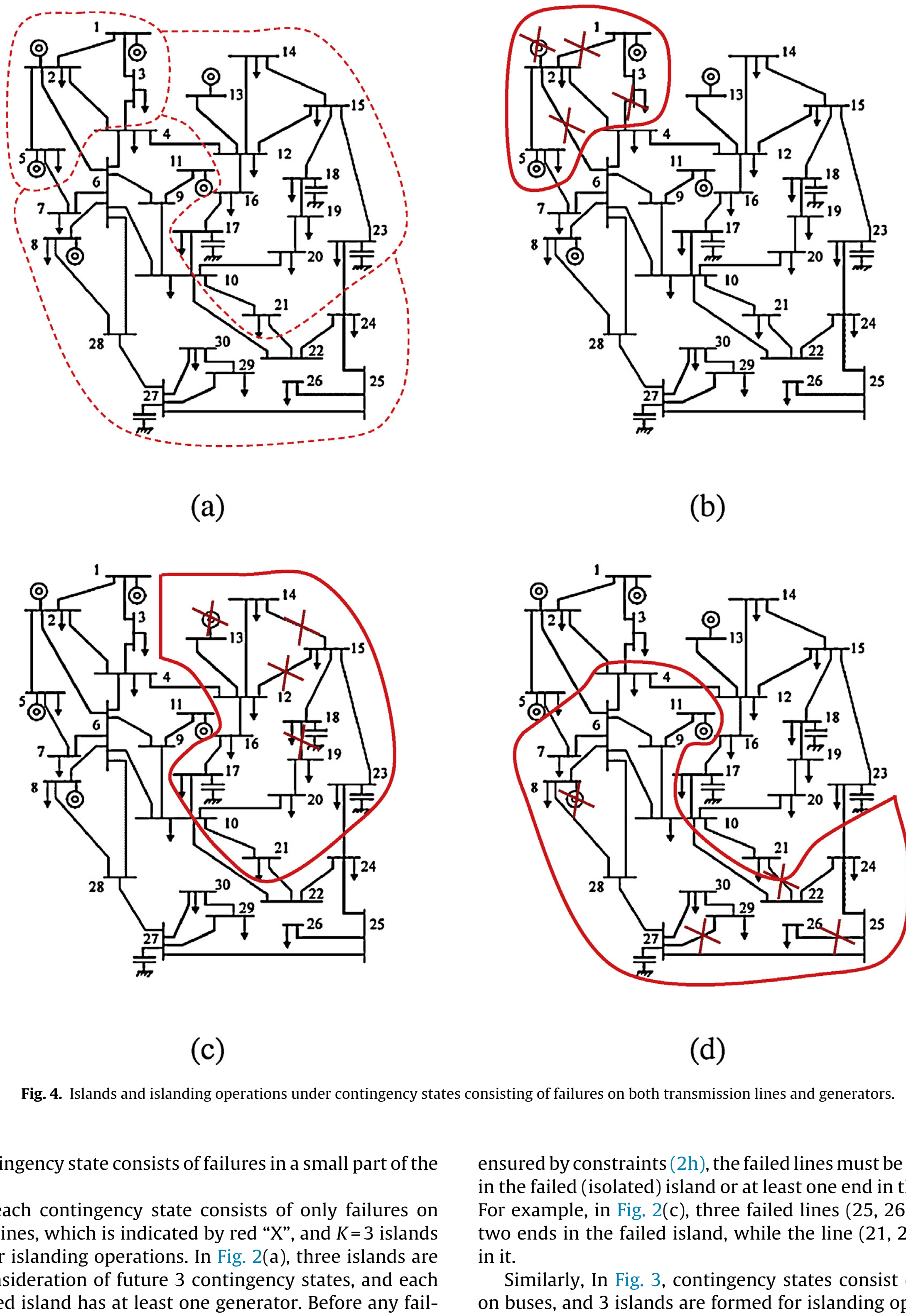

Fig. 4. Islands and islanding operations under contingency states consisting of failures on both transmission lines and generators.

assume a contingency state consists of failures in a small part of the grid.

