The group SE(2) of planar rigid motions is our first example of a **semi-direct product**. This structure—where one subgroup "twists" another—is fundamental to robotics.

The Translation Group $\mathbb{T}(2)$: The set of 2D translations forms a group $\mathbb{T}(2)$ under composition. As a matrix Lie group:

$$\mathbb{T}(2) = \left\{ \begin{bmatrix} I & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} : \mathbf{t} \in \mathbb{R}^2 \right\} \cong (\mathbb{R}^2, +)$$

This group is abelian (translations commute) and isomorphic to $\mathbb{R}^2$ with vector addition. Its Lie algebra is $\mathfrak{t}(2) \cong \mathbb{R}^2$.