University of Canterbury

Mid Year Examinations 2014

Prescription number: COSC363

Paper title: Computer Graphics

Time allowed: 3 hours

Number of pages: 8

Maximum marks: 100. Contribution to the final grade: 50%

Number of questions: 10 (Each question carries a total of 10 marks)

Suggested time for each whole question: 15 mins.

- **Answer *all* questions**.
- This test is worth a total of 100 marks.
- This is a **closed book** exam. No written or printed material is allowed**.**
- **UC approved calculators** are allowed.
- Please answer all questions carefully and to the point. Check carefully the number of marks allocated to each question. This suggests the degree of detail required in each answer, and the amount of time you should spend on the question.
- Use the separate answer booklet provided for answering all questions.

Important Formulae

If $\boldsymbol{v}_1 = (x_1, y_1, z_1)$ and $\boldsymbol{v}_2 = (x_2, y_2, z_2)$, then

$\boldsymbol{v}_1 \times \boldsymbol{v}_2 = (y_1z_2 - y_2z_1,\ z_1x_2 - z_2x_1,\ x_1y_2 - x_2y_1)$.

Rotation matrices:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Diffuse reflection: $L_d\ M_d\ \max(0, \mathbf{L}\bullet\mathbf{N})$

Specular reflection: $L_s\, M_s\, (\mathbf{R}\bullet\mathbf{V})^{f}$

Equation of a ray through $\boldsymbol{p}_0$ and having a direction $\boldsymbol{d}$: $\boldsymbol{p} = \boldsymbol{p}_0 + k\boldsymbol{d}$, $k > 0$.

Second order Bezier curve defined using three control points P_0 , P_1 , P_2:

$$P(t) = (1-t)^2\, P_0 + 2t(1-t)\, P_1 + t^2\, P_2\,, \text{ where } P(t) = (x(t), y(t)), \quad 0 \le t \le 1.$$

Fractal dimension: (s = scale factor)

$$D = \frac{\log N}{\log\left(\dfrac{1}{s}\right)}$$

Question 1. Vectors [10 marks for the whole question]

(a) Show that the vectors $(6, -2, 7)$ and $(3, -5, -4)$ are perpendicular to each other. [2 Marks]

(b) Convert the vector $(6, -3, 6)$ to a unit vector. [2 Marks]

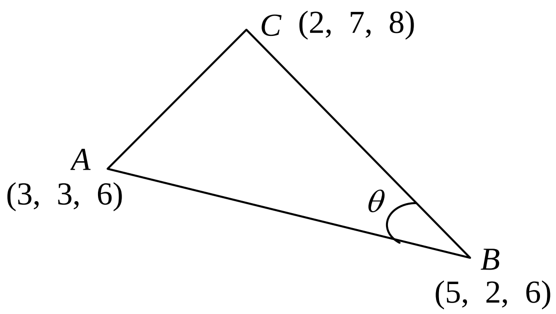


Figure 1

(c) Compute the normal vector of the plane of the triangle shown in Fig. 1. It is not necessary to convert the vector to a unit vector. [4 Marks]

(d) Write the equation for obtaining the angle θ of the triangle in Fig. 1. Give the expressions for vectors in the equation in terms of vertices *A*, *B*, *C*. You are <u>not</u> required to compute the numerical value of the angle. [2 Marks]

Question 2. Model Transformation [10 marks for the whole question]

(a) A rectangular object *A* of length 4 units is placed at the origin along the *x*-axis as shown in Fig. 2(a).

