

Inter-airline Equity in Airport Scheduling Interventions

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Abstract

In the absence of opportunities for capacity expansion or operational enhancements, air traffic congestion mitigation may require scheduling interventions aimed to control the extent of over-capacity scheduling at busy airports. Previous research has shown that large delay reductions could be achieved through comparatively small changes in the schedule of flights. While existing approaches have focused on minimizing the overall impact of scheduling interventions across the airlines, this paper designs, optimizes, and assesses a novel approach for airport scheduling interventions that incorporates inter-airline equity objectives. It relies on a lexicographic modeling architecture based on efficiency (i.e., meeting airline scheduling preferences), equity (i.e., balancing scheduling adjustments fairly among the airlines), and on-time performance (i.e., mitigating airport congestion) objectives, subject to scheduling and network connectivity constraints. Theoretical results show that, under some scheduling conditions, equity and efficiency can be jointly maximized. Computational results suggest that, under a wide range of current and hypothetical scheduling settings, ignoring inter-airline equity can lead to highly inequitable outcomes, but that our modeling approach achieves inter-airline equity at no, or small, losses in efficiency.

Keywords: airport demand management, inter-airline equity, efficiency-equity trade-off, integer programming, dynamic programming, queuing model

1. Introduction

The development of air transportation systems worldwide has been supported by airport and air traffic management infrastructure. However, limitations on infrastructure capacity, coupled with significant growth in air traffic, have resulted in severe congestion at many of the world's busiest airports. This congestion typically materializes in the form of flight delays and cancellations. The costs of air traffic congestion in the United States were estimated at over \$30 billion for the year 2007 (Ball et al., 2010) and this issue is likely to become even more pressing over the medium- and long-term horizons as demand for air traffic is expected to increase nationally and internationally.

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Most of the air traffic delays in the United States originate from imbalances between demand and capacity at the busiest airports (Bureau of Transportation Statistics, 2013). In the absence of opportunities for capacity expansion or operating enhancements, such imbalances can only be significantly mitigated through scheduling interventions that limit the extent of over-capacity scheduling at peak hours. This paper proposes and evaluates a quantitative approach to optimize such interventions in a way that achieves on-time performance objectives, while minimizing interference with airlines’ competitive scheduling and, for the first time, balancing the impact of such interventions equitably among the airlines. Before presenting the contributions of this paper (Section 1.3), we elaborate on the approaches for airport scheduling interventions (Section 1.1) and review existing work on the trade-off between efficiency and equity in resource allocation (Section 1.2).

1.1. Airport Scheduling Interventions

Scheduling interventions refer to the demand management measures that impose limits, or constraints, on the number of flights scheduled at an airport. They are implemented months in advance of the day of operations (before flight schedules get published and tickets get marketed). Most airports outside the United States operate under slot control policies that limit the number of flights scheduled per hour (or other units of time) and distribute a corresponding number of slots across the different airlines through an administrative procedure (International Air Transport Association, 2015). In contrast, no demand management is applied at a large majority of US airports. A few of the busiest airports were subject to slot restrictions under the High Density Rule, but, since its phase-out in 2007, airline schedules of flights in the United States have been subject to limited constraints. Given the high delays that ensued in 2007, “flight caps” have been imposed at the three major airports in the New York Metroplex, but these were found too high to effectively alleviate congestion (Office of Inspector General, 2010; Government Accountability Office, 2012; de Neufville and Odoni, 2013). Given these regulatory differences, European airports may reject flight requests that could be accommodated, resulting in smaller throughput than their US counterparts. On the other hand, US airports face larger imbalances between demand and capacity and hence, larger and less predictable delays (Morisset and Odoni, 2011; Odoni et al., 2011).

Recent research has showed the potential to improve current scheduling intervention practices to mitigate congestion and satisfy airline scheduling requests as closely as possible. First, market mechanisms based on congestion pricing (Carlin and Park, 1970; Daniel, 1995; Brueckner, 2002; Vaze and Barnhart, 2012a) or slot auctions (Rassenti et al., 1982; Ball et al., 2006; Harsha, 2009) have been proposed to allocate airport capacity to the users that assign the highest value to it. However, they have not been successfully implemented in the current institutional environment, most likely due to the monetary transfers they involve. A second line of research has focused on improving current slot allocation procedures at slot-controlled airports by optimizing the matching of airlines’ scheduling requests (Zografos et al., 2012). Third, several studies have investigated the

potential of demand management to mitigate congestion at US airports, based on the well-known result from queuing theory that the relationship between air traffic demand, airport capacity and on-time performance is highly non-linear at airports operating close to capacity (de Neufville and Odoni, 2013; Pyrgiotis et al., 2013; Nikoleris and Hansen, 2012; Jacquillat and Odoni, 2015b). Using a game-theoretic framework of airline frequency competition, Vaze and Barnhart (2012b) showed that small reductions in allocated airport capacity can reduce delays and improve airline profitability. By modeling the trade-off between flight delays and passenger schedule delay (i.e., schedule inconvenience), Swaroop et al. (2012) found that a reduction in allocated capacity of 10% to 20% would improve passenger welfare at a majority of busy US airports. Using a different approach, Pyrgiotis and Odoni (2016) and Jacquillat and Odoni (2015a) modeled and optimized the intra-day scheduling interventions, and found that limited changes in airline timetabling of flights could yield significant delay reductions. In summary, evidence suggests that performance improvements could be achieved at the busiest US airports through limited scheduling interventions that involve only temporal shifts in demand (i.e., changes in the intra-day timetabling of flights), and no reduction in overall demand (i.e., no change in the set of flights scheduled in the day).