In Fig. 2, each contingency state consists of only failures on transmission lines, which is indicated by red "X", and $K = 3$ islands are formed for islanding operations. In Fig. 2(a), three islands are formed in consideration of future 3 contingency states, and each pre-determined island has at least one generator. Before any failures happen, the system operates as a whole part, and in case of a contingency with failures on lines (1, 2), (2, 6) and (3, 4), an island is formed to isolate these failures by switching off lines (4, 12), (11, 16), (16, 14), (18, 19), (21, 22), (10, 22), (6, 28), (7, 28), as shown in Fig. 2(b). Outside the isolated island, the reminder of the system operates normally with at least $1-\epsilon = 90\%$ of the total demand is satisfied. Similarly, In Figs. 2(c) and 2(d), two other islands are isolated to respond different contingency states, (21, 22), (25, 26) and (27, 29), and (12, 15), (14, 15) and (18, 19) respectively. As ensured by constraints (2h), the failed lines must be either two ends in the failed (isolated) island or at least one end in the failed island. For example, in Fig. 2(c), three failed lines (25, 26), (27, 29) have two ends in the failed island, while the line (21, 22) has one end in it.

Similarly, In Fig. 3, contingency states consist of only failures on buses, and 3 islands are formed for islanding operations under different contingency states. Complex contingency states are considered in Figs. 4 and 5. In Fig. 4, 3 islands are formed and islanding operations under 3 contingency states, each of which consists of failures on both transmission lines and generators. In Fig. 5, each contingency state consists of all types of failures on transmission lines, generators and buses.

In the above numerical experiments, we assume $K = 3$ islands are formed in consideration of 3 future contingency states. As discussed in Section 2, the number of islands is bounded by the number

