No electronic/communication devices are permitted.

Students may take exam question paper away after the exam.

Computer Science and Software Engineering

EXAMINATION

Mid-year Examinations, 2017

COSC363-17S1 (C) Computer Graphics

Examination Duration: 180 minutes

Exam Conditions:

Closed Book exam: Students may not bring in anything apart from writing instruments and calculators (if approved).

Calculators with a 'UC' sticker approved.

Materials Permitted in the Exam Venue:

None

Materials to be Supplied to Students:

1 x Standard 16-page UC answer book

Instructions to Students:

- Answer *all* questions.
- This is a closed book exam. No written or printed material is allowed.
- Check carefully the number of marks allocated to each question. This suggests the degree of detail required in each answer, and the amount of time you should spend on the question.
- Use the separate answer booklet provided for answering all questions.
- No form of collaboration is permitted.
- This question paper carries a total of 100 marks with 50% contribution to the final grade.

Questions Start on Page 3

Important Formulae

If $\boldsymbol{v}_1 = (x_1, y_1, z_1)$ and $\boldsymbol{v}_2 = (x_2, y_2, z_2)$, then the cross-product of the two vectors is given by $\boldsymbol{v}_1 \times \boldsymbol{v}_2 = (y_1 z_2 - y_2 z_1,\ z_1 x_2 - z_2 x_1,\ x_1 y_2 - x_2 y_1)$.

Length of projection of a vector $\boldsymbol{v}$ on a unit vector $\boldsymbol{n} = \boldsymbol{v} \bullet \boldsymbol{n}$

Rotation matrices:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation of a ray through $\boldsymbol{p}_0$ and having a unit direction $\boldsymbol{d}$: $\quad \boldsymbol{p} = \boldsymbol{p}_0 + t\boldsymbol{d}, \quad t > 0.$

Third degree Bezier curve defined using four control points P_0, P_1, P_2, P_3

$$P(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3, \quad 0 \le t \le 1.$$

Question 1. [10 marks for the whole question] *Transformations*

Assume that you are given three functions: (i) `drawCylinder()` that draws a cylinder centred at the origin with axis along the y-axis, height 1 unit and radius 1 unit (ii) `drawCone()` that draws a cone with the centre of the base at the origin and axis along *z*-axis, radius 1 unit and height 1 unit, and (iii) `drawFin()` that draws a fin shaped object near the origin (Fig. 1).

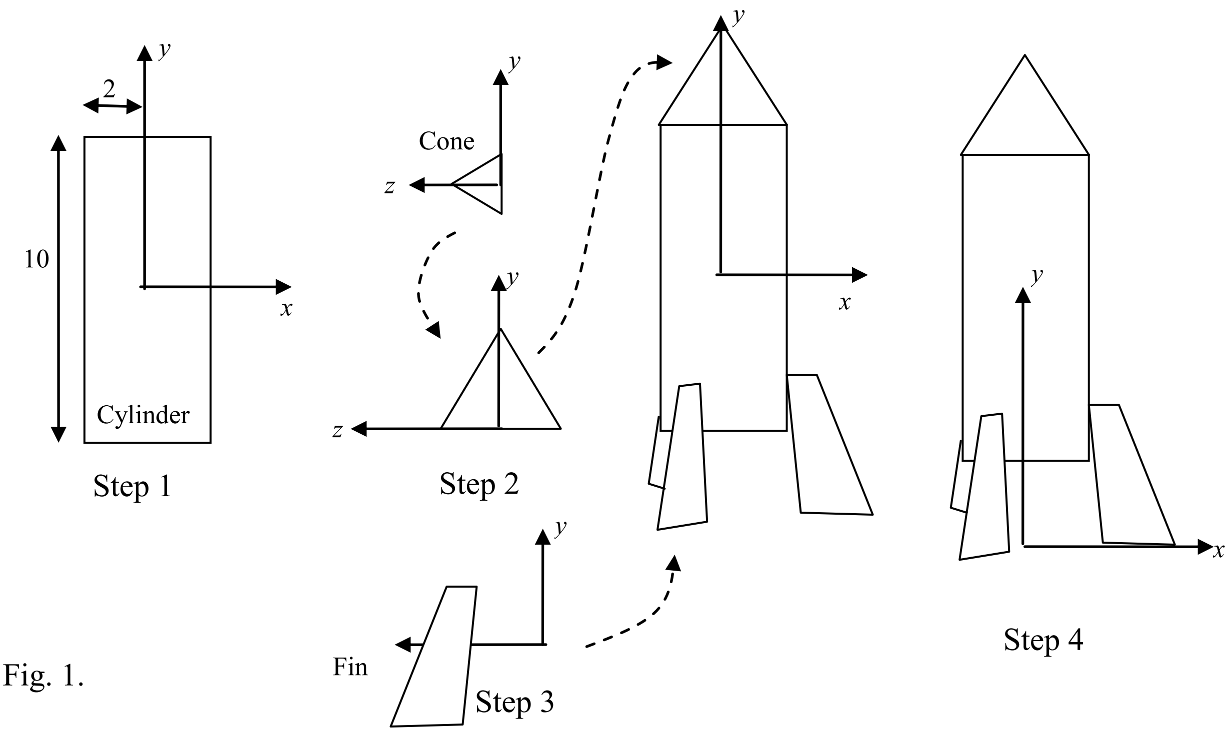


Fig. 1.

