Basic Properties of Logarithmic

1) $a^x = n \implies \log_a a^x = \log_a n$

$x = \log_a n$

2) $\log_b 1 = 0$

3) $\log_a a = 1 \Rightarrow$

4) $\log_a (P/q) = \log_a P - \log_a q$

5) $\log_a pq = \log_a p + \log_a q$

6) $\log_a p^n = n \log_a p$

7) $a^{(\log_a P)} = P$

$P^{\cancel{\log_a a}^{\,1}} = P$

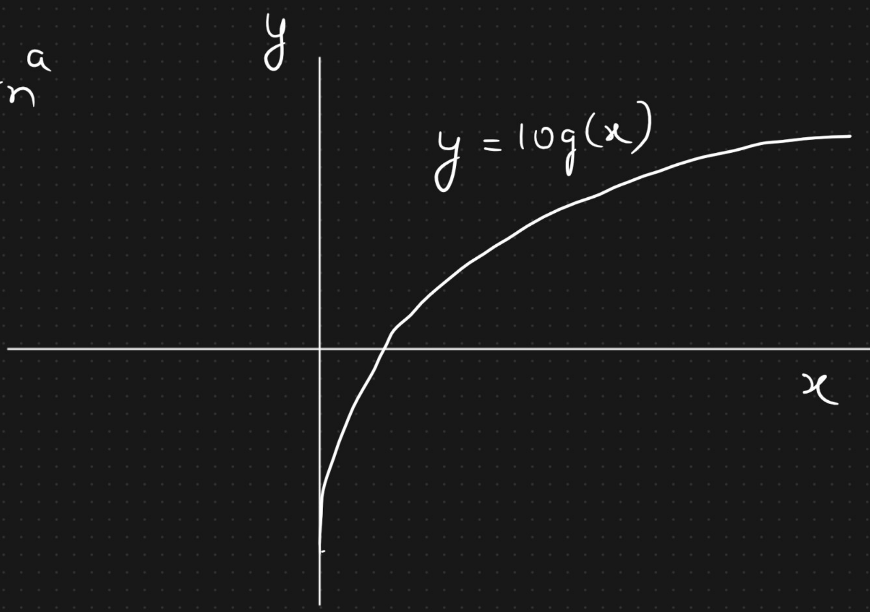
8) $\log_a n = \log_b n * \log_a b$

$a = 2$

$b = 3$

$\log_2 n = \log_3 n * \log_2 3$

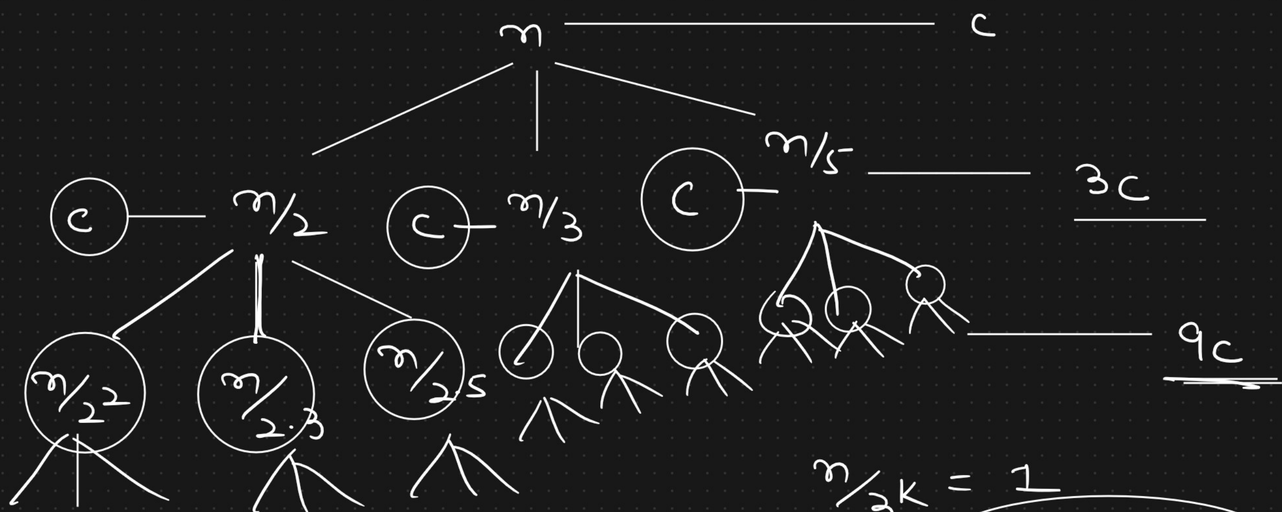
9) $\log_a b = \log_n b / \log_n a$



10) Lower the base in logarithmic → Higher the value

Assignment Problem

2) $T(n) = T(n/2) + T(n/3) + T(n/5) + c$



$n/3^k = 1$

$k = \log_3 n$

$\frac{n}{2^k} = 1 \qquad n = 2^k \qquad k = \log_2 n$ — * Higher value

$n/5^k = 1$

$k = \log_5 n$

$(3)^0 c + (3)^1 c + (3)^2 c + \dots + (3)^k c$

$c\left((3)^0 + (3)^1 + (3)^2 + \dots + (3)^{\log_2 n}\right)$

↳ GP series

$r = 3 \qquad r > 1$

$a = 1$

Property Number 7

$S = \frac{a(r^n - 1)}{r - 1} = c\left(\frac{1(3^{\log_2 n} - 1)}{3-1}\right)$

$= c\left(\frac{n^{\log_2 3} - 1}{2}\right)$

$$\Rightarrow \underline{\underline{O(n^{1.5})}}$$

1)

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + n & n > 1 \end{cases}$$

<u>Substitution Method</u>;

$$\boxed{T(n) = 2T\left(\frac{n}{2}\right) + n}$$

$$= 2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$\Rightarrow 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

$$\Rightarrow 2^2\left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + 2n$$

