NEURAL NETWORKS AND DECISION-MAKING

Bita Moharrami
University of Tehran

Supervised by: Mohammad Ganjtabesh

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Abstract

Neurons do not work in isolation, but are embedded in networks of neurons with similar properties. Such networks of similar neurons can be organized as distributed assemblies or as local pools of neurons. Groups of neurons with similar properties can be approximated as homogeneous or weakly heterogeneous populations of neurons. In mathematical models, the connectivity within the population is typically all-to-all or random. The population activity is defined as the number of spikes fired in a short instant of time, averaged across the population. Since each neuron in a population receives input from many others (either from the same and/or from other populations) its total input at each moment in time depends on the activity of the presynaptic population(s). Hence the population activity A(t) controls the mean drive of a postsynaptic neuron.

An influential computational model describes decision making as the competition of several populations of excitatory neurons which share a common pool of inhibitory neurons. Under suitable conditions, the explicit model of inhibitory neurons can be replaced by an effective inhibitory coupling between excitatory populations.

Contents

Chapter 1

Synapse

1.1 The synapse

How do neurons "talk" to one another? The action happens at the synapse, the point of communication between two neurons or between a neuron and a target cell, like a muscle or a gland. At the synapse, the firing of an action potential in one neuronâthe presynaptic, or sending, neuronâcauses the transmission of a signal to another neuronâthe postsynaptic, or receiving, neuronâmaking the postsynaptic neuron either more or less likely to fire its own action potential.

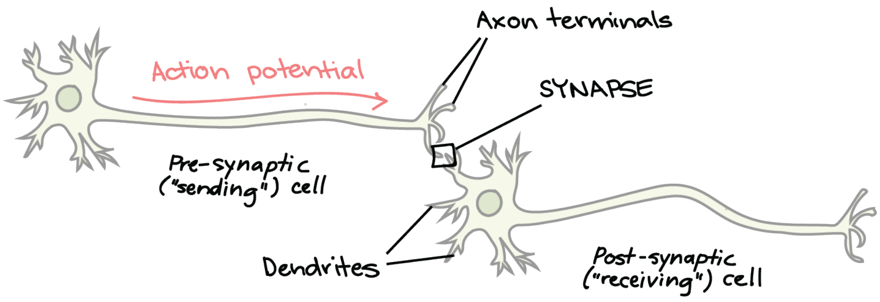

Figure 1.1: Presynaptic neuron, synapse cleft and postsynaptic neuron

If we want to describe action at the synapse, we can simply consider following steps:

1. Arrival of action potential at the terminal button causes vesicles to move toward synaptic gap.

2. Those reaching the cleft burst open, dumping neurotransmitter into the cleft.

3. Neurotransmitter locks onto receptor proteins, activating them.

4. Resting potential of receiving neuron changes temporarily.

5. Neurotransmitter is removed from receptor proteins, terminating the action.

Synapses can be thought of as converting an electrical signal (the action potential) into a chemical signal in the form of neurotransmitter release, and then, upon binding of the transmitter to the postsynaptic receptor, switching the signal back again into an electrical form, as charged ions flow into or out of the postsynaptic neuron.
When a neurotransmitter binds to its receptor on a receiving cell, it causes ion channels to open or close. This can produce a localized change in the membrane potential of the receiving cell. In some cases, the change makes the target cell more likely to fire its own action potential. In this case, the shift in membrane potential is called an excitatory postsynaptic potential, or EPSP. In other cases, the change makes the target cell less likely to fire an action potential and is called an inhibitory postsynaptic potential, or IPSP.

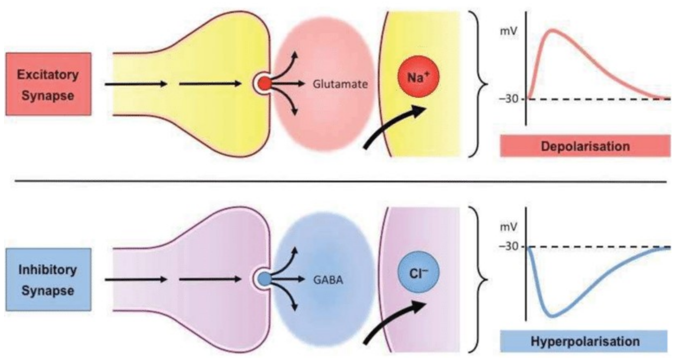

Figure 1.2: Inhibitory and excitatory synapses

About 80% of neurons are excitatory and 20% of them are inhibitory.

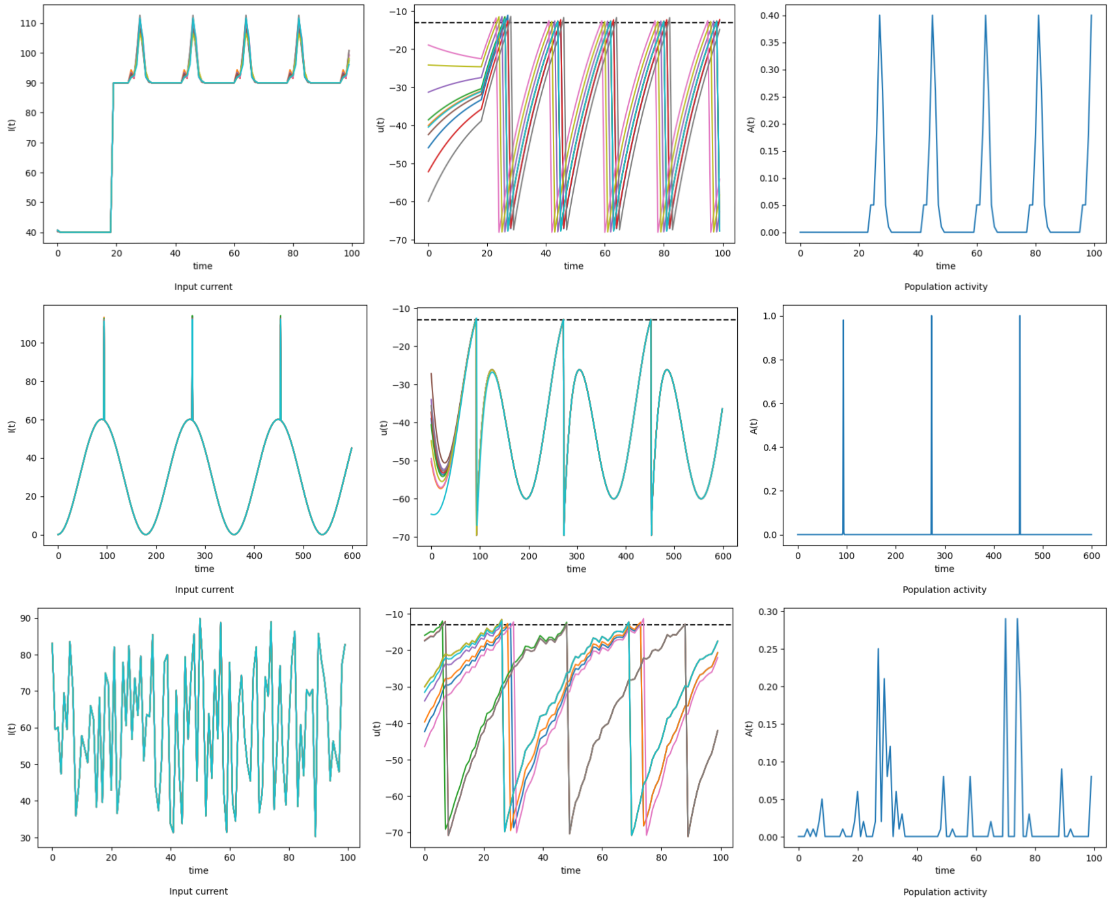

Figure 1.3: LIF neural population with various input currents

In figure 1.3, we can see behavior of a population of connected neurons due to the three different types of input currents: step function, trigonometric and noisy input.

