ECE-GY 9243 / ME-GY 7973
Optimal and learning control for robotics

Exercise series 3

For questions requesting a written answer, please provide a detailed explanation. Typesetted answers are required (e.g. using LaTeX[1]). Include plots where requested, either in a Jupyter Notebook or in the typesetted answers. For questions requesting a software implementation, please provide your code in a python file or in a Jupyter Notebook such that it can be run directly. Include comments explaining how the functions work and how the code should be run if necessary. Any piece of code that does not run out of the box or does not contain instructions to execute it will be considered invalid.

Exercise 1

The exercise is described in the file *Exercise 1 - Model Predictive Control.ipynb*.

Exercise 2

Consider the grid-world shown in Figure 1. In each (non grey) cell, it is possible to perform five actions: move up, down, left, right or do nothing as long as the resulting move stays inside the grid world. Grey cells are obstacles and are not allowed. We would like to find the optimal value function and optimal policy that minimize the following cost

$$\min \sum_{n}^{\infty} \alpha^n g_n(x_n)$$

with discount factor $\alpha = 0.99$ and where the instantaneous cost is defined as

$$g_n(x_n) = \begin{cases} -1 & \text{if } x_n \text{ is a violet cell} \\ 0 & \text{if } x_n \text{ is a white cell} \\ 1 & \text{if } x_n \text{ is a green cell} \\ 10 & \text{if } x_n \text{ is a red cell} \end{cases}$$

a) In Python, implement the value iteration algorithm to solve the problem (initialize the value function to 0). How many iterations does it take to attain convergence? (we assume here that convergence happens when all the elements of the value function do not change more than 10^{-6} in a new iteration.

b) In Python, implement the policy iteration algorithm to solve the problem (use the version that solves the linear equation $I - \alpha A)J_\mu = \bar{g}$). Start with an initial policy that does not move. How many iterations does it take to converge?

c) Compare the solutions and convergence/complexity of each algorithm to solve this problem.

[1] `https://en.wikibooks.org/wiki/LaTeX`, NYU provides access to Overleaf to all the community `https://www.overleaf.com/edu/nyu`

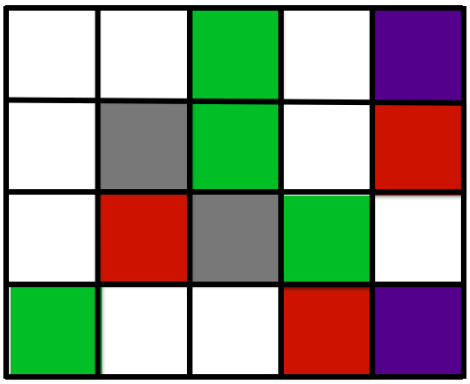

Figure 1: *Grid world*