The risk sensitive control policy is

$$\min_{\mu_0:T-1} \mathcal{L}_\theta,\ \theta \in \mathbb{R}$$
$$\text{where } \mathcal{L}_\theta = \begin{cases} \frac{1}{\theta}\log(\mathbb{E}(\exp(\theta C_{0:T}))) & \text{if } \theta \neq 0 \\ \mathbb{E}(C_{0:T}) & \text{if } \theta = 0 \end{cases}$$

Assume $\theta \neq 0$ then $\mathcal{L}_\theta = \frac{1}{\theta}\log(\mathbb{E}(\exp(\theta C_{0:T})))$. The Maclaurin series in θ of $\exp(a\theta)$ is

$$\exp(a\theta) = 1 + a\theta + \frac{a^2\theta^2}{2} + \dots$$

The Maclaurin series in x of $\log(1+x)$ is

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

Substituting the Macluarin series for $\exp(a\theta)$ into $\mathcal{L}_\theta$ gives

$$\mathcal{L}_\theta = \frac{1}{\theta}\log\left(\mathbb{E}(1 + \theta C_{0:T} + \frac{(\theta C_{0:T})^2}{2} + \dots)\right)$$

Using linearity of expectation,

$$\mathcal{L}_\theta = \frac{1}{\theta}\log\left(1 + (\theta \mathbb{E}C_{0:T} + \frac{\theta^2}{2}\mathbb{E}(C_{0:T})^2 + \dots)\right)$$

Defining $x = (\theta \mathbb{E}C_{0:T} + \frac{\theta^2}{2}\mathbb{E}(C_{0:T})^2 + \dots)$, the Maclaurin seris for $\log(1+x)$ can be used

$$\mathcal{L}_\theta = \frac{1}{\theta}((\theta \mathbb{E}C_{0:T} + \frac{\theta^2}{2}\mathbb{E}(C_{0:T}^2) + \dots) - \frac{1}{2}(\theta(\mathbb{E}C_{0:T})^2 + \text{H.O.T}) + \text{H.O.T}) + \text{H.O.T}$$

Therefore, using the first two leading terms $\mathcal{L}_\theta$ can be approximated by

$$\mathcal{L}_\theta \approx \mathbb{E}C_{0:T} + \frac{1}{2}(\mathbb{E}(C_{0:T}^2) - (\mathbb{E}C_{0:T})^2)\theta$$

Noticing that $\text{Var}(C_{0:T}) = \mathbb{E}(C_{0:T}^2) - (\mathbb{E}C_{0:T})^2$

$$\mathcal{L}_\theta \approx \mathbb{E}C_{0:T} + \frac{1}{2}\text{Var}(C_{0:T})\theta$$

It is now obvious that

$$\begin{cases} \theta > 0 & \text{means that } \text{Var}(C_{0:T}) \text{ is penalized (risk averse)} \\ \theta < 0 & \text{means that } \text{Var}(C_{0:T}) \text{ is rewarded (risk seeking)} \end{cases}$$

In solving this I referenced https://laurentlessard.com/teaching/me7247/lectures/lecture