You are given the following instructions for creating the model of a rocket:

Step 1: Create a cylinder using `drawCylinder()` and scale it along both *x* and *z* axes by a factor 2, and along the *y* axis by a factor 10.

Step 2: Create a cone using `drawCone()` and rotate it to make its axis vertical as shown in the figure above. You need to find the angle and axis of rotation. Scale the rotated cone by a factor of 2 along all three axes, and then translate it to position (0, 5, 0).

Step 3: Create three fins calling the function `drawFin()` thrice. Rotate the second fin by 120 degs about the *y*-axis, and the third fin by 240 degs about *y*-axis. Translate the three fins to (0, –5, 0).

Step 4: Steps 1-3 above create the rocket model. Translate the whole model by (0, 8, 0) .

Using the given functions, OpenGL transformation functions and glPushMatrix-glPopMatrix blocks, write a code segment for defining only the transformations as outlined above, so that a model as depicted in the Fig. 1 (Step 4) is generated. You need not write any other OpenGL code or functions as part of the answer.

Question 2. [15 marks for the whole question] *Vectors and Matrices*

(a) [4 Marks] Consider the following code segment:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslated(5.0, 3.0, 1.0);
glScaled(0.5, -1.0, 0.5);
glRotated(30, 0, 0, 1);
glBegin(GL_POINTS);
      glVertex3d(6., 4., 3.);      //Point P
glEnd();
```

Write the transformation of the point P as a matrix expression, where each transformation is represented by a 4x4 matrix containing only numerical values. You are not required to multiply the matrices. Rotation matrices are given on Page 2.

(b) Consider the triangle formed by three points A, B, C as shown below (Fig. 2):

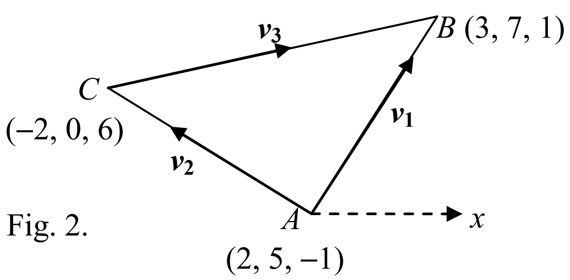


Fig. 2.

(i) [2 Marks] Compute the vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ shown in the figure above.

(ii) [2 Marks] Compute the unit vector along the direction of $\boldsymbol{v}_1$

(iii) [2 Marks] Show that the vector $\boldsymbol{v}_1$ is perpendicular to $\boldsymbol{v}_2$.

(iv) [3 Marks] Using the vector cross-product formula (see Page 2), compute the normal vector of the triangle. You need not convert this vector to a unit vector.

(v) [2 Marks] Compute the length of the projection of the vector $\boldsymbol{v}_3$ on the x-axis.

Question 3. [10 marks for the whole question] *Illumination*

The following figure (Fig. 3) gives the components of unit vectors used for lighting calculations at a point P on a sphere, and also the ambient, diffuse, specular properties of the light and the sphere material ($\boldsymbol{l}$ = light source vector, $\boldsymbol{n}$ = normal vector, $\boldsymbol{v}$ = view vector, $\boldsymbol{r}$ = reflection vector).

$\boldsymbol{l}$ = (0.8, 0.6, 0)

$\boldsymbol{n}$ = (0, 1, 0)

$\boldsymbol{r}$ = (−0.8, 0.6, 0)

$\boldsymbol{v}$ = (−1, 0, 0)

f (Phong's constant) = 10.

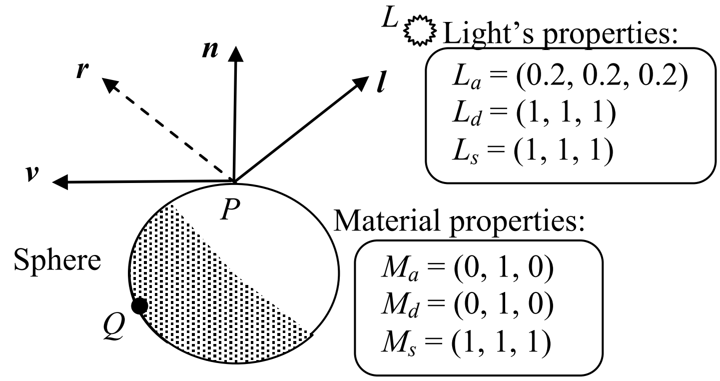


Fig. 3.

(a) [2 Marks] Assume that L is the only light source in the scene. .A point Q lies in the shadow region of the sphere. Write the condition satisfied by the light source vector and the normal vector at this point. Write the colour components of net reflection from this point.