Chapter 2

Neural population

2.1 Population activity

In a population of N neurons, we calculate the proportion of active neurons by counting the number of spikes n_{act}(t;t+Δt) in a small time interval Δt and dividing by N. Further division by Δt yields the population activity

$$A(t) = lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{n_{act}(t; t + \Delta t)}{N} = \frac{1}{N} \sum_{j=1}^{N} \sum_{f} \delta(t - t_j^{(f)})$$

where δ denotes the Dirac δ function. The double sum runs over all firing times $t_j^{(f)}$ of all neurons in the population. In other words the activity A is defined by a population average.

2.2 Connectivity schemes

The real connectivity between cortical neurons of different types and different layers, or within groups of neurons of the same type and the same layer is still partially unknown, because experimental data is limited. At most, some plausible estimates of connection probabilities exist. In some cases the connection probability is considered as distance-dependent, in other experimental estimates as uniform in the restricted neighborhood of a cortical column. In simulations of spiking neurons, there are a few coupling schemes that are frequently adopted. Most of these assume random connectivity within and between populations. In this section we discuss these schemes with a special focus on the scaling behavior induced by each choice of coupling scheme. Here, scaling behavior refers to a change in the number N of neurons that participate in the population.

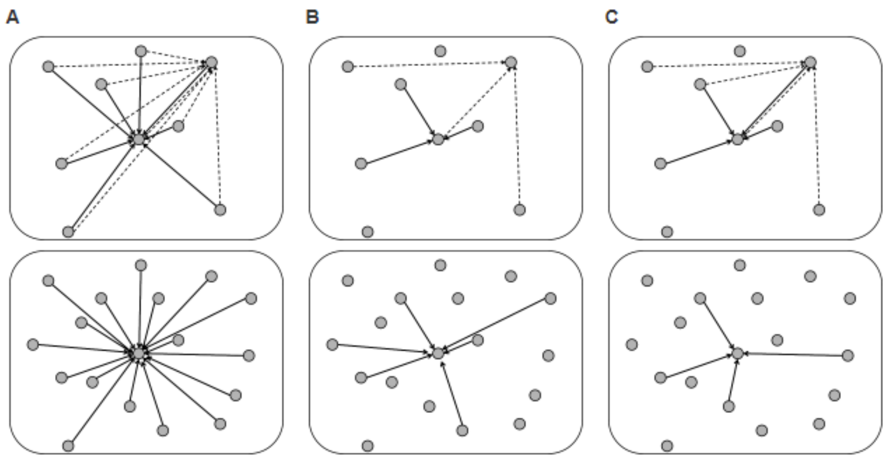

Figure 2.1: Coupling Schemes. **A.** Full connectivity: Top: A network of 9 neurons with all-to-all coupling. The input links are shown for two representative neurons. Self-couplings are not indicated. Bottom: The number of input links (indicated for one representative neuron) increases, if the size of the network is doubled. **B.** Random coupling with fixed connection probability. In a network of 18 neurons (bottom) the number of input links is larger than in a network of 9 neurons (top). **C.** Random coupling with fixed number of inputs. The number of links from presynaptic neurons (top: input links to two representative neurons) does not change when the size of the network is increased (bottom: input links to one representative neuron).

2.2.1 Full connectivity

The simplest coupling scheme is all-to-all connectivity within a population. All connections have the same strength. If we want to change the number N of neurons in the simulation of a population, an appropriate scaling law is:

$$w_{ij} = \frac{J_0}{N}$$

parameter	neuron model	N	J_0
	LIF	100	500

Table 2.1: Full connectivity scheme with synaptic weight 5

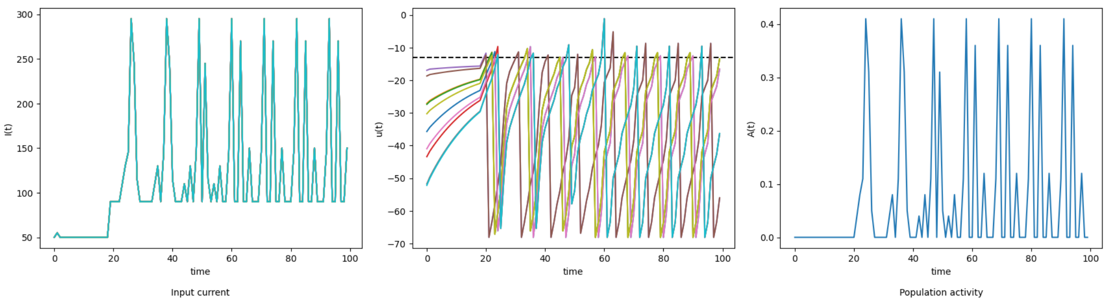

Figure 2.2: Population behavior with the parameters given in table 2.1

parameter	neuron model	N	J_0
	LIF	100	1000

Table 2.2: Full connectivity scheme with synaptic weight 10

parameter	neuron model	N	J_0
	LIF	100	100

Table 2.3: Full connectivity scheme with synaptic weight 1

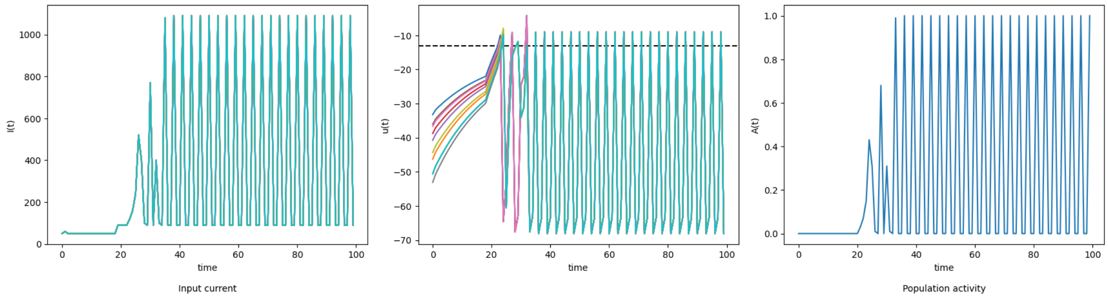

Figure 2.3: Population behavior with the parameters given in table 2.2; In this figure we can see the effect of increasing synaptic weights.

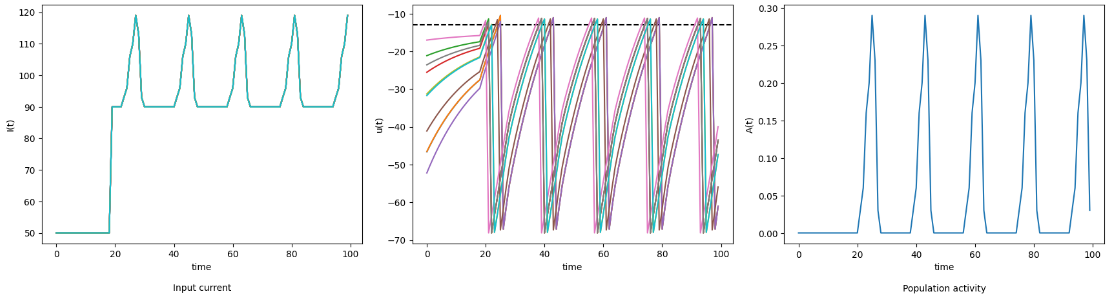

Figure 2.4: Population behavior with the parameters given in table 2.3; In this figure we can see the effect of decreasing synaptic weights.

