COMPUTATIONAL MODELS OF THE SINGLE NEURON

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Abstract

Computational neuroscience is the field of study in which mathematical tools and theories are used to investigate brain function; It employs computational simulations to validate and solve mathematical models and as a final step, to explain in computational terms how brains generate behaviors. Neurons are the cells that make up the brain and the nervous system. They are the fundamental units that send and receive signals that allow us to move our muscles, feel the external world, think, form memories, and much more. Spikes are the mechanism that allows neurons to pass information over relatively long distances within themselves; And as spikes are how neurons communicate, they are how we do anything. So understanding the brain means understanding spikes, for they are the brainâs own language. In this report, we tried to investigate the simplest neuron model, called the Leaky Integrate-and-Fire (LIF) model, implementing it and some of its expansions and searching how they function regarding the changes we make on the model's different parameters, and different behaviours we define for it.

Contents

Chapter 1

LIF model

1.1 Leaky Integrate-and-Fire model

Leaky Integrate-and-Fire neuron model takes the sum of weighted inputs, much like the artificial neuron. But rather than passing it directly to an activation function, it will integrate the input over time with a leakage, much like an RC circuit. If the integrated value exceeds a threshold, then the LIF neuron will emit a voltage spike.

$$\tau \frac{du}{dt} = -(u - u_{rest}) + R.I(t)$$

$$\text{if u(t)=}\theta \Rightarrow Fire + Reset(u = u_{rest})$$

1.2 LIF behavior on step function input

Now, we are going to analyse how the LIF model behaves with the information defined in table 1.1.

input type	τ	u_{rest}	u_{reset}	threshold	R	primary current
step function	20	-65	-73	-13	1	50

Table 1.1: information

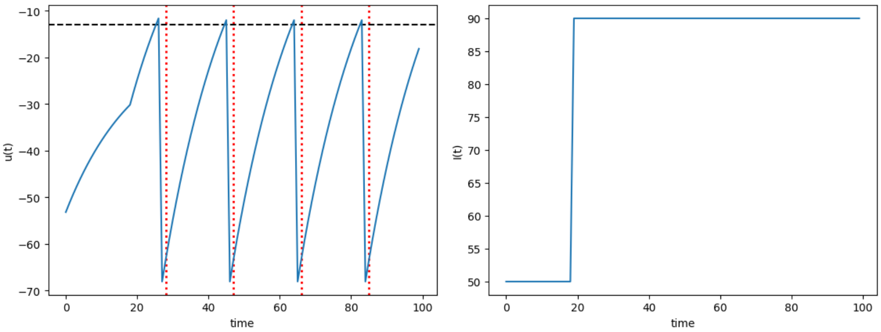

Figure 1.1: LIF behavior on information given in table 1.1

As we can see, our single neuron starts to spike once its voltage reaches the threshold value. Now, we are going to decrease the value of τ and see what happens in that case.

input type	τ	u_{rest}	u_{reset}	threshold	R	primary current
step function	10	-65	-73	-13	1	50

Table 1.2: information

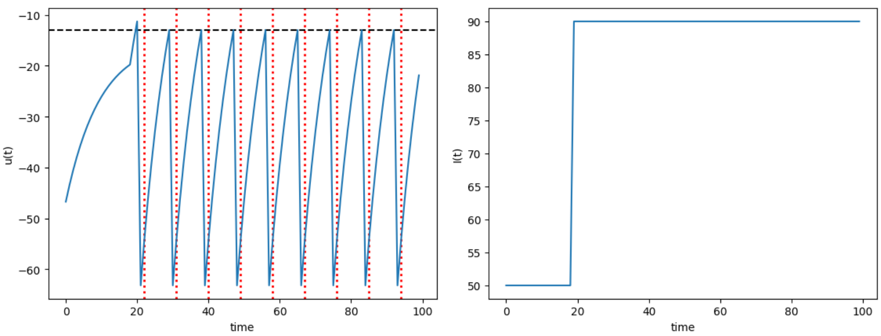

Figure 1.2: LIF behavior on information given in table 1.2

As expected, the neuron spikes sharper since τ defines the membrane time constant of the neuron.
Now, we are going to change the amount of resistance to analyse its effect on our model's behaviour.

input type	τ	u_{rest}	u_{reset}	threshold	R	primary current
step function	20	-65	-73	-13	2	50
step function	20	-65	-73	-13	0.5	50

Table 1.3: information

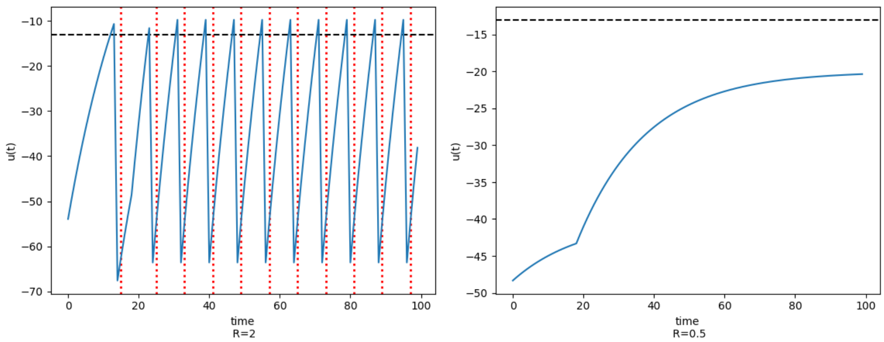

Figure 1.3: LIF behavior on information given in table 1.3

As we can see in figure 1.3, by increasing the amount of resistor, the amount of current increases and our neuron spikes more immediately; While decreasing the resistance causes a decrease in the current and it may prevent reaching action potential.