(b) [3 Marks] Write the mathematical expression for the colour of diffuse reflection at P, and compute the numerical values of its components.

(c) [3 Marks] Write the mathematical expression for the colour of specular reflection (including the shininess term) at P along the view direction $\boldsymbol{v}$, and compute the numerical value of the colour components.

(d) [2 Mark] Write the expression for the half-way vector at P, and compute its components. You need not normalize this vector.

Question 4. [10 marks for the whole question] *Texture Mapping*

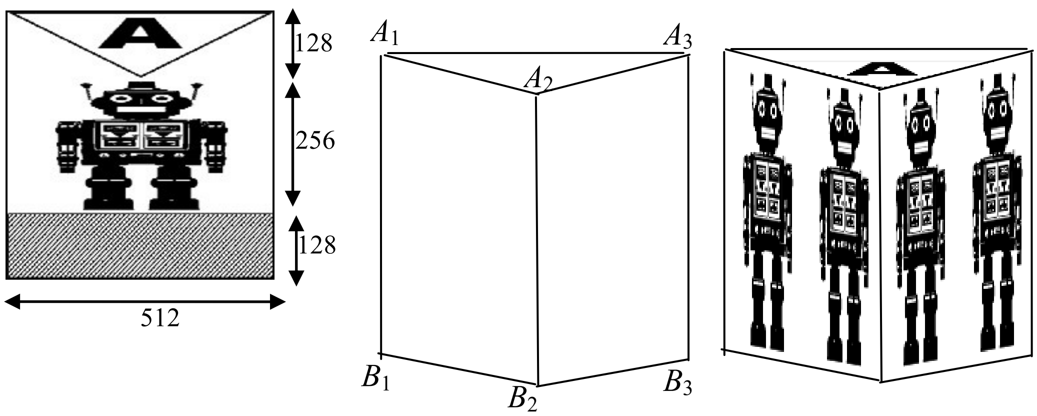


Fig. 4 (a) Fig. 4(b) Fig. 4(c)

A single texture image of size 512x512 pixels is shown in Fig. 4(a). The top part of the image has a triangle with the letter 'A' inside it, the middle section has a robot image, and the bottom part has a striped pattern. A polygonal model is shown in Fig. 4(b). It consists of a quad strip {A_1, B_1, A_2, B_2, A_3, B_3}, and a triangle {A_1, A_2, A_3}. The texture is required to be mapped onto the model as shown in Fig. 4(c) with each of the two sides of the quadstrip containing two robot images and the triangle on top containing the 'A' image. The shaded/striped part of the texture is not used.

(a). [7 Marks] Write the texture coordinates for the six vertices of the quadstip in the following format in your answer booklet:

$A_1 = (\quad , \quad)$
$B_1 = (\quad , \quad)$ .. etc.

(b) [3 Marks] Write the texture coordinates for the vertices of the triangle $\{A_1, A_2, A_3\}$.

Question 5. [10 marks for the whole question] *View Transformation*

A view transformation matrix transforms points from world coordinates to eye coordinates. Consider the following view transformation matrix:

$$\begin{bmatrix} 0.8 & 0 & 0.6 & -11 \\ 0 & 1 & 0 & -7 \\ -0.6 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) [3 Marks] Compute the eye coordinates of the point P (10, 7, 5) using the above matrix. From your answer, what can you conclude about the point P ?

(b) [4 Marks] Consider a vector $\boldsymbol{v}$ in homogeneous coordinates (6, 0, −8, 0). Transform the vector using the above view transformation matrix. From your answer, what can you conclude about the vector $\boldsymbol{v}$?

(c) [3 Marks] A point Q has world coordinates (−5, 3, 10). Using its eye coordinates, determine if this point is behind or in front of the camera .

Question 6. [10 marks for the whole question] *Projections*

A view frustum is specified using the following statement:

```
glFrustum(-40.0,  40., -30., 30.,  50.0, 100.0);
```

(a) [6 Marks] Draw a sketch of the view frustum, clearly showing what each of the six parameters of the above function represent. The figure should also show the camera position, camera coordinate axes directions, and the view axis direction.

(b) [4 Marks] Using the parameters of the function, compute the aspect ratio of the near plane, and the field of view along the y-direction.

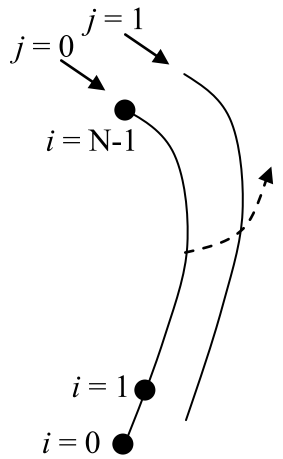
Question 7. [8 marks for the whole question] *Surfaces of revolution*

Consider the following code segment which generates a surface of revolution by transforming points `(vx[i], vy[i], vz[i]), i = 0..N-1,` of a base polygonal curve.

