DSA Interview Question

1) ↳ Bit Manipulation ⟶ Divide two Integer

↳ Left shift

Right shift

No use → { 1) Multiplication
2) Divison
3) Mod }

count = 0

Dividend = 9

Divisor = 3

9 - 3 = 6

count = 1

6 - 3 = 3

count = 2

3 - 3 = 0

count = 3 *

Strassen's Matrix Multiplication

A B

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \longrightarrow \text{Matrix Multiplication}$$

2×2 ($i \times k$) 2×2 ($k \times j$) 2×2

Small Problem

↳ $\Theta(1)$ (constant)

$$\boxed{\begin{aligned} c_{11} &= ae + bg \quad (1) \\ c_{12} &= af + bh \quad (2) \\ c_{21} &= ce + dg \quad (3) \\ c_{22} &= cf + dh \quad (4) \end{aligned}}$$

$\Theta(n^3)$

```
for i = 0 to n:
    for j = 0 to n:
        c(i,j) = 0
        for k = 0 to n:
            c(i,j) += A(i,k) * B(k,j)
```

Matrix addition $\rightarrow \Theta(n^2)$

ae bg

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} * \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 5 & 12 \\ 21 & 32 \end{pmatrix}$$

$A_{21} \rightarrow$ r $\rightarrow$ 2, 3
c $\rightarrow$ 0, 1

$A_{22} \rightarrow$ 2, 3 $\rightarrow$ r
2, 3 $\rightarrow$ c

$$
\begin{array}{c|cc|cc|}
 & 0 & 1 & 2 & 3 \\
 & A_{11} & & A_{12} & \\
\hline
0 & a_{11} & a_{12} & a_{13} & a_{14} \\
 & & a & & b \\
1 & a_{21} & a_{22} & a_{23} & a_{24} \\
\hline
2 & a_{31} & a_{32} & a_{33} & a_{34} \\
 & & c & & d \\
3 & a_{41} & a_{42} & a_{43} & a_{44} \\
\hline
 & A_{21} & & A_{22} &
\end{array}_{4\times 4}
\; * \;
\begin{array}{|cc|cc|}
B_{11} & & & B_{12} \\
\hline
b_{11} & b_{12} & b_{13} & b_{14} \\
 & e & & f \\
b_{21} & b_{22} & b_{23} & b_{24} \\
\hline
b_{31} & b_{32} & b_{33} & b_{34} \\
 & g & & h \\
b_{41} & b_{42} & b_{43} & b_{44} \\
\hline
B_{21} & & & B_{22}
\end{array}_{4\times 4}
$$

A   B

$n/2 \Rightarrow 2$

$A_{11} \rightarrow$

rows $\rightarrow$ 0 to 1

cols $\rightarrow$ 0 to 1

Matrix multiplication

$c_{11} = ae + bg \rightarrow mm$

matrix addition

$A_{12} \rightarrow$

$c_{12} = af + bh$

row $\rightarrow$ 0, 1

$c_{21} = ce + dg$

Cols $\rightarrow$ 2, 3

$c_{22} = cf + dh$

Psuedocode

matrixMul(A, B, n):

small problem

if $n \leq 2$:

$$
c \;—\; \boxed{\begin{aligned}
c_{11} &= ae + bg && (1)\\
c_{12} &= af + bh && (2)\\
c_{21} &= ce + dg && (3)\\
c_{22} &= cf + dh && (4)
\end{aligned}}
$$

else:

$mid = n/2$

matrix addition

$c_{11} = \underbrace{matrixMul}_{T(n/2)}(a, e, n/2) +$
$\underbrace{matrixMul}_{T(n/2)}(b, g, n/2)$

$c_{12} = \underbrace{matrixMul}_{T(n/2)}(a, f, n/2) +$
$\underbrace{matrixMul}_{T(n/2)}(b, h, n/2)$

$c_{21} = \underbrace{matrixMul}_{T(n/2)}(c, e, n/2) +$
$\underbrace{matrixMul}_{T(n/2)}(d, g, n/2)$

$c_{22} = \underbrace{matrixMul}_{T(n/2)}(c, f, n/2) +$
$\underbrace{matrixMul}_{T(n/2)}(d, h, n/2)$

Divide & conquer approach

Recurrence Relation

$$T(n) = \begin{cases} c & n \leq 2 \\ 8T\left(\frac{n}{2}\right) + n^2 & n > 2 \end{cases}$$

