

(2.7) Stochastic LQG

Define the value function

$$J_t(\pi_t) = \min_{\mu_{t:T-1}} \sum_{k=t}^{T-1} \mathbb{E}(\|x_k\|_{Q_k}^2 + \|u_k\|_{R_k}^2) + \mathbb{E}\|x_T\|_{Q_T}^2$$

Prove (2.69) by showing that

$$J_t(\pi_t) = \text{Tr}(S_t P_{t|t}) + \sum_{k=t}^{T-1} (\text{Tr}(W_k S_{k+1}) + \text{Tr}(\Theta_k P_{k|k}))$$

for each $t = 0, 1, \dots, T$

Note: (2.68) says the optimal policy of the value function is

$$\begin{aligned}\hat{x}_{t|t} &= \hat{x}_{t|t-1} + L_t(y_t - C_t \hat{x}_{t|t-1}) \\ u_t &= K_t \hat{x}_{t|t} \\ \hat{x}_{t+1|t} &= A_t \hat{x}_{t|t} + B_t u_t\end{aligned}$$

So Kalman filter + LQR