Mathematical Justification of Technical Parameters: Cold-Welding Robotic Attachment

1. Required Pressure for Cold Welding

Material: Aluminum 6061
Work of adhesion: $W_a = 2\gamma = 2 \times 1.14 = 2.28\,\mathrm{J/m^2}$
Pressure must exceed the material's yield strength at asperities. For Al6061 annealed:

$$\sigma_y = 95\,\mathrm{MPa} \Rightarrow \boxed{P = 100\,\mathrm{MPa}} \text{ (rounded)}$$

2. Welding Force

$$A = 1\,\mathrm{cm^2} = 1 \times 10^{-4}\,\mathrm{m^2}$$
$$F = P \cdot A = (1 \times 10^8)(1 \times 10^{-4}) = \boxed{10\,000\,\mathrm{N}}$$

3. Energy to Initiate Bond (Griffith Criterion)

$$E_c = \frac{\pi \sigma^2 a}{E}$$
$$\sigma = 100 \times 10^6\,\mathrm{Pa}, \quad a = 10^{-6}\,\mathrm{m}, \quad E = 69 \times 10^9\,\mathrm{Pa}$$
$$E_c = \frac{\pi (10^8)^2 \cdot 10^{-6}}{6.9 \times 10^{10}} = \boxed{0.455\,\mathrm{J}}$$

4. Frictional Work to Break Oxide Layer

$$W_f = \mu F_n d = 0.5 \cdot 10^4 \cdot 0.001 = \boxed{5\,\mathrm{J}}$$

5. Power Requirement

$$W = F \cdot d = 10^4 \cdot 0.001 = 10\,\mathrm{J}$$
$$t = 10\,\mathrm{s} \Rightarrow P = \frac{10}{10} = 1\,\mathrm{W} \quad \text{(ideal)}$$

$$P_{\text{actual}} = \frac{1}{0.25} = \boxed{4\,\text{W}} \quad (25\%\ \text{efficiency})$$

$$P_{\text{total}} \approx \boxed{20\,\text{W} - 30\,\text{W}} \quad (\text{with controls})$$

6. Thermal Operating Range (Stefan-Boltzmann Law)

$$Q = \epsilon\sigma A(T^4 - T_{env}^4)$$

$$T_{env} = 30\,\text{K}, \quad T = [90\,\text{K}, 410\,\text{K}] \Rightarrow \boxed{[-183°C, +137°C]}$$

7. Mass Estimation

$$\rho = 2700\,\text{kg/m}^3, \quad V = 800\,\text{cm}^3 = 8 \times 10^{-4}\,\text{m}^3$$

$$m = \rho \cdot V = 2700 \cdot 8 \times 10^{-4} = \boxed{2.16\,\text{kg}}$$

8. Joint Strength (Shear Criterion)

$$\sigma_y = 95\,\text{MPa} \Rightarrow \tau_y = \frac{\sigma_y}{\sqrt{3}} = \frac{95}{1.732} = \boxed{54.85\,\text{MPa}}$$

$$\tau_{\text{target}} = 1.5 \cdot \tau_y = 1.5 \cdot 54.85 = \boxed{82.28\,\text{MPa}} \Rightarrow \boxed{\tau = 80{-}150\,\text{MPa}}$$

9. Energy per Weld

$$P = 20\,\text{W}, \quad t = 10\,\text{s} \Rightarrow E = P \cdot t = 200\,\text{J}$$

$$E = \frac{200}{3600} = \boxed{0.055\,\text{Wh}}$$

10. Autonomy Metric

$$\eta = \frac{n_{AI}}{n_{\text{total}}} \Rightarrow \boxed{\eta_{\text{min}} = 0.70, \quad \eta_{\text{goal}} = 0.95}$$

11. Summary Table

Metric	Value	Derived From
Pressure	100 MPa	Yield strength of Al6061
Force	10 000 N	$F = P \cdot A$
Bond Energy	0.455 J	Griffith fracture criterion
Work to Fracture Oxide	5 J	$W_f = \mu F_n d$
Power (ideal)	1 W	$P = \frac{W}{t}$
Power (actual)		Efficiency-adjusted + control
Thermal Range	$[-183, +137]°C$	Stefan-Boltzmann estimation
Mass	2.16 kg	$m = \rho \cdot V$
Shear Strength	80–150 MPa	$\tau = \frac{\sigma_y}{\sqrt{3}}$
Energy per Weld	0.055 Wh	$E = P \cdot t$
Autonomy	70–95%	$\eta = \frac{n_{AI}}{n_{\text{total}}}$