AAE 590ACA: Applied Control in Astronautics

Problem Set 8

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Contents

Code Listing

Control Techniques	Optimality	Analytical Effort	Computational Effort	Convergence	Constraint Freedom
Pontryagin-based	Local Optimality	High	High	Low	Medium
Optimization-based	Local Optimality within Discrete Limits	High	Medium	High	High
Lyapunov-based	None	Medium	Low	N/A	Low

Table 1: Comparison of the general characteristics of different control methods

Problem 1a

Discuss the relative advantages and limitations of the three control techniques covered in class: (i) Pontryagin-based optimal control, (ii) optimization-based control, and (iii) Lyapunov-based control.

Solution:

A comparison of the characteristics of Pontryagin-based, optimization-based, and lyapunov-based control is given in table 1. One important caveat is that there are extensions to all of these methods that allow them to tackle larger classes of control problems with increased optimality, accuracy, and speed.

Problem 1b

Let us first consider controlling the orbit of deputy spacecraft to rendezvous with chief spacecraft. Define $\mathbf{x}_d = \begin{pmatrix}\mathbf{r}_d^T & \mathbf{v}_d^T\end{pmatrix}^T$ and $\mathbf{x}_d^* = \mathbf{x}_c \in \mathbb{R}^6$ to represent the deputy orbital state and its target (= chief orbit) in Cartesian coordinates, respectively. The control input is thruster acceleration, $\mathbf{u} \in \mathbb{R}^3$, in the ECI frame. Denote the relative state by $\delta\mathbf{x} = \mathbf{x}_d - \mathbf{x}_c$. Table 2 summarize the initial orbital elements.

Table 2: Earth dynamical parameters

parameter	Symbol	Value	Unit
Earth gravitational parameter	μ	3.9860×10^5	km^3/s^2
Earth equatorial radius	r_o	6378.1	km

Table 3: Keplerian orbital elements at epoch ($t = 0$) for deputy and chief about Earth (ECI frame)

Orbital element	Deputy	Chief	Unit
Semi-major axis	$a_d = 11500$	$a_c = 10000$	km
Eccentricity	$e_d = 0.15$	$e_c = 0.3$	-
Inclination	$i_d = 35$	$i_c = 50$	degree
Right ascension of ascending node	$\Omega_d = 50$	$\Omega_c = 50$	degree
Argument of periapsis	$\omega_d = 40$	$\omega_c = 40$	degree
True anomaly at epoch	$\nu_d = 0$	$\nu_c = 0$	degree

(b.1)

Derive the error dynamics of our system in ECI frame under the influence of $\mathbf{u}$. Solution:
Gauss planetary equation:

$$\dot{\mathbf{x}} = f_0(\mathbf{x}) + B\mathbf{u}$$
$$f_0 = \begin{pmatrix}\mathbf{v} \\ -\frac{\mu}{r^3}\mathbf{r}\end{pmatrix}, \quad B = \begin{pmatrix}0_{3\times3} \\ I_3\end{pmatrix}$$

Therefore the error dynamics are (assuming no control by the chief)

$$\delta\dot{\mathbf{x}} = \dot{\mathbf{x}_d}(\mathbf{x}_d, \mathbf{u}, t) - \dot{\mathbf{x}}_d^*(\mathbf{x}_d^*, t), \quad \delta\mathbf{v} = \mathbf{v}_d - \mathbf{v}_d^*, \quad \delta\mathbf{a} = \mathbf{a}_d(\mathbf{x}_d, t) - \mathbf{a}_d^*(\mathbf{x}_d^*, t)$$
$$\delta\dot{\mathbf{x}} = f_0(\mathbf{x}_d, t) + B\mathbf{u} - f_0(\mathbf{x}_d^*, t) = \begin{pmatrix}\delta\mathbf{v} \\ \delta\mathbf{a} + \mathbf{u}\end{pmatrix}$$

(b.2)

Consider a candidate Lyapunov function $V = \frac{1}{2}\delta\mathbf{r}^T K_r \delta\mathbf{r} + \frac{1}{2}\delta\mathbf{v}^T\delta\mathbf{v}$, where $K_r^T = K_r$ and $K_r \succ 0$. **Discuss** the positive definiteness of V, and **derive** the Lyapunov rate of this system.
Solution:
Positive Definiteness:
When $\delta\mathbf{x} = \begin{pmatrix}\delta\mathbf{r}^T & \delta\mathbf{v}^T\end{pmatrix}^T = 0$, $V = \frac{1}{2}\begin{pmatrix}0 \\ 0\end{pmatrix}^T K_r \begin{pmatrix}0 \\ 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 \\ 0\end{pmatrix}^T \begin{pmatrix}0 \\ 0\end{pmatrix} = 0$

$V = \frac{1}{2}\delta\mathbf{r}^T K_r \delta\mathbf{r} + \frac{1}{2}\delta\mathbf{v}^T\delta\mathbf{v} = \frac{1}{2}\delta\mathbf{x}^T K \delta\mathbf{x} > 0$ when $\delta\mathbf{x}_{\text{slow}} \neq 0$ therefore V is positive definite.
Lyapunov Rate

$$\dot{V} = \frac{1}{2}\delta\mathbf{x}^T K \delta\mathbf{x} = \delta\mathbf{x}^T K \delta\dot{\mathbf{x}} = \frac{1}{2}\delta\mathbf{x}^T K \delta\mathbf{x} = \delta\mathbf{x}^T K \begin{pmatrix}\delta\mathbf{v} \\ \delta\mathbf{a} + \mathbf{u}\end{pmatrix} = \delta\mathbf{v}^T(K_r\delta\mathbf{r} + \delta\mathbf{a} + \mathbf{u})$$

(b.3)

Derive a stabilizing controller such that satisfies $\dot{V} = -\delta\mathbf{v}^T P \delta\mathbf{v}$ where $P \succ 0$, and **discuss** the stability property of the controlled system based on $\dot{V}$ (Lyapunov/asymptotic? local/global?)

Solution:

Want $\dot{V} = -\delta\mathbf{v}^T P \delta\mathbf{v} = \delta\mathbf{v}^T(K_r\delta\mathbf{r} + \delta\mathbf{a} + \mathbf{u})$

$$-P\delta\mathbf{v} = K_r\delta\mathbf{r} + \delta\mathbf{a} + \mathbf{u}$$
$$\therefore \mathbf{u} = -K_r\delta\mathbf{r} - P\delta\mathbf{v} - \delta\mathbf{a}$$

The Lyapunov function is positive definite and the Lyapunov rate is negative semidefinite. The Lyapunov function is radially unbounded because if $|\delta\mathbf{x}| \to \infty \implies V \to \infty$. Also, the domain of the Lyapunov function is all of $\mathbb{R}^6$. Therefore, the Lyapuonv theorem says the system is globally stable. The Lyapunov function is continuously differentiable so the system is Lyapunov stable. Because of the semidefiniteness of the Lyapunov function, the system can't be shown to be asymptotically stable without further analysis such as Theorem 1 or Theorem 2.