These existing approaches suffer from two main limitations. First, they are focused exclusively on overall scheduling levels at the airports, without considering explicitly the impact of the interventions on the different airlines. In turn, they may penalize one airline (or a small number of airlines) disproportionately. Second, they do not investigate the potential for strategic behaviors from the airlines when providing their scheduling inputs. This paper addresses the first of these concerns by integrating inter-airline equity considerations into the decision-making framework underlying airport scheduling interventions. We leave the second one for further research.

Our scheduling process uses, as a starting point, capacity estimates at an airport under consideration, and the preferred schedule of flights. Airport capacity estimates can be obtained from historical records of operations (Gilbo, 1993; Simaiakis, 2012). As in current practice, the preferred schedule is typically provided by the airlines to a central decision-maker (e.g., administratively appointed schedule coordinators at slot-controlled airports, the Federal Aviation Administration (FAA) in the United States), who then produces a schedule of flights to reduce anticipated delays at the considered airport. Per the discussion above, we focus primarily on the case where these adjustments involve only temporal shifts in demand. We also discuss the case where the adjustments involve reductions in demand (i.e., the elimination of some flights), as may be required at a few of the busiest slot-controlled airports worldwide, where unconstrained airline demand may be so high that acceptable delay levels cannot be attained with existing levels of capacity. In addition, we consider the general case where each flight is assigned a weight characterizing the cost, or the inconvenience, of this flight being rescheduled (or eliminated). This captures the standard “a flight is a flight” paradigm (with equal weights assigned to all flights, as currently practiced at slot-controlled airports and assumed in other previously proposed mechanisms), as well as extensions

of existing mechanisms in which the airlines can signal the relative rescheduling costs of different flights through non-monetary credit allocation or a monetary auction-based mechanism. In turn, this paper introduces inter-airline equity in a wide range of settings representing current practice as well as potential extensions of previously proposed mechanisms.

1.2. Equity in Resource Allocation

Airport scheduling interventions fall into a broader class of problems involving the allocation of scarce resources by a central decision-maker to distributed agents (here, airlines). One major challenge in this class of problems involves defining the objective of resource allocation to balance the preferences and requirements of various stakeholders (Sen et al., 2014). This may create trade-offs between efficiency (i.e., maximizing the sum of agents’ utilities), equity (i.e., balancing utilities fairly among the agents), and, possibly, other objectives (e.g., maximizing outcome predictability, ensuring incentive-compatibility, etc.). The trade-off between efficiency and equity was first studied by Nash (1950) and Kalai and Smorodinsky (1975) for the two-player bargaining problem. It has been recently extended by Bertsimas et al. (2011, 2012) to general problems of resource allocation, who obtained theoretical bounds on the “price of fairness” and the “price of efficiency”, i.e., the relative loss in efficiency if equity is maximized, and vice versa. These bounds were derived with general utility functions, and in special instances involving compact and convex utility sets. However, no study till date has incorporated inter-airline equity considerations into the design of airport scheduling interventions.

In a related area, equitable mechanisms have been developed for allocating air transportation capacity on the day of operations through Air Traffic Flow Management (ATFM) initiatives. ATFM consists of optimizing the flows of aircraft at airports or through air traffic control sectors over the day of operations to reduce local imbalances between demand and capacity. Whereas early ATFM developments were exclusively based on efficiency objectives (minimizing total congestion costs), recent studies have incorporated inter-airline equity considerations into the objective of ATFM models, aiming to make the outcome of centralized decision-making more acceptable to each of the individual airlines (Vossen et al., 2003; Barnhart et al., 2012; Bertsimas and Gupta, 2016). This paper aims to integrate similar objectives into the optimization of scheduling interventions. However, the problem of scheduling interventions exhibits several differences from the ATFM problem. First, unlike ATFM, no standard of equity has been accepted in the industry with respect to scheduling interventions. Second, scheduling interventions may result in flights being rescheduled later or earlier than their preferred times requested by the airlines. This contrasts with the situation in ATFM where flights cannot be moved earlier than their scheduled time. Thus, the ATFM schemes of ration-by-schedule and schedule compression do not have any direct analogs in the context of scheduling interventions. It is thus necessary to propose new metrics of inter-airline equity and to develop new modeling frameworks for scheduling interventions.

1.3. Contributions

The main contribution of this paper consists of developing and solving a set of optimization models that incorporate inter-airline equity considerations in airport scheduling interventions. Our approach builds upon the *Integrated Capacity Utilization and Scheduling Model (ICUSM)* from Jacquillat and Odoni (2015a) that optimizes such interventions through temporal shifts in demand, but extends it in a way that balances scheduling adjustments equitably among the airlines. We name the resulting model the Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E).

Specifically, this paper makes the following contributions:

- *Quantifying and optimizing the trade space between performance attributes for scheduling interventions.* We identify efficiency (i.e., meeting airline scheduling preferences), equity (i.e., balancing scheduling adjustments fairly among the airlines), and on-time performance (i.e., mitigating airport congestion) as three performance attributes. We develop quantitative indicators for each of them, using a unified framework of scheduling interventions. We then formulate a tractable lexicographic architecture to characterize and optimize the trade space between efficiency, equity, and on-time performance in airport scheduling interventions.
- *Characterizing conditions under which efficiency and equity can be jointly maximized.* We show that, in the absence of network connections and in the case where all flights are equally costly (or equally inconvenient) to reschedule, efficiency and equity can be jointly maximized when the interventions involve only reductions in demand, or when the interventions involve only temporal shifts in demand and one of the following conditions is satisfied: (i) the imbalances are limited to non-consecutive periods in the day, or (ii) the schedules of flights of the different airlines exhibit the same intra-day patterns. We then describe instances where the schedules of flights, network connections, or unequal flight valuations can give rise to a trade-off between efficiency and equity.
- *Generating and solving real-world full scale computational scenarios at the John F. Kennedy Airport (JFK).* We show that, under a wide range of realistic and hypothetical scheduling conditions, the consideration of efficiency-based objectives exclusively in airport scheduling interventions may lead to highly inequitable outcomes, but that inter-airline equity can be achieved at no (or minimal) efficiency losses. This suggests that existing approaches for scheduling interventions can be extended to include inter-airline equity considerations.