As we can in figures 2.3 and 2.4, the synaptic weights and spikes in a fixed population are in a direct relationship; The effect of synapses increases by increasing synaptic weights and it causes more action potentials in our neural population; This happens simply because of we don't have any inhibitory synapses, yet and neurons' activities only reinforce one another.
Now we are going to see the effect of noise in the population activity; First, we will put a noise current on our population with synaptic weight of 1:

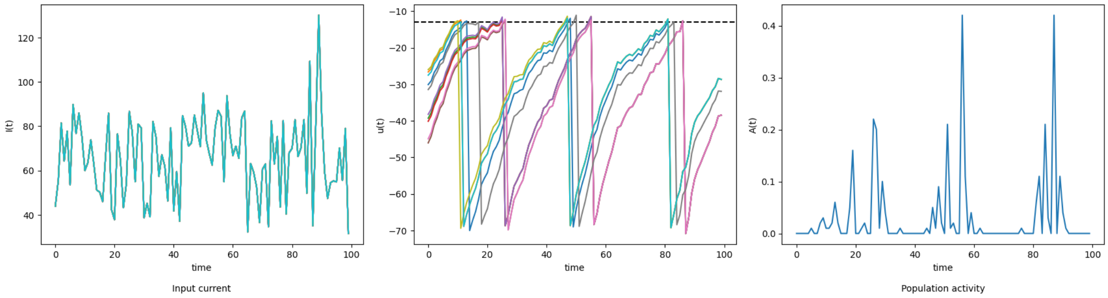

Figure 2.5: LIF population behavior with noise and synaptic weight of 1

Now, we will put a noise current on the same population with synaptic weight of 10:

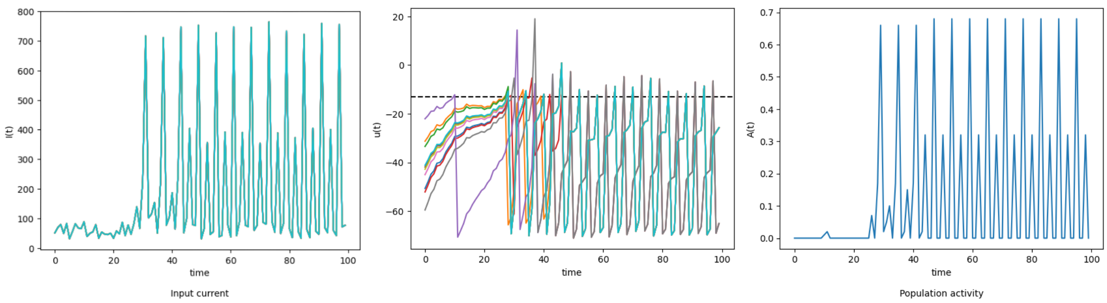

Figure 2.6: LIF population behavior with noise and synaptic weight of 10

As expected, the sensibility of population to the noise increases by increasing synaptic weights.

2.2.2 Random coupling: Fixed coupling probability

In this connectivity scheme, we choose connections randomly with probability p among all the possible N^2 connections and scale the strength of the connections as:

$$w_{ij} = \frac{J_0}{C} = \frac{J_0}{pN}$$

parameter	neuron model	N	J_0	p
	LIF	100	500	0.4

Table 2.4: Random coupling scheme with fixed coupling probability p and synaptic weight 12.5

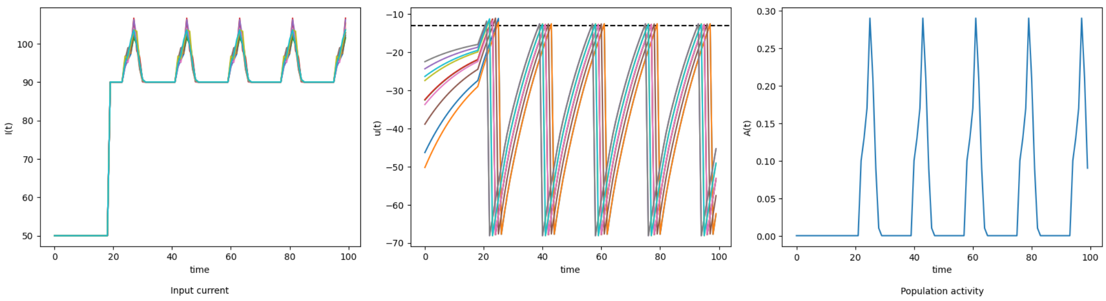

Figure 2.7: Population behavior with the parameters given in table 2.4

Now, we are going to change the coupling probability and analyse what happens in each case.

parameter	neuron model	N	J_0	p
	LIF	100	500	1

Table 2.5: Random coupling scheme with fixed coupling probability p and synaptic weight 5

parameter	neuron model	N	J_0	p
	LIF	100	500	0.1

Table 2.6: Random coupling scheme with fixed coupling probability p and synaptic weight 50

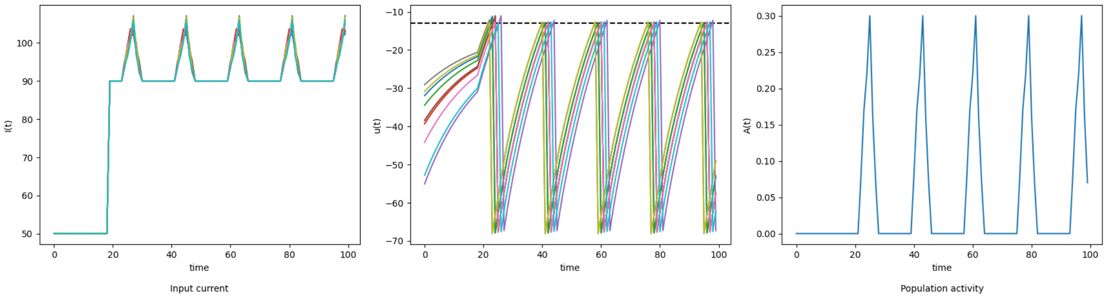

Figure 2.8: Population behavior with the parameters given in table 2.5

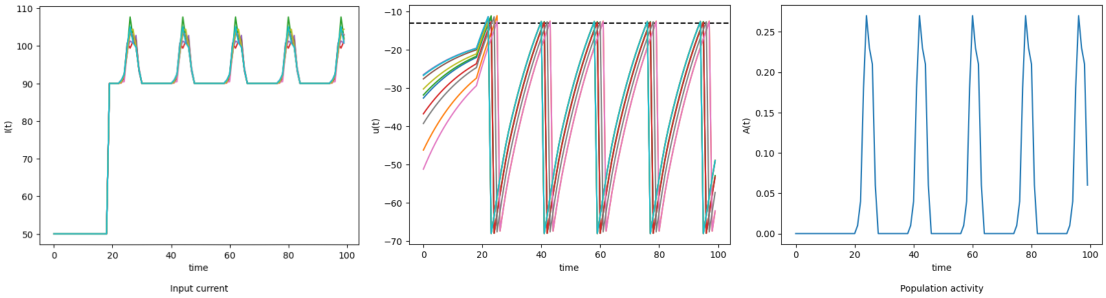

Figure 2.9: Population behavior with the parameters given in table 2.6

As expected, by decreasing the coupling probability, synaptic weight increases, which means every neuron has fewer connections with the other neurons but the effect of its connections is stronger. On the other hand, by increasing the coupling probability we are more likely to have a full connectivity scheme, but with not such effective connections.

2.2.3 Random coupling: Fixed number of presynaptic partners

In this connectivity scheme, we construct a random network with a fixed number C of inputs; For each neuron, we choose randomly its C presynaptic partners. Here no scaling of the connections with the population size N is necessary. We expect the more effect of population's activity on each single neuron by increasing the number of presynaptic partners.