(b.4)

Show the asymptotic stability of the system by applying either Theorem 1 or Theorem 2.

Solution:

I will use Theorem 1 to show that the system is asymptotically stable under the controller. Consider $\delta\dot{\delta\mathbf{x}}$ under $\mathbf{u}$ for $\delta\mathbf{x} \in \Omega$. For $\delta\mathbf{x}$ to remain in an invariant set M, $\dot{\delta\mathbf{x}}$ must be 0. Therefore,

$$\dot{\delta\mathbf{x}} = \begin{pmatrix} \delta\mathbf{v} \\ -K_r\delta\mathbf{r} - P\delta\mathbf{v} \end{pmatrix} = 0 \implies \delta\mathbf{v} = 0$$
$$\implies -K_r\delta\mathbf{r} = 0, \quad (K_r \succ 0 \implies K_r \text{ invertible}) \implies \delta\mathbf{r} = 0$$

To stay in M, $\delta\mathbf{x} = 0$. M is the only invariant set so it is the largest invariant set. The largest invariant set only contains the origin, thus, by LaSalle's Invariance Principle Theorem (Theorem 1) the system is asymptotically stable about the origin.

(b.5)

For diagonal K_r and P, **design** three types of controller gains to achieve critically, under, or over damped systems, and **report** the relation K_r and P needs to satisfy for each controller.

Solution:

If K_r and P are diagonal then the position and velocity equations in the different directions can be decoupled and all of the diagonal terms are positive (because the diagonal terms are the eigenvalues of a diagonal matrix and $K_r, P \succ 0$),

$$K_r = \begin{pmatrix} K_{r_1} & 0 & 0 \\ 0 & K_{r_2} & 0 \\ 0 & 0 & K_{r_3} \end{pmatrix}, \quad K_r = \begin{pmatrix} K_{r_1} & 0 & 0 \\ 0 & K_{r_2} & 0 \\ 0 & 0 & K_{r_3} \end{pmatrix}$$
$$\dot{\delta\mathbf{x}} = \begin{pmatrix} \delta\mathbf{v} \\ -K_r\delta\mathbf{r} - P\delta\mathbf{v} \end{pmatrix} = \begin{pmatrix} 0 & I_3 \\ -K_r & -P \end{pmatrix}\delta\mathbf{x} \implies \begin{pmatrix} \delta\dot{\mathbf{r}}_i \\ \delta\dot{\mathbf{v}}_i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -K_{r_i} & -P_i \end{pmatrix}\begin{pmatrix} \delta\mathbf{r}_i \\ \delta\mathbf{v}_i \end{pmatrix}, \ i = 1, 2, 3$$

The eigenvalues for this system are $\lambda_i = \frac{1}{2}(-P_i \pm \sqrt{P_i^2 - 4K_{k_i}})$ for $i = 1, 2, 3$ Note that because $K_{r_i}, P_i > 0$, making the real component of all of the eigenvalues negative. Therefore, from linear stability theory, the eigenvlues can be used to deteremine the behviour of the closed-loop system:

1. under damped: complex eigenvalues with negative real part so $P_i^2 - 4K_{r_i} < 0 \implies 2\sqrt{K_{r_i}} > P_i$
2. critically damped: term inside square root is zero $P_i^2 - 4K_{r_i} = 0 \implies 2\sqrt{K_{r_i}} = P_i$
3. over damped: real eigenvlues so positive square root term $P_i^2 - 4K_{r_i} > 0 \implies 2\sqrt{K_{r_i}} < P_i$

(b.6)

For both part (1.b.6) and part (1.b.7), $K = I_3$

Underdamped

Using $P_i = 0.2P_{i_{\text{crit}}} = 0.4$

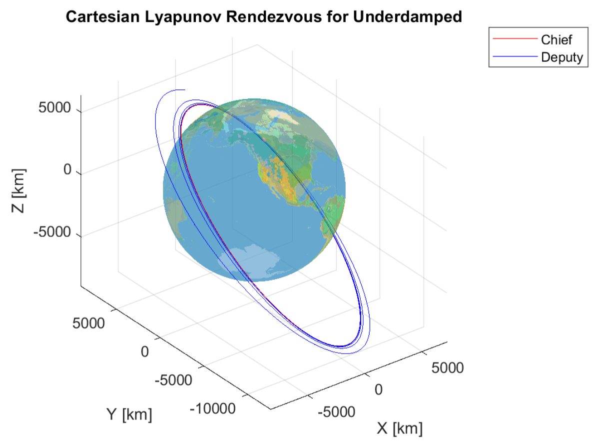

Figure 1: Underdamped cartesian Lyapunov controlled orbital rendezvous orbital plot

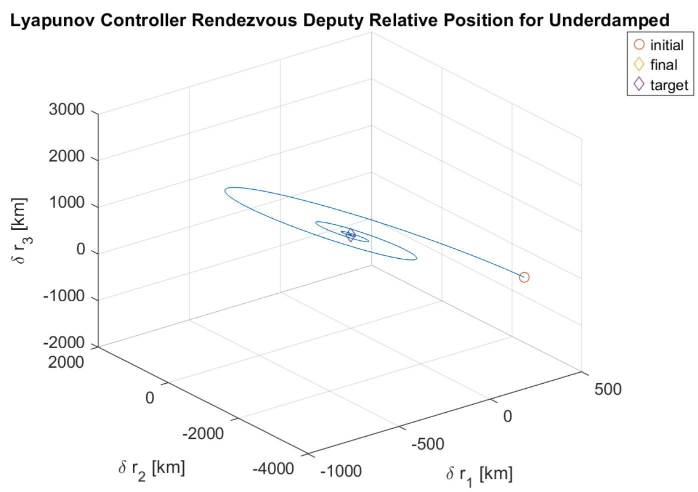

Figure 2: Undamped cartesian Lyapunov controlled orbital rendezvous relative orbit plot