1.4. Outline

The remainder of this paper is organized as follows. In Section 2, we summarize the Integrated Capacity Utilization and Scheduling Model (ICUSM) from Jacquillat and Odoni (2015a), and discuss its limitations related to inter-airline equity. In Section 3, we formulate the Integrated

Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E) that builds upon the ICUSM to account for inter-airline equity. Section 4 provides a theoretical discussion of the trade-off, or lack of it, between efficiency and equity in the context of airport scheduling interventions. In Section 5, we show computational results from a case study at JFK Airport. Section 6 concludes.

2. Base Model of Scheduling Interventions

We first summarize the Integrated Capacity Utilization and Scheduling Model (ICUSM) from Jacquillat and Odoni (2015a), which provides the baseline for the optimization of our scheduling interventions. Moreover, this section introduces notations that will be used throughout this paper.

2.1. Formulation

The ICUSM considers a two-step process, under which the airlines provide a schedule of flights to a central decision-maker, who may then propose scheduling adjustments to reduce above-capacity scheduling at an airport, and hence reduce anticipated delays. We denote by Π the airport where the scheduling interventions are considered. No flight is eliminated, and delays are reduced by distributing flights more evenly over the day. The model takes as inputs each airline's preferred schedule of flights (e.g., the schedule in the absence of demand management) and estimates of the capacity of airport Π (i.e., the expected number of movements that can be operated per unit of time in various operating conditions). It determines which flights to reschedule to later or earlier times to minimize the displacement from the airlines' preferred schedule of flights, subject to scheduling and network connectivity constraints and on-time performance constraints. Scheduling and network connectivity constraints ensure that the airlines' flight networks are minimally affected, and on-time performance constraints ensure that expected arrival and departure queue lengths do not exceed pre-specified targets. The modeling framework of the ICUSM integrates into an Integer Programming model of scheduling interventions a Stochastic Queuing Model of airport congestion and a Dynamic Programming model of airport capacity utilization.

Inputs.

\mathcal{T}	=	set of 15-minute time periods, indexed by $t = 1, \dots, T$
\mathcal{F}	=	set of flights, indexed by $i = 1, \dots, F$
$\mathcal{F}^{\text{arr}}/\mathcal{F}^{\text{dep}}$	=	set of flights $i \in \mathcal{F}$ scheduled to land/take off at airport Π
$\mathcal{C} \subset \mathcal{F} \times \mathcal{F}$	=	set of ordered flight pairs $(i, j) \in \mathcal{F} \times \mathcal{F}$ such that there is a connection from i to j
$\mathcal{S}_{it}^{\text{arr}}/\mathcal{S}_{it}^{\text{dep}}$	=	$\begin{cases} 1 & \text{if flight } i \text{ is scheduled to land/take off no earlier than period } t \\ 0 & \text{otherwise} \end{cases}$
$t_{ij}^{\text{min}}/t_{ij}^{\text{max}}$	=	minimum/maximum connection time between flight i and flight $j \quad \forall (i, j) \in \mathcal{C}$

A connection refers to any pair of flights between which a minimum and/or a maximum time must be maintained to enable an aircraft, passengers, or a crew to connect. Note that the set of flights considered in the model may include flights that are not scheduled to land or take off at the airport Π where the scheduling interventions are applied, i.e., $\mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$ may be a strict subset of \mathcal{F} . This arises from the need to maintain feasible connections in a network of airports.

Variables.

$$\begin{aligned} w_{it}^{\text{arr}}/w_{it}^{\text{dep}} &= \begin{cases} 1 & \text{if flight } i \text{ is rescheduled to land/take off no earlier than period } t \\ 0 & \text{otherwise} \end{cases} \\ u_i &= \text{displacement (positive or negative) of flight } i, \text{ as number of 15-minute periods} \\ \lambda_t^{\text{arr}}/\lambda_t^{\text{dep}} &= \text{number of arrivals/departures scheduled at airport } \Pi \text{ during period } t, \text{ after rescheduling} \end{aligned}$$

By convention, we assume that $w_{i,T+1}^{\text{arr}} = w_{i,T+1}^{\text{dep}} = 0, \forall i \in \mathcal{F}$.

Objective. The model minimizes, first, the *largest* schedule displacement that any flight will sustain, denoted by δ (Equation (1)), and, second, the *total* schedule displacement, denoted by Δ_0 (Equation (2)).

$$\delta = \max_{i \in \mathcal{F}} |u_i| \quad (1)$$

$$\Delta_0 = \sum_{i \in \mathcal{F}} |u_i| \quad (2)$$

Constraints. For notational ease, a parameter κ refers either to the “arr” or the “dep” superscript of the inputs and variables defined above.

