# AAE 590: Lie Group Methods for Control and Estimation

## Course Introduction

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Spring 2026

# Welcome to AAE 590

## Lie Group Methods for Control and Estimation

**Instructor:**

- Dr. James Goppert
- `jgoppert@purdue.edu`
- Office: ARMS 3214

**Course Details:**

- Tuesday/Thursday 1:30–2:45 PM
- Location: Physics 331
- 3 Credit Hours

**Office Hours**

Tuesday/Thursday 3:00–4:00 PM (after lecture), ARMS 3214

# Course Description

## Official Description

This course presents the mathematical theory of Lie groups with a focus on applications to aerospace systems.

**What you'll learn:**

- How to represent rotations and rigid body poses **correctly**
- Why traditional methods (Euler angles, quaternions) have problems
- Modern geometric approaches used in cutting-edge research
- Practical algorithms: geometric control, IEKF, Equivariant Filters

**Key mathematical objects:**

- Special Orthogonal Groups: $\mathrm{SO}(2)$, $\mathrm{SO}(3)$
- Special Euclidean Groups: $\mathrm{SE}(2)$, $\mathrm{SE}(3)$
- Extended Pose Group: $\mathrm{SE}_2(3)$

# Course Scope

**Lie groups appear in many fields:**

- **Quantum mechanics:** SU(2), SU(3), symmetry groups
- **Robotic manipulators:** Product of SE(3) for each joint
- **Particle physics:** Gauge symmetries, Lorentz group
- **Computer graphics:** Rotation interpolation, animation

### Our Focus: Rigid Bodies

This course focuses on **rigid body motion**—rotations (SO(3)) and poses (SE(3))—as these are most relevant for aerospace applications: spacecraft, aircraft, drones, and navigation systems.

*The mathematical foundations we develop will transfer to other domains, but our examples and applications will be aerospace-focused.*

# Why This Course?

**The problem:** Rotations and rigid body motion are **everywhere** in aerospace

- Spacecraft attitude determination and control
- Aircraft flight dynamics
- Drone navigation
- Robot manipulation and SLAM

**The challenge:** Traditional approaches have fundamental limitations

- Euler angles: gimbal lock
- Quaternions: double cover, unit constraint
- MRPs: singularity at $\pm 2\pi$

### The Solution

Lie group methods work **directly** on the space of rotations/poses, avoiding coordinate singularities entirely.

# Prerequisites

**Required background:**

- **Calculus:** Derivatives, Taylor series, basic ODEs
- **Linear Algebra:** Matrix operations, eigenvalues, vector spaces
- **Programming:** Python (Modelica will also be introduced)

**Helpful but not required:**

- Control theory (state-space, feedback)
- State estimation (Kalman filtering)
- Robotics or flight dynamics experience

**Note**

We will cover the necessary abstract algebra and differential geometry from scratch. No prior exposure to Lie groups is assumed!

# Course Learning Objectives

Upon successful completion of this course, you will be able to:

1. **Leverage Lie groups** for nonlinear control, estimation, and optimization of rigid bodies
2. **Describe properties** of abstract groups and algebras (CAIN axioms)
3. **Apply Lie theory tools:**
   - Exponential and logarithmic maps
   - Wedge/vee operators ($\wedge/\vee$)
   - Adjoint representations ($\mathrm{Ad}/\mathrm{ad}$)
4. **Model dynamics** using group affine systems
5. **Implement filters:** IEKF and Equivariant Filter
6. **Design controllers** that respect state space geometry

# What Makes This Course Different

**Traditional approach:**

- Pick a parameterization (Euler angles, quaternions)
- Derive equations in those coordinates
- Deal with singularities as special cases
- Hope nothing goes wrong...

**Lie group approach:**

- Work directly with rotation matrices / transformation matrices
- Use intrinsic geometric operations (exp, log, adjoint)
- **No singularities by construction**
- Algorithms that work uniformly across all configurations

# Course Schedule Overview

*Subject to change based on class progress*

**Foundations (Weeks 1–3):**

- W01: Motivation, Groups, Topology
- W02: SO(2), SE(2), Group Affine Intro
- W03: SE(2) Control, Stability

**3D Groups (Weeks 4–7):**

- W04–06: SO(3), Attitude Control
- W07: SE(3) Kinematics

**Estimation (Weeks 8–14):**

- W08–09: Kalman Filter, $\mathrm{SE}_2(3)$
- W10–12: IEKF
- W13–14: Equivariant Filter

