University of Canterbury

Mid-Year Examinations 2015

Prescription number: COSC363

Paper title: Computer Graphics

Time allowed: 3 hours

Number of pages: 7

Maximum marks: 100. Contribution to the final grade: 50%

Number of questions: 10

- **Answer *all* questions**.
- This test is worth a total of 100 marks.
- This is a **closed book** exam. No written or printed material is allowed**.**
- **UC approved calculators** are allowed.
- Please answer all questions carefully and to the point. Check carefully the number of marks allocated to each question. This suggests the degree of detail required in each answer, and the amount of time you should spend on the question.
- Use the separate answer booklet provided for answering all questions.

Important Formulae

If $\boldsymbol{v}_1 = (x_1, y_1, z_1)$ and $\boldsymbol{v}_2 = (x_2, y_2, z_2)$, then

$\boldsymbol{v}_1 \times \boldsymbol{v}_2 = (y_1z_2 - y_2z_1,\ z_1x_2 - z_2x_1,\ x_1y_2 - x_2y_1)$.

Rotation matrices:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation of a ray through $\boldsymbol{p}_0$ and having a unit direction $\boldsymbol{d}$: $\boldsymbol{p} = \boldsymbol{p}_0 + t\boldsymbol{d}$, $t > 0$.

Equation of a plane passing through $\boldsymbol{a}$ and having a normal direction $\boldsymbol{n}$: $(\boldsymbol{p}-\boldsymbol{a}) \bullet \boldsymbol{n} = 0$

The parameter t on the ray at the point of intersection with the above plane:

$$t = \frac{(a - p_0) \bullet n}{d \bullet n}$$

Second order Bezier curve defined using three control points P_0, P_1, P_2:

$P(t) = (1-t)^2P_0 + 2t(1-t)\,P_1 + t^2P_2$, where $P(t) = (x(t), y(t))$, $0 \le t \le 1$.

Question 1. [8 marks for the whole question] *Transformations*

Assume that you are given a function `drawCylinder()` that draws a cylinder of radius1 unit and height 2 units with the centre of its base at the origin, and axis along the *z*-axis (Fig. 1a)

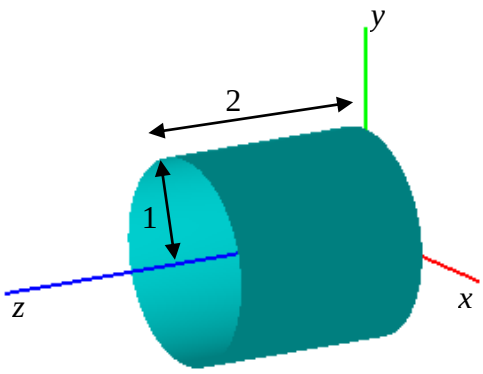


Fig. 1a

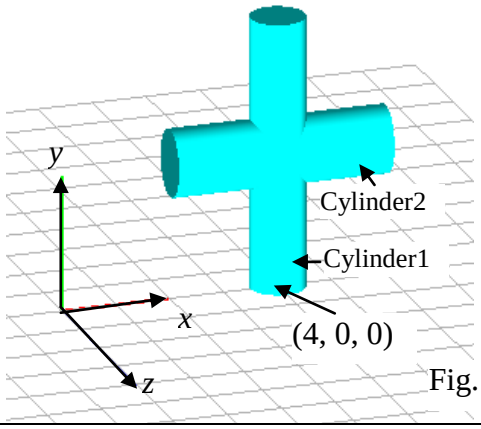
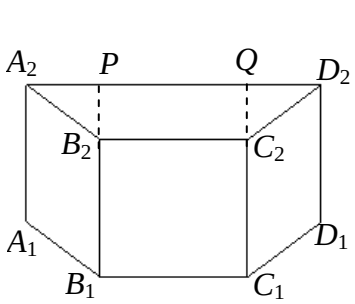


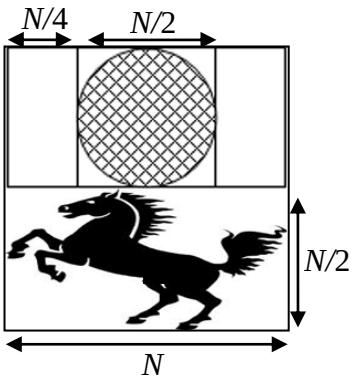
Fig. 1b

Write a code segment containing only calls to `drawCylinder()`, OpenGL transformation functions and glPushMatrix/glPopMatrix functions for creating two cylinders and transforming them as shown in Fig. 1b. Both cylinders have radius 0.5 units, and height 4 units. The first cylinder has its base positioned at (4, 0, 0) and axis along the *y*-direction. The second cylinder has its axis horizontal along the *x*-direction with its centroid coinciding with that of the first cylinder. (The centroid is the centre of the circular section at half-way between the two ends of the cylinder).