1.3 LIF behavior on trigonometric function input

In this section, we are going to give an trigonometric function as input current and analyse the model's behaviour.

input type	τ	u_{rest}	u_{reset}	threshold	R	given current
trigonometric function	10	-65	-73	-13	1.5	40

Table 1.4: information

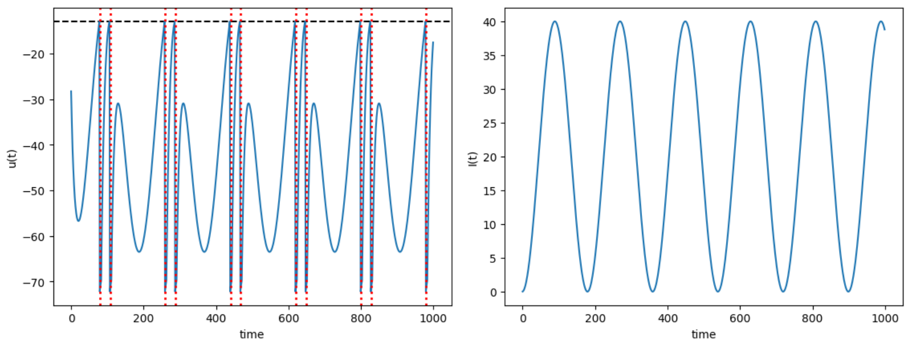

Figure 1.4: LIF behavior on information given in table 1.4

As we can see, once the input current has reached the threshold voltage, the neuron got into a bursting phase.

1.4 adding noise to the LIF model's input

In this section, we are going to define a random noise function as our neuron's input current.

input type	τ	u_{rest}	u_{reset}	threshold	R
random function	10	-65	-73	-13	1.5

Table 1.5: information

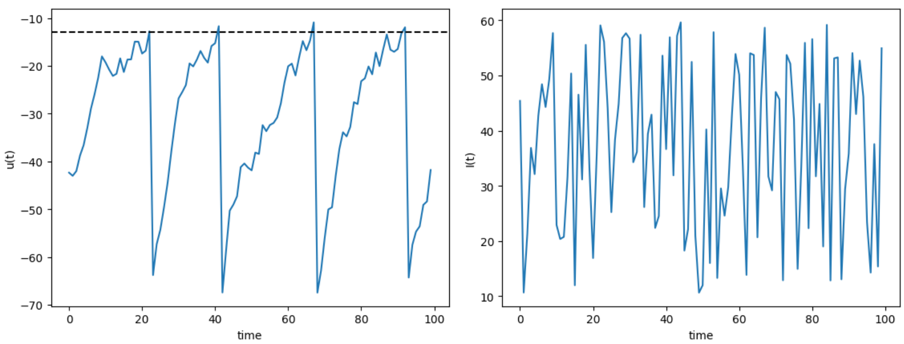

Figure 1.5: LIF behavior on information given in table 1.5

Now, we are going to change the noise value and see what happens.

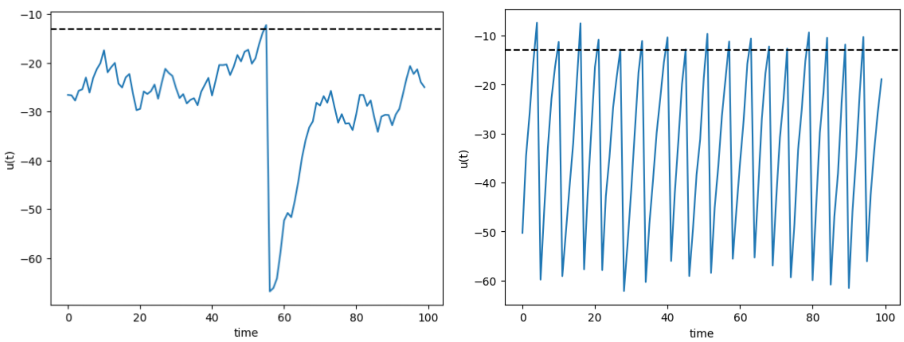

Figure 1.6: LIF behavior with low and high noise currents

As we can see, if the random noise function has a sufficiently low value, it will prevent firing and the neuron could rarely spike as its voltage would not easily exceed the threshold voltage. On the other hand, if we consider a greatly high value of noise, our neuron may continuously spike.

1.5 adding refractory period to the LIF model

In this section, we are going to add the refractory period to our LIF model. The absolute refractory period occurs immediately after an action potential is fired and it is not possible for another action potential to be produced. A relative refractory period takes place after the absolute refractory period. During relative refractory period, another action potential could possibly occur, but only if a neuron receives a much stronger stimulus than the previous action potential. However, here we will only discuss the absolute refractory period.