Critically Damped

Using $P_i = P_{i_{\text{crit}}} = 1$

Overdamped

Using $P_i = 2P_{i_{\text{crit}}} = 4$

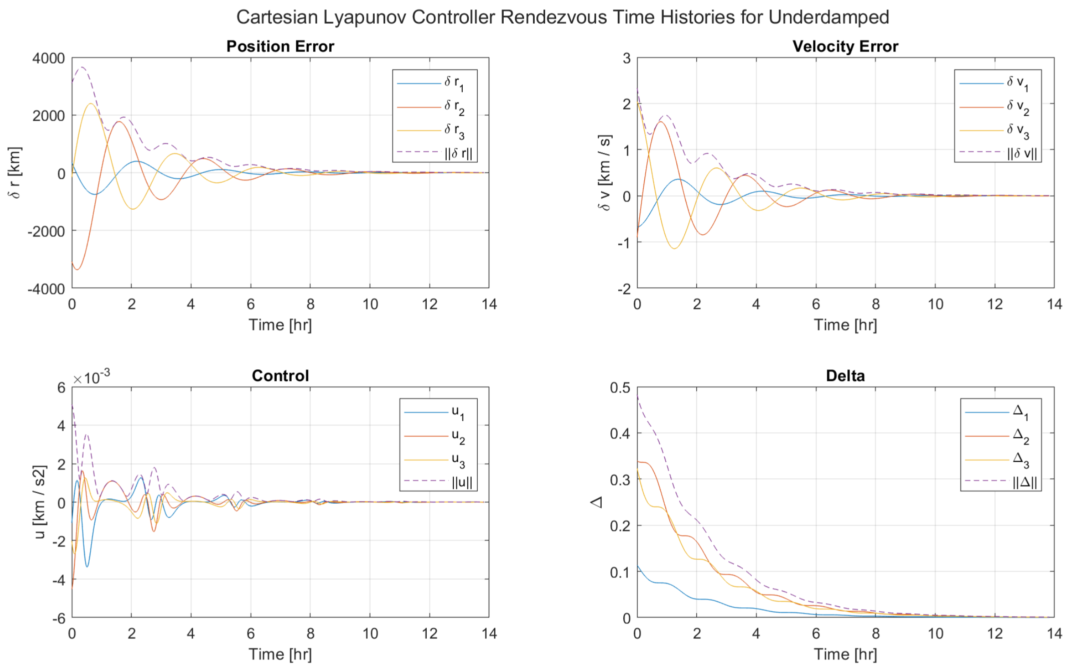

Figure 3: Underdamped cartesian Lyapunov controlled orbital rendezvous time histories

(b.7)

Bounded Underdamped

Using $P_i = 0.2P_{i_{\text{crit}}} = 0.4$

Bounded Critically Damped

Using $P_i = P_{i_{\text{crit}}} = 1$

Bounded Overdamped

Using $P_i = 1.5P_{i_{\text{crit}}} = 3$. I had to use a smaller value than for the nonbounded overdamped case because the controller would not converge to the chief.

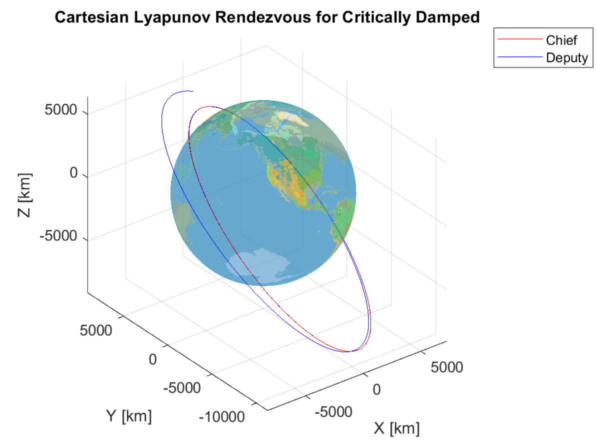

Figure 4: Critically damped cartesian Lyapunov controlled orbital rendezvous orbital plot

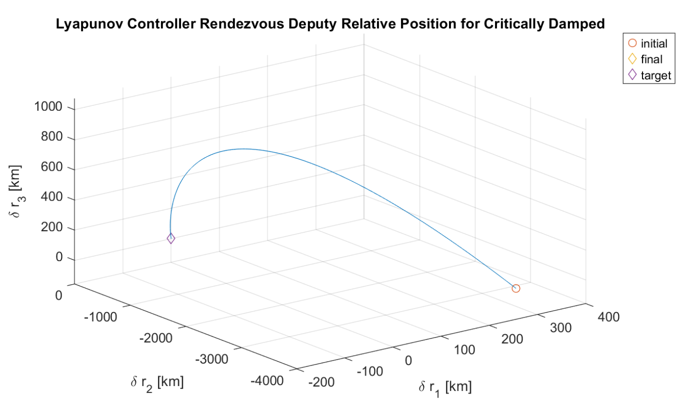

Figure 5: Critically damped cartesian Lyapunov controlled orbital rendezvous relative orbit plot

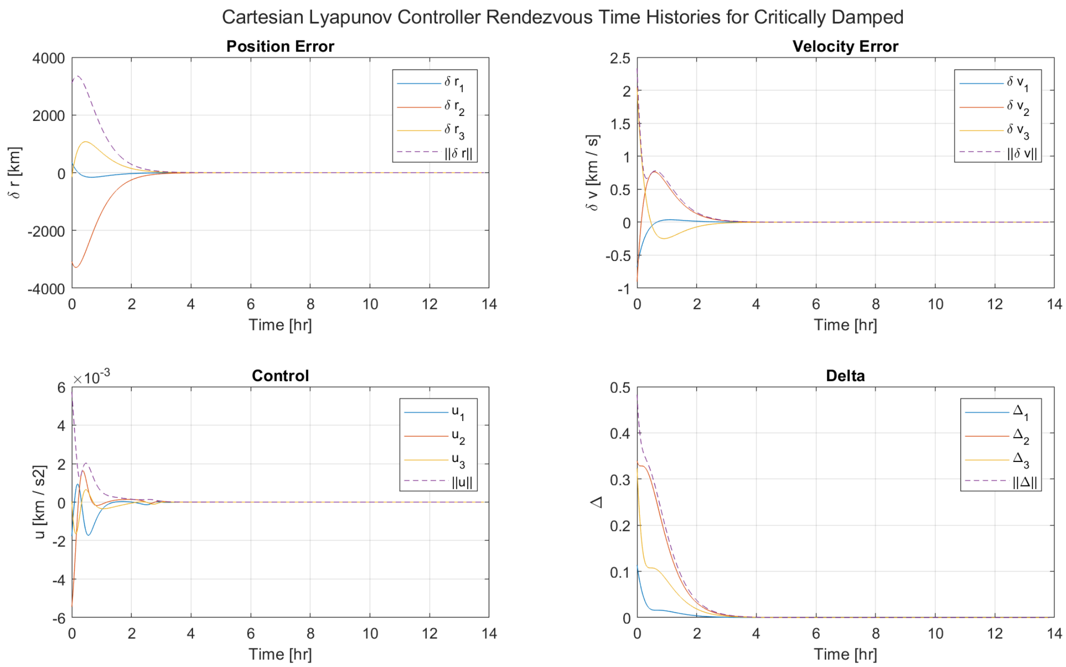

Figure 6: Critically damped cartesian Lyapunov controlled orbital rendezvous time histories

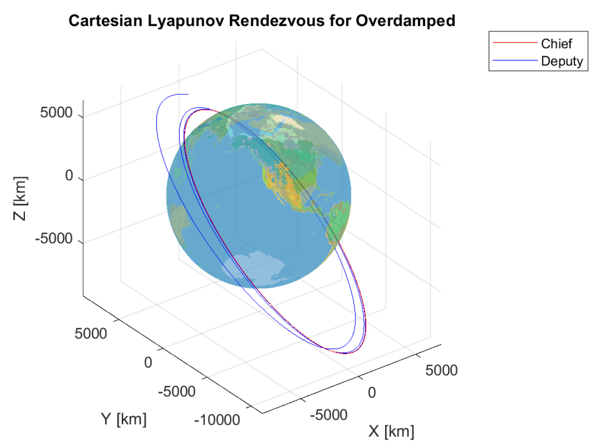

Figure 7: Overdamped cartesian Lyapunov controlled orbital rendezvous orbital plot