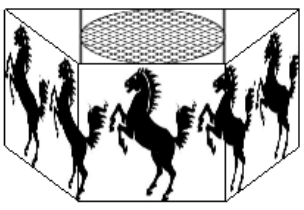
Question 2. [10 marks for the whole question] *Texture Mapping*



(a)



(b)



(c)

Fig. 2.

(a) [6 Marks] The polygonal object shown in Fig. (2a) is created using a quad-strip $\{A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2\}$. The top side consists of two triangles A_2B_2P, C_2D_2Q and a quad PB_2C_2Q. Consider the texture image of size NxN pixels given in Fig. (2b). Write the texture coordinates (s, t) that must be assigned to the vertices, so that the quad-strip displays 5 repetitions of the horse image

distributed evenly across the three segments; and the quad PB_2C_2Q on the top displays the disc (Fig. 2c). Assume that the texture wrap mode is set to GL_REPEAT along *s* and *t*. Please provide your answer in the answer-booklet in the following format:

Quad-strip	*s*	*t*
A_1		
A_2		
B_1		
B_2		
C_1		
C_2		
D_1		
D_2		

Quad	*s*	*t*
P		
B_2		
C_2		
Q		

(b) [4 Marks] Briefly describe (in 3 or 4 sentences) what is meant by "mipmapping", and its usefulness in texture mapping. You are not required to give any equations.

Question 3. [12 marks for the whole question] *Mathematical Aspects*

(a) [6 Marks] A circle on the *xy*-plane is transformed into an ellipse as shown in the figure below (Fig. 3).

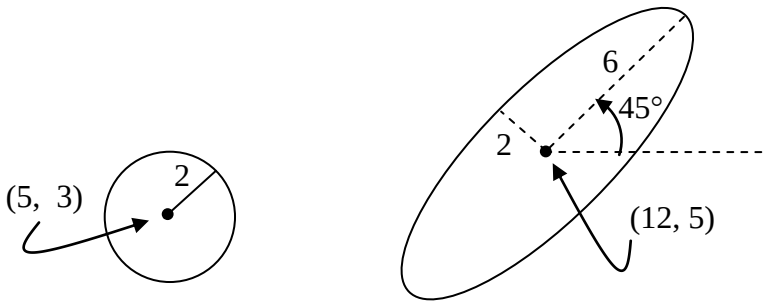


Fig. 3.

Write the matrix product expression (containing 4x4 matrices) for the above transformation. You are <u>not</u> required to multiply the matrices. The matrices should contain only numerical values. (cos(45) = 0.707, sin(45) = 0.707).

(b) [4 Marks] Compute the surface normal vector of a triangle *ABC* with vertices *A*=(1, 0, 2), *B*=(3, 5, 2), *C*= (1, 5, 5). The vector need not be normalized.

(c) [2 Marks] What transformation does the following matrix represent?

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 4. [12 marks for the whole question] *Illumination Model*

The OpenGL illumination model is given by the following equation:

$$I_p = L_a M_a + L_d M_d \max(\boldsymbol{l}.\boldsymbol{n}, 0) + L_s M_s \{\max(\boldsymbol{h}.\boldsymbol{n}, 0)\}^f$$

(a) [2 Marks] What do the terms L_a, M_a, L_d, M_d, L_s, M_s represent?

(b) [2 Marks] Write the colour values that you would assign to L_a, M_a, L_d, M_d, L_s, M_s to render a teapot with material colour cyan under a white light.

(c) [4 Marks] With the help of a diagram showing vectors $\boldsymbol{l}$, $\boldsymbol{v}$, $\boldsymbol{n}$, briefly describe how the vector $\boldsymbol{h}$ is computed and what it represents.

(d) [2 Marks] What does the term f represent, and how does its value affect the colour values generated by the illumination model?

(e) [2 Marks] Why is the function max() used in the second term on the right-hand side of the equation?

Question 5. [10 marks for the whole question] *Shadows*.

A shadow transformation matrix useful for generating planar shadows of objects on a floor plane with $y = 0$, is given below.

$$\begin{bmatrix} s_x \\ s_y \\ s_z \\ w \end{bmatrix} = \begin{bmatrix} l_y & -l_x & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -l_z & l_y & 0 \\ 0 & -1 & 0 & l_y \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(a) [6 Marks] An OpenGL application creates an object and transforms it using a transformation matrix M. Briefly describe or give a pseudo-code for the process of generating the display of both the object and its shadow using the shadow matrix given above. You are not required to use camera or projection matrices in the method.

(b) [4 Marks] If a point light source is specified at location (5, 10, 5), compute the three-dimensional coordinates (x', y', z') of the shadow of an object point (6, 2, 4).

Question 6. [10 marks for the whole question] *Projections*.

(a) [6 Marks] Draw a sketch of the perspective view volume given by `glFrustum(−40., 40., −30., 30., 30., 500.)` and mark the frustum parameters, coordinate axes directions, and the camera position.