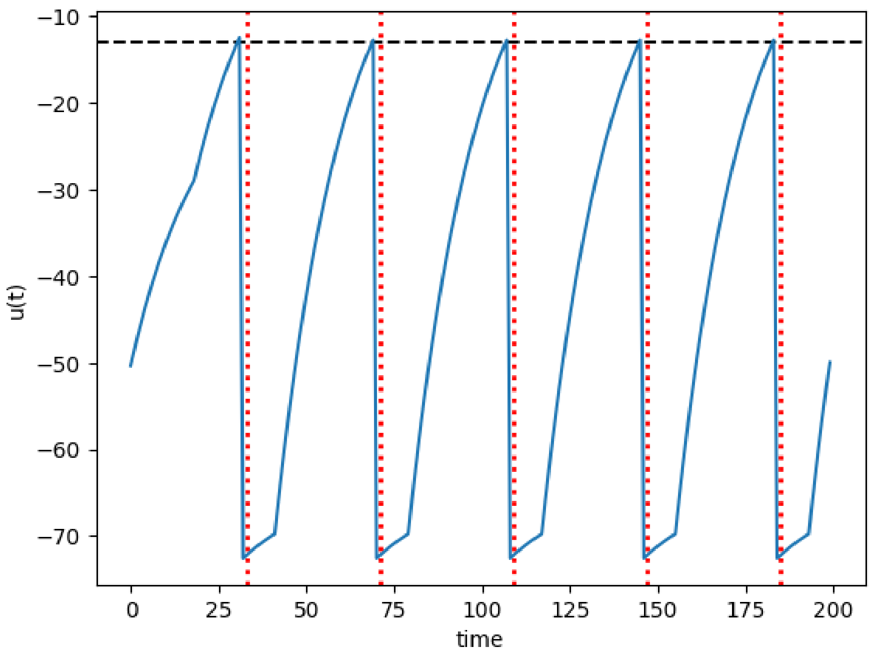

Figure 1.7: LIF model's behaviour with refractory period

As we can see in the figure 1.7, the neuron is not capable of firing an action potential during its refractory period, which occurs immediately after each spike.

1.6 F-I curve on LIF model

In neuroscience, a frequency-current curve is the function that relates the net synaptic current (I) flowing into a neuron to its firing rate (F). Here we are going to plot the frequency-current curve for LIF model with a fixed current, and then considering a noise function. We can see that adding noise to the input current causes more fluctuation on the F-I curve.

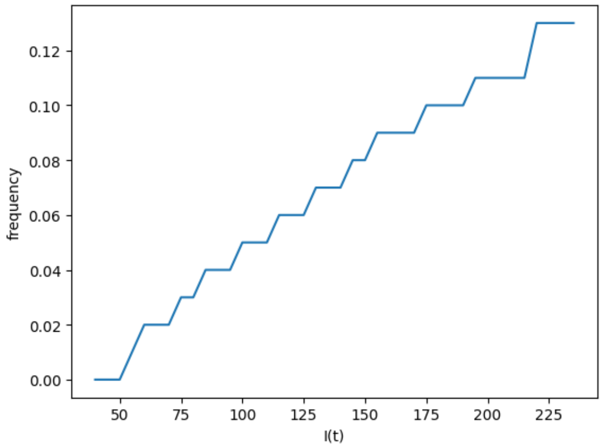

Figure 1.8: F-I curve on LIF model with fixed input current

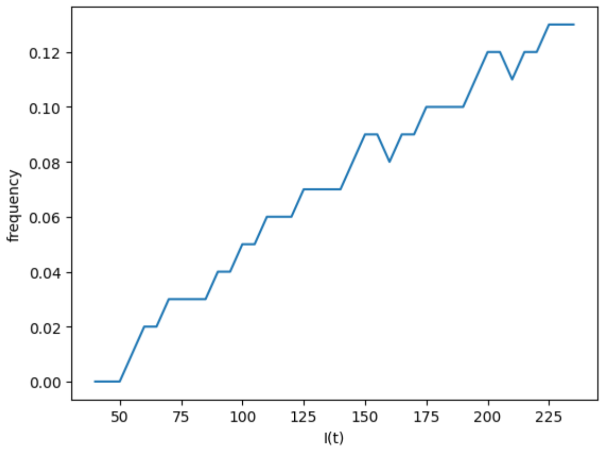

Figure 1.9: F-I curve on LIF model with noise

Chapter 2

ELIF model

2.1 Exponential LIF model

The exponential LIF model is a biological neuron model, a simple modification of the classical leaky integrate-and-fire model describing how neurons produce action potentials. In the exponential integrate-and-fire model, spike generation is exponential.

$$\tau\frac{du}{dt} = -(u - u_{rest}) + \Delta_T \, exp(\frac{u-\theta_{rh}}{\Delta_T}) + R.I(t) \; ; \; \; if \; firing : (u = u_{reset})$$

- The first term describes the leak of a passive membrane.
- The second term is an exponential nonlinearity.
- Δ_T is the sharpness parameter.
- θ_{rh} is the firing threshold.

2.2 ELIF behavior on step function input

First, we are going to analyse how the ELIF model behaves on information defined in table 2.1.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	primary current
step function	10	-65	-73	20	-13	1	30

Table 2.1: information

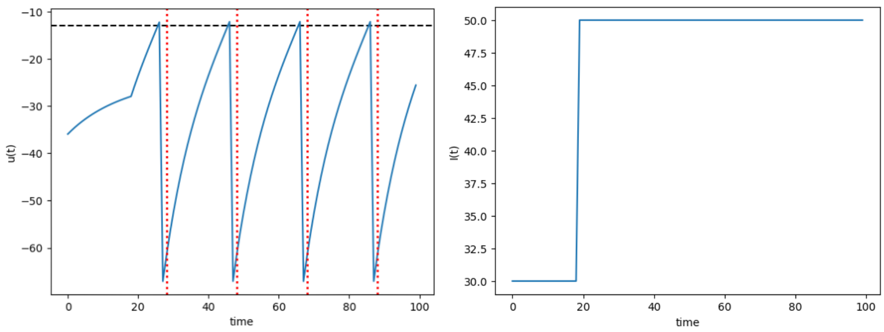

Figure 2.1: ELIF behavior on information given in table 2.1

Now, we are going to lower the sharpness of the threshold, Δ_T, and analyse the model's function. We will use the information given in the table below.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	primary current
step function	10	-65	-73	5	-13	1	30
step function	10	-65	-73	1	-13	1	30
step function	10	-65	-73	0.5	-13	1	30
step function	10	-65	-73	0.05	-13	1	30