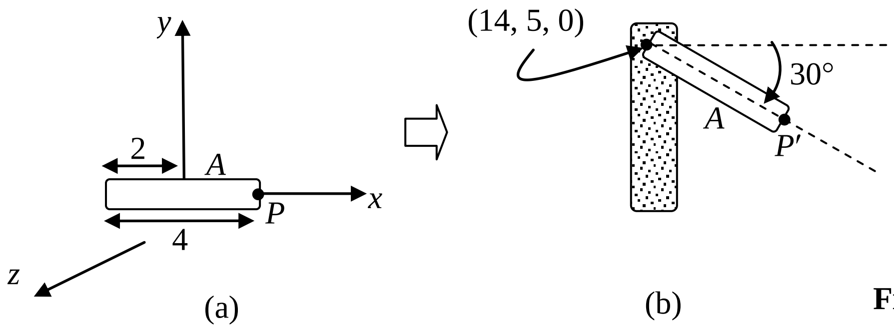


Figure 2

The object is required to be transformed on the *xy*-plane as shown in Fig. 2(b). Write the matrix expression (product of matrices) for the transformation. You are <u>not</u> required to multiply the matrices. The matrices should contain only numerical values. (cos(30) = 0.866, sin(30) = 0.5). [5 Marks]

(b) Consider the following code segment:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslatef(3.0, 2.0, 5.0);
glRotatef(90, 1, 0, 0);
glScalef(0.5, 0.5, 0.5);
glTranslatef(-3.0, -2.0, -5.0);
glBegin(GL_TRIANGLES);
   ...
   glVertex3f(-1.0, 4.0, -1.0);    //a point P
 glEnd();
```

Write the transformation for the point P as a matrix expression, and calculate its transformed coordinates. [5 Marks]

Question 3. View Transformation [10 marks for the whole question]

A view transformation matrix transforms points from world coordinates to eye coordinates. Consider the following view transformation matrix:

$$\begin{bmatrix} 0.8 & 0 & 0.6 & -11 \\ 0 & 1 & 0 & -7 \\ -0.6 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Compute the eye coordinates of the point P (10, 7, 5) using the above matrix. From your answer, what can you conclude about the point P ? [3 Marks]

(b) Consider a vector v in homogeneous coordinates (6, 0, –8, 0). Transform the vector using the above view transformation matrix. From your answer, what can you conclude about the vector v ? [4 Marks]

(c) A point Q has world coordinates (–5, 3, 10). Using its eye coordinates, determine if this point is behind or in front of the camera . [3 Marks]

Question 4. Projections [10 marks for the whole question]

A view frustum is specified using the following statement:

```
gluPerspective(45.0,   2.0,  30.0, 100.0);
```

(a) Draw a sketch of the view frustum, clearly showing what each of the four parameters of the above function represent. The figure should also show the camera position, coordinate axes directions, and the view axis direction. [6 Marks]

(b) Using the parameters of the function gluPerspective(fov, asp, N, F), write the formulae for computing h, w, L, R where h is the height of the near plane, w the width of the near plane, L the distance to the left plane, and R the distance to the right plane. [4 Marks]

Question 5. **Illumination [10 marks for the whole question]**

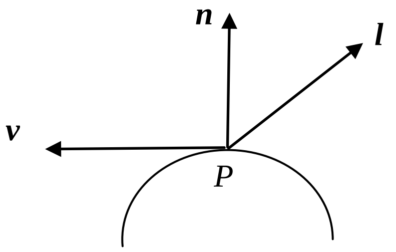


Figure 3

Fig. 3 shows the light source vector ***l***, the view vector ***v***, and the normal vector ***n*** at a point P on a sphere.

(a) Redraw this figure in your answer booklet and include the reflection vector ***r***, half-way vector ***h*** and the angles used for lighting computations. [3 Marks]

(b) Give the formulae for computing the half-way vector ***h*** and the specular reflection using this vector. (L_s = Light's specular colour, M_s = Material's specular colour) [2 Marks]

(c) Draw the above figure again, changing the view vector ***v*** to a direction where the viewer perceives maximum specular reflection from the point P. Show the direction of ***h*** for this case. [2 Marks]

(d) Consider the following code segment. Assume that lighting is enabled using GL_LIGHT0, and its position is defined by the variable `lgt_pos`.

