



TODAY:

- prefix sum
- problems on prefix sum.

Given a array representing daily profit or loss from a particular stock. function that calculate total profit or loss in given range of days.

→ Stock price = [ -5, 10, 20, 40, 50, -10, 80, -90, -20, -10 ]

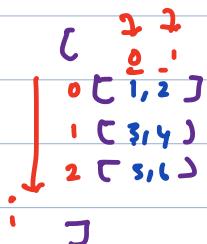
→ Queries

[ [ start, end ], [ —, — ] ]

start	end	Profit/Loss	sb [ —, —, — ] and c[ —, —, — ].
1	1	-5	
1	2	-5 + 10 = 5	
2	5	10 + 20 + 40 + 50 = 120.	
8	10	-120	
3	8	90.	
Range → L R			

→ Given N elements and Q queries. for each query, calculate sum of all elements from L to R / S to E [0 based index].

A = [	0	1	2	3	4	5	6	7	8	9	]
L	R	Sum									
→ 4	8	9									
→ 3	7	10.									



Pseudocode

```

function querysum ( A[], Q[][], querySize, size ) {
    for ( i → 0 to querySize ) { // This will iteration Q[][]
        L = A[i][0], R = A[i][1]
        sum = 0;
        for ( j → L to R ) {
            sum += A[j];
        }
        print( sum );
    }
}

```

$A = [ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -3, 6, 2, 4, 5, 2, 8, -9, 3, 1 \end{matrix} ] \leftarrow$

$Q = [ \begin{matrix} L & R \end{matrix} ]$

- 1.  $\{ \underline{0}, \underline{2}, 4 \} \rightarrow 11$
- 2.  $\{ \underline{1}, 6 \} \rightarrow 15$
- 3.  $\{ \underline{2}, \underline{7}, 8 \} \rightarrow -6$
- 4.  $\{ \underline{3}, \underline{7} \} \rightarrow 10$
- 5.  $\{ \underline{6}, 9 \} \rightarrow 3$

$Q.length.$

]

func querySum( $A[]$ ,  $Q[Q[]]$ )  $\rightarrow$   $O(Q \cdot N)$

int qlength =  $Q.length$ ;

int alength =  $A.length$ ;

for ( $i \rightarrow 0$  to  $qlength$ )  $\rightarrow Q$

$L = Q[i][0];$

$R = Q[i][1];$

$sum = 0$

for ( $j \rightarrow L$  to  $R$ )  $\rightarrow N$

$sum = sum + A[j];$

$print(sum);$

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	<u>L</u>	<u>R</u>
1 2 3 4 5 6 7 8 9 10		
cumulation → 2 8 14 29 31 49 65 79 88 97		
sum		

Prefix sum       $\text{Sum } 3-5 = \underline{\text{Sum}[1-5]} - \text{Sum}[1-2]$

$$31 - 8$$

$$65 - 49 = 16 //$$

$$\text{Score in 7^{th} pos} = \underline{\text{Sum}[1-7]} - \text{Sum}[1-6]$$

$$\text{Score } 6-10^{th} = 97 - 31 = 66 //$$

$$\text{Score } 10^{th} \in [10-10] = R - [L-1]$$

$$\text{Score } [3-6] = \underline{\underline{R}} - \underline{[L-1]}$$

$$49 - 8 = 41 //$$

$$\text{Score } [4-9] = \underline{R} - \underline{[L-1]}$$

$$88 - 14 = 74 //$$

### prefix sum

$$A = \{ \underline{2} \ 5 \ -1 \ 7 \ 1 \}$$

0 1 2 3 4	Pfsum[i]	cumulative sum [0-i].
0 1 2 3 4	0 → 2	
0 1 2 3 4	1 → 2 + 5	
0 1 2 3 4	2 → 2 + 5 - 1	
0 1 2 3 4	3 → 2 + 5 - 1 + 7	
0 1 2 3 4	4 → 2 + 5 - 1 + 7 + 1	

$$A = \{10, 32, 6, 12, 20, 1\}$$

$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

$$PFsum = \{10, 42, 48, 60, 80, 81\}$$

pseudo code

AC[N], PF[N].