$$w_{it}^{\kappa} \geq w_{i,t+1}^{\kappa} \quad \forall i \in \mathcal{F}, \forall \kappa \in \{\text{arr}, \text{dep}\}, \forall t \in \mathcal{T} \quad (3)$$

$$w_{i1}^{\kappa} = 1 \quad \forall i \in \mathcal{F}, \forall \kappa \in \{\text{arr}, \text{dep}\} \quad (4)$$

$$\sum_{t \in \mathcal{T}} (w_{it}^{\kappa} - S_{it}^{\kappa}) = u_i \quad \forall i \in \mathcal{F}, \forall \kappa \in \{\text{arr}, \text{dep}\} \quad (5)$$

$$\sum_{t \in \mathcal{T}} (w_{jt}^{\text{dep}} - w_{it}^{\text{arr}}) \geq t_{ij}^{\min} \quad \forall (i, j) \in \mathcal{C} \quad (6)$$

$$\sum_{t \in \mathcal{T}} (w_{jt}^{\text{dep}} - w_{it}^{\text{arr}}) \leq t_{ij}^{\max} \quad \forall (i, j) \in \mathcal{C} \quad (7)$$

$$\sum_{i \in \mathcal{F}^{\kappa}} (w_{it}^{\kappa} - w_{i,t+1}^{\kappa}) = \lambda_t^{\kappa} \quad \forall t \in \mathcal{T}, \forall \kappa \in \{\text{arr}, \text{dep}\} \quad (8)$$

Constraint (3) ensures that w^{arr} and w^{dep} are non-increasing in t . Constraint (4) ensures that no flight is eliminated. Constraint (5) defines flight displacement as the difference between rescheduled and original scheduled times, and ensures that the scheduled block-times are left unchanged. Con-

straints (6) and (7) maintain connection times within the specified ranges. Constraint (8) defines the aggregate schedule of flights (i.e., the number of scheduled arrivals and departures per time period). We summarize next the model of airport congestion that quantifies arrival and departure queue lengths as a function of the schedule of flights (i.e., of λ_t^{arr} and λ_t^{dep}).

Arrival and departure queues are quantified by means of stochastic $M(t)/E_3(t)/1$ queuing systems, i.e., the demand processes are modeled as time-varying Poisson processes, and the service processes are modeled as time-varying Erlang processes of order 3. For each period t , the arrival and departure demand rates are determined by flight schedules, i.e., they are equal to λ_t^{arr} and λ_t^{dep} , respectively. Service rates are constrained by airport capacity. To capture the endogeneity of the service rates with respect to airport capacity and flight schedules, a control of capacity utilization procedures is integrated into the Stochastic Queuing Model of congestion. It is formulated as a finite-horizon Dynamic Programming model, that minimizes congestion costs for a given schedule of flights (Jacquillat et al., 2016). At the beginning of each 15-minute period, the control selects the runway configuration and the balance of arrival and departure service rates for that period, under capacity constraints, as a function of observed arrival and departure queue lengths, the runway configuration in use, and wind and weather conditions. The combination of the Stochastic Queuing Model and the Dynamic Programming model of capacity utilization provides an integrated model of airport congestion that is computationally efficient and that approximates well the magnitude and dynamics of delays at busy US airports (Jacquillat and Odoni, 2015b). This quantifies, in turn, the relationship (denoted by q) between flight schedules and flight delays, represented as follows (where A_t and D_t denote the random variables that represent the arrival and departure queue lengths at the end of period t):

$$q : (\lambda_1^{\text{arr}}, \dots, \lambda_T^{\text{arr}}, \lambda_1^{\text{dep}}, \dots, \lambda_T^{\text{dep}}) \mapsto (A_1, \dots, A_T, D_1, \dots, D_T) \quad (9)$$

Based on this relationship, the ICUSM aims to ensure that, at any time of the day, the expected arrival and departure queue lengths do not exceed the prespecified limits, denoted by A_{MAX} and D_{MAX} , respectively (Constraints (10) and (11) below).

$$E(A_t) \leq A_{\text{MAX}} \quad \forall t \in \mathcal{T} \quad (10)$$

$$E(D_t) \leq D_{\text{MAX}} \quad \forall t \in \mathcal{T} \quad (11)$$

However, these constraints cannot be directly formulated into the Integer Programming scheduling model described above, as the queuing dynamics (Equation (9)) depend nonlinearly on the schedule of flights, hence on the model's decision variables. The solution of the ICUSM therefore relies on an algorithm that iterates between the Integer Programming model of scheduling interventions, the Dynamic Programming model of capacity utilization, and the Stochastic Queuing Model of airport congestion outlined above, until it converges to the optimal value of the schedule

displacement. For any given expected queue length targets, this algorithm terminates in 10-15 iterations and in 90 minutes to several hours on a modern computer, depending on model parameters. For more details, we refer the reader to Jacquillat and Odoni (2015a).

2.2. Inter-airline Equity Concerns

The ICUSM provides a modeling framework for optimizing congestion-mitigating scheduling interventions. Its implementation quantifies the trade-off between schedule displacement (i.e., δ and Δ_0) and peak expected queue length limits (i.e., A_{MAX} and D_{MAX}). However, it does not account for inter-airline equity considerations. Its two-stage lexicographic formulation (characterized by Equations (1) and (2)) ensures equity at the flight level, i.e., no flight is disproportionately displaced. However, it does *not* ensure equity at the airline level. In turn, its solution may penalize one airline (or a small subset of airlines) disproportionately.