Table 2.2: information

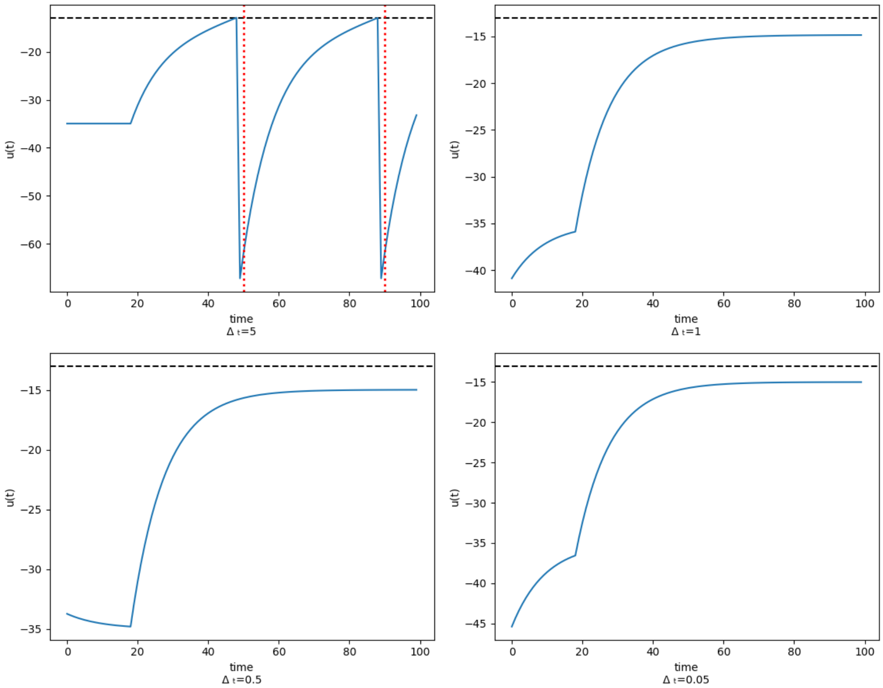

Figure 2.2: ELIF behavior on different values of Δ_T

As we can see, in the limit $\Delta_T \to 0$ the exponential integrate-and-fire model becomes equivalent to a leaky integrate-and-fire model.

2.3 ELIF behavior on trigonometric function input

In this section, we are going to give an trigonometric function as input current to the ELIF model.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	given current
trigonometric function	10	-65	-73	30	-13	1	40

Table 2.3: information

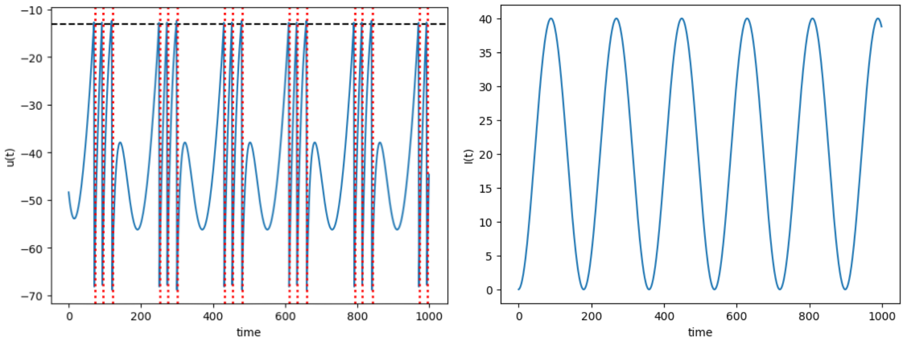

Figure 2.3: LIF behavior on information given in table 2.3

Now, we are going to increase our current and see what happens.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	given current
trigonometric function	10	-65	-73	30	-13	1	100

Table 2.4: information

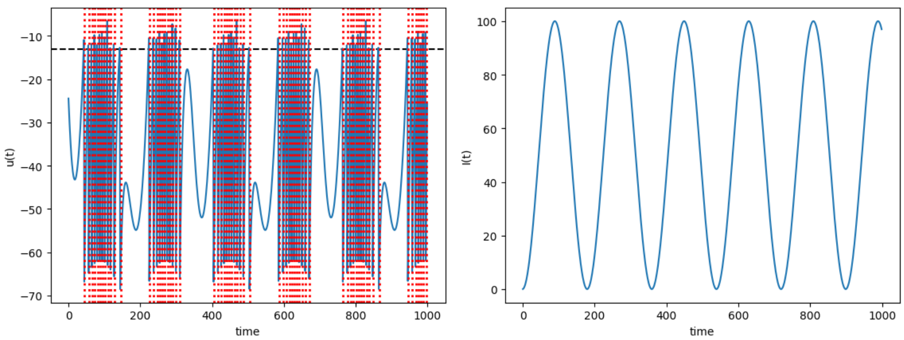

Figure 2.4: LIF behavior on information given in table 2.4

Now, we put $\Delta_T = 0$ with the high current.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	given current
trigonometric function	10	-65	-73	0	-13	1	100

Table 2.5: information

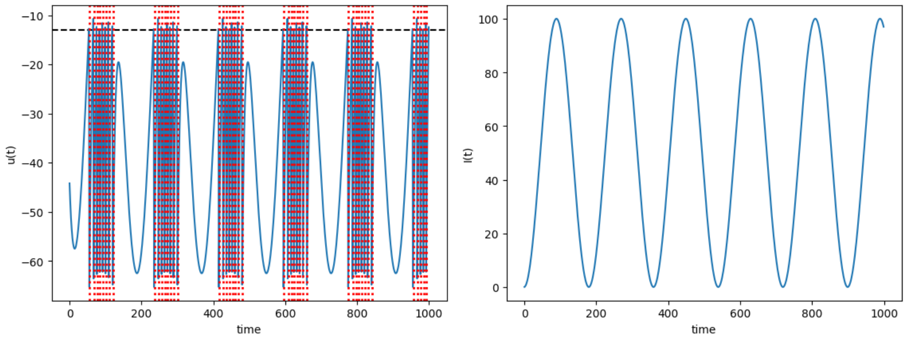

Figure 2.5: LIF behavior on information given in table 2.5

As we can see, if our current is strong enough, the effect of Δ_T will almost disappear.