for ( $i \rightarrow 0$  to  $N$ ) {  $\longrightarrow N$   $O(N^2)$

    sum = 0;

    for ( $j \rightarrow 0$  to  $i$ ) {  $\longrightarrow N$

        sum = sum + A[i]

    }

    PFsum[i] = sum;

}

Optimise... ???.

$$PF[0] = A[0]$$

$$PF[1] = A[0] + A[1]$$

$$PF[2] = A[0] + A[1] + A[2].$$

$$PF[3] = A[0] + A[1] + A[2] + A[3]$$

$$PF[3] = PF[2] + A[3]$$

$$PF[4] = PF[3] + A[4].$$

$$PF[5] = PF[4] + A[5]$$

:

:

$$PF[i] = PF[i-1] + A[i].$$

## pseudocode to calculate Pfsum

$A[N]$ ,  $Pfsum[N]$ ,  $Pfsum[0] = A[0]$ .  
 for ( $i \rightarrow 1$  to  $N$ ) {  $\longrightarrow N$   
 |  $Pfsum[i] = Pfsum[i-1] + A[i];$   
 |  
 }  $0$   $0-1 + A[0]$

T.C : O(N)

### optimized approach

$\rightarrow$  stock price = [ -3, 6, 2, 4, 5, 2, 8, -9, 3, 1 ]

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$   
 $psum = [-3, 3, 5, 9, 14, 16, 24, 15, 18, 19]$

Sum[L-R] = Pfsum[R] - Pfsum[L-1]

Sum[0-S] = Pfsum[S] - Pfsum[-1]. XY

if L == 0

= Pfsum[R].

function querySum(stock[], qrc[]) { TC: O(N+Q) }

$n = stock.length; q.length = Q.length;$

$Pfsum[n];$  // create Pfsum array of size n.

$Pfsum[0] = A[0]$ .

for ( $i \rightarrow 1$  to  $N$ ) {  $\longrightarrow N$

|  $Pfsum[i] = Pfsum[i-1] + A[i];$   
 |  
 }  $0$   $0-1 + A[0]$

for ( $i \rightarrow 0$  to  $q.length$ ) { // loop QRC[]  $\rightarrow Q$

$L = Q[i][0], R = Q[i][1];$

$if (L == 0) \{ print(Pfsum[R])\}$

$else \{ print(Pfsum[R] - Pfsum[L-1])\}$

10:31pm



10:40pm

Q → Given an array of N, & Q queries with s and e index for every query, return the sum of all even indexed elements from s to e.

$$A = \{ \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 6 & 6 & 5 \end{array} \}$$

Query C

$$[1, 3] \rightarrow x \checkmark x : A(2) = 1 //$$

$$[2,5] \rightarrow \frac{2}{x} + \frac{5}{x} : A(2) + A(4) = 5_{11}$$

$$[ \quad 0, 4 \quad ] \rightarrow \quad \frac{0}{x} \frac{1}{x} \frac{2}{x} \frac{3}{x} \frac{4}{x} = A[0] + A[2] + A[4] = 7$$

$$[3, 3] \rightarrow \frac{3}{x} = 0 \leftarrow$$

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Brute force :- we will iterate over q and get s and e and from s and e check if index is even the add .

for (i → 0 to Q.length) { → Q

$s = Q[i][0]$ ,  $e = Q[i][C^i]$ , sum = 0

for(j → s to c) { → N}

if ( $j \neq 2 == 0$ ) {

$\text{sum} + = AC[j]$

print (sum);

$$T.C = O(N \times Q)$$

PRO TIP : Range sum  $\rightarrow$  use Pfsum

## Prefix sum of Even elements

$A = \{ 2, 3, 1, 6, 4, 5 \}$

$$Psum = \{ 2, 2, 3, 3, 7, 7 \}$$

$A = \{ 2, 4, 9, 3, 1, 5 \}$

$$Psum = \{ 1, 2, 2, 5, 5, 10 \}$$

function evenSumqr(stockC[], q[],) & TC:  $O(N+Q)$

$n = stock.length; q.length = Q.length;$

Pfsum[n]; // create Pfsum array of size n.