To illustrate this, Table 1 shows an example with 18 flights scheduled by two airlines over a one-hour interval, with twice as many flights scheduled by Airline 1 (12 flights) as by Airline 2 (6 flights). We consider simple on-time performance constraints that impose that no more than five flights (arrivals and departures) can be scheduled during any 15-minute period. The original schedule indicates the preferred schedule of each flight, as requested by the airlines. A total of 6 flights (3 flights between 8:00 and 8:14, and 3 flights between 8:30 and 8:44) need to be rescheduled to comply with the limit of five flights per period. Schedule 1 provides a solution where 6 flights from Airline 1 are rescheduled. This solution is clearly inequitable, as it assigns all the rescheduling to one airline, and leaves the schedule of the other airline unchanged. In contrast, Schedule 2 provides a solution where 3 flights from Airline 1 and 3 flights from Airline 2 are rescheduled. This solution does not appear equitable either, as it assigns a similar displacement to the two airlines, even though Airline 1 has more flights scheduled at the airport than Airline 2. It thus imposes a greater per-flight displacement to the schedule of Airline 2 than to that of Airline 1. Schedule 3, then, provides an equitable solution that displaces exactly twice as many flights from Airline 1 (4 flights) as from Airline 2 (2 flights). The modeling architecture presented in the next section formalizes the measurement of inter-airline equity and integrates it into the optimization model for airport scheduling interventions.

3. Multi-criteria Modeling Architecture

We now present our Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E). The model structure, the decision variables, and the scheduling and network connectivity constraints are identical to those in the ICUSM, but the main difference lies in the objectives of scheduling interventions. In addition to the notations introduced in Section 2, we partition the set of flights scheduled at airport Π , i.e., $\mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$, into subsets scheduled by the

Table 1: Example of inequitable and equitable scheduling interventions

Movement	Airline	Original schedule	Schedule 1	Schedule 2	Schedule 3
Arrival	Airline 1	8:00	<u>8:15</u>	<u>8:15</u>	<u>8:15</u>
Departure	Airline 1	8:00	8:00	8:00	8:00
Arrival	Airline 2	8:00	8:00	8:00	8:00
Departure	Airline 1	8:05	<u>8:20</u>	8:05	8:05
Arrival	Airline 2	8:05	8:05	<u>8:20</u>	<u>8:20</u>
Departure	Airline 1	8:10	<u>8:25</u>	<u>8:25</u>	<u>8:25</u>
Arrival	Airline 1	8:10	8:10	8:10	8:10
Departure	Airline 2	8:10	8:10	8:10	8:10
Arrival	Airline 1	8:15	8:15	8:15	8:15
Arrival	Airline 1	8:30	<u>8:45</u>	8:30	<u>8:45</u>
Departure	Airline 1	8:30	8:30	8:30	8:30
Arrival	Airline 2	8:30	8:30	<u>8:45</u>	8:30
Departure	Airline 1	8:35	<u>8:50</u>	<u>8:50</u>	<u>8:50</u>
Arrival	Airline 1	8:35	8:35	8:35	8:35
Departure	Airline 1	8:40	8:40	8:40	8:40
Arrival	Airline 1	8:40	<u>8:55</u>	8:40	8:40
Departure	Airline 2	8:40	8:40	<u>8:55</u>	<u>8:55</u>
Departure	Airline 2	8:45	8:45	8:45	8:45

different airlines.

$$\begin{aligned}
\mathcal{A} &= \text{set of airlines, indexed by } \{1, \dots, A\} \\
\mathcal{F}_a &= \text{set of flights scheduled by airline } a \text{ at airport } \Pi
\end{aligned}$$

With these notations, we have: $\mathcal{F}_{a_1} \cap \mathcal{F}_{a_2} = \emptyset, \forall a_1, a_2 \in \mathcal{A}, a_1 \neq a_2$ and $\cup_{a \in \mathcal{A}} \mathcal{F}_a = \mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$.

We also introduce parameters $v_i, \forall i \in \mathcal{F}$ to characterize “flight valuations”, reflecting airlines’ preferences regarding which flights to reschedule. Flights with lower valuations can be thought of as less “costly” to reschedule, or as the flights that exhibit more timetabling flexibility. Note that the current setting where “a flight is a flight” is a special case, where $v_i = 1, \forall i \in \mathcal{F}$. Even though the valuations v_i are not available to the central decision-makers in current mechanisms for airport scheduling interventions, they could be considered in future extensions of these mechanisms. For instance, they could be the result of non-monetary processes that would allow the airlines to indicate their preferences through ranking or credit allocation. Alternatively, they could result from an auction-based mechanism where airlines would submit a bid for each flight i , and pay an access fee that is discounted by a fixed percentage for each period of displacement (e.g., if an airline bids x for an 8:00 flight, it would pay an access fee equal to x if it is scheduled at 8:00, $(1 - \alpha)x$ if it is scheduled at 7:45 or 8:15, $(1 - 2\alpha)x$ if it is scheduled at 7:30 or 8:30, etc., where $\alpha < 1$). This is similar to mechanisms proposed in energy (Newbery, 2003; Stern and Turvey, 2003), railway transportation (Pena-Alcaraz, 2015) or telecommunications markets (Hoffman, 2010). While the design of such mechanisms is beyond the scope of this paper, our modeling approach incorporates

inter-airline equity objectives in instances with either identical or differentiated flight valuations.

$$v_i = \text{valuation of flight } i, \forall i \in \mathcal{F}$$

3.1. Performance Attributes

We consider the following three performance attributes of scheduling interventions: efficiency, inter-airline equity, and on-time performance. Efficiency and on-time performance extend the notions of schedule displacement and expected queue length limits, respectively, that are considered in the ICUSM, while the notion of equity is added to this framework by us.