$$\Rightarrow 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

k times

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k$$

$$2^k T\left(\frac{n}{2^k}\right) + k \cdot n$$

$$2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot n$$

$$n^{\cancel{\log_2^2}} \; T\left(\frac{\cancel{n}\,1}{\cancel{n^{\log_2^2}}1}\right) + \log_2^n \cdot n$$

$\underline{T(1) = 1}$

$$n \cdot \underline{T(1)} + \log_2^n * n$$

$$\underline{n} + \underline{n * \log_2^n}$$

$$\underline{\underline{O(n \log_2^n)}}$$

3)
$$T(n) = \begin{cases} 1 & n = 1 \\ 8T(n/2) + n^2 & n > 1 \end{cases}$$

$$T(n) = 8T(n/2) + n^2$$

$$= 8\left(8T\left(n/2^2\right) + \left(\frac{n}{2}\right)^2\right) + n^2$$

$$\overset{2}{\cancel{8}} * \frac{n^2}{\cancel{4}} + n^2$$

$$= 8^2 T\left(\frac{n}{2^2}\right) + \underline{3n^2}$$

$$\overset{4}{\cancel{8^2}} * \frac{n^2}{\cancel{16}} = 4n^2$$

$$= 8^2\left(8\,T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 3n^2$$

$$= 8^3 T\left(\frac{n}{2^3}\right) + 7n^2$$

$$\frac{n}{2^k} = 1$$

$\downarrow$ k times

$$K = \log_2 n$$

$$8^k \, T\left(\frac{n}{2^k}\right) + (2^k - 1)\, n^2$$

$$\log_2 8 = \log_2 2^3 = 3$$

$$8^{\log_2 n} \, T\left(\frac{n}{2^{\log_2 n}}\right) + \left(2^{\log_2 n} - 1\right) n^2$$

$$\Downarrow$$

$$n^{\cancel{\log_2 8}^{\,3}} \, T\left(\frac{n}{n^{\cancel{\log_2 2}}}\right) + \left(n^{\cancel{\log_2 2}} - 1\right) n^2$$

$$T(1) = 1$$

$$n^3 \, T\left(\frac{\cancel{n}\,1}{\cancel{n}}\right) + n^3 - n^2$$

$$n^3 + n^3 - n^2$$

$$O(n^3)$$

1)