```
for(int j = 0; j < 36; j++)   //36 slices
{
      for(int i = 0; i < N; i++)   //Vertex transformations
      {
          wx[i] = cos(0.1745)*vx[i] + sin(0.1745)*vz[i];
          wy[i] = vy[i];
          wz[i] = -sin(0.1745)*vx[i] + cos(0.1745)*vz[i];
      }

      glBegin(GL_QUAD_STRIP);          //Create quads
      for(int i = 0; i < N; i++)
      {
            glTexCoord2f(s1,  t1);
            glVertex3f(vx[i], vy[i], vz[i]);
            glTexCoord2f(s2,  t2);
            glVertex3f(wx[i], wy[i], wz[i]);
       }
       glEnd();

      ...   //Update vertices  (code removed)
 }
```



(a) [3 Marks] Briefly describe the transformation applied to the vertices, giving the angle and axis of rotation used in this transformation.

(b) [3 Marks] Give the expressions for the texture coordinates `s1, t1, s2, t2` in terms of the parameters `i, j,` such that values of `i` are mapped along the vertical axis of the texture and `j` along the horizontal axis.

(c) [2 Marks] How are the vertex coordinates updated after construction of a quad strip? Why is the update operation necessary?

Question 8. [12 marks for the whole question] *Ray Tracing*

A shadow ray is traced from a point *P* towards the light source *L* as shown in the following figure (Fig. 5).

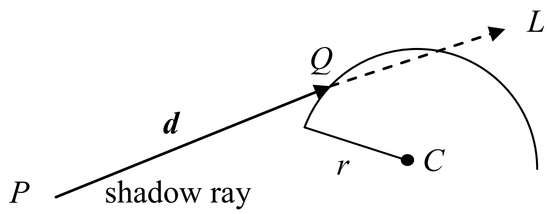


Fig. 5.

We define two vectors $\boldsymbol{s} = P - C$, and $\boldsymbol{l} = L - P$.

The points of intersection of the ray $\boldsymbol{p} = P + t\boldsymbol{d}$ and a sphere centered at a point C with radius r is given by

$$t = -(s \bullet d) \pm \sqrt{(s \bullet d)^2 - (s \bullet s) + r^2}$$

(a) [2 Marks] Write the conditions for the existence of a valid point of intersection of the ray with the sphere.

(b) [2 Marks] If a valid point of intersection exists and is given by the ray parameter t, what other condition should t satisfy in order that the point P is in shadow?

(c) [2 Marks] What does the term $(s \bullet s)$ in the above equation represent?

(d) [6 Marks] If $P = (0, 2, -10)$, $C = (0, 12, -15)$, $\boldsymbol{d} = (0, 0.6, -0.8)$, $r = 5$, compute the values of the ray intersection parameter t using the formula given above. What can you conclude from the two values of t ?

Question 9. [7 marks for the whole question] *Bezier Curves.*

A two-dimensional cubic Bezier curve through four control points P_0, P_1, P_2, P_3 is given by the equation $P(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3$, $0 \le t \le 1$.

(a) [5 Marks] If $P_0=(0, 0)$, $P_1 = (0, 1)$, $P_2 = (1, 1)$, $P_3 = (1, 0)$, write the parametric equations of the Bezier curve in the form $x(t) = a_0 + a_1t + a_2t^2 + a_3t^3$; $y(t) = b_0 + b_1t + b_2t^2 + b_3t^3$, where some of the coefficients may be zeros.

(b) [2 Marks] Show that for the curve in the previous question (Q.9a), the point on the curve at $t = 0.5$ is $(0.5, 0.75)$.

Question 10. [8 marks for the whole question] *OpenGL* 4 *Shaders*.

(a) [2 Marks] What are the two important computations or operations commonly performed in a vertex shader?

(b) [1 Mark] Give an example of an input (a variable with qualifier `in`) to a vertex shader.

(c) [1 Mark] What does the built-in output variable of a vertex shader, `gl_Position` represent?

(d) [2 Marks] Give an example each of a uniform variable that is commonly used in a vertex shader and a fragment shader.

(e) [2 Marks] Give a brief outline of two operations that can be performed in a tessellation control shader.

End of Examination