2.4 adding noise to the ELIF model's input

In this section, we are going to define a random noise function as our neuron's input current.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R
random function	10	-65	-73	20	-13	1

Table 2.6: information

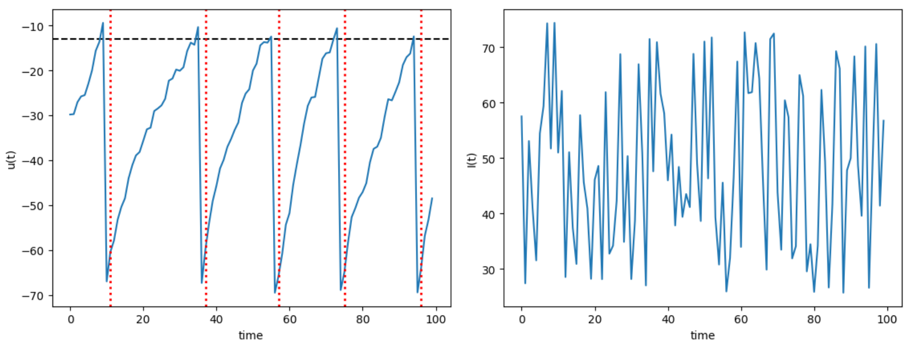

Figure 2.6: ELIF behavior on information given in table 2.6

Now we are going to change the amount of different variables and observe what happens in each case.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R
random function	30	-65	-73	20	-13	1

Table 2.7: information

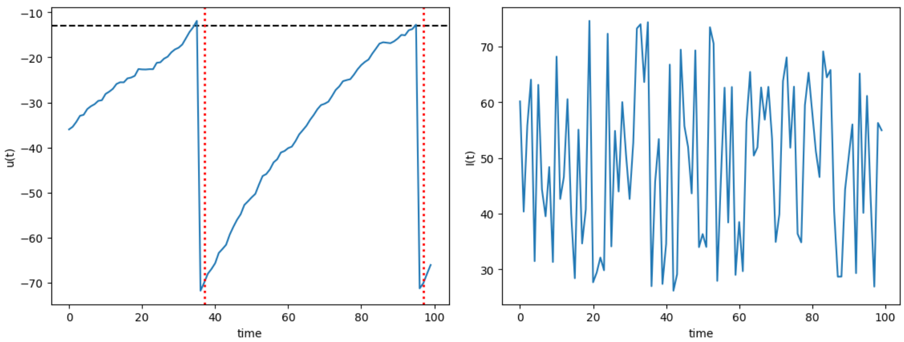

Figure 2.7: ELIF behavior on information given in table 2.7

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R
random function	30	-65	-73	60	-13	1

Table 2.8: information

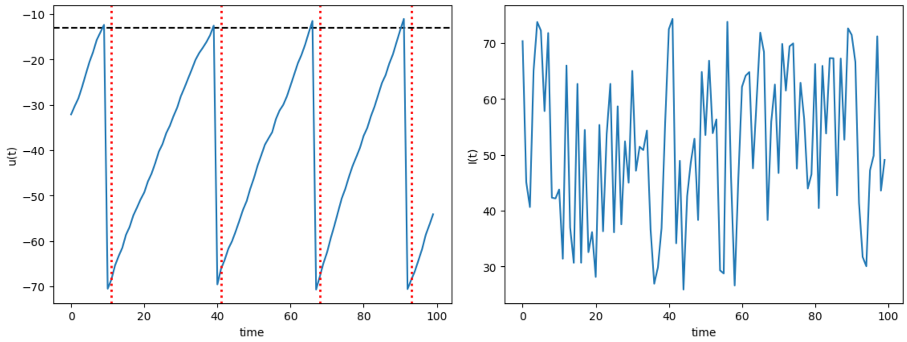

Figure 2.8: ELIF behavior on information given in table 2.8

As we can see in figure 2.8, by increasing Δ_T, our neuron spikes more often; Also the effect of noise decreases as we see less fluctuation in u(t)-time curve.

2.5 adding refractory period to the ELIF model

In this section, we are going to add the refractory period to our ELIF model. After doing this, we expect our neuron not to fire during a while just after firing an action potential.

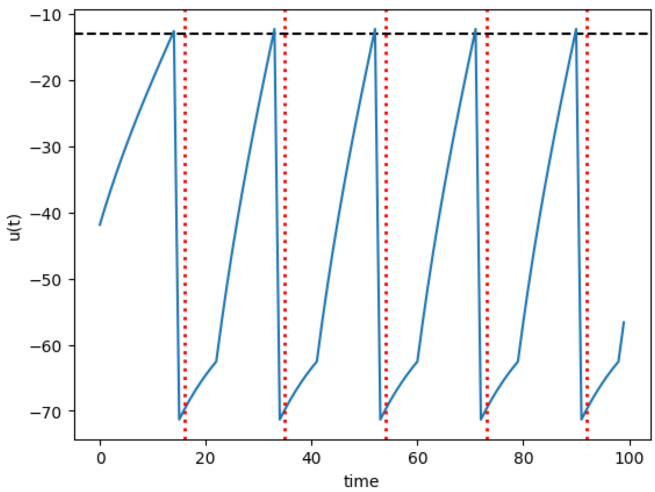

Figure 2.9: LIF model's behaviour with refractory period

2.6 F-I curve on ELIF model

In this section we are going to plot the frequency-current curve for ELIF model with a fixed current, and then considering a noise function.