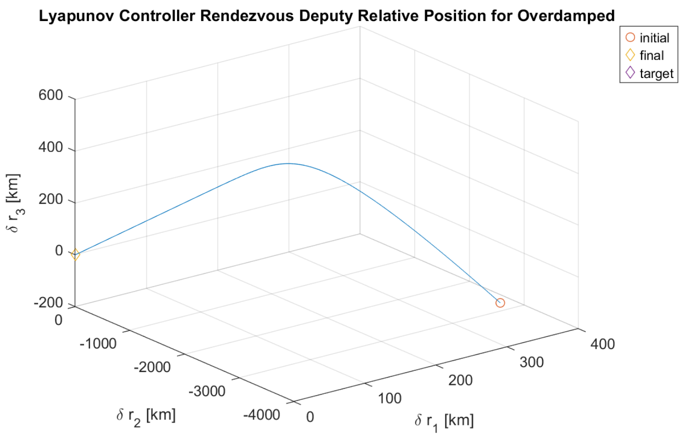

Figure 8: Overdamped cartesian Lyapunov controlled orbital rendezvous relative orbit plot

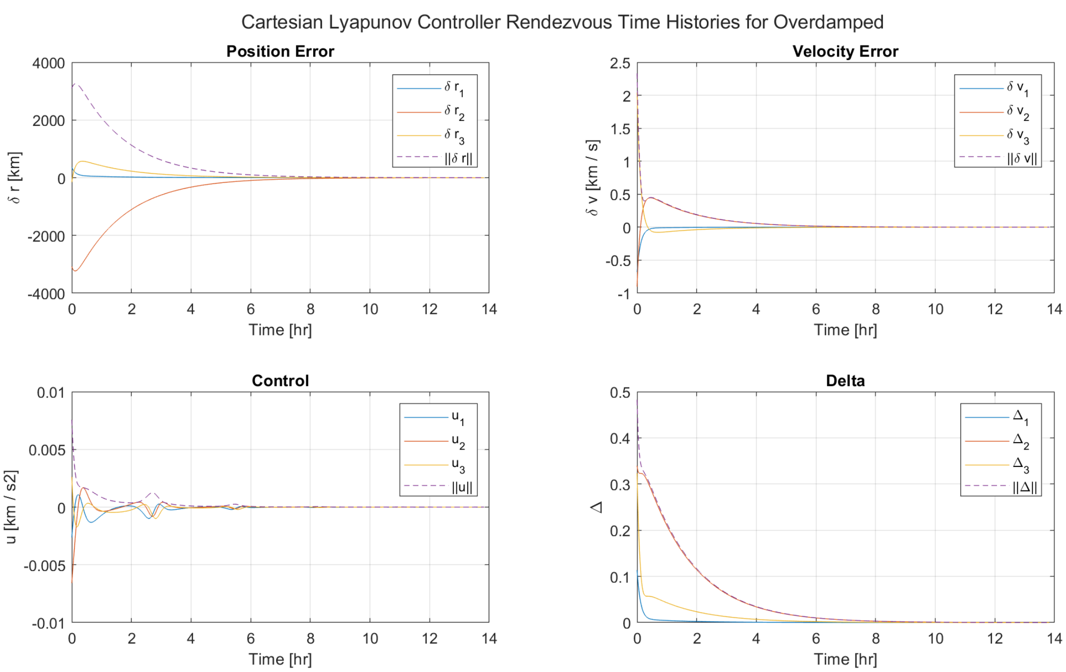

Figure 9: Overdamped cartesian Lyapunov controlled orbital rendezvous time histories

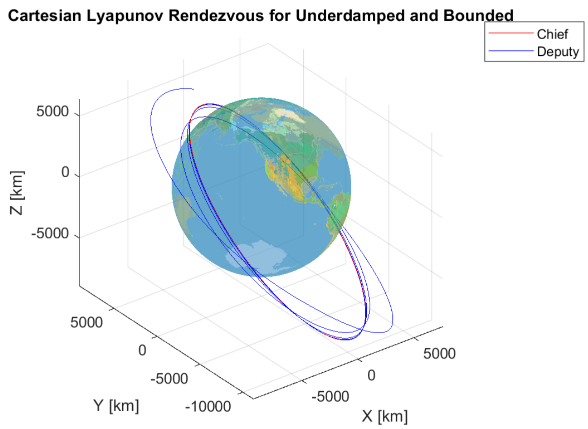

Figure 10: Bounded underdamped cartesian Lyapunov controlled orbital rendezvous orbital plot

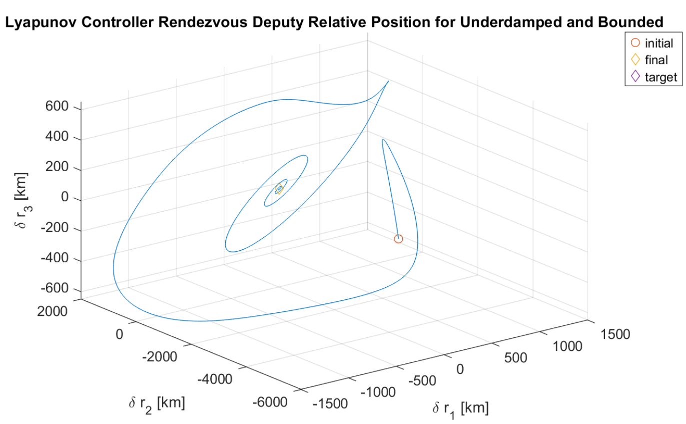

Figure 11: Bounded underdamped cartesian Lyapunov controlled orbital rendezvous relative orbit plot

