ECE-GY 9243 / ME-GY 7973
Optimal and learning control for robotics

Exercise series 2

For questions requesting a written answer, please provide a detailed explanation. Typesetted answers are required (e.g. using LaTeX[1]). Include plots where requested, either in a Jupyter Notebook or in the typesetted answers. For questions requesting a software implementation, please provide your code in a python file or in a Jupyter Notebook such that it can be run directly. Include comments explaining how the functions work and how the code should be run if necessary. Any piece of code that does not run out of the box or does not contain instructions to execute it will be considered invalid.

Exercise 1

Consider the following three dynamical systems

$$1) \quad \mathbf{x}_{n+1} = \begin{bmatrix} 0.5 & 0. & 0.5 \\ 0. & 0. & -2. \\ 4. & 2. & 1. \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}_n$$

$$2) \quad \mathbf{x}_{n+1} = \begin{bmatrix} 0.5 & 0. & 0.5 \\ 0. & 0. & -0.5 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}_n$$

$$3) \quad \mathbf{x}_{n+1} = \begin{bmatrix} 2 & 0. & 0. \\ 0. & 0. & -2. \\ 1. & 1. & 0. \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}_n$$

For each of them answer the following questions:

a) Is the uncontrolled system (i.e. $\mathbf{u}_n = \mathbf{0}$) stable? (justify)

b) Is the system controllable? (justify)

c) Given the previous answers, do you expect a LQR design to be able to drive the system to the origin from any initial conditions? For which horizon length? (justify)

d) In Python, compute the optimal control policy to minimize the following cost function

$$\sum_{n=0}^{\infty} 100\mathbf{x}_n^T\mathbf{x}_n + \mathbf{u}_n^T\mathbf{u}_n$$

What are the optimal gains? Simulate the uncontrolled and controlled system (use $\mathbf{x}_0 = [10, 10, 10]^T$) and plot the time evolution of $\mathbf{x}_n$ and $\mathbf{u}_n$. Discuss the simulation results with respect to the answer to the previous question.

[1] `https://en.wikibooks.org/wiki/LaTeX`, NYU provides access to Overleaf to all the community `https://www.overleaf.com/edu/nyu`

Exercise 2

The exercise is described in the file *exercise 2 - cart pole.ipynb*.