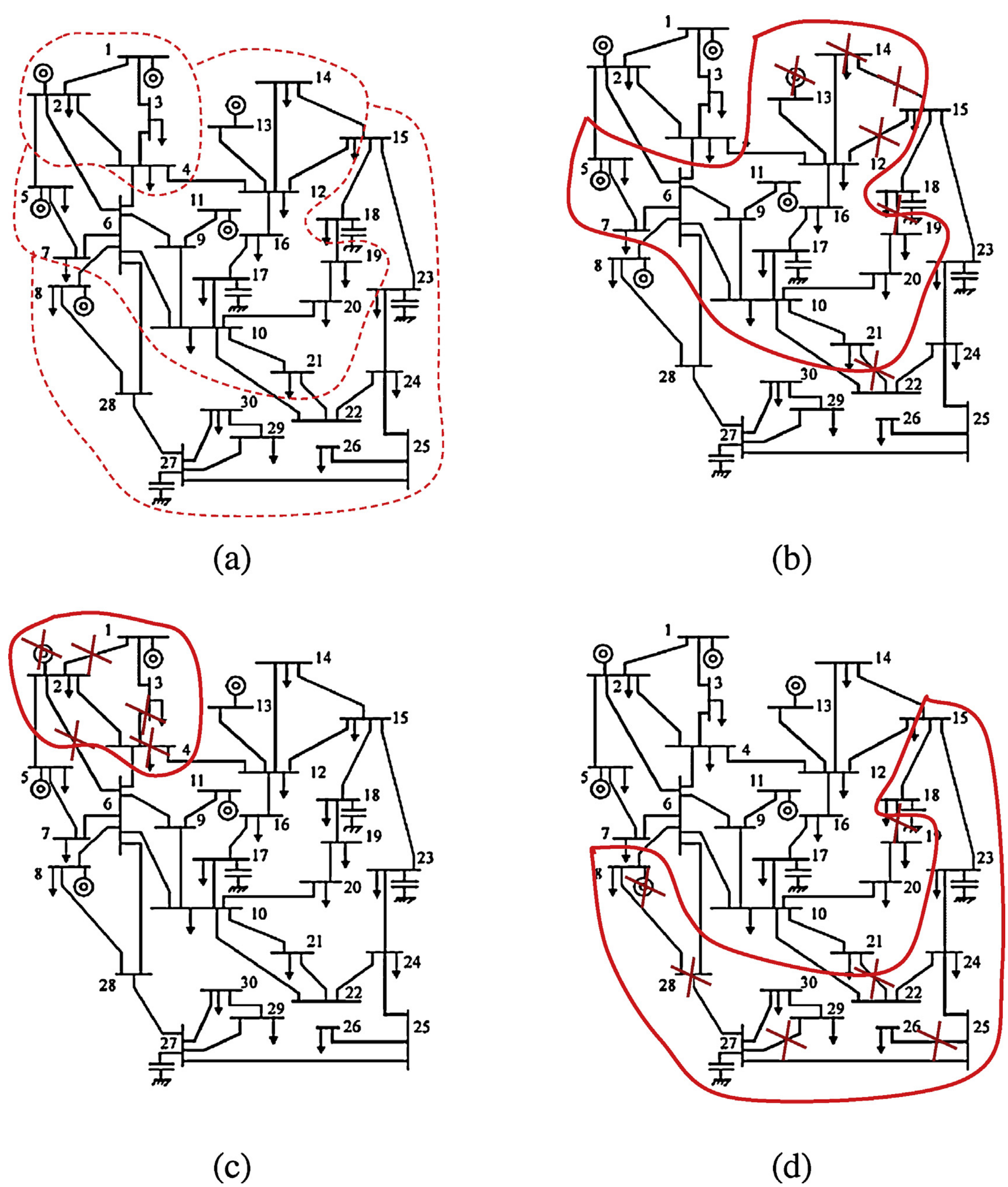

Fig. 5. Islands and islanding operations under contingency states consisting of failures on transmission lines, generators, and buses.

of generating buses. In the following, we perform the numerical experiments to find the relations between expected load shedding costs and the number of islands. In Fig. 6, we consider the relations in many cases, such as for different numbers of contingency states, the different types of failures in contingency states. In most cases, the choice for the number of islands, as the number of contingency states +1 (i.e., $|\mathcal{C}|+1$), has the least cost. It suggests that the choice K as the minimum of $|\mathcal{C}|+1$ and the number of generating buses be the best.

Based on numerical experiments on the IEEE 30-Bus system, within one-hour limit, we also perform numerical experiments on other modified test systems regarding different parameters (see Table 4). Additionally, by the proposed two-stage stochastic programming model in Section 2, all islands are predefined in the first stage, in consideration of future failures (contingencies) in the second stage. In practice, the system operator can predefine the islands offline, and the second stage decisions to isolate failed part can

Table 4
Performances of computational complexity.

Name	$\|V\|$	$\|E\|$	$\|G\|$	$\|\mathcal{C}\|$	K	Computational time (seconds)
IEEE 30-Bus	30	41	11	1	2	0.21
				3	2	8.46
					3	44.10
				5	2	51.05
					3	351.08
					4	380.73
					5	3110.18
IEEE 57-Bus	57	90	22	1	2	26.23
				3	2	300.92
					3	3938.71
				5	2	2002.72
					3	>3600.00
RTS-96	73	108	99	1	2	29.38
				3	2	2113.37
					3	>3600.00