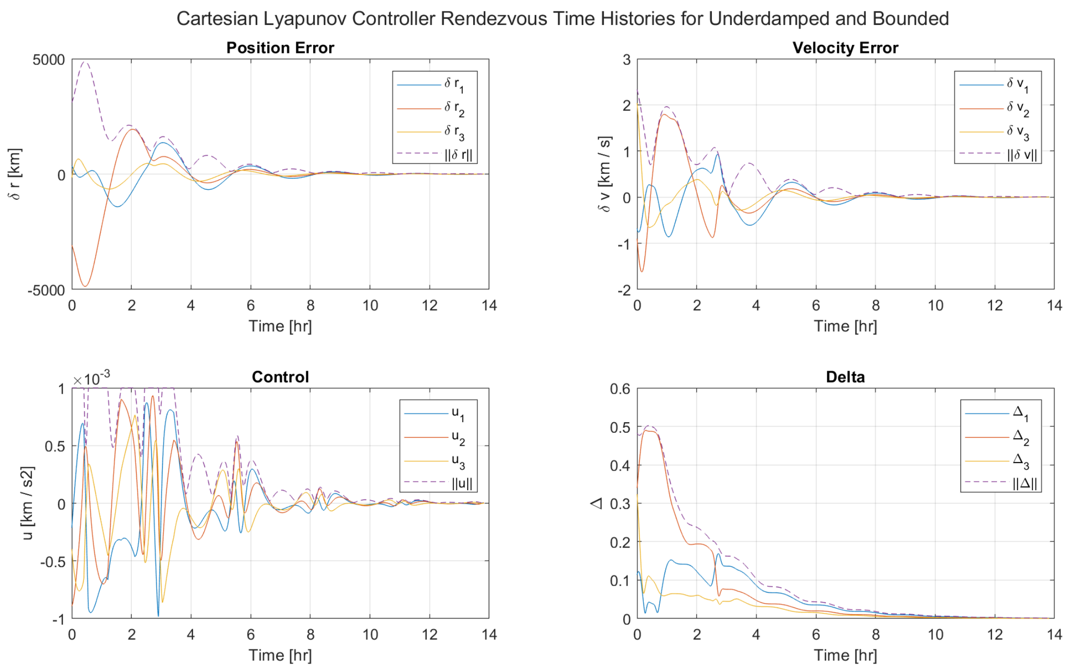

Figure 12: Bounded underdamped cartesian Lyapunov controlled orbital rendezvous time histories

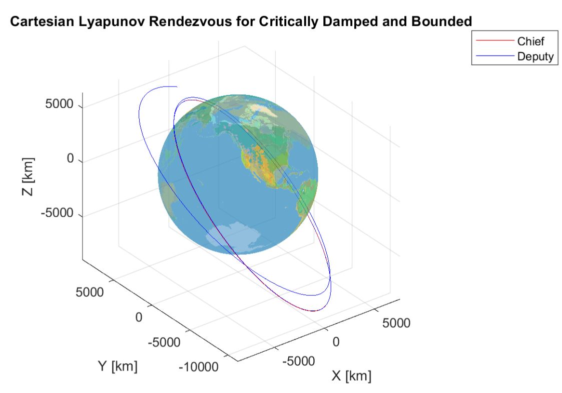

Figure 13: Bounded critically damped cartesian Lyapunov controlled orbital rendezvous orbital plot

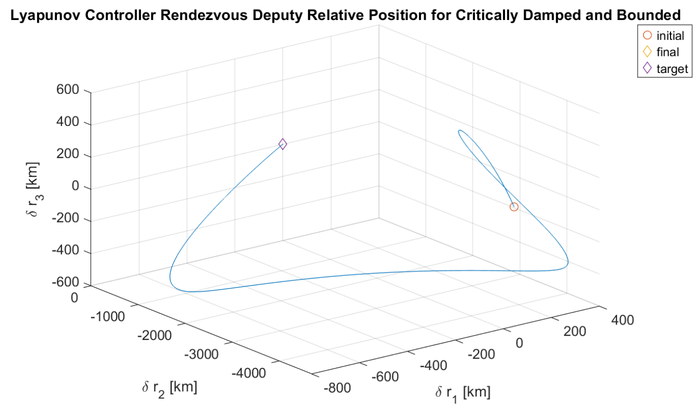

Figure 14: Bounded critically damped cartesian Lyapunov controlled orbital rendezvous relative orbit plot

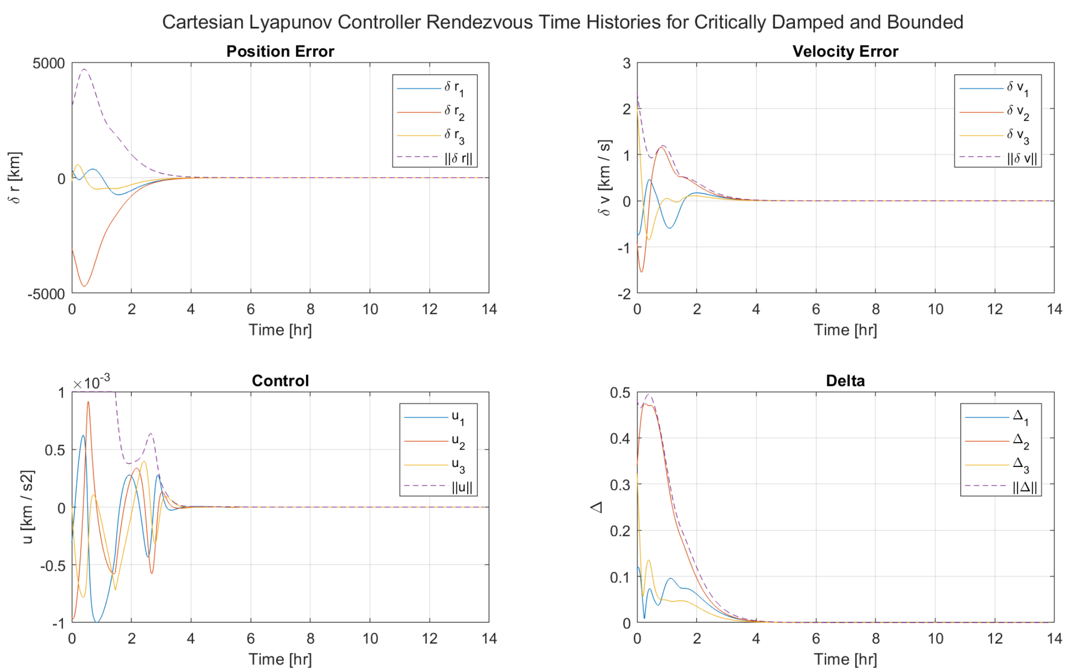

Figure 15: Bounded critically damped cartesian Lyapunov controlled orbital rendezvous time histories

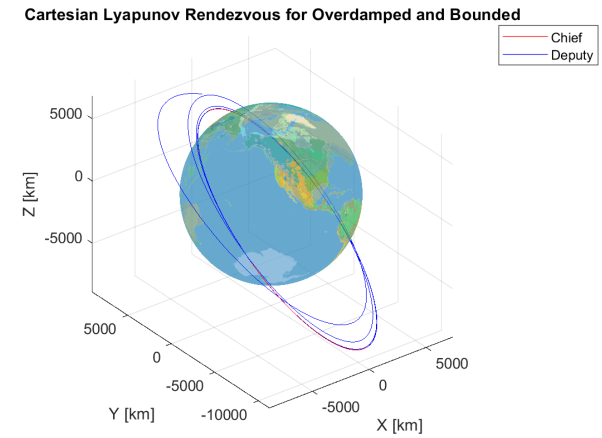

Figure 16: Bounded overdamped cartesian Lyapunov controlled orbital rendezvous orbital plot