Efficiency. This refers to the ability to meet airline scheduling preferences. Since no flight is eliminated, efficiency is measured by the displacement from the schedule of flights requested by the airlines. We consider two efficiency objectives. First, we define *min-max efficiency* as the largest displacement sustained by any flight. As in Section 2, we denote it by δ . Second, we define *weighted efficiency* as the weighted sum of schedule displacements sustained by all flights, and we denote it by Δ . Weighted efficiency generalizes the total displacement Δ_0 considered in the ICUSM in a way that accounts for flight valuations. Directionally, maximizing efficiency involves minimizing δ and/or Δ .

$$\delta = \max_{i \in \mathcal{F}} |u_i| \implies \min \delta \quad (12)$$

$$\Delta = \sum_{i \in \mathcal{F}} v_i |u_i| \implies \min \Delta \quad (13)$$

Inter-airline Equity. This refers to the ability to balance schedule displacement fairly among the airlines. We describe each airline's disutility as the weighted average of per-flight displacements, denoted by σ_a . Perfect equity is achieved when the weighted sum of displacements borne by any airline is proportional to its number of flights scheduled at airport Π , i.e., when the weighted average of per-flight displacements is the same for all airlines. In order to maximize inter-airline equity, we minimize airline disutilities lexicographically, i.e., we first minimize the largest airline disutility, then the second-largest, etc.

$$\sigma_a = \frac{1}{|\mathcal{F}_a|} \sum_{i \in \mathcal{F}_a} v_i |u_i|, \forall a \in \mathcal{A} \implies \text{lex min } \sigma \quad (14)$$

We denote the largest airline disutility by Φ :

$$\Phi = \max_{a \in \mathcal{A}} \sigma_a \quad (15)$$

This lexicographic approach to inter-airline equity maximization extends the min-max formulation proposed by Bertsimas et al. (2011, 2012), which maximizes the largest utility, i.e., mini-

mizes the largest weighted average per-flight displacements borne by any airline in this case. Note that other equity formulations could be considered. For instance, Bertsimas et al. (2012) also propose a broader class of welfare functions $W_\alpha(u) = \sum_{i=1}^n \frac{u_i^{1-\alpha}}{1-\alpha}$, for each $\alpha \geq 0, \alpha \neq 1$, and $W_1(u) = \sum_{i=1}^n \log(u_i)$, where u_i denotes the utility of agent $i = 1, \dots, n$. This establishes a continuum between the efficiency-maximizing outcome ($\alpha \rightarrow 0$) and the mix-max equity scheme considered in this paper ($\alpha \rightarrow \infty$). Other formulations could also be based on the *dispersion* of agent utilities (e.g., by minimizing functions like $\sum_{i,j=1}^n |u_i - u_j|$, $\max_{i,j=1,\dots,n} |u_i - u_j|$ or $\sum_{i=1}^n (u_i - \bar{u})^2$, where \bar{u} denotes the average utility across all agents i (Leclerc et al., 2012)). The choice of a lexicographic inter-airline equity maximization is motivated by three main factors. From a theoretical standpoint, it extends the solution of Kalai and Smorodinsky (1975) for the two-player bargaining problem, which is the only solution that satisfies the axioms of Pareto optimality (i.e., no other solution can improve the utility of one airline, without reducing that of another airline), symmetry (i.e., all airlines are treated equivalently), affine invariance (i.e., it does not depend on the choice of equivalent utility representations), and monotonicity (i.e., if the total displacement is reduced, then the utility of any of the airlines should not decrease)—see Bertsimas et al. (2011) for a more detailed discussion. Note that the condition of Pareto optimality is not satisfied by dispersion-based metrics, which would favor, for instance, a displacement of 100 flights for Airline 1 and 100 flights for Airline 2 over a displacement of 100 flights of Airline 1 and 90 flights of Airline 2 (for two symmetric airlines). From a computational standpoint, the lexicographic approach considered in this paper can be formulated using linear mixed-integer programming models, which ensures far greater computational efficiency than alternative approaches based on non-linear objectives. From a practical standpoint, it has broad application in multi-criteria decision-making problems, for instance in the context of resource allocation (Klein et al., 1992; Luss, 1999), telecommunications (Ogryczak et al., 2005), and electricity (Sun, 2011) with significant equity gains at moderate losses in efficiency.

On-time performance. This refers to the ability to mitigate airport congestion. We quantify on-time performance by a non-decreasing function of the arrival and departure queue lengths A_t and D_t (which depend on the schedule of flights according to Equation (9)), denoted by $g(A_1, \dots, A_T, D_1, \dots, D_T)$. Examples of such functions include the peak expected arrival and departure queue lengths, the total delay experienced over a day of operations, the 95th percentile of the peak arrival and departure queue lengths, etc. Maximizing on-time performance involves minimizing the function g .

$$\min \{g(A_1, \dots, A_T, D_1, \dots, D_T)\} \quad (16)$$

The optimization of scheduling interventions is a multi-objective optimization problem. First, each of these three performance attributes comprises several dimensions (e.g., minimizing min-max efficiency vs. weighted efficiency; minimizing the largest airline disutility vs. variations in airlines' utilities for equity; minimizing arrival vs. departure delays for on-time performance). Moreover,

there exists a trade-off between efficiency and on-time performance, quantified by the ICUSM: the larger the schedule displacement, the larger the potential delay reductions (up to a limit). Finally, there may be, for given on-time performance objectives, a trade-off between efficiency and equity.

3.2. Lexicographic Modeling Approach

We characterize the trade space between efficiency, equity, and on-time performance in airport scheduling interventions. In order to provide a transparent and optimal characterization of this trade space, we aim to find its Pareto frontier, i.e., the set of solutions such that no other feasible solution could improve at least one of the three objectives without worsening the others. This representation of the trade space is flexible enough to be used by system managers and policy makers to select the most appropriate level of compromise between these objectives. To this end, we develop a lexicographic optimization approach that (i) fixes on-time performance targets; (ii) maximizes efficiency under on-time performance targets; and (iii) maximizes equity under on-time performance and efficiency targets. This lexicographic structure is also consistent with industry practice. For instance, air traffic flow management typically aims, first, to maximize system safety, then to minimize total delays in the system, then the delay costs borne by various airlines, etc.