(b) [4 Marks] A perspective view volume could also be defined using the function `gluPerspective(fov, aspect, near, far)`. Compute the values of the function parameters `fov, aspect, near, far` that would produce the same frustum as given in the previous question (6(a)).

Question 7. [14 marks for the whole question] *Ray tracing*.

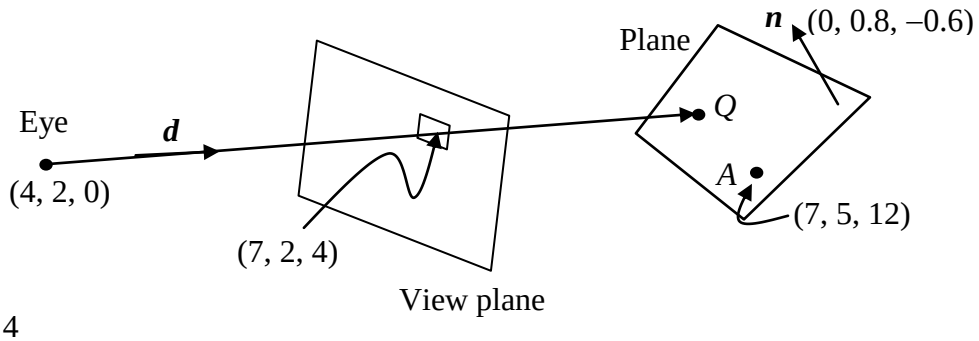


Fig. 4

A primary ray is traced from the eye position (4, 2, 0) through the point (7, 2, 4), as shown in the Fig. 4. The ray hits a plane that contains the point A = (7, 5, 12). The plane's unit normal direction is (0, 0.8, –0.6).

(a) [2 Marks] What are the values of $\boldsymbol{p}_0$ and $\boldsymbol{d}$ in the equation of the ray : $\boldsymbol{p} = \boldsymbol{p}_0 + t\boldsymbol{d}$?

(b) [5 Marks] Compute the value of the ray parameter t at the point Q where the ray meets the plane. What does this value of t represent?

(c) [3 Marks] Compute the coordinates of the point of intersection Q.

(b) [4 Marks] The direction of the reflected ray is given by $\boldsymbol{r} = -2\,(\boldsymbol{d} \bullet \boldsymbol{n})\,\boldsymbol{n} + \boldsymbol{d}$. Compute the direction of the reflected ray from the point Q. You need not normalize this direction.

Question 8. [6 marks for the whole question] *Bezier Curves*.

A two-dimensional Bezier curve is given by the equation

$$P(t) = \sum_{i=0}^{2} B_i(t).P_i \quad , \text{ where, } \quad B_0(t) = (1-t)^2, \quad B_1(t) = 2t(1-t) \quad , \quad B_2(t) = t^2$$

$$\text{and} \quad P(t) = (x(t),\ y(t)), \quad P_i = (x_i\ , y_i).$$

Given three control points (0, 0), (1, −1), (2, 0), derive the parametric equations $(x(t), y(t))$ for the Bezier curve, and then show that the curve's equation is

$y = -x + (x^2/2)$.

Question 9. [10 marks for the whole question] *Tessellation Shaders*

(a) [5 Marks] Outline two uses of a tessellation control shader. Briefly describe one application where a tessellation control shader could be effectively used.

(b) [5 Marks] A triangle domain is tessellated as shown in the figure below (Fig. 5). The tessellation coordinates of two points *A*, *B* on the domain are also given.

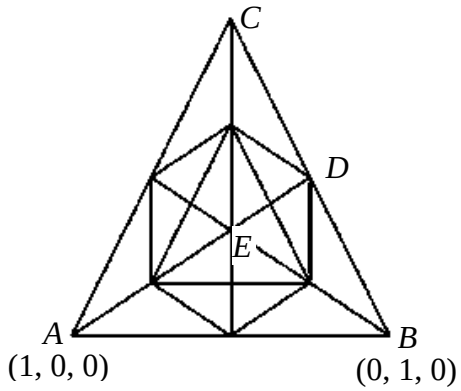


Fig. 5

b(i). Write the tessellation coordinates of the points *C*, *D* and *E*.

b(ii). Write the inner and outer tessellation levels used in generating the figure.

Question 10. [8 marks for the whole question] *OpenGL-4 Primitives*

Choose any ***one*** of the following two primitive types, and answer the questions below.

GL_PATCHES with 9 patch vertices

GL_TRIANGLES_ADJACENCY

(a) [3 Marks] How is the chosen primitive type different from a commonly used primitive type such as GL_TRIANGLES? Outline one important property of a primitive of the chosen type.

(b) [5 Marks] Briefly describe one application containing either a tessellation or a geometry shader, where a primitive of the chosen type could be effectively used for modelling/rendering an object. Please explain how the primitive is processed inside the shader. You are not required to write any shader code.

END OF PAPER