

Problem: Task 1

Determine the actual height of a fish seen by a person in an aquarium tank if the person sees the fish 1 ft above the person's eye level. The refractive index of air is 1.00, the refractive index of glass is 1.69, and the refractive index of water is 1.33. The person is standing 2 feet away from the glass, the fish is 1 foot away from the glass, and the glass is 6 inches thick.

Air

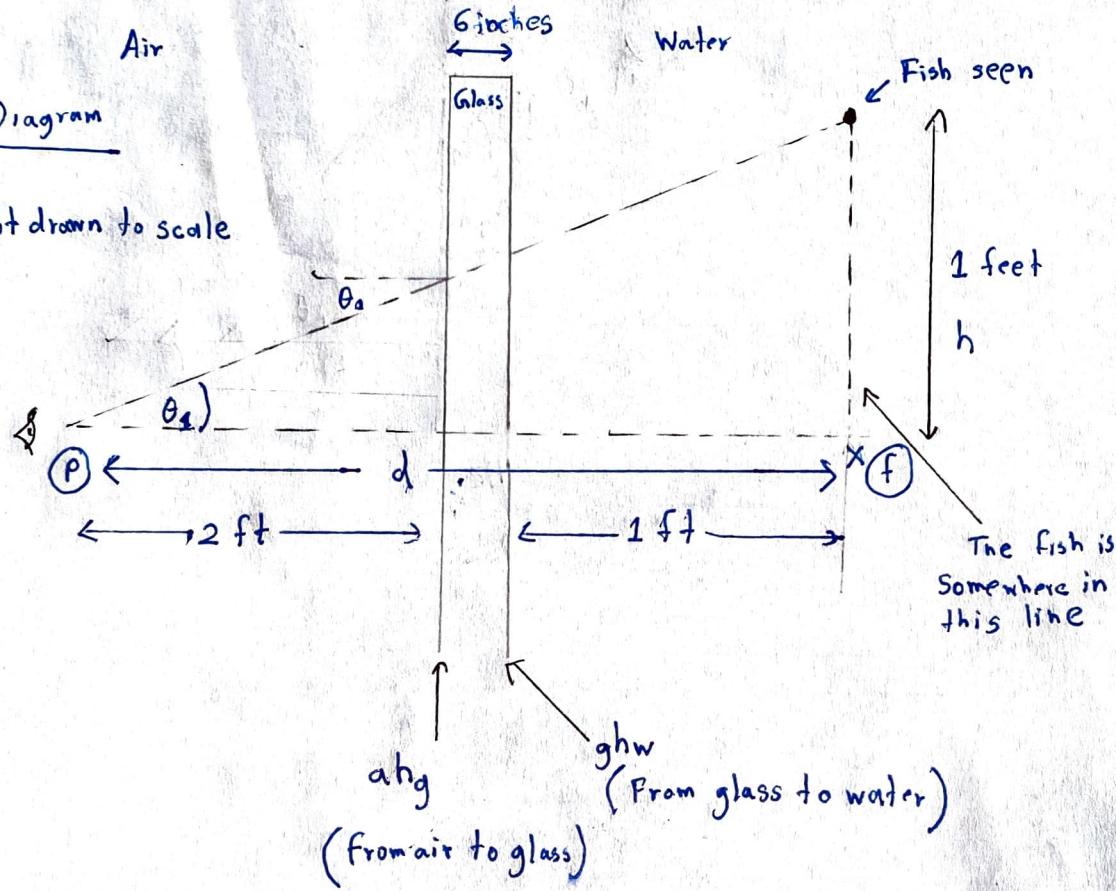
Diagram

6 inches

Water

Fish seen

Not drawn to scale

Assumptions

- 1) The fish and the person is lined up perfectly.
- 2) Given measurements are exact and accurate.
- 3) Fish is stationary.
- 4) Glass is perfectly flat.
- 5) Everything is perfectly perpendicular to the ground.
- 6) Person has good vision.