$a = 8 \qquad k = 2$

$b = 2 \qquad p = 0$

$\log_b a = \log_2 8 = \underline{\underline{3}}$

$\log_b a > k \rightarrow \underline{\text{case 1}}$

$\hookrightarrow \underline{\Theta(n^3)}$

Strassen's approach

$+$ – Matrix addition

$*$ – Matrix multiplication

$$\left\{\begin{aligned}
P &= (A_{11} + A_{22}) * (B_{11} + B_{22}) \\
Q &= (A_{21} + A_{22}) * B_{11} \\
R &= A_{11} * (B_{12} - B_{22}) \\
S &= A_{22} * (B_{21} - B_{11}) \\
T &= (A_{11} + A_{12}) * B_{22} \\
U &= (A_{21} - A_{11}) * (B_{11} + B_{12}) \\
V &= (A_{12} - A_{22}) * (B_{21} + B_{22})
\end{aligned}\right.$$

$C_{11} = P + S - T + V$

$C_{12} = R + T$

$C_{21} = Q + S$

$C_{22} = P + R - Q + U$

$$T(n) = \begin{cases} c & n \le 2 \\ 7T(n/2) + n^2 & n > 2 \end{cases}$$

$a = 7$, $b = 2$

$k = 2$, $p = 0$

Case 1

$\log_b a = \log_2 7 = 2.81 > k$

$\Theta(n^{2.81})$

Power of an element

$x = 2$

$n = -2$

x^n

$n < 0$

$\hookrightarrow 2^{-2}$

$2 \hookrightarrow x$

$\hookrightarrow \frac{1}{x^n}$

$= \frac{1}{2^2} = \frac{1}{4} = 0.25$

Divide & conquer $\rightarrow \Theta(\log n)$

$n < 0$

$\hookrightarrow x = 1/x$

$n = -n$

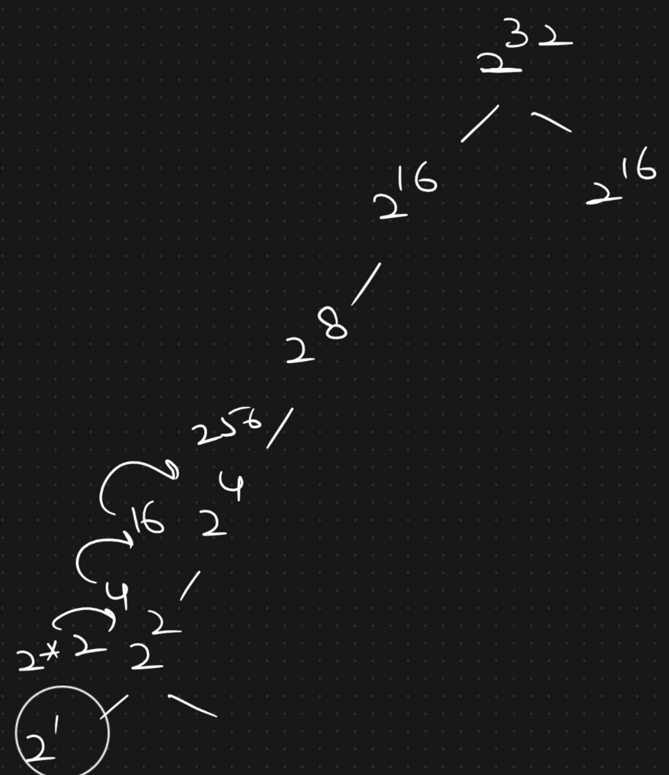
$n = 32 \qquad x = 2$

$mid = 32/2 = 16$

Small problem

$n = 1$

$\hookrightarrow x$



Median of nums1 & nums2

$\underbrace{\text{nums1} \ \& \ \text{nums2}}$

$\Theta(\log(m+n))$

after merging

nums1 → $\begin{array}{cc} 0 & 1 \\ [1, & 2] \end{array}$ — $2/2 = 1-1 = 0$

nums2 → $\begin{array}{cc} 0 & 1 \\ (3, & 4] \end{array}$ ⟹ $\begin{array}{cccc} 0 & 1 & 2 & 3 \\ [1, & 2, & 3, & 4) \end{array}$

/ → float

// → Lower Bound

$n = 4$ ⟶ $\frac{2+3}{2} = 2.5$

Approach 1 → Merge Procedure

mid = 4//2 = 2

sorted(nums1 + num2)

$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ (1 & 2 & 3 & 4) \\ & \uparrow & \uparrow & \end{array}$

$(2+3)/2$

$$\frac{\text{nums(mid)} + \text{nums(mid-1)}}{2}$$