**Projects (Weeks 15–16):**

- W15: Project work
- W16: Poster Session

# Detailed Schedule: First Half

| Week | Topic | Assignment |
|---|---|---|
| W01 | Motivation; Groups, Manifolds, Topology | PS01 Assigned |
| W02 | SO(2) & SE(2); Group Affine Systems Intro | PS01 Due, PS02 Assigned |
| W03 | SE(2) Control: Geometric Error, Stability Analysis | |
| W04 | SO(3): Representations, Singularities, Proofs | PS02 Due, PS03 Assigned |
| W05 | SO(3) Attitude Control: Lyapunov Stability | |
| W06 | SO(3) Almost-Global Stability; Quaternions | PS03 Due, PS04 Assigned |
| W07 | SE(3): Kinematics, Adjoint Derivation | |
| W08 | Kalman Filter Review; Group Affine for Estimation | PS04 Due, PS05 Assigned |

# Detailed Schedule: Second Half

| Week | Topic | Assignment |
|---|---|---|
| W09 | $\mathrm{SE}_2(3)$ Extended Pose; Mixed Invariance | |
| W10 | IEKF: Derivation from First Principles | PS05 Due, PS06 Assigned |
| W11 | IEKF: Consistency, Observability Analysis | |
| W12 | IEKF: IMU Navigation, VIO Applications | PS06 Due, PS07 Assigned, Proposals Due |
| W13 | Equivariant Filter: Theory & Derivation | |
| W14 | Equivariant Filter: Comparison & Analysis | PS07 Due, PS08 Assigned |
| W15 | Advanced Topics / Project Work | PS08 Due |
| W16 | Poster Session | Final Report Due |

# Problem Sets

**Eight problem sets** (biweekly, lowest grade dropped):

- PS01: Group axioms, SO(2)
- PS02: SE(2) trajectory tracking
- PS03: SO(3) representations
- PS04: SO(3) attitude control
- PS05: SE(3) kinematics
- PS06: Group affine systems
- PS07: IEKF implementation
- PS08: Equivariant filter

**Each problem set includes:**

- **Theory:** Proofs, derivations (pencil-and-paper)
- **Implementation:** Python code to verify theory

# Problem Set Grading

**Grading approach:**

- **Random selection:** One problem selected at random for detailed grading
- **Completion:** Remaining problems graded on completion (must attempt all)

**Spot Checks:**

- Several students randomly selected per PS to explain their solution at office hours
- Grade multiplier: **1.0** (clear), **0.85** (partial), **0.5** (unable to explain)
- Expect to be called at least once during the semester

**Why this approach?**

- Large class (30 students), no TA
- Problem sets are for **learning**, not gatekeeping
- **Quizzes** and **poster session Q&A** validate understanding

**Collaboration:**

- Discussion with classmates is **encouraged**
- Code and write-ups must be your own work

Important

# In-Class Quizzes

**Purpose:** Validate understanding of lecture material

**Format:**

- Administered via Brightspace during class
- Short questions on recent topics
- Open notes (your own notes, not the internet)

**Frequency:** Periodic throughout semester (approximately weekly)

### Why Quizzes?

With AI tools permitted on homework, quizzes ensure you personally understand the material—not just your AI assistant!

# Final Project

**Apply course material to a research problem of your interest**

**Deliverables:**

1. **Proposal** (Week 12): 1-page problem statement & approach
2. **Paper** (Week 16): 6–10 pages, IEEE format
   - Literature review (min. 10 references)
   - Theoretical contribution or rigorous analysis
   - Implementation with quantitative results
   - Code submitted for reproducibility
3. **Poster Session** (Week 16): Present at poster with individual Q&A

### Poster Q&A Validates Understanding

The instructor will spend 5–10 minutes at each poster asking questions about your project **and** course concepts. This 1-on-1 format allows deeper assessment of understanding.

# Project Ideas

**Some possible directions:**

**Control:**

- Geometric attitude control
- Spacecraft reorientation
- Quadrotor trajectory tracking
- Robot arm control

**Estimation:**

- Visual-inertial odometry
- IMU-based navigation
- SLAM on Lie groups

**Theory/Analysis:**

- Comparison of filter variants
- Stability analysis
- New application domains

**Other:**

- Computer vision applications
- Neural networks on Lie groups
- Optimization on manifolds

**Discuss ideas with instructor early!**

# Final Report Rubric (35%)

| Criterion | Description | Points |
|---|---|---|
| Problem Formulation | Clear problem statement using Lie group notation; well-motivated | 15 |
| Literature Review | Minimum 10 relevant references; demonstrates understanding of prior work | 15 |
| Technical Depth | Correct derivations; appropriate use of course concepts | 25 |
| Results | Quantitative evaluation; meaningful comparisons; proper figures | 20 |
| Writing Quality | Clear exposition; proper mathematical notation; well-organized | 15 |
| Reproducibility | Code submitted; documentation sufficient to reproduce results | 10 |

# Poster Session Rubric (15%)

| Criterion | Description | Points |
|---|---|---|
| Poster Quality | Clear layout; key results visible; appropriate detail level | 20 |
| Technical Content | Correct explanation of methods; demonstrates understanding | 30 |
| Q&A Performance | **Demonstrates mastery of project and course concepts**; answers questions clearly | 40 |
| Results Display | Compelling figures/demos; validates the work | 10 |

## Q&A is Critical (40%)

I will spend 5–10 minutes at each poster asking about your project **and** general course concepts. This 1-on-1 Q&A is the primary assessment of understanding.