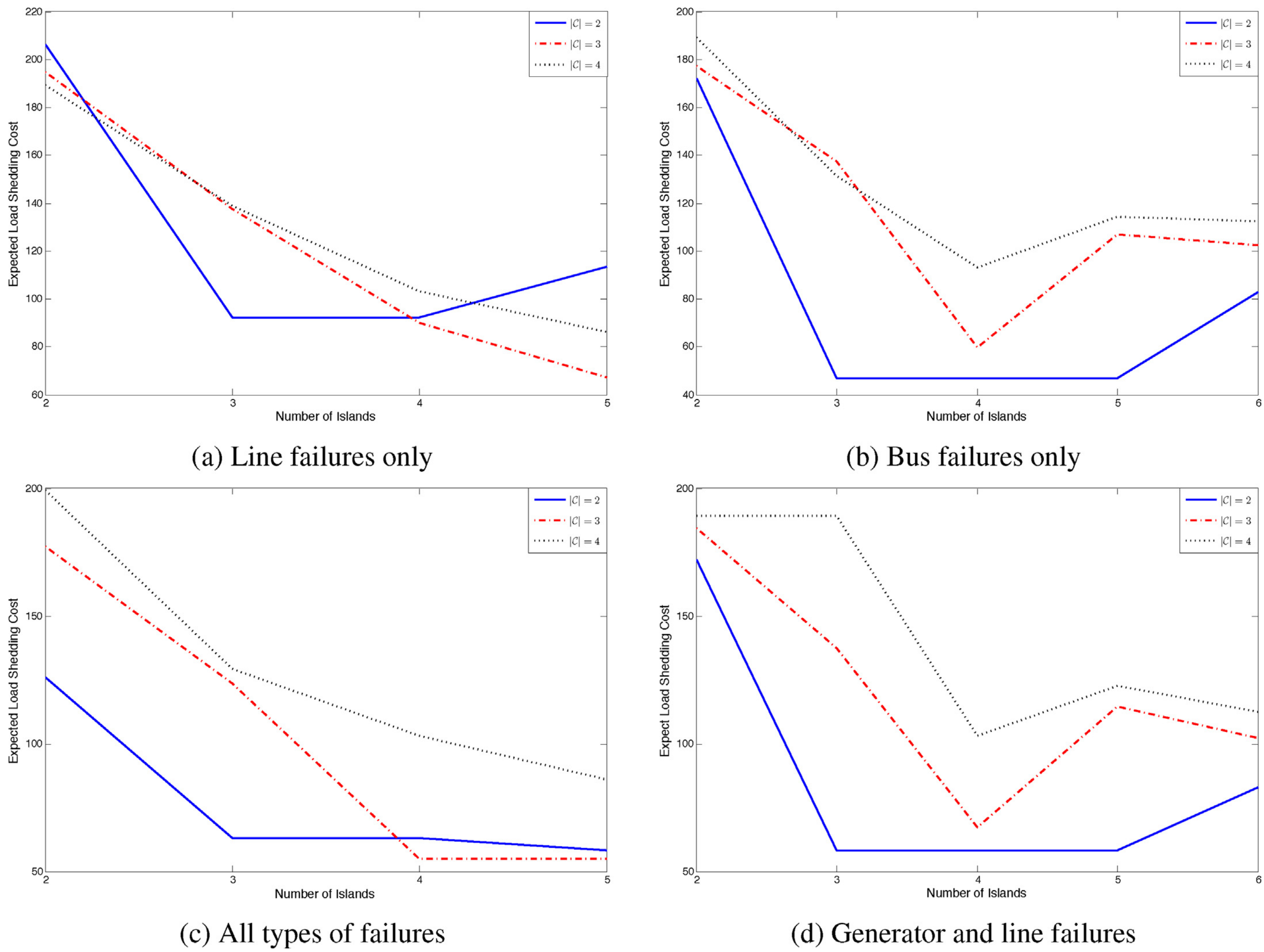

Fig. 6. Expected load shedding cost versus number of islands ($|\mathcal{C}|$ denotes the number of contingency states considered in an experiment).

be performed immediately once we have decisions from the first stage.

4. Conclusions

In this paper, a two-stage stochastic mixed integer program is proposed for optimal islanding of power grids considering severe contingencies. In the first stage, the optimal islanding is defined simulating the severe multiple contingency scenarios in the second stage. After the realization of a contingency in the second stage, which includes multiple failed components of the power grid, the islands are divided into one non-operational island and the reminder stays operational such that the total load shedding cost is minimized. Controlled load shedding is allowed and all power flow constraints are satisfied in the operational part. The optimal islanding is robust in the sense that it hedges against all severe multiple contingency scenarios and in each case the optimal line switching operations are available. Additionally, numerical experiments suggest the number of islands should be predetermined by considering possible future severe contingency states in a power grid. Future research directions include identification of highly-probable severe contingency states leading to cascading blackouts for the set $\mathcal{C}$, and design of efficient algorithms to solve this large-scale mixed integer program.

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