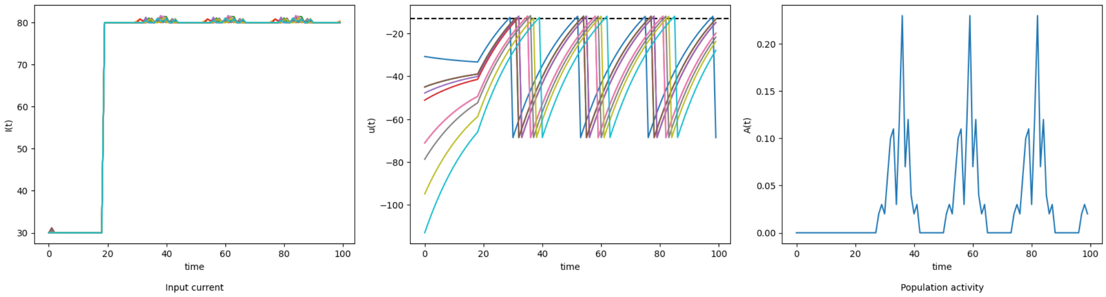

Figure 2.10: Random coupling scheme with fixed number of presynaptic partners; Population with N = 100 and C = 10.

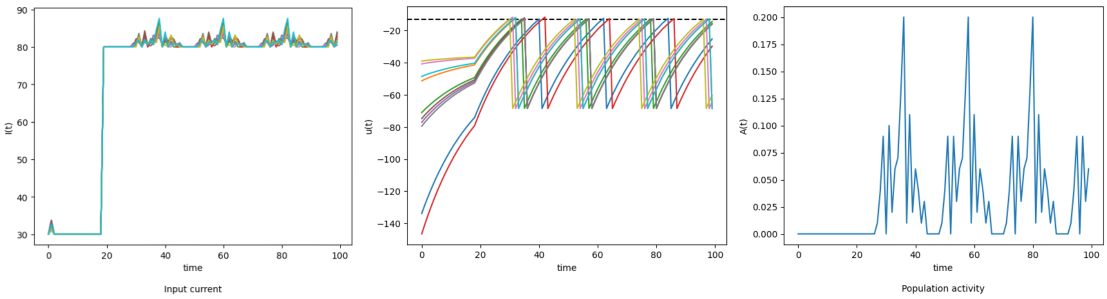

Figure 2.11: Random coupling scheme with fixed number of presynaptic partners; Population with N = 100 and C = 50.

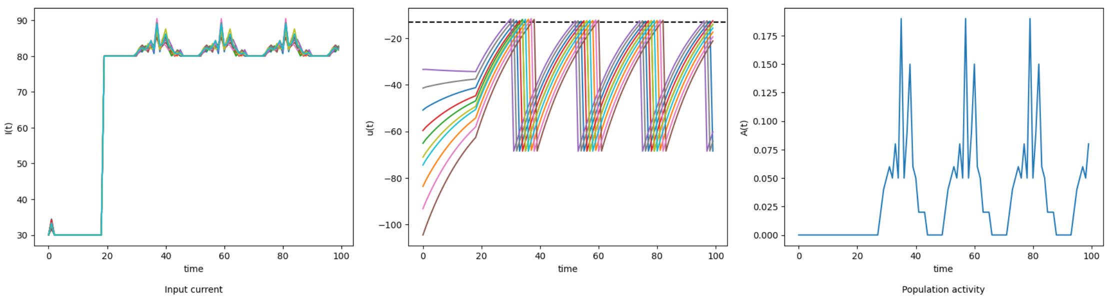

Figure 2.12: Random coupling scheme with fixed number of presynaptic partners; Population with N = 100 and C = 80.

Chapter 3

Homogeneous network

3.1 Homogeneous networks and their behavior

In this chapter, we study a large and homogeneous population of neurons. By homogeneous we mean that:

1. All neurons $1 \leq i \leq N$ are identical.
2. All neurons receive the same external input $I_i^{ext}(t) = I^{ext}(t)$.
3. The interaction strength w_{ij} for the connection between any pair j,i of pre- and postsynaptic neurons is statistically uniform.

Note that **not** all neurons need to be coupled with each other; Connections can, for example, be chosen randomly. Now, we are going to see the homogeneous network's activity according to various parameters.

parameter	neuron model	connectivity scheme	N	J_0
excitatory neuron group	LIF	full connectivity	80	500
inhibitory neuron group	LIF	full connectivity	20	500

Table 3.1: Homogeneous network with full connectivity scheme

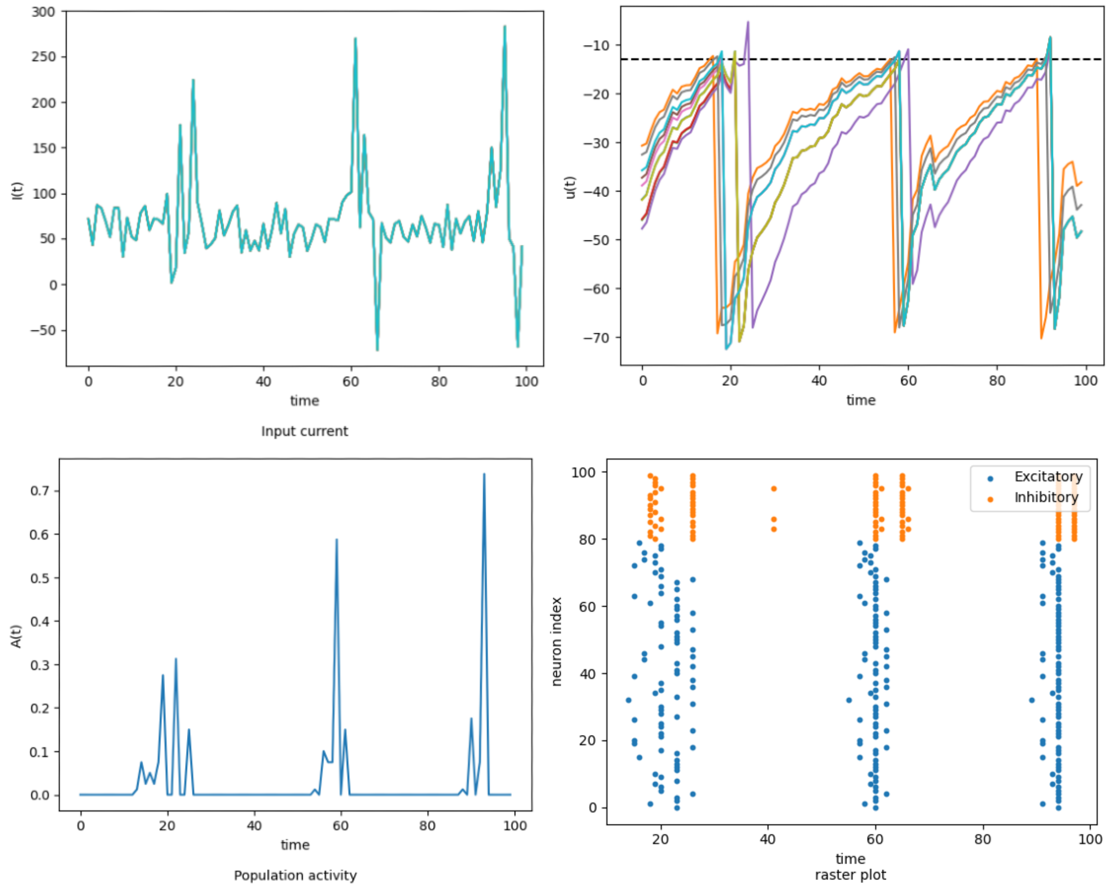

Figure 3.1: Homogeneous network with the parameters given in table 3.1

Now, we are going to increase the synaptic weight of inhibitory-to-excitatory synapses; As a result, we expect fewer spikes of excitatory neurons in our network because this time our inhibitory-to-excitatory synapses are stronger, and their effect of preventing firing potentials would be more impressive:

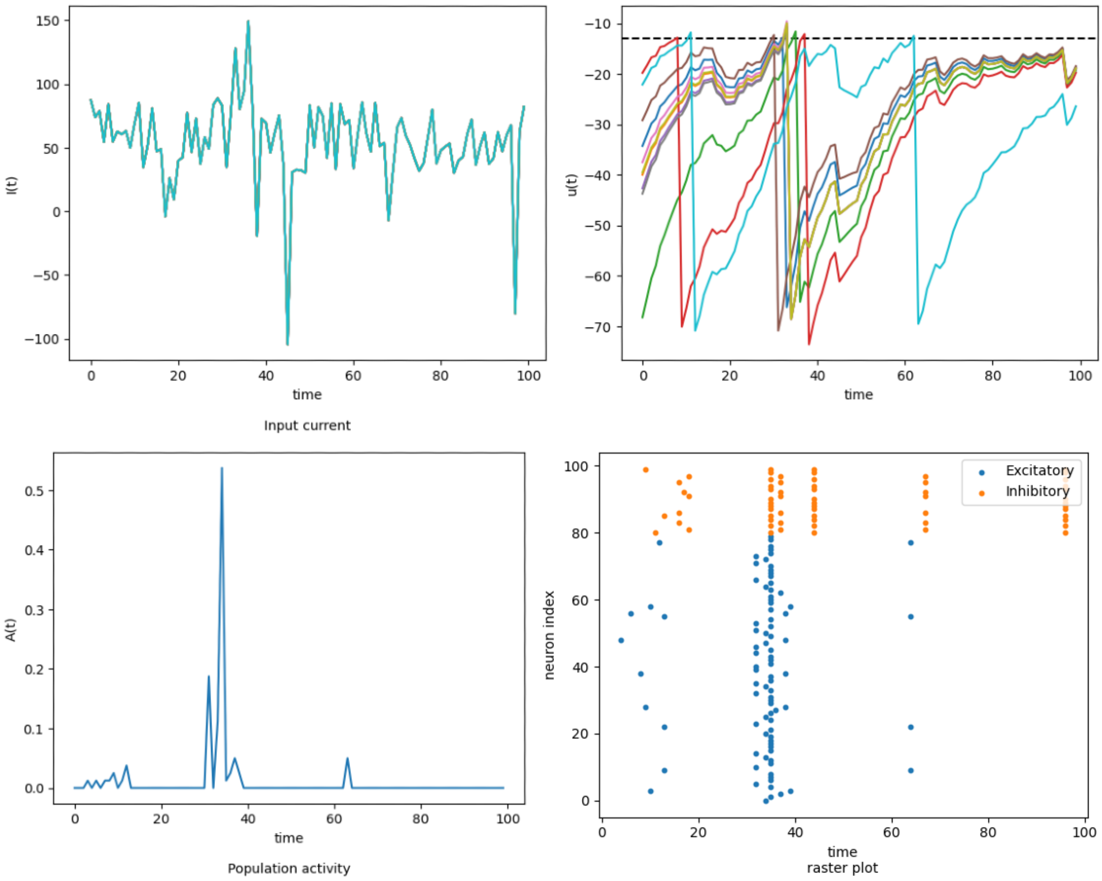

Figure 3.2: Homogeneous network with inhibitory-to-excitatory synaptic weight of 12.5

Now, we are going to see the homogeneous network's activity on random coupling with fixed coupling probability connectivity scheme:

parameter	neuron model	connectivity scheme	N	J_0	p
excitatory neuron group	LIF	fixed coupling probability	80	500	0.3
inhibitory neuron group	LIF	fixed coupling probability	20	500	0.3

Table 3.2: Homogeneous network with random coupling: fixed coupling probability connectivity scheme

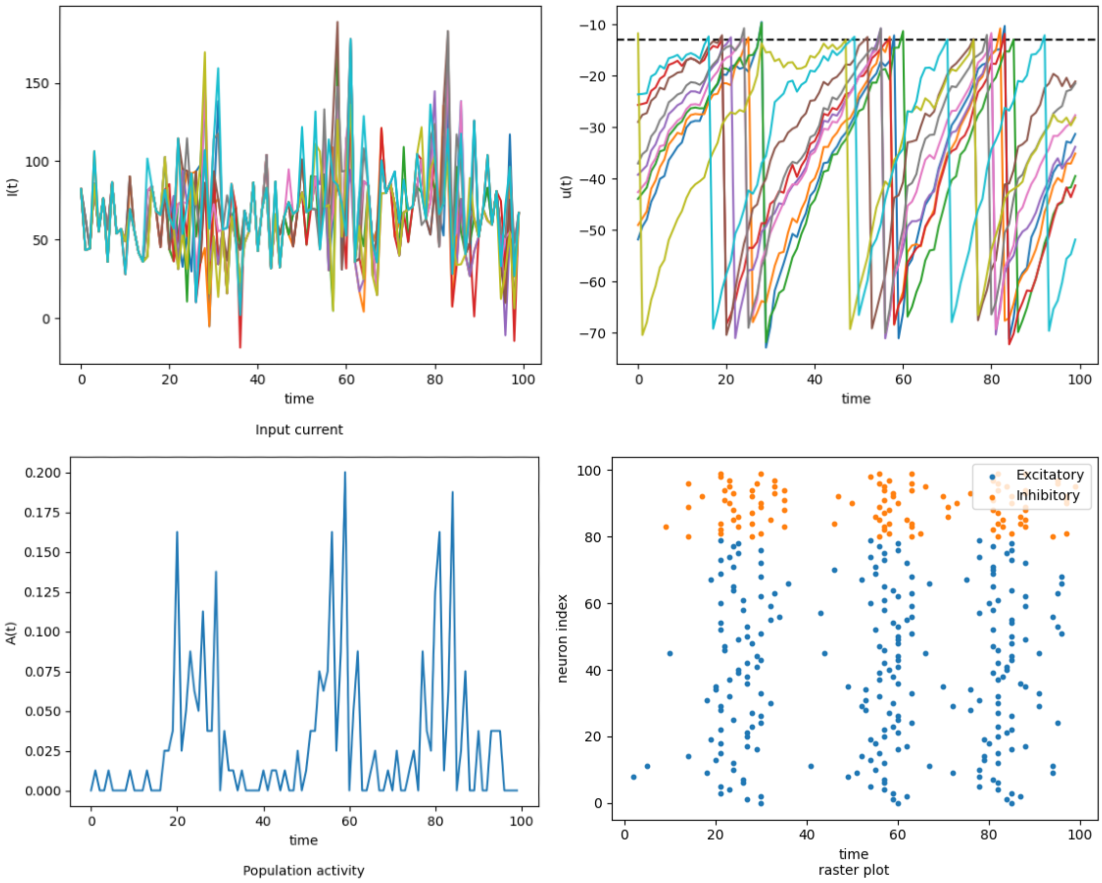

Figure 3.3: Homogeneous network with the parameters given in table 3.2

Now, we are going to increase the connectivity probability and analyse what happens in that case.

parameter	neuron model	connectivity scheme	N	J_0	p
excitatory neuron group	LIF	fixed coupling probability	80	500	0.8
inhibitory neuron group	LIF	fixed coupling probability	20	500	0.8

Table 3.3: Homogeneous network with random coupling: fixed coupling probability connectivity scheme

