2.a) Matrix Representation:

Question:
Write $X \in \mathrm{SE}(()2)$ as a 3×3 matrix:

$$X = \begin{bmatrix} R & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad R \in \mathrm{SO}(2),\ \mathbf{t} \in \mathbb{R}^2$$

Tuple notation: We can also write $X = (\mathbf{t}, R)$ as a compact tuple. The correspondence is:

$$(\mathbf{t}, R) \longleftrightarrow \begin{bmatrix} R & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$$

We order as $(\mathbf{t}, R)$ (translation first) to match the semi-direct product $\mathbb{T}(2) \rtimes \mathrm{SO}(2)$ convention. Both notations represent the same rigid motion: rotate by R, then translate by $\mathbf{t}$ (in the world frame).

- Derive the composition $X_1 X_2$ and inverse X^{-1} formulas using block matrix multiplication. You don't need to expand 2×2 blocks—leave products like $R_1 R_2$ as is.
- Express your results in tuple form: $(\mathbf{t}_1, R_1) \cdot (\mathbf{t}_2, R_2) = (?, ?)$ and $(\mathbf{t}, R)^{-1} = (?, ?)$.
- Implement `se2_compose(X1, X2)` and `se2_inverse(X)`.

Check your work: $(\mathbf{t}_1, R_1) \cdot (\mathbf{t}_2, R_2) = (\mathbf{t}_1 + R_1 \mathbf{t}_2,\ R_1 R_2)$ and $(\mathbf{t}, R)^{-1} = (-R^T \mathbf{t},\ R^T)$.
Solution:
Code snippet 1 shows the implementation of `se2_compose(X1, X2)` and `se2_inverse(X)`