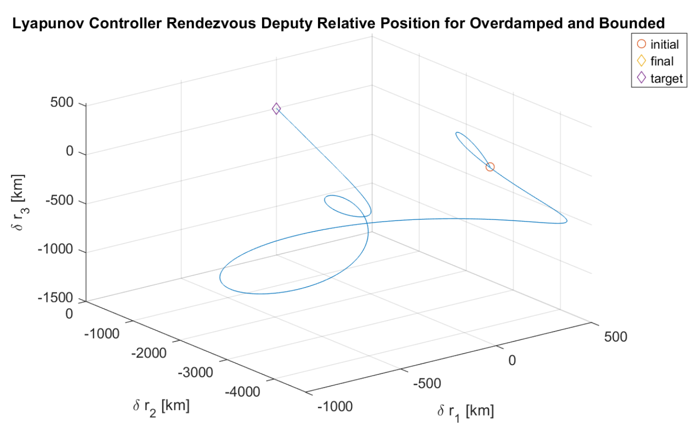

Figure 17: Bounded overdamped cartesian Lyapunov controlled orbital rendezvous relative orbit plot

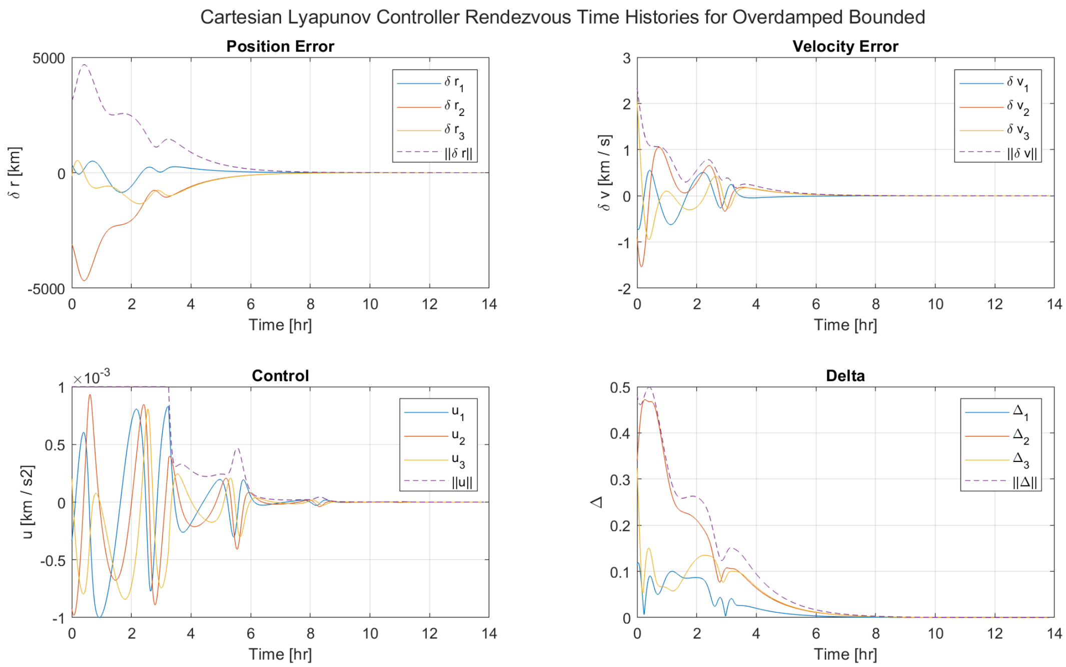

Figure 18: Bounded overdamped cartesian Lyapunov controlled orbital rendezvous time histories

Problem 1c

Let us first consider controlling the orbit of deputy spacecraft to rendezvous with chief spacecraft. Define $\mathbf{x}_d = \begin{pmatrix}\mathbf{r}_d^T & \mathbf{v}_d^T\end{pmatrix}^T$ and $\mathbf{x}_d^* = \mathbf{x}_c \in \mathbb{R}^6$ to represent the deputy orbital state and its target (= chief orbit) in Cartesian coordinates, respectively. The control input is thruster acceleration, $\mathbf{u} \in \mathbb{R}^3$, in the ECI frame. Denote the relative state by $\delta\mathbf{x} = \mathbf{x}_d - \mathbf{x}_c$. Table 2 summarize the initial orbital elements.

Table 4: Earth dynamical parameters

parameter	Symbol	Value	Unit
Earth gravitational parameter	μ	3.9860×10^5	$\mathrm{km}^3/\mathrm{s}^2$
Earth equatorial radius	r_o	6378.1	km

Table 5: Keplerian orbital elements at epoch ($t = 0$) for deputy and chief about Earth (ECI frame)

Orbital element	Deputy	Chief	Unit
Semi-major axis	$a_d = 11500$	$a_c = 10000$	km
Eccentricity	$e_d = 0.15$	$e_c = 0.3$	-
Inclination	$i_d = 35$	$i_c = 50$	degree
Right ascension of ascending node	$\Omega_d = 50$	$\Omega_c = 50$	degree
Argument of periapsis	$\omega_d = 40$	$\omega_c = 40$	degree
True anomaly at epoch	$\nu_d = 0$	$\nu_c = 0$	degree

(c.1)

Consider a candidate Lyapunov function $V = \frac{1}{2}\delta\mathbf{x}_{\text{slow}}^T K \delta\mathbf{x}_{\text{slow}}$, where $K \succ 0$. **Discuss** the positive definiteness of V, and **derive** the Lyapunov rate of this system.

Solution:

Gauss planetary equation:

$$\dot{\mathbf{x}} = f_0(\mathbf{x}) + B\mathbf{u}$$

$$f_0 = \begin{pmatrix}0\\0\\0\\0\\0\\n\end{pmatrix}, \quad B = \begin{pmatrix} 2a^2 e\sin(\nu) & \frac{2a^2p}{r} & 0 \\ p\sin(\nu) & (p+r)\cos(\nu) + re & 0 \\ 0 & 0 & r\cos(\nu+\omega) \\ 0 & 0 & \frac{r\sin(\nu+\omega)}{\sin(i)} \\ -\frac{p\cos(\nu)}{e} & \frac{(p+r)\sin(\nu)}{e} & -\frac{r\sin(\nu+\omega)}{\tan(i)} \\ \frac{bp\cos(\nu)}{ae} - \frac{2br}{a} & -\frac{b(p+r)\sin(\nu)}{ae} & 0 \end{pmatrix}$$

where n is the mean motion of the orbit $\sqrt{\frac{a^3}{\mu}}$

The error state is now only considering the slow variables because those are the ones that matter for orbit transfer control. $\mathbf{x}_{\text{slow}} = \begin{pmatrix}a & e & i & \Omega & \omega\end{pmatrix}^T$. Only looking at the slow elements gives

$$f_{0_{\text{slow}}} = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}, \quad B_{\text{slow}} = \begin{pmatrix} 2a^2 e\sin(\nu) & \frac{2a^2p}{r} & 0 \\ p\sin(\nu) & (p+r)\cos(\nu) + re & 0 \\ 0 & 0 & r\cos(\nu+\omega) \\ 0 & 0 & \frac{r\sin(\nu+\omega)}{\sin(i)} \\ -\frac{p\cos(\nu)}{e} & \frac{(p+r)\sin(\nu)}{e} & -\frac{r\sin(\nu+\omega)}{\tan(i)} \end{pmatrix}$$

Positive Definiteness:

When $\delta\mathbf{x}_{\text{slow}} = 0$, $V = \frac{1}{2}\begin{pmatrix}0\\0\end{pmatrix}^T K \begin{pmatrix}0\\0\end{pmatrix} = 0$

$V = \frac{1}{2}\delta\mathbf{x}_{\text{slow}}^T K \delta\mathbf{x}_{\text{slow}} > 0$ when $\delta\mathbf{x}_{\text{slow}} \neq 0$ because $K \succ 0$ so V is positive definite.