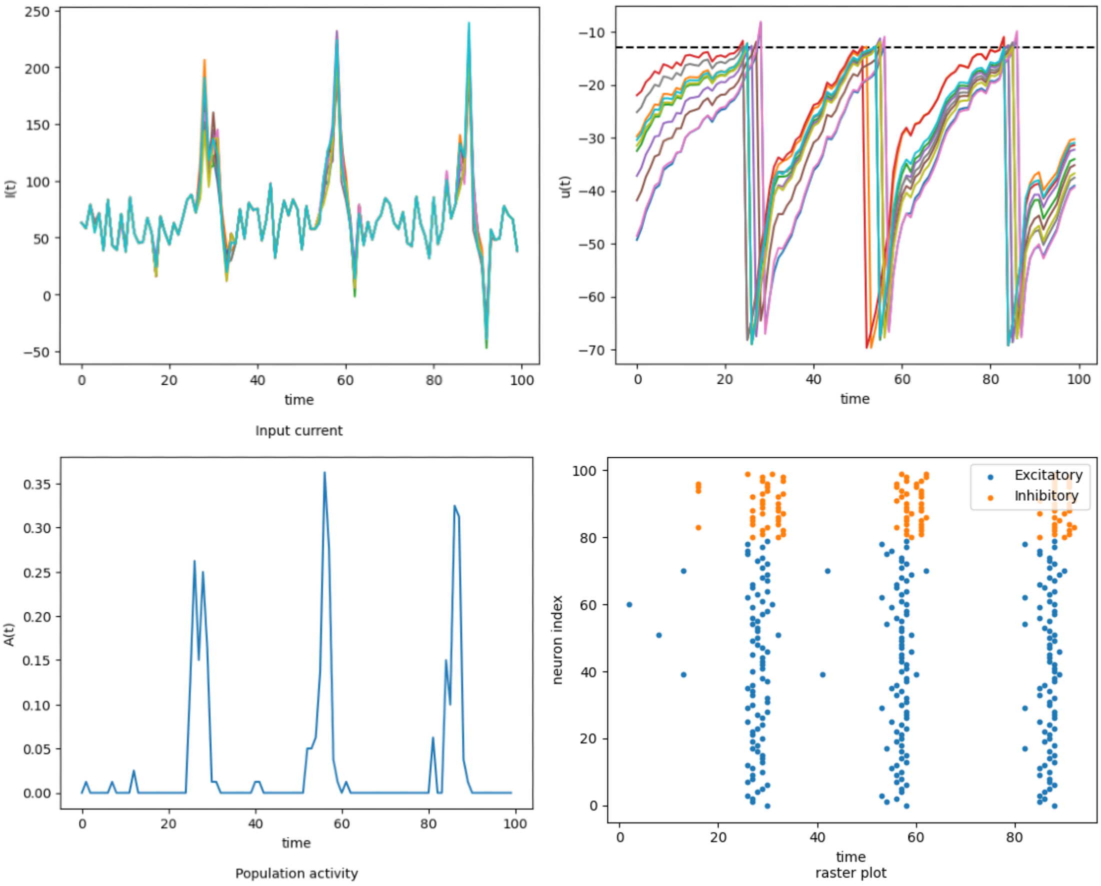

Figure 3.4: Homogeneous network with the parameters given in table 3.3

As we can see, increasing the connectivity probability causes more similarity in the behavior of our network's neurons. In other words, our network is more likely to be **homogeneous**.

Now let's try the random coupling with fixed number of presynaptic partners connectivity scheme on our network.

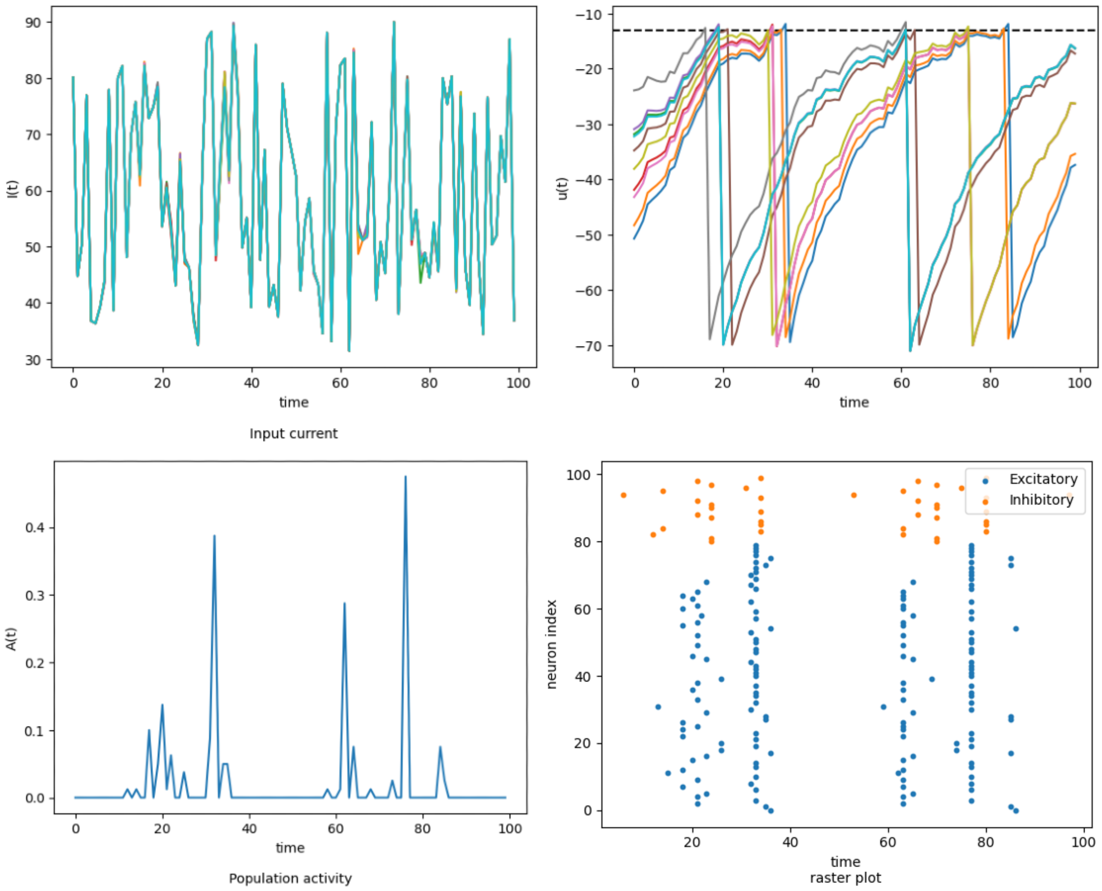

Figure 3.5: Homogeneous network with random coupling scheme: fixed number of presynaptic partners; C = 20.

Now we are going to increase the number of presynaptic partners then analyse what happens in our network.