Pfsum[0] = A[0]

for (i → 1 to N) &

if (i % 2 == 0) &

Psum[i] = Psum[i-1] + A[i];

else &

Psum[i] = Psum[i-1];

for (i → 0 to q.length) & // loop q[i] → Q

s = Q[i][0], E = Q[i][1];

if (s == 0) & print(Pfsum[E]);

else & print(Pfsum[E] - Psum[s-1]);

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Another clever: sum of odd elements from s - e.

Given an array of size  $N$ , count the number of special index in the array.

Note: special index :-

those indices after removing which,

Sum of all even index elements = Sum of all odd index elements.

	0	1	2	3	4	5
A[ ]:	4	3	2	7	6	-2

i	$A[i]$	$S_e$	$S_o$	Special Index
0	3	2	7	$\checkmark \quad c=0 \rightarrow 1$
1	4	2	7	$\times$
2	4	3	7	$\checkmark \quad c=1 \rightarrow 2$
3	4	3	2	$\times$
4	4	3	2	$\times$
5	4	3	2	$\times$

Ans = 2/1.

	0	1	2	3	4	$S_e$	$S_o$
$A = [$	4	1	3	7	10	7	
0	0	1	3	7	10	8	13
1	4	3	7	10		11	13
2	4	1	7	10		11	11
3	4	1	2	10		7	11
4	4	1	3	7		7	2

$A =$	0	1	2	3	4	5	6	7	8	9
	2	3	1	9	0	-1	2	-2	10	8
	2	3	1	9	0	-1	2	-2	10	8

$i = 3$

$$0 \quad \underline{i-1} \quad i+1 \quad N$$

$i = 3 +$  sum of all even elements

$i=2$

from 4-9.

$$A = \begin{array}{r|rrrr} 0 & 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 7 & 10 \\ \hline 4 & 1 & 7 & 10 & 34 \\ & & \hline & & \end{array}$$

$0 - i-1$

$i+1 \rightarrow N$

sum of all odd  
indices

sum of all even  
indices.

$$A = \begin{array}{ccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 9 & 0 & -1 & 2 & -2 & 10 & 8 \\ \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 0 & -1 & 2 & -2 & 10 & 8 \end{array}$$

$0 - i-1$

$i+1 \rightarrow n$

sum of all even  
indices

sum of all odd  
indices

$2^0$

$i-1$

$$A = \begin{array}{ccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 9 & 0 & -1 & 2 & -2 & 10 & 8 \\ \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 0 & -1 & 2 & -2 & 10 & 8 \end{array}$$

$0 - i-1$

$i+1 \rightarrow n$

sum of all even  
indices

sum of all odd  
indices

## Optimised approach

- 1) calculate prefix sum of even indices
- 2) calculate prefix sum of odd indices

- 3) loop from  $i \rightarrow 0$  to  $N-1$

evensum; count=0;

odd sum.

$$\text{evensum} = \frac{\text{sum of even indices } [0-i-1]}{} + \frac{\text{sum of odd indices. } [i+1-n]}{}$$

$$\text{oddsum} = \frac{\text{sum of odd indices } [0-i-1]}{} + \frac{\text{sum of even indices } [i+1-n]}{}$$

if ( oddsum == evensum ) {

|     count++;

## pseudo code

```
functions count_spread_index ( arr, n ) {  
    Pfsumc [ ] // calculate  
    Psumo [ ] // calculate  
    count = 0;  
    for ( i = 0 to n - 1 ) {  
        if ( i == 0 ) {  
            // Todo 2 Hw.  
            s = Pfsumc [ 0 → i - 1 ] + Psumo [ i + 1 → N ]  
            S0 = Psumo [ 0 → i - 1 ] + Pfsumc [ N ] - Pfsumc [ i ];  
            // Psumo [ 0 → i - 1 ] + Pfsumc [ i + 1 → N ].  
            if ( s == S0 ) {  
                count++;  
            }  
        }  
    }  
    print( count );  
}
```

## Next class

- carry forward Technique.
- Subarrays
- problems on books