First, we quantify on-time performance by the peak expected arrival and departure queue lengths, i.e. $g(A_1, \dots, A_T, D_1, \dots, D_T) = (\max_{t \in \mathcal{T}} E(A_t), \max_{t \in \mathcal{T}} E(D_t))$. It is motivated by the objective of controlling the largest delays experienced over the day. Corresponding on-time performance constraints are identical to those in the ICUSM (Constraints (10) and (11)). We then aim to find the “best” schedule (in terms of efficiency and equity) that meets these constraints.

Second, we determine the schedule of flights that maximizes efficiency, subject to scheduling constraints, network connectivity constraints, and on-time performance constraints. We formulate the efficiency-maximizing problem by lexicographically maximizing, first, min-max efficiency δ , and, second, weighted efficiency Δ . This is motivated by the objective of avoiding large flight displacements, and consistent with the literature on this topic (Pyrgiotis and Odoni, 2016; Jacquillat and Odoni, 2015a). This is expressed in Problems **P1** and **P2** described below:

P1. We minimize min-max efficiency metric δ , subject to scheduling, network connectivity and on-time performance constraints. We denote by δ^* its optimal value.

$$\begin{aligned} \min \quad & \delta \text{ (Equation (12))} \\ \text{s.t.} \quad & \text{Scheduling and network connectivity constraints: (3) to (8)} \\ & \text{On-time performance constraints: (9) to (11)} \end{aligned}$$

P2. We minimize weighted efficiency metric Δ , subject to scheduling, network connectivity and on-time performance constraints, and subject to the constraint that no flight may be displaced by

more than δ^* . We denote by Δ^* its optimal value.

$$\begin{aligned}
& \min \quad \Delta \text{ (Equation (13))} \\
& \text{s.t.} \quad \text{Scheduling and network connectivity constraints: (3) to (8)} \\
& \quad \text{On-time performance constraints: (9) to (11)} \\
& \quad \text{Min-max efficiency objectives: } |u_i| \leq \delta^*, \forall i \in \mathcal{F}
\end{aligned}$$

Third, we maximize inter-airline equity, subject to scheduling constraints, network connectivity constraints, on-time performance constraints, and efficiency targets. This is formulated in the class of problems **P3**(ρ) described below:

P3(ρ). We fix efficiency targets, and we lexicographically minimize airline disutilities, subject to scheduling, network connectivity, on-time performance, and efficiency constraints. We characterize the trade space between efficiency and equity by varying the efficiency target. Specifically, we impose that min-max efficiency must be optimal (i.e., no flight may be rescheduled by more than δ^*) and we denote by $\rho \in [0, \infty)$ the relative loss in weighted efficiency that is allowed (i.e., the weighted displacement must not exceed $(1+\rho)\Delta^*$). When $\rho = \infty$, we only maximize equity (without any weighted efficiency consideration). When $\rho = 0$, we maximize equity, under optimal min-max and optimal weighted efficiency.

$$\begin{aligned}
& \text{lex min} \quad \sigma \text{ (Equation (14))} \\
& \text{s.t.} \quad \text{Scheduling and network connectivity constraints: (3) to (8)} \\
& \quad \text{On-time performance constraints: (9) to (11)} \\
& \quad \text{Min-max efficiency objectives: } |u_i| \leq \delta^*, \forall i \in \mathcal{F} \\
& \quad \text{Weighted efficiency objectives: } \sum_{i \in \mathcal{F}} v_i |u_i| \leq (1 + \rho) \Delta^*
\end{aligned}$$

Problems **P1**, **P2**, and **P3**(ρ) together determine the Pareto frontier of the trade space between efficiency, equity, and on-time performance. First, variations in the on-time performance targets A_{MAX} and D_{MAX} quantify the trade-off between the costs of scheduling interventions (in terms of inefficiency and inequity) and delay reductions. Second, for any on-time performance targets A_{MAX} and D_{MAX} , varying the parameter ρ quantifies the potential trade-off between weighted efficiency and inter-airline equity (under optimal min-max efficiency).

We denote by $\sigma^*(\rho)$ the equity-maximizing vector of airline per-flight displacements, as a function of ρ , and $\Phi^*(\rho) = \max_{a \in \mathcal{A}} \sigma_a^*(\rho)$. We denote by Δ^{eq} the smallest equity-maximizing value of Δ , and by ρ^* the minimum loss in weighted efficiency required to attain optimal equity (i.e., $\Delta^{\text{eq}} = (1 + \rho^*)\Delta^*$). With these notations, the “price of efficiency” and the “price of equity” will be characterized by $P_{\text{eff}} = \frac{\Phi^*(0) - \Phi^*(\infty)}{\Phi^*(\infty)}$, and by $P_{\text{eq}} = \frac{\Delta^{\text{eq}} - \Delta^*}{\Delta^*} = \rho^*$, respectively.

Figure 1 illustrates our approach to maximizing weighted efficiency and inter-airline equity, for given on-time performance targets A_{MAX} and D_{MAX} , and the optimal value of min-max efficiency δ^* . Specifically, it shows hypothetical variations in three airlines' disutilities (σ_1 , σ_2 , and σ_3) as a function of the weighted efficiency target $\Delta = (1 + \rho)\Delta^*$. By construction, the region on the left side of Δ^* is infeasible, i.e., the weighted schedule displacement must be at least Δ^* . Moreover, the largest airline disutility Φ is a non-increasing function of the value of weighted efficiency Δ (i.e., of ρ). Note that the other airlines' utilities (here, σ_2 and σ_3) may increase or decrease as Φ is reduced. As the largest airline disutility Φ attains its optimal value, the second-largest disutility may still be larger than its optimal value. In this case, further increases in ρ may yield further improvements in the lexicographic minimization of airline disutilities. Optimal equity is attained when the largest, second largest, third largest, etc., airline disutilities have all reached their optimal values (i.e., the values that would be obtained without any efficiency consideration, or with $\rho = \infty$). This representation shows the price of efficiency and the price of equity as the relative difference between $\Phi^*(\infty)$ and $\Phi^*(0)$ and between Δ^* and Δ^{eq} , respectively. Note that Figure 1 shows an instance where the order of airline disutilities remains identical for all values of ρ (i.e., in this case, $\sigma_1^*(\rho) > \sigma_2^*(\rho) > \sigma_3^*(\rho), \forall \rho \geq 0$), but this need not be the case (i.e., the curves may intersect).