Lyapunov Rate

$$\dot{V} = \frac{1}{2}\delta\mathbf{x}_{\text{slow}}^T K \delta\mathbf{x}_{\text{slow}} = \delta\mathbf{x}_{\text{slow}}^T K(\delta f_{0_{\text{slow}}} + B_{\text{slow}}\mathbf{u}) = \delta\mathbf{x}_{\text{slow}}^T K B_{\text{slow}}\mathbf{u}$$

(c.2)

Derive a stabilizing controller for the system, and discuss the stability property of the controlled system (Lyapunov/asymptotic? local/global?), where assume that B_{slow}, i.e., a matrix form of Gauss planetary equations for the slow variables, is full rank.

Solution:

Want $\dot{V} = \delta\mathbf{x}_{\text{slow}}^T P B_{\text{slow}}\mathbf{u} = -\delta\mathbf{x}_{\text{slow}}^T \delta\mathbf{x}_{\text{slow}}$

$$-P\delta\mathbf{x}_{\text{slow}} = K B_{\text{slow}}\mathbf{u}$$
$$\therefore \mathbf{u} = -(B_{\text{slow}}^T K^T K B_{\text{slow}})^{-1} B_{\text{slow}}^T K^T P \delta\mathbf{x}_{\text{slow}}$$

The last step is possible because K being positive definite and $\text{rank}(B_{\text{slow}}) = 3$ so KB_{slow} is invertible.

The Lyapunov function is positive definite and the Lyapunov rate is negative semidefinite. The Lyapunov function is radially unbounded because if $|\delta\mathbf{x}| \to \infty \implies V \to \infty$. Also, the domain of the Lyapunov function is all of $\mathbb{R}^5$. Therefore, the Lyapuonv theorem says the system is globally stable. The Lyapunov function is continuously differentiable so the system is Lyapunov stable. Because of the semidefiniteness of the Lyapunov function, the system can't be shown to be asymptotically stable without further analysis such as Theorem 1 or Theorem 2.

Now looking at the Lyapunov rate for the stabilizing controller

$$\delta\mathbf{x}_{\text{slow}}^T P B_{\text{slow}} - (B_{\text{slow}}^T K^T K B_{\text{slow}})^{-1} B_{\text{slow}}^T K^T P \delta\mathbf{x}_{\text{slow}}$$
$$\text{let } \mathbf{y} = B_{\text{slow}}^T K^T \delta\mathbf{x}_{\text{slow}}$$
$$\dot{V} = -\mathbf{y}^T (B_{\text{slow}}^T K^T K B_{\text{slow}})^{-1}\mathbf{y}$$

$B_{\text{slow}}^T K^T K B_{\text{slow}}$ is invertible and positive definite so therefore $\dot{V}$ is negative definite. The Lyapunov function is positive definite and the Lyapunov rate is negative definite, therefore, the controlled system is locally asymptotically stable about the origin.

(c.3)

Under no control magnitude constraint, perform the numerical integrations of the controlled system with K = I5, and discuss the results with relevant plots. Be sure to scale the length and time units appropriately as discussed in class, and propagate the dynamics for at least 10 days.

Solution:

Assuming $K = I_5$, numerically propagating the orbit with $P = 2I_5$ results in the transfer seen in Fig 19 This controller is able to make the spacecraft's orbit converge to the target orbit in about 150 hours and it takes a total delta V of 15.3 km/s.

(c.4)

Let us then consider the same orbit transfer problem with a control magnitude constraint $||\mathbf{u}||_2 \leq u_{\max}$ with $u_{\max} = 0.1\frac{\text{m}}{\text{s}^2}$. Perform the numerical integrations of the controlled system with $K = I_5$, and discuss the results with relevant plots

Solution:

Making the controller saturate at the maximum control magnitude results in the transfer seen in Fig 20 This controller is able to make the spacecraft's orbit converge to the target orbit while obeying the control magnitude constrait in about 175 hours and it takes a total delta V of 25.7 km/s. For roughly the first 100 hours, the control input is saturated. Another interesting observation is that the bounded controller uses almost twice as much total control as the unbounded one.

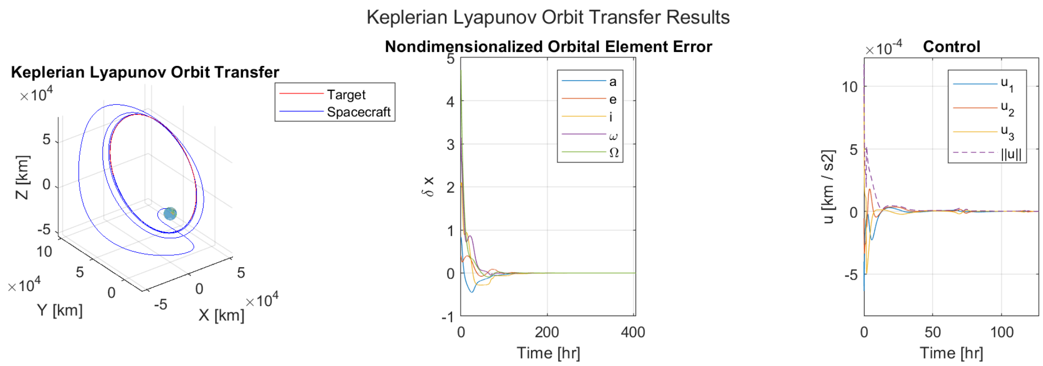

Figure 19: Keplerian Lyapunov controlled orbital transfer result plots

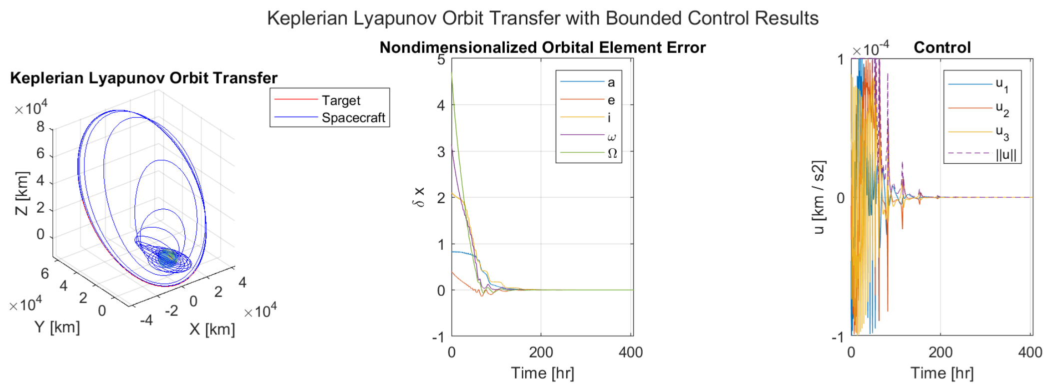

Figure 20: Keplerian Lyapunov controlled orbital transfer with control bounds result plots