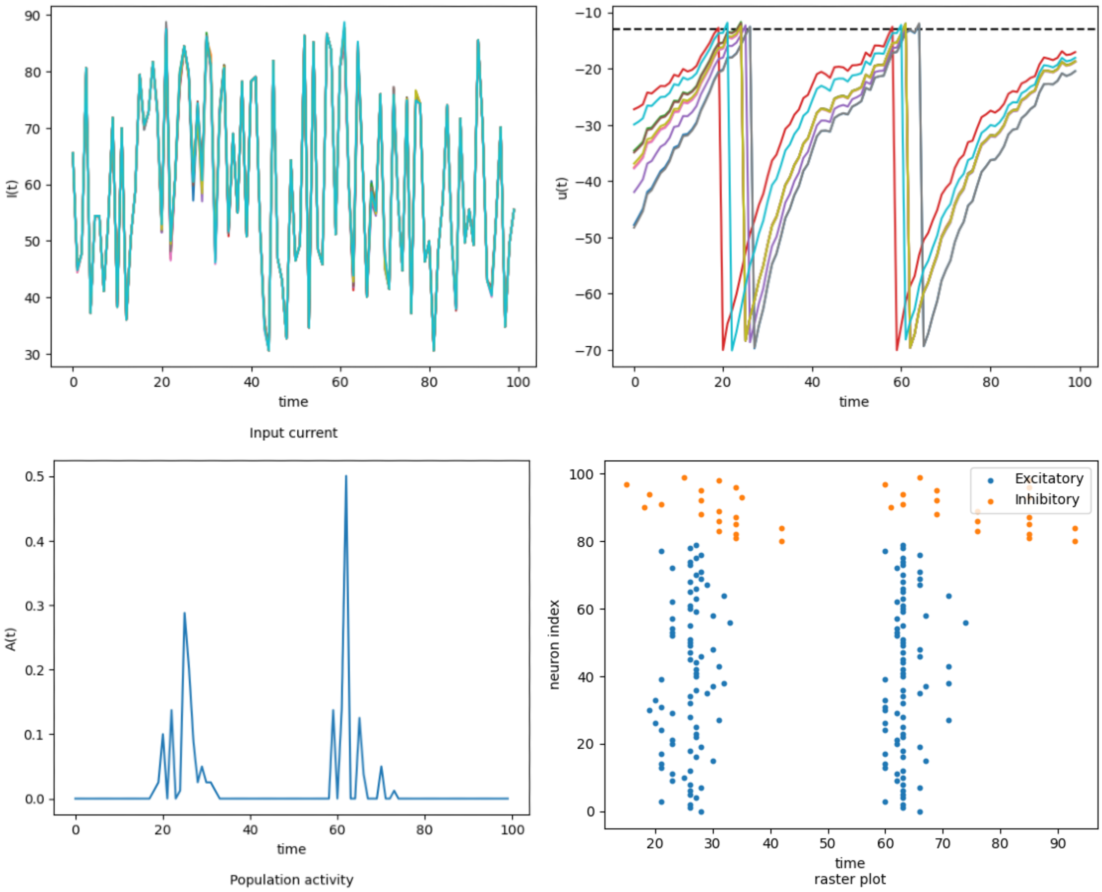

Figure 3.6: Homogeneous network with random coupling scheme: fixed number of presynaptic partners; C = 60.

As we can understand from figures 3.5 and 3.6, there is not such a remarkable difference between the two situations; The only thing to notice is the greater number of spikes according to the increment in the number of presynaptic partners, and so in inputs of each neuron (This is because of the greater percentage of excitatory neurons in the network).

Chapter 4

Decision making

4.1 Competition through common inhibition

Consider a network consisting of two excitatory populations interacting with a common inhibitory population. Now, if we have a strong, but unbiased stimulus, then immediately after the onset of stimulation, both excitatory populations increase their firing rates. Soon after that, one of the activities grows further and the other one is suppressed. The prior population is called the winner of the competition.

4.2 Dynamics of decision making

Let $A_{E,k} = g_E(h_{E,k})$ denote the population activity of an excitatory population k driven by an input potential $h_{E,k}$. $A_{inh} = g_{inh}(h_{inh})$ is the activity of the inhibitory population. g_E and g_{inh} are the gain functions. The input potentials evolve according to:

$$\tau_E \frac{dh_{E,1}}{dt} = -h_{E,1} + w_{EE} g_E(h_{E,1}) + w_{EI} g_{inh}(h_{inh}) + RI_1$$

$$\tau_E \frac{dh_{E,2}}{dt} = -h_{E,2} + w_{EE} g_E(h_{E,2}) + w_{EI} g_{inh}(h_{inh}) + RI_2$$

$$\tau_{inh}\frac{dh_{inh}}{dt} = -h_{inh} + w_{IE}g_E(h_{E,1}) + w_{IE}g_E(h_{E,2})$$

4.3 Competing populations and decision making

In this section, we have a network containing one inhibitory and two excitatory neuron groups and we want to simulate the decision making process through it.

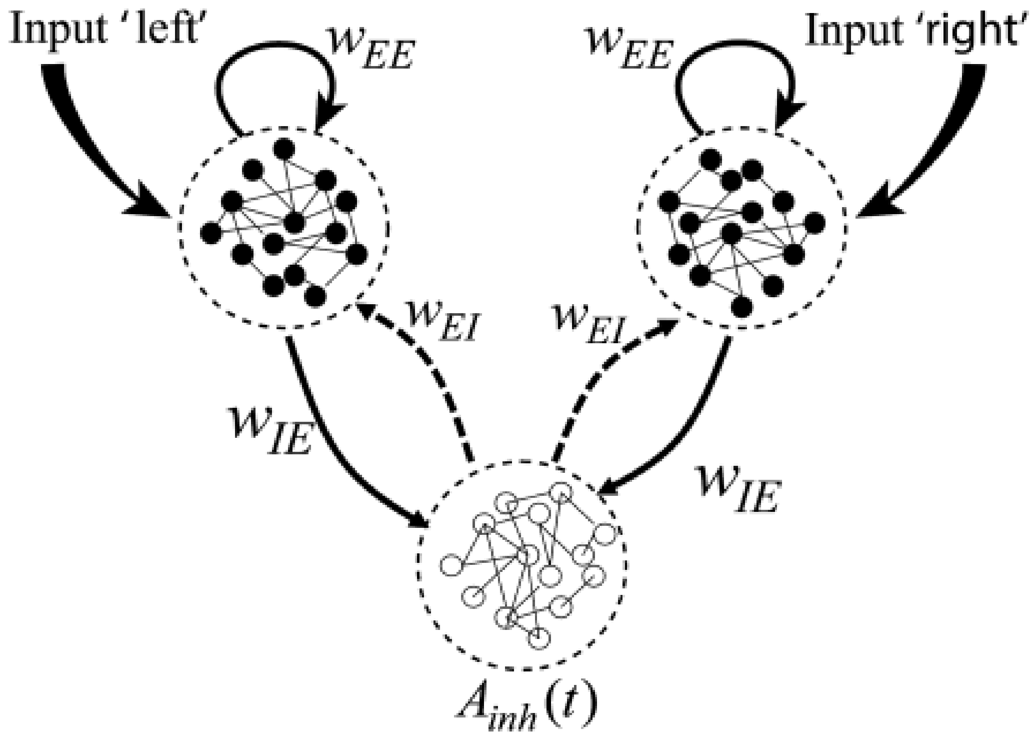

Figure 4.1: Competition between neuronal pools. Two populations of excitatory neurons interact with a common pool of inhibitory neurons. Input signals indicating movement to the left are fed into population 1 with activity $A_{E,1}$(t). Each population of excitatory neurons makes excitatory connections of strength W_{EE} onto itself. The inhibitory population receives input of strength W_{IE} from the two excitatory populations and sends back inhibition of strength W_{EI}.

First, we are going to simulate the decision making process with the full connectivity scheme.

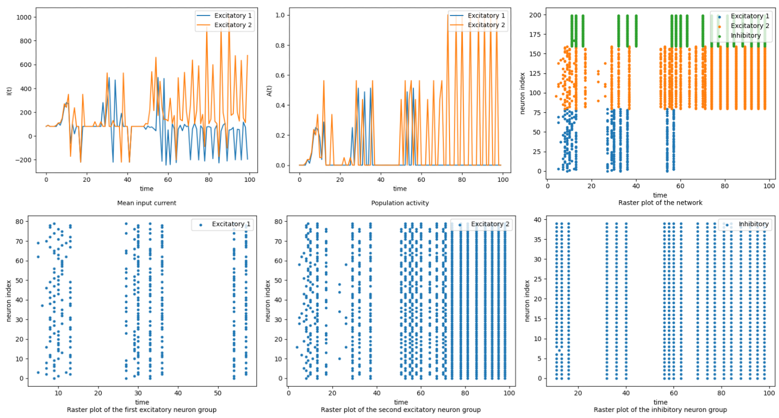

Figure 4.2: Decision making process with the full connectivity scheme; N = 200.

As we can see in figure 4.2, once the input current of the second excitatory neuron group gets stronger, its population activity increases, while the first excitatory neuron group's activity decreases; It is saying, the second excitatory neuron group is the winner of the decision making process.