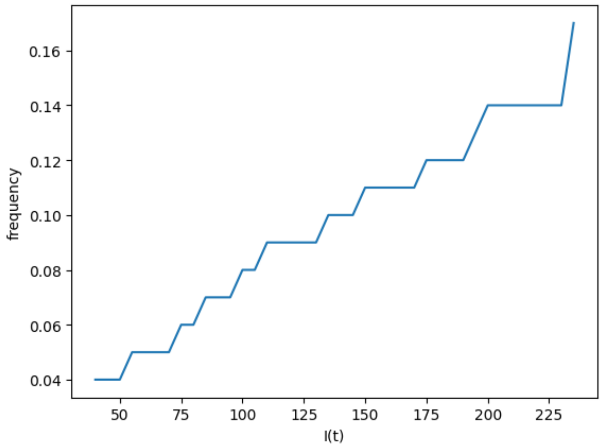

Figure 2.10: F-I curve on ELIF model with fixed input current

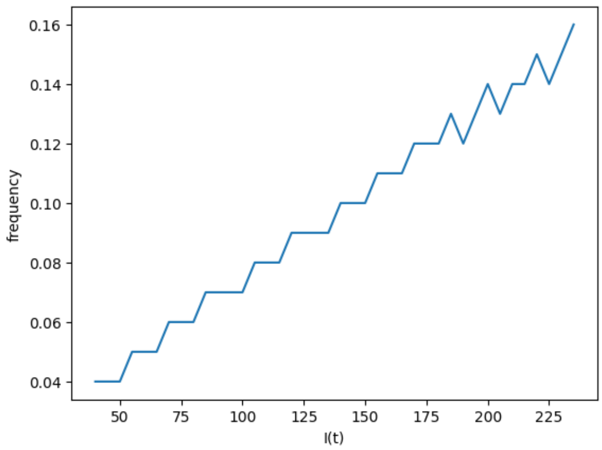

Figure 2.11: F-I curve on ELIF model with noise

One thing to notice is that the ELIF's F-I curve is increasing by a less gradient in comparison with the LIF's F-I curve, which we plotted in section 1.6.

Chapter 3

AELIF model

3.1 Adaptive LIF model

In the previous chapter we have explored nonlinear integrate-and-fire neurons where the dynamics of the membrane voltage is characterized by a function f(u). A single equation is not sufficient to describe the variety of firing patterns that neurons exhibit in response to a step current. We therefore couple the voltage equation to abstract current variables w_k, each described by a linear differential equation. The set of equations is

$$\tau_m \frac{du}{dt} = f(u) - R\sum_k w_k + R.I(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k(u - u_{rest}) - w_k + b_k\tau_k \sum_{t^{(f)}} \delta(t - t^{(f)})$$

The coupling of voltage to the adaptation current w_k is implemented by the parameter a_k and evolves with time constant τ_k. The adaptation current is fed back to the voltage equation with resistance R. Just as in other integrate-and-fire models, the voltage variable u is reset if the membrane potential reaches the numerical threshold Θ_{reset}. The moment u(t)=Θ_{reset} defines the firing time

$t^{(f)}$=t. After firing, integration of the voltage restarts at u=u_r. The δ-function in the w_k equations indicates that, during firing, the adaptation currents w_k are increased by an amount b_k. For simplicity, the voltage equation of exponential LIF model could be coupled to a single adaptation variable w:

$$\tau_m \frac{du}{dt} = -(u - u_{rest}) + \Delta_T \, exp(\frac{u - \vartheta_{rh}}{\Delta_T}) - Rw + R.I(t)$$

$$\tau_w \frac{dw}{dt} = a(u - u_{rest}) - w + b\tau_w \sum_{t^{(f)}} \delta(t - t^{(f)})$$

At each threshold crossing the voltage is reset to u=u_r and the adaptation variable w is increased by an amount b. Adaptation is characterized by two parameters: a couples adaptation to the voltage and is the source of subthreshold adaptation. Spike-triggered adaptation is controlled by a combination of a and b. The choice of a and b largely determines the firing patterns of the neuron and can be related to the dynamics of ion channels.

3.2 AELIF behavior on step function input

First, we are going to analyse how the ELIF model behaves with the information defined in the table 3.1.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	a	b	τ_w	primary current
step function	10	-65	-73	5	-13	1	1.3	0.7	30	0

Table 3.1: information

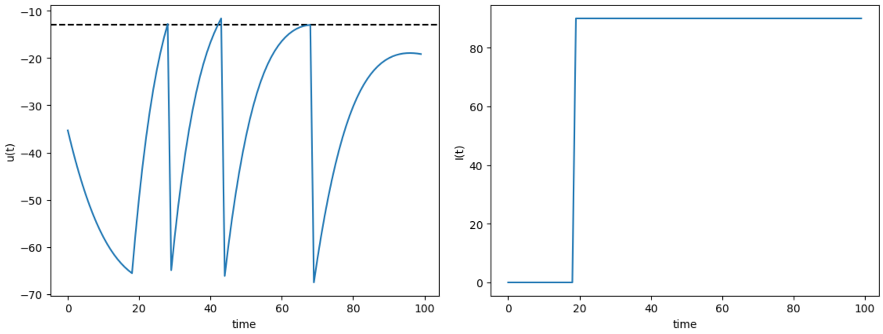

Figure 3.1: AELIF behavior on information given in table 3.1

We can obviously see the adapting process in the figure 3.1. Now, we are going to increase τ_w.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	a	b	τ_w	primary current
step function	10	-65	-73	5	-13	1	1.3	0.7	60	0