$f(n) = n - 10$

$g(n) = n + 10$

$f(n) = \Theta(g(n))$

$\Downarrow$

Big O &

omega

Large values of n

Big O

$f(n) \leq c \cdot g(n)$

$n - 10 \leq c \cdot (n+10)$

$c = 1$

$f(n) = O(g(n))$ ✓

Omega

$f(n) \geq c \cdot g(n)$

$n - 10 \geq c \cdot (n+10)$

$c = 1/2$

$n = 1000$

$990 \geq \frac{\cancel{1010}^{505}}{\cancel{2}}$

$f(n) = \Omega(g(n))$

$f(n) = \Theta(g(n))$

2) $f(n) = n$ $\quad$ $g(n) = n$

Big O

$n <= c \cdot n$

$n >= c \cdot n$ $\quad$ $c = 1$ $\rightarrow$ omega

$f(n) = \Theta(g(n)) \longrightarrow$ True $\hookrightarrow$ $c = 1$

$f(n) <= c \cdot g(n)$

$f(n) >= c \cdot g(n)$

$\log_2 64 = \log_2 2^6$

$= 6$

$\log_2 32 = \log_2 2^5$

$= 5$

3) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$ $\hookrightarrow$ false

$\Downarrow$

$n^{\log_2 64} \cdot n^{\log_2 32}$

$n^6 \cdot n^5 = n^{11}$

$$f(n) <= c \cdot g(n)$$

$$\boxed{n^{11} <= c \cdot n^{5}} \rightarrow \text{Not a Valid Big O}$$

$$\hookrightarrow c = n^6$$

4) $\frac{4^n}{2^n} = O(2^n) \longrightarrow$ True

$$\frac{2^n * \cancel{2^n}}{\cancel{2^n}} = O(2^n)$$

$$2^n <= c \cdot 2^n$$

$$c = 1$$

5) $\underbrace{128^{\log_2^n} \cdot n^2}_{f(n)} = \Theta(\underbrace{n^9}_{g(n)})$

$$n^{\log_2^{128}} \cdot n^2$$

$$n^{\log_2^{2^7}} \cdot n^2$$

$$n^7 \cdot n^2 = n^9$$

$$f(n) = \Theta(g(n))$$

$$n^9 = \Theta(n^9)$$

Big O	omega
$n^9 <= c \cdot n^9$	$n^9 >= c \cdot n^9$
$c = 1$	$c = 1$

Master's Theorem

$$f(n) = \Theta(n^k \log^p n)$$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$\boxed{T(n) = 2T\left(\frac{n}{2}\right) + n^2}$$