```
float lgt_pos[4] = {0, 50, 20, 1};
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
                           I
gluLookAt(10, 50, 1, 0, 0, 0, 0, 1, 0);
glTranslatef(5, 3, 7);
glRotatef(angle, 0, 1, 0);
                           II
glutSolidTeapot(1.0);
```

Briefly explain the difference in the way the light's position would be transformed if the statement

`glLightfv(GL_LIGHT0, GL_POSITION, lgt_pos);`

is placed at position (I), and position (II) of the code. [3 Marks]

Question 6. Texture Mapping [10 marks for the whole question]

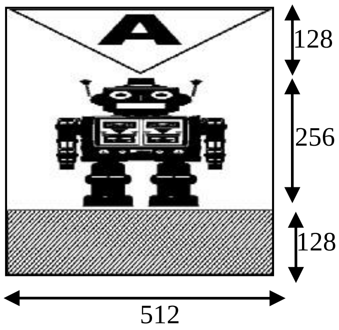


Fig. 4 (a)

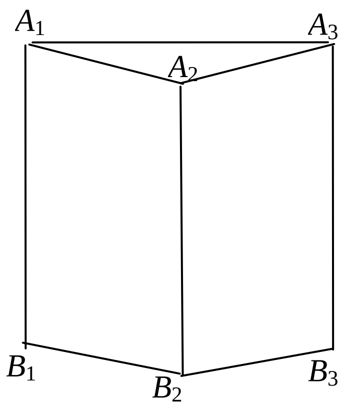


Fig. 4(b)

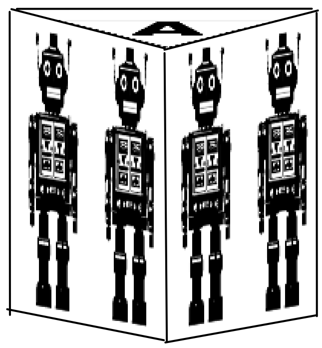


Fig. 4(c)

A single texture image of size 512x512 pixels is shown in Fig. 4(a). The top part of the image has a triangle with the letter 'A' inside it, the middle section has a robot image, and the bottom part has a striped pattern. A polygonal model is shown in Fig. 4(b). It consists of a quad strip $\{A_1, B_1, A_2, B_2, A_3, B_3\}$, and a triangle $\{A_1, A_2, A_3\}$. The texture is required to be mapped onto the model as shown in Fig. 4(c) with each of the two sides of the quadstrip containing two robot images and the triangle on top containing the 'A' image. The striped part of the texture is not used.

(a). Write the texture coordinates for the six vertices of the quadstip in the following format in your answer booklet:

A_1 = (,   )
B_1 = (,   ) .. etc. [7 Marks]

(b) Write the texture coordinates for the vertices of the triangle $\{A_1, A_2, A_3\}$. [3 Marks]

Question 7. Ray Tracing [10 marks for the whole question]

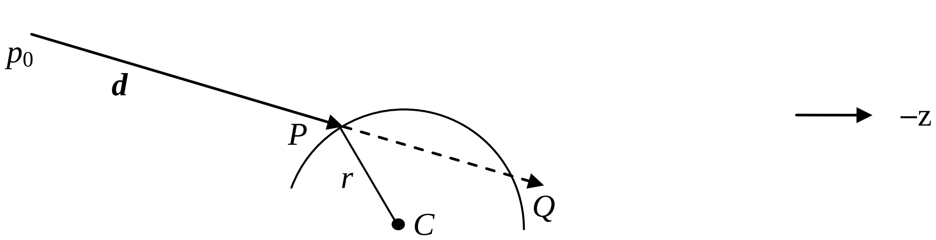


Figure 5.

The points of intersection of a ray $\boldsymbol{p} = \boldsymbol{p}_0 + t\boldsymbol{d}$ and a sphere centered at a point C with radius r (Fig. 5) is given by

$$t = -(s \bullet d) \pm \sqrt{(s \bullet d)^2 - (s \bullet s) + r^2}$$

where $s = \boldsymbol{p}_0 - C$.

(a) A primary ray originates at $\boldsymbol{p}_0 = (0, 0, 0)$ and has a direction $\boldsymbol{d} = (0, -0.6, -0.8)$. If a sphere is defined with centre $C = (0, -10, -5)$, and radius $r = 5$, compute the value of t for the closest point of intersection of the ray with the sphere. [5 Marks]