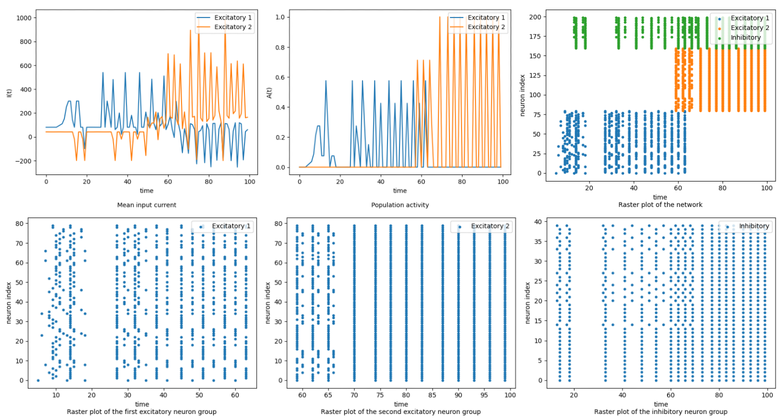

Figure 4.3: Decision making process with the full connectivity scheme; N = 200.

In figure 4.3, we can see a change of mind in the decision making process! This happens because this time, we decreased the second excitatory neuron group's

primary input current to the extent that the second excitatory neuron group's current overcame it.
Now, let's simulate the decision making process with the random coupling: fixed coupling probability connectivity scheme and analyse what happens.

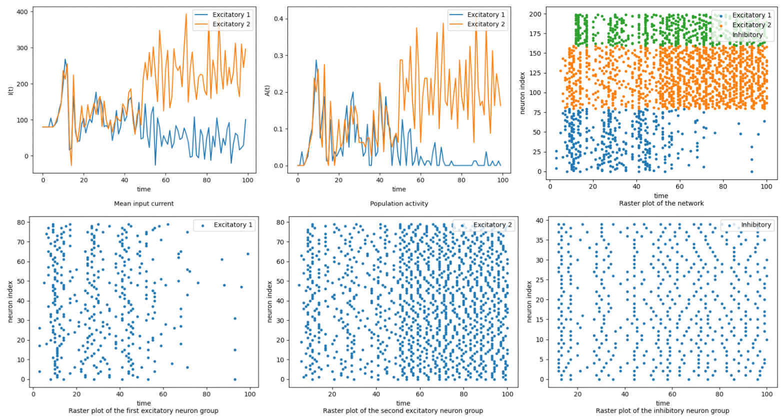

Figure 4.4: Decision making process with the random coupling: fixed coupling probability connectivity scheme; N = 200, p = 0.3.

As we can see in figure 4.4, the simulation of the decision making process using this connectivity scheme is much more realistic than the previous simulations in which we used the full connectivity scheme.
Now let's see how would mind changing look like in this simulation:

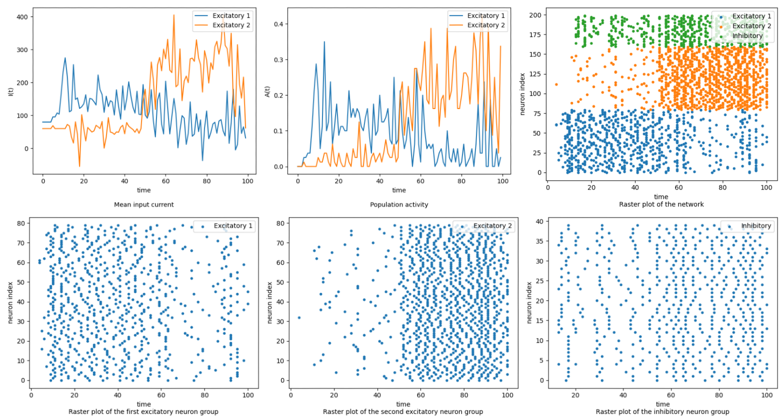

Figure 4.5: Changing of mind process with the random coupling: fixed coupling probability connectivity scheme; N = 200, p = 0.3.

This time, let's increase the connectivity probability and analyse what happens:

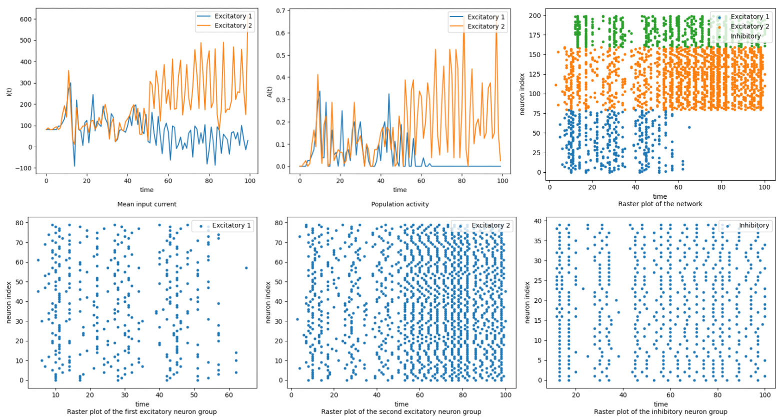

Figure 4.6: Decision making process with the random coupling: fixed coupling probability connectivity scheme; N = 200, p = 0.7.

As expected, by increment of the coupling probability amount, the process would occur more likely to the simulations in which we had full connectivity between our neurons.

At the end, we are going to simulate the decision making process with the random coupling: fixed number of presynaptic partners connectivity scheme.

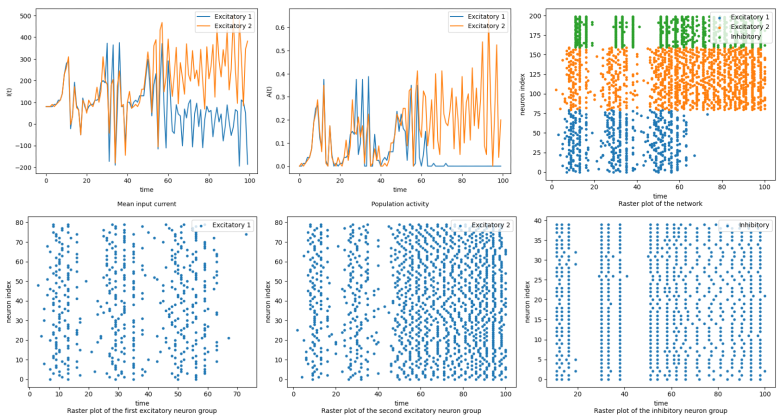

Figure 4.7: Decision making process with the random coupling: fixed number of presynaptic partners connectivity scheme; N = 200, C = 30.

As we can clearly see in figure 4.7, this simulation is so close to the simulation with fixed coupling probability connectivity which we showed in figure 4.4; Now let's increase the number of presynaptic partners and see what happens in that case.

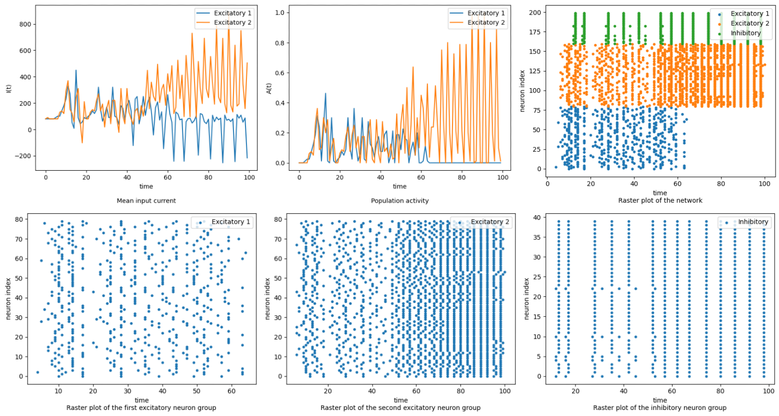

Figure 4.8: Decision making process with the random coupling: fixed number of presynaptic partners connectivity scheme; N = 200, C = 60.

We can see that by increasing the number of presynaptic partners, the simulation's behavior gets close to the process in a full connective network.

Chapter 5

Conclusions

In this report, we studied the mechanism of synapse and with the help of that, we constructed neuronal populations; After that, we constructed neural networks via three different schemes of neuronal connectivity: full connectivity, random coupling with fixed coupling probability, and random coupling with fixed number of presynaptic partners.
Using our neural network, we simulated the decision making process and analyzed the process according to different situations.