3.1.i)

Show: that $||J_k - J^*||_\infty \leq \gamma^k ||J_0 - J^*||_\infty$
Solution:
From Lemma 3.1, we know $||TJ - T\bar{J}||_\infty \leq \gamma ||J - \bar{J}||_\infty$. Also, $TJ^* = J^*$.
Proof by induction.
Base Case: $k = 1$. Let $J = J_0, \bar{J} = J^* \implies ||J_1 - J^*||_\infty \leq \gamma ||J - J^*||_\infty$
Inductive Hypothesis: $||J_k - J^*||_\infty \leq \gamma^k ||J_0 - J^*||_\infty \implies ||J_{k+1} - J^*||_\infty \leq \gamma^{k+1} ||J_0 - J^*||_\infty$
Inductive Step: Assume $||J_k - J^*||_\infty \leq \gamma^k ||J_0 - J^*||_\infty$
Let $J = J_k$, $\bar{J} = J^*$

$$
\begin{gathered}
TJ = TJ_k = J_{k+1},\ T\bar{J} = TJ^* = J^* \\
||J_{k+1} - J^*||_\infty \leq \gamma ||J_k - J^*||_\infty \\
||J_{k+1} - J^*||_\infty \leq \gamma(\gamma^k ||J_0 - J^*||_\infty) \\
||J_{k+1} - J^*||_\infty \leq \gamma^{k+1} ||J_0 - J^*||_\infty
\end{gathered}
$$

Which proves our inductive hypothesis. Therefore, by induction $||J_k - J^*||_\infty \leq \gamma^k ||J_0 - J^*||_\infty$.

3.1.ii)

Show: that the stopping condition $||J_{k+1} - J_k||_\infty \leq \epsilon \implies ||J_{k+1} - J^*||_\infty \leq \frac{\epsilon\gamma}{1-\gamma}$
Solution:
Note the identity: $\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^k + \ldots$. This also means $\frac{x}{1-x} = x + x^2 + \cdots + x^k + \ldots$
The triangle inequality says $||a + b||_\infty \leq ||a||_\infty + ||b||_\infty$
From Lemma 3.1 and the argument in 3.1.i) we know $||J_{k+a+1} - J_{k+a}||_\infty \leq \gamma^a ||J_{k+1} - J_k||_\infty$

$$
\begin{gathered}
||J_{k+1} - J^*||_\infty = ||J_{k+1} - J_{k+2} + J_{k+2} - J^*||_\infty \leq ||J_{k+2} - J_{k+1}||_\infty + ||J_{k+2} - J^*||_\infty \\
\implies ||J_{k+1} - J^*||_\infty \leq ||J_{k+2} - J_{k+1}||_\infty + ||J_{k+3} - J_{k+2}||_\infty + ||J_{k+3} - J^*||_\infty \\
||J_{k+1} - J^*||_\infty \leq ||J_{k+2} - J_{k+1}||_\infty + ||J_{k+3} - J_{k+2}||_\infty + ||J_{k+4} - J_{k+3}||_\infty + \ldots \\
||J_{k+1} - J^*||_\infty \leq \gamma ||J_{k+1} - J_k||_\infty + \gamma^2 ||J_{k+1} - J_k||_\infty + \gamma^3 ||J_{k+1} - J_k||_\infty + \ldots \\
||J_{k+1} - J^*||_\infty \leq (\gamma + \gamma^2 + \gamma^3 + \ldots) ||J_{k+1} - J_k||_\infty = \frac{\gamma}{1-\gamma} ||J_{k+1} - J_k||_\infty \\
\therefore ||J_{k+1} - J_k||_\infty \leq \epsilon \implies ||J_{k+1} - J^*||_\infty \leq \frac{\epsilon\gamma}{1-\gamma}
\end{gathered}
$$