(b) From the values of t, what can you conclude about the point of intersection? [1 Mark]

(c) Using the value of t, write the coordinates of the point P where the ray meets the sphere. [2 Marks]

(d) What does the term $(s \bullet s)$ in the above equation represent? [2 Marks]

Question 8. Visibility/BSP Trees [10 marks for the whole question]

A set of 11 polygons (A..K) and their surface normal directions are shown in Figure 6.

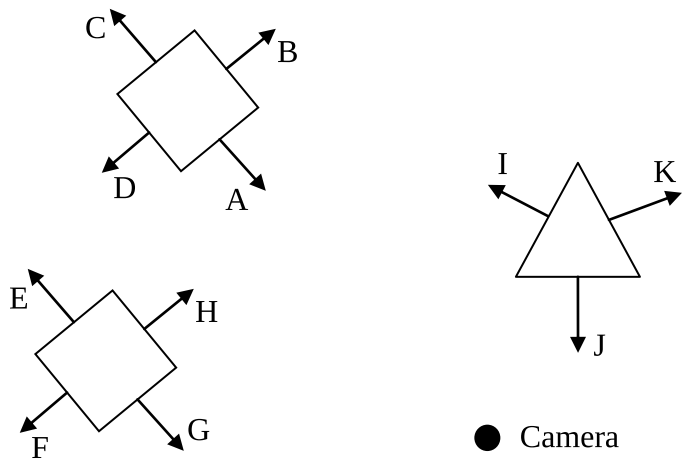


Figure 6.

(a) Draw a Binary Space Partitioning (BSP) tree for the configuration in the figure. Select polygon 'A' as the root node. The left branch at a node should represent the front side of the current polygon (indicated by the normal vector). [6 Marks]

(b) Traverse the BSP tree and write the rendering order of the polygons, for the camera position shown in the figure. [4 Marks]

Question 9. Curves and Surfaces [10 marks for the whole question]

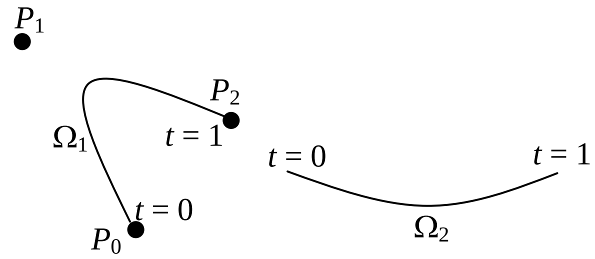


Figure 7.

A quadratic Bezier curve Ω_1 is defined using three control points $P_0 = (10, 0)$, $P_1 = (0, 30)$, and $P_2 = (30, 20)$ on the xy-plane. Another quadratic Bezier curve Ω_2 is given by the following parametric equations:

$x(t) = 30 + 30\,t$

$y(t) = 20 - 10\,t + 10\,t^2$, $\quad 0 \le t \le 1$.

(a) Write the parametric equations of the first Bezier curve Ω_1. [5 Marks]

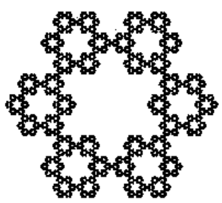
(b) Determine if the two curves have C_0-continuity, C_1-continuity and G_1-continuity at the point P_2. [5 Marks]

Question 10. Fractals [10 marks for the whole question]

The dragon curve is produced by an L-System with the following grammar:

S $\rightarrow$ FX (zero-order dragon)
F $\rightarrow$ F
X $\rightarrow$ X+YF+
Y$\rightarrow$ −FX−Y
Angle = 90 degs, F = move forward, Initial direction: $\longrightarrow$

(a) Write the strings generated by the grammar for the second and third order dragons. [4 Marks]
(b) Draw the third order dragon. [4 Marks]
(c) A figure of the Sierpinski Hexagon is given below. If the scale factor s used in each iteration is 1/3, compute the shape's fractal dimension. [2 Marks]



$s = 1/3$.

Figure 8.

END OF PAPER