Table 3.2: information

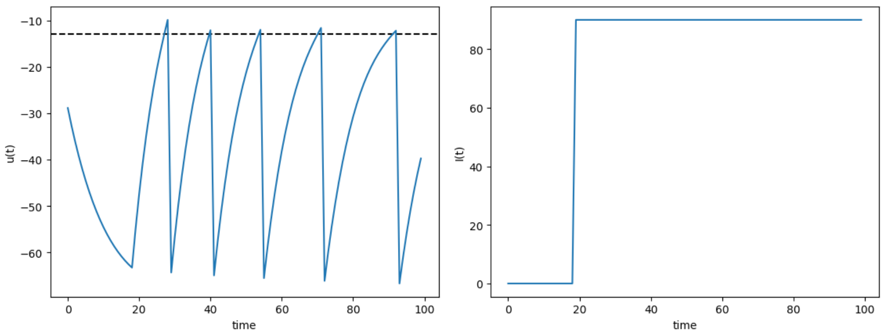

Figure 3.2: AELIF behavior on information given in table 3.2

We can see that by increasing τ_w, the neuron spikes faster, as we were expecting.

Now, let's increase the amount of a and b, and see what happens in each case.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	a	b	τ_w	primary current
step function	10	-65	-73	5	-13	1	2	0.7	30	0
step function	10	-65	-73	5	-13	1	1.5	5	30	0

Table 3.3: information

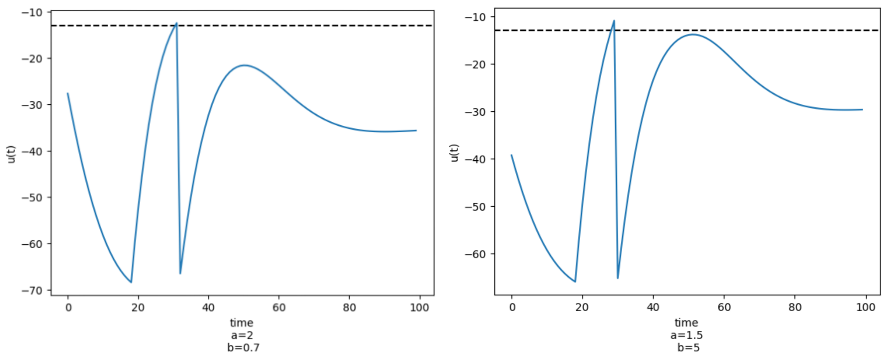

Figure 3.3: AELIF behavior on information given in table 3.3

We can observe here that parameter a effects firing more than parameter b regarding defined values; And increasing each of them causes reduction in action potentials.

3.3 AELIF behavior on trigonometric function input

Now we are going to give an trigonometric function as input current and see AELIF model's behaviour.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	a	b	τ_w	given current
trigonometric function	10	-65	-73	30	-13	1	2	0.7	40	100

Table 3.4: information

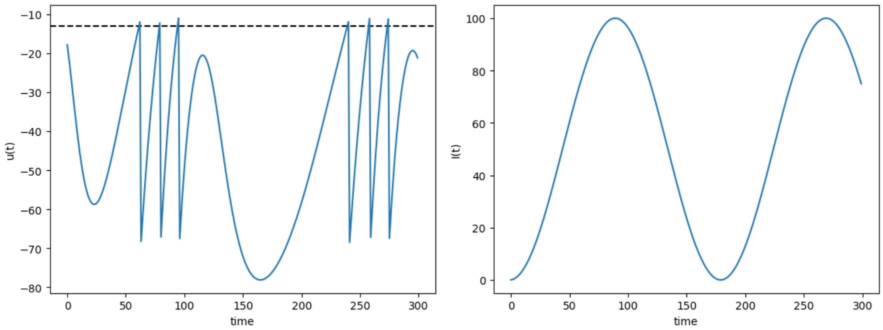

Figure 3.4: AELIF behavior on information given in table 3.4

3.4 adding noise to the AELIF model's input

In this section, we are going to define a random noise function as our model's input current.

input type	τ	u_{rest}	u_{reset}	Δ_T	threshold	R	a	b	τ_w	given current
random function	10	-65	-73	30	-13	1	2	0.7	40	100

Table 3.5: information

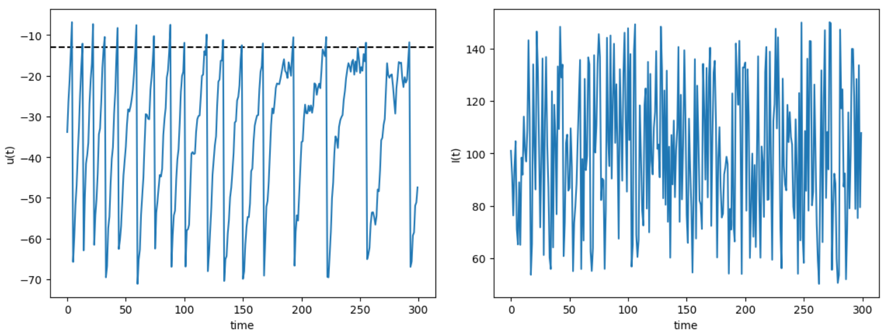

Figure 3.5: AELIF behavior on information given in table 3.5

3.5 adding refractory period to the AELIF model

In this section, we are going to add the refractory period to our AELIF model. After doing this, we expect the model not to fire during a while just after firing an action potential.