# Grade Breakdown

| Weight | Component | Notes |
|---|---|---|
| 10% | In-Class Quizzes | Validates understanding |
| 40% | Problem Sets (8) | Random + completion, drop lowest |
| 35% | Final Report | Conference paper format |
| 15% | Poster Session | 1-on-1 Q&A assessment |

## Grading Philosophy

Problem sets are for **learning**. The **quizzes** and **poster session Q&A** are where you demonstrate personal mastery. Use AI tools wisely, but make sure *you* understand the material.

# Grading Scale

| Grade | Percentage |
| --- | --- |
| A+ | 97–100% |
| A | 94–96% |
| A- | 90–93% |
| B+ | 87–89% |
| B | 84–86% |
| B- | 80–83% |

| Grade | Percentage |
| --- | --- |
| C+ | 77–79% |
| C | 74–76% |
| C- | 70–73% |
| D+ | 67–69% |
| D | 64–66% |
| D- | 60–63% |
| F | 0–59% |

# AI Use Policy

## Policy

The use of Large Language Models (LLMs) and other AI tools is **permitted** for coursework.

**Why allow AI?**

- AI tools are part of modern engineering practice
- They can help you learn and explore faster
- Focus should be on understanding, not memorization

**Validation:**

- In-class quizzes (no AI access)
- In-class presentations (you must explain your work)
- These ensure **you** understand, not just your AI

## Warning

If you rely entirely on AI without understanding, you will struggle on quizzes and presentations!

# Academic Integrity

**Purdue University Academic Integrity Policy applies**

**Allowed:**

- Discussing concepts with classmates
- Using AI tools for homework
- Referencing textbooks and online resources

**Not allowed:**

- Copying code/solutions from classmates
- Submitting work that isn't yours
- Using unauthorized aids during quizzes

**When in doubt, ask!**

# Accommodations

**Students with disabilities:**

- Contact the instructor at the beginning of the semester
- Work with Disability Resource Center (DRC)
- Accommodations will be provided as needed

**Other accommodations:**

- Religious observances
- University-sponsored activities
- Contact instructor in advance to arrange alternatives

# Reference Materials

**Primary References:**

- Solà, Deray, Atchuthan. *A micro Lie theory for state estimation in robotics*. arXiv:1812.01537
- Barfoot. *State Estimation for Robotics*. Cambridge, 2017

**Additional Resources:**

- Murray, Li, Sastry. *A Mathematical Introduction to Robotic Manipulation*
- Chirikjian. *Stochastic Models, Information Theory, and Lie Groups*

**Course Materials:**

- Lecture slides on Brightspace
- Homework starter code provided
- Example implementations

# Getting Help

**Resources available:**

1. **Office Hours:** T/Th 3:00–4:00 PM, ARMS 3214 (after lecture)
2. **Piazza: Primary channel for course questions**
   - Post questions about homework, concepts, etc.
   - **Open discussion is strongly encouraged!**
   - Students are welcome to answer each other's questions
   - Helps all students see answers
   - Course content questions via email will be redirected here
3. **Email:** `jgoppert@purdue.edu`
   - **Personal matters only** (accommodations, illness, etc.)
   - NOT for homework help or course content
4. **AI Tools: Encouraged!**
   - LLMs do well with Lie group theory
   - Like a TA you can talk to anytime, anywhere
   - Use to clarify concepts and check understanding
   - **Always double-check against course materials!**

# Tips for Success

1. **Attend class**
    - Quizzes happen during class
    - Discussion helps understanding
2. **Start homework early**
    - Implementation takes time
    - Get help before the deadline
3. **Understand, don't just copy**
    - AI can write code, but you need to explain it
    - Quizzes test your understanding
4. **Build intuition**
    - Visualize the geometry
    - Think about physical meaning
5. **Ask questions!**
    - In class, office hours, Piazza
    - There are no dumb questions
6. **Help improve the course!**
    - These slides are a **living document**
    - Spotting typos and errors is encouraged—report on Piazza!

# What's Next?

**Today (Rest of Week 1):**

- Why Lie groups? Motivation and applications
- Mathematical foundations: groups, manifolds, topology
- Rotation representations and their problems
- **PS01: Group axioms, SO(2)**

**Week 2:**

- SO(2): The simplest Lie group
- SE(2): Planar rigid body motion
- All the Lie theory tools in 2D

**Week 3:**

- Geometric control on SE(2)
- Fixed-wing 2D trajectory tracking
- **PS02: SE(2) trajectory tracking**

# Questions about the course?

**Piazza** (course questions)

jgoppert@purdue.edu (personal matters only)

Office Hours: T/Th 3–4 PM, ARMS 3214