Theory

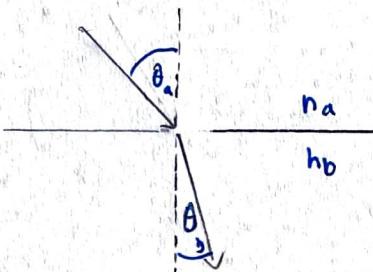
1) The law of refraction

$$h_a \sin(\theta_a) = h_b \sin(\theta_b)$$

$$2) \sin(a) = \frac{A}{C}$$

$$3) \tan(a) = \frac{A}{B}$$

$$4) Eh = h_1 + h_2 + h_3$$



Solution

$$1) \tan(\theta_1) = \frac{h}{d} = \left(\frac{1 \text{ ft}}{3.5 \text{ ft}} \right) \rightarrow \theta_1 = \tan^{-1} \left(\frac{1}{3.5} \right) = 15.95^\circ$$

$$2) (1) \sin(15.95) = (1.69) \sin(\theta_2) \rightarrow \theta_2 = \sin^{-1} \left(\frac{\sin(15.95)}{1.69} \right) = 9.36^\circ$$

$$3) (1.69) \sin(9.36) = (1.33) \sin(\theta_3) \rightarrow \theta_3 = \sin^{-1} \left(\frac{(1.69) \sin(9.36)}{1.33} \right) = 11.92^\circ$$

$$4) \tan(\theta_1) = \frac{h_1}{d_1} \rightarrow h_1 = (2) \tan(15.95) = 0.57 \text{ ft}$$

$$\tan(\theta_2) = \frac{h_2}{d_2} \rightarrow h_2 = d_2 \tan \theta_2 = (0.5) \tan(9.36) = 0.08 \text{ ft}$$

$$\tan(\theta_3) = \frac{h_3}{d_3} \rightarrow h_3 = (1) \tan(11.92) = 0.21 \text{ ft}$$

$$5) h = h_1 + h_2 + h_3 = 0.865 \text{ ft} //$$

Verification

Since the glass has the highest refractive index, it would obstruct the view first, so the lowest possible angle would be that of the glass.

The range would be

$$\tan(\theta_2) = \frac{h_{\text{lowest}}}{d}$$

$$\begin{aligned} \Rightarrow h_{\text{lowest}} &= \tan(\theta_2)d \\ &= \tan(9.36) (3.5) \\ &= 0.577 \text{ ft} \end{aligned}$$

And since our answer is $0.865 \text{ ft} > 0.577 \text{ ft} < 0.865 \text{ ft} < 1 \text{ ft}$
it falls into the range and is a valid answer

Conclusion: The fish is actually only 0.865 ft above the person's eye level.

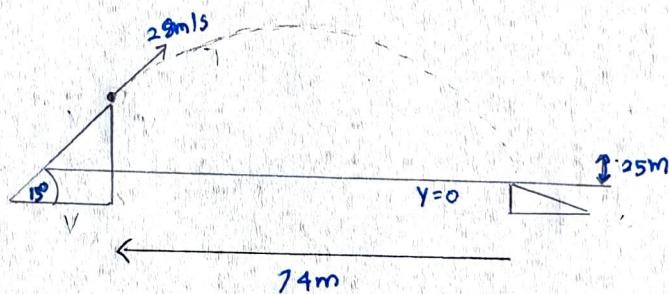
Problem: Task 2

A stunt woman is jumping across a canyon that is 74m wide. The ramp on the far side of the canyon is 25m lower than the ramp from which she will leave. The takeoff ramp is built with a 15° angle from the horizontal. The stuntwoman is planning to leave the takeoff ramp with a speed of 28m/s. Determine if she would make it to the other side, and how many seconds will she be in the air.

Diagram

Not drawn

to scale

Assumptions

- 1) No external forces (wind, drag)
- 2) Stuntwoman is leaping perfectly straight, not at an angle.

