# (1.2) Three-Body Problem

Next, we consider the spacecraft orbital motion in a three-body system.

## (a) Circular Restricted Three-Body Problem (CR3BP)

Consider the Earth-Moon system using the circular restricted three-body problem (CR3BP). The parameter values for this problem are provided in Table 1.

Table 1: Assumed dynamical parameter values (Problem (1.2))

| Parameter | Symbol | Value | Unit |
|---|---|---|---|
| Earth-Moon distance | $d_{\text{Earth-Moon}}$ | $3.8475 \times 10^5$ | km |
| Earth-Moon barycenter GM | $G(M_{\text{Earth}} + M_{\text{Moon}})$ | $4.0350 \times 10^5$ | $\text{km}^3/\text{s}^2$ |
| Mass ratio | $\mu$ | $1.2151 \times 10^{-2}$ | - |

Simulate the spacecraft's orbital motion in the CR3BP system with the initial conditions and propagation times given in Table 2. Plot each trajectory in the synodic frame in the dimensional system, marking the positions of the Earth and Moon.

Table 2: Initial conditions for CR3BP simulations (non-dimensional)

| IC # | $x_0$ | $y_0$ | $z_0$ | $\dot{x}_0$ | $\dot{y}_0$ | $\dot{z}_0$ | Propagation time |
|---|---|---|---|---|---|---|---|
| IC-1 | 1.2 | 0 | 0 | 0 | -1.06110124 | 0 | 6.20628 |
| IC-2 | 0.85 | 0 | 0.17546505 | 0 | 0.2628980369 | 0 | 2.5543991 |
| IC-3 | 0.05 | -0.05 | 0 | 4.0 | 2.6 | 0 | 15.0 |

## (b) Third-Body Perturbation in the ECI Frame

The orbital motion under the influence of Earth and Moon gravity can also be modeled as a perturbed two-body problem, where the Moon exerts a third-body perturbation. To do this, re-define the ECI frame such that:

- $\hat{\boldsymbol{n}}_1$ is aligned with the Earth-Moon line at the epoch.
- $\hat{\boldsymbol{n}}_3$ is aligned with the normal vector of the Earth-Moon orbital plane.

The Moon's third-body perturbation in the ECI frame is given by:

$$\boldsymbol{a}_{\text{moon}} = -\mu_{\text{moon}} \left( \frac{\boldsymbol{r} - \boldsymbol{r}_{\text{moon}}}{\|\boldsymbol{r} - \boldsymbol{r}_{\text{moon}}\|_2^3} + \frac{\boldsymbol{r}_{\text{moon}}}{\|\boldsymbol{r}_{\text{moon}}\|_2^3} \right),$$

where:

- $\mu_{\text{moon}} = 4.9028 \times 10^3 \text{ km}^3/\text{s}^2$,
- $\boldsymbol{r}_{\text{moon}}$ is the Moon's position relative to the ECI frame origin.

For simplicity, assume that the Moon's orbit is circular (similar to the CR3BP model), but about Earth instead of the barycenter.

Convert the initial conditions in Table 4 to position and velocity vectors in the ECI frame and propagate the system using the perturbed two-body equations under the Moon's third-body perturbation. Show the results in both:

- The ECI frame.
- The synodic frame.

## (c) Consistency of Results

Compare the results obtained in parts (a) and (b). Discuss the consistency between the two approaches. If discrepancies are observed, explain the possible causes.

## (d) Extra Credit: Additional Perturbation

For extra credit, choose another perturbing force of your choice and derive its explicit mathematical expression. Include it in the three-body simulation (either in the CR3BP or ECI frame, at your discretion). Run the simulation with the same initial conditions from Table 4 and compare the results to the previous cases.

The chosen perturbation does not need to be covered in class.