(c.5)

$$\hat{V} = V + V_p$$
$$\text{Consider} V_p = wV(\mathbf{x}_{\text{slow}})P(g(\mathbf{x}_{slow})) \implies \hat{V} = V(1 + wP)$$

where w is the penalty weight, g is the constraint function, and P is the penalty function

The Lyapunov rate is

$$\dot{\hat{V}} = \dot{V}(1 + wP) + wV\dot{P} = ((1 + wP)\frac{\partial V}{\partial \mathbf{x}_{\text{slow}}} + wV\frac{\partial P}{\partial g}\frac{\partial g}{\partial \mathbf{x}_{\text{slow}}})\dot{\mathbf{x}}_{\text{slow}}$$
$$\dot{\hat{V}} = (2(1 + wP)\delta\mathbf{x}_{\text{slow}}^T K + w\delta\mathbf{x}_{\text{slow}}^T K\delta\mathbf{x}_{\text{slow}}\frac{\partial P}{\partial g}\frac{\partial g}{\partial \mathbf{x}_{\text{slow}}})B_{\text{slow}}\mathbf{u}$$
$$\dot{\hat{V}} = \delta\mathbf{x}_{\text{slow}}^T = (2(1 + wP)K + wK\delta\mathbf{x}_{\text{slow}}\frac{\partial P}{\partial g}\frac{\partial g}{\partial \mathbf{x}_{\text{slow}}})B_{\text{slow}}\mathbf{u}$$
$$\text{define } L = (2(1 + wP)K + wK\delta\mathbf{x}_{\text{slow}}\frac{\partial P}{\partial g}\frac{\partial g}{\partial \mathbf{x}_{\text{slow}}})B \in \mathbb{R}^{5\times 3}$$
$$\implies \dot{\hat{V}} = \delta\mathbf{x}_{\text{slow}}^T L\mathbf{u} = -\delta\mathbf{x}_{\text{slow}}^T Q\delta\mathbf{x}_{\text{slow}}$$

Assuming $Q = I_5$,

$$L\mathbf{u} = -\delta\mathbf{x}_{\text{slow}} \implies \mathbf{u} = -(L^T L)^{-1}L^T\delta\mathbf{x}_{\text{slow}} \text{ (assuming } \operatorname{rank}(L) = 3)$$
$$\implies \dot{\hat{V}} = \delta\mathbf{x}_{\text{slow}}L\mathbf{u} = -\delta\mathbf{x}_{\text{slow}}L(L^T L)^{-1}L^T\delta\mathbf{x}_{\text{slow}} \prec 0$$

The Lyapunov rate under the derived controller is negative definite and the Lyapuonv function is positive definite, therefore, the controller is locally asymptotically stabilizing.

I choose to constrain the spacecraft's orbit to have an eccentricity which is always less than or equal to the target orbit's. Using a quadratic penalty function, a constraint of $g(\mathbf{x}_{\text{slow}}) = e_{s/c} - e_{\text{target}}$ which has $\frac{\partial g}{\partial \mathbf{x}_{\text{slow}}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix}$, $w = 100$, and $k = 100$, results in Fig 21 Looking at the eccentricity of

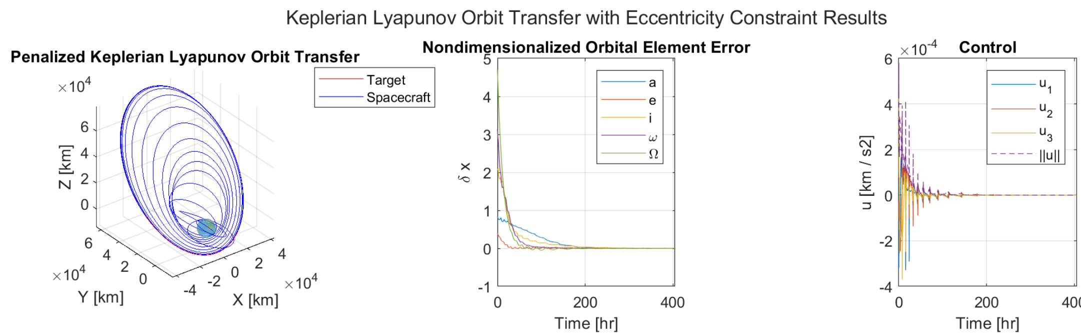

Figure 21: Keplerian Lyapunov controlled orbital transfer with eccentricity constraint result plots

the spacecraft's orbit and the target orbit in Fig 22, constraint is almost always satisfied, with clear sharp changes in the eccentricity when the constraint was almost being violated but the penalty lyapunov function kicked in. However, after around 150 hours there are some small violations of the constraint but this is to be expected as this method of constraining the transfer brings no guarentees and this solution is pretty close. The total delta V used for the constrained transfer was 23.26 km/s and it took about 200 hours to converge

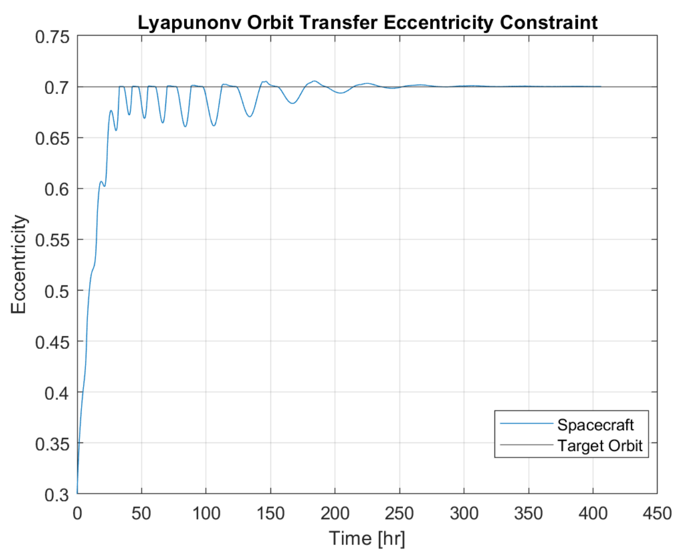

Figure 22: Eccentricity constraint plot

to the target orbit. This is longer than any of the previous parts so this shows that this constraint is quite constraining on the way the controller can influence the orbit.

(c.6)