Theory

$$x = v_0 \cos(\theta) t$$

$$y = -\frac{1}{2} g t^2 + v_0 \sin(\theta) t + y_0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin(\alpha) = \frac{A}{C}$$

$$\cos(\alpha) = \frac{B}{C}$$

$$\tan(\alpha) = \frac{A}{B}$$

Solutions

$$y = -\frac{1}{2}gt^2 + v_0 \sin(\theta)t + y_0$$

$$0 = -\frac{1}{2}(9.81)t^2 + 28 \sin(15)t + 25$$

$$0 = -4.905t^2 + 7.247t + 25$$

$$t = \frac{-7.247 \pm \sqrt{7.247^2 - 4(25) - 4.905}}{2(-4.905)}$$

$$t = 3.12s$$

$$t = -1.64s$$

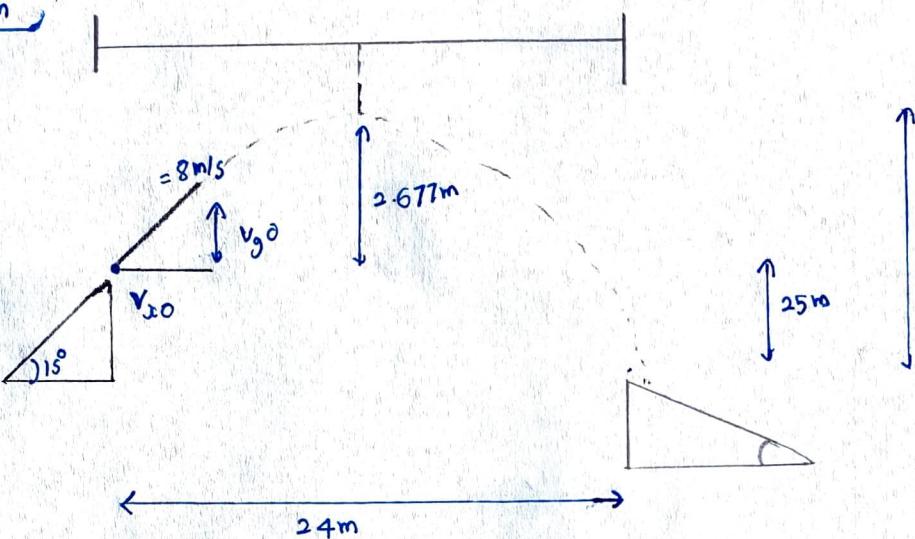
$$x = v_0 \cos \theta t$$

$$= (28) \cos(15)(3.12)$$

$$x = 84.38$$

$$x > 74m$$

= She does make the jump and she spent 3.12s airbone

Verification

$$\textcircled{1} \quad \sin(15^\circ) = \frac{V_{y0}}{V_0} \Rightarrow V_{y0} = V_0 \sin(15^\circ) = 28 \sin(15^\circ) = 7.247 \text{ m/s}$$

$$\textcircled{2} \quad V_{y\text{top}} = V_{y0} + g t_{\text{top}}$$

$$t_{\text{top}} = 0.739 \text{ s.}$$

$$\textcircled{3} \quad \Delta y_{\text{top}} = V_{y0} t + \frac{1}{2} g t^2$$

$$= 2.677 \text{ m}$$

$$\textcircled{4} \quad \Delta y_{\text{total}} = 25 + 2.677 \\ = 27.677 \text{ m}$$

$$\textcircled{5} \quad \Delta y_{\text{total}} = V_{y\text{top}} t_2 + \frac{1}{2} g t_2^2 \\ - 27.677 = 0 t_2 + \frac{1}{2} (-9.81) t_2^2$$

$$\textcircled{6} \quad t_1 + t_2 = t \\ t = 3.44 \text{ s}$$

And since the time we got are vertically the same ($3.14 \approx 3.12$) differences due to rounding in calculations), the solution is valid

Conclusion

The stuntwoman spends 3.12s airbourne and makes it to the other side.