$\log_b a$

$\hookrightarrow \log_2 2 = 1$

$1 < 2$

$a = 2$

$b = 2$

$k = 2$

$p = 0$

cormen

case 1 $\longrightarrow$ $\log_b a > k$ ✓

$\Theta(n^{\log_b a})$

$1 > 0$

$\log_b a > k$

$T(n) = 2T(n/2) + 1$

① $a = 2$ $\quad k = 0$

$b = 2$ $\quad p = 0$

② $\log_b a = \log_2 2 = 1$

$\Theta(n^1)$

$\Theta(n)$

case 2 $\quad \log_b a = k$

$p > -1$ —— $\Theta(n^k \log^{p+1} n)$ ✓

$p = -1$ —— $\Theta(n^k \log \log n)$

$p < -1$ —— $\Theta(n^k)$

$\} \rightarrow$ Rare

case 3 $\quad \log_b a < k$

$p \geq 0 \quad \Theta(n^k \log^p n)$ ——

$p < 0 \quad \Theta(n^k)$

$\Theta(n) \Rightarrow O(n)$

$\Omega(n)$

$\log_b^a = 3$

2) $T(n) = 8T(n/2) + n^2$

$k = 2$

1 $a = 8$ $\quad k = 2$

$b = 2$ $\quad p = 0$

$\Theta(n^{\log_b^a})$

2 $\log_b^a = \log_2^8 = 3$

$\Theta(n^{\log_b^a})$

$\Rightarrow \log_b^a > k \Rightarrow 3 > 2$

case 1

$\Theta(n^3)$

$\log_b^a = k$

$P = 0$

case 2

$\Theta(n \log n)$

1) $T(n) = 2T(n/2) + n$

$\Theta(n^k \log^{P+1} n)$

① $a = 2$ $\quad k = 1$

$b = 2$ $\quad P = 0$

② $\log_2^2 = 1$

$\Theta(n \log n)$

$p > -1$

③ $\log_b^a = k$

$0 > -1$

$1 = 1$

(2) $T(n) = 8T(n/2) + n^3$ $\qquad p = 0$

(1) $a = 8 \qquad k = 3$

$b = 2 \qquad p = 0$

(2) $\log_b a = \log_2 8 = \log_2 2^3 = 3$

(3) $\log_b a = k \iff 3 = 3$

$p > -1$

$\Theta(n^3 \log n)$

(3) $T(n) = 2T(n/2) + \frac{n}{\log n}$ — $n \log^{-1} n$

$a = 2 \qquad p = -1$

$b = 2$

$k = 1$

$\Theta(n^k \log \log n)$

$\Theta(n \log \log n)$

(4) $T(n) = 2T(n/2) + \frac{n}{\log^2 n}$

$a = 2 \qquad p < -1$

$b = 2$

$k = 1$

$p = -2 \qquad \Theta(n)$

most frequent $\log_b a = k$

$p > -1$ $\quad$ $p = -1$ $\quad$ $p < -1$

$\Theta(n^k \log^{p+1} n)$ $\quad$ $\Theta(n^k \log\log n)$ $\quad$ $\Theta(n^k)$

case 3 $\qquad$ $\log_b a = 1$

$k = 2$

① $T(n) = 2T(n/2) + n^2 \log^2 n$

$f(n)$

$a = 2 \quad k = 2$

$b = 2 \quad p = 0$

$\Theta(n^2 \log^2 n)$

$\log_b a = \log_2 2 = 1$

$\log_b a < k$ ——— case 3

$\Theta(n^k \log^p n)$

$\Rightarrow \Theta(n^2)$

② $T(n) = 4T(n/2) + \dfrac{n^3}{\log n}$ $\quad$ Numerator

$p < 0$

$a = 4 \quad k = 3$

$b = 2 \quad p = -1$

$p < 0$

$\Theta(n^k)$

$\log_b a = 2 < k \quad \Theta(n^3)$

Simple

$\log_b a < k$

$\hookrightarrow \Theta(f(n))$

Case 1

$\log_b a > k$

$\Theta(n^{\log_b a})$ ✓

Case 3

$\log_b a < k$ ✓

$\Theta(f(n))$ skip

$\hookrightarrow$ Denominator

$\hookrightarrow$ NO Denominator

Case 2

$\log_b a = k$

$p > -1$ $\Theta(f(n) \log n)$ ✓

Monday → 6pm - 9pm ⟸

6 - 12 pm

↳ Doubt class

(Monday - Sunday) ⟸

↳ Data Science

Practice Problem

1) $T(n) = 2T\left(\frac{n}{2}\right) + n$

2) $T(n) = 2T(n/2) + n\log n$

3) $T(n) = 2T(n/2) + n^2$

4) $T(n) = 8T(n/2) + n^2$