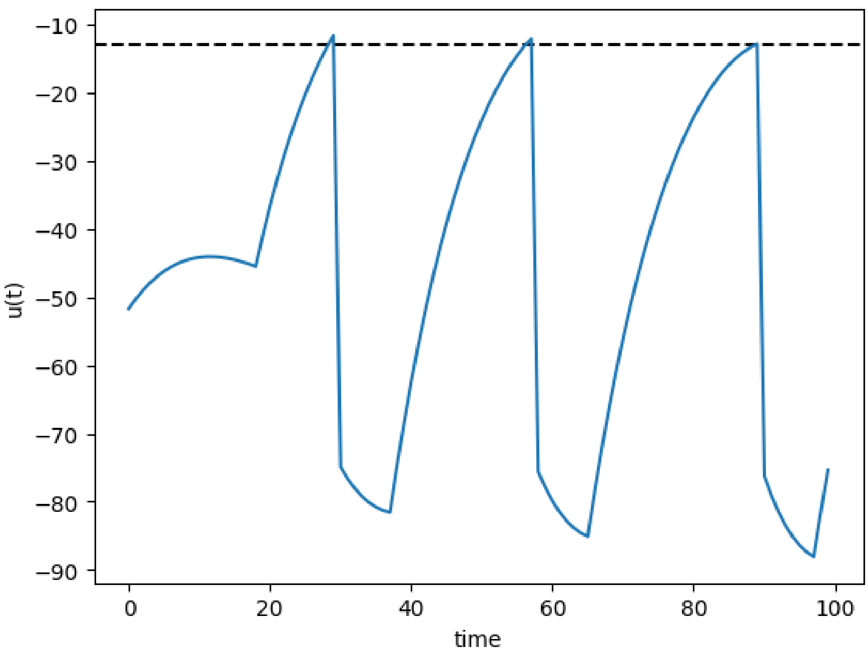

Figure 3.6: AELIF behavior with refractory period

3.6 F-I curve on AELIF model

In this section we are going to plot the frequency-current curve for AELIF model with a fixed current, and after that considering a noise function.

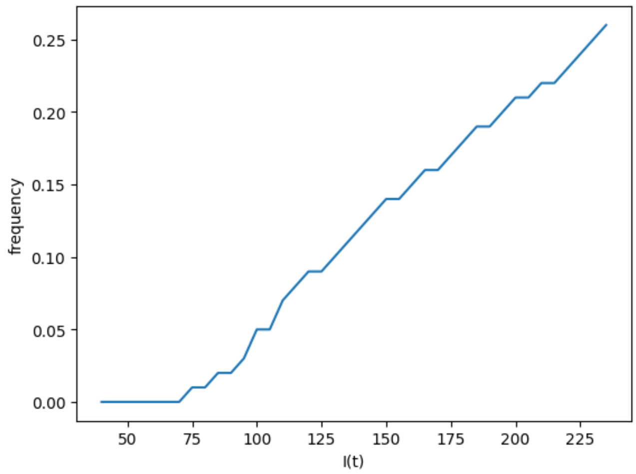

Figure 3.7: F-I curve on AELIF model with fixed input current

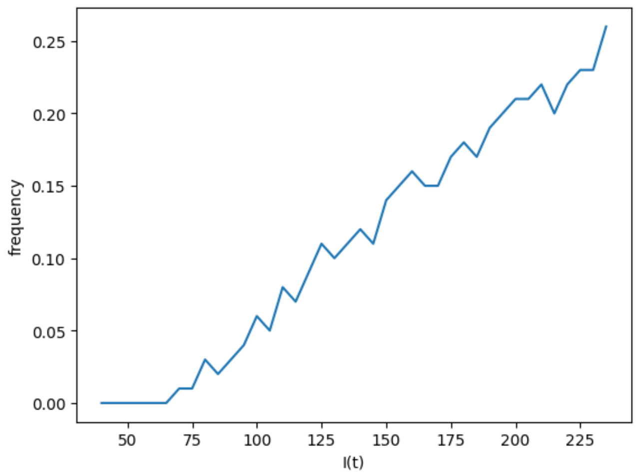

Figure 3.8: F-I curve on AELIF model with noise

It's a matter of great importance to notice the AELIF's F-I curve is increasing by a less gradient in comparison with the ELIF's F-I curve, which itself increases again by a less gradient in comparison with the LIF's F-I curve. Also we can easily see the effect of adaption in figure 3.7 if we compare it with the figure 2.10 in chapter 2, which shows the F-I curve on ELIF model.

Chapter 4

Conclusions

In this report we tried to review the leaky integrate-and-fire model and some of its expansions for a single neuron, which probably is the best-known example of a formal spiking neuron model. We studied the model's behaviour on different types of current inputs, parameters and different functions we defined. We observed how adding the refractory period effects the neuron's function, and analyzed frequency-current curve for different expansions of the model.
However, it's good to remark that LIF model has some limitations; The leaky integrate-and-fire model is highly simplified and neglects many aspects of neuronal dynamics. In particular, input, which may arise from presynaptic neurons or from current injection, is integrated linearly, independently of the state of the postsynaptic neuron. Furthermore, after each output spike the membrane potential is reset; So that no memory of previous spikes is kept and a neuron model that has no memory beyond the most recent spike, cannot describe bursting. It also can't describe adaption and inhibitory rebound.