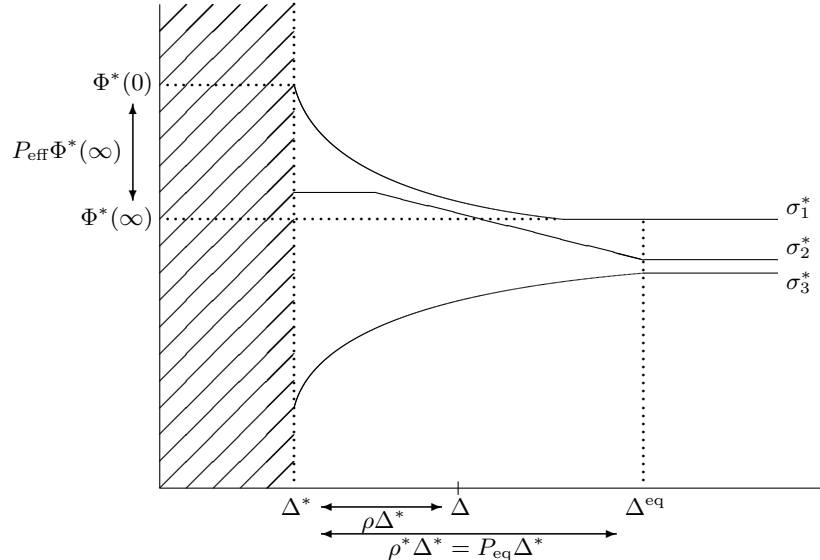


Figure 1: A schematic trade space between weighted efficiency and equity

3.3. Solution Architecture

As discussed in Section 2, the on-time performance constraints are not linear. Solving Problems **P1**, **P2**, and **P3**(ρ) thus requires an iterative solution algorithm. To solve Problem **P1**, we update iteratively a lower bound of the optimal maximum flight displacement δ^* , i.e., we increase the value of the maximum flight displacement from 0 15-minute period to 1 period, then 2 periods, until a feasible schedule that meets the on-time performance targets is found. To solve Problem **P2**,

we iteratively update an upper bound $\overline{\Delta}$ and a lower bound $\underline{\Delta}$ on the optimal weighted efficiency Δ^* , using binary search. At each iteration, we consider a value of $\Delta = \frac{\overline{\Delta} + \underline{\Delta}}{2}$, and we update $\overline{\Delta}$ (respectively $\underline{\Delta}$) to $\frac{\overline{\Delta} + \Delta}{2}$ if the resulting delay estimates meet (respectively do not meet) the on-time performance constraints. We repeat the process until the following stopping criteria is reached: $\frac{\overline{\Delta} - \underline{\Delta}}{\underline{\Delta}} \leq \varepsilon_{\Delta}$. This ensures that the schedule displacement obtained is within ε_{Δ} of the optimal schedule displacement. We use a value of 1% for ε_{Δ} . This algorithm is adopted from Jacquillat and Odoni (2015a).

When solving the equity-maximizing problem (Problem **P3**(ρ)), one candidate approach is to design a similar iterative algorithm. This would consist of iteratively updating lower and upper bounds on the optimal value of each airline disutility σ_a until convergence. However, the computational requirements of such an algorithm prevent it from being applied repeatedly for several airlines, for several values of the parameter ρ , and with different sets of inputs. For this reason, we develop an alternative approach that approximates Problem **P3**(ρ) while ensuring computational tractability.

Specifically, we consider, instead of the on-time performance constraints (Constraints (10) and (11)), scheduling limit constraints (Constraints (17) and (18), defined below). These constraints ensure that, for any period t , the number of scheduled arrivals and departures does not exceed limits denoted by $\widehat{\lambda}_t^{\text{arr}}$ and $\widehat{\lambda}_t^{\text{dep}}$, respectively. We refer to these constraints as “time-dependent schedule limit constraints”.

$$\lambda_t^{\text{arr}} \leq \widehat{\lambda}_t^{\text{arr}}, \forall t \in \mathcal{T} \quad (17)$$

$$\lambda_t^{\text{dep}} \leq \widehat{\lambda}_t^{\text{dep}}, \forall t \in \mathcal{T} \quad (18)$$

The resulting model is formulated below, and we refer to it as $\widehat{\mathbf{P3}}(\rho)$:

lex min σ (Equation (14))

s.t. Scheduling and network connectivity constraints: (3) to (8)

Time-dependent schedule limits constraints: (17) and (18)

Min-max efficiency objectives: $|u_i| \leq \delta^*, \forall i \in \mathcal{F}$

Weighted efficiency objectives: $\sum_{i \in \mathcal{F}} v_i |u_i| \leq (1 + \rho) \Delta^*$

Unlike Problem **P3**(ρ), Problem $\widehat{\mathbf{P3}}(\rho)$ is an Integer Programming model and can be solved directly using a commercial solver. Its solution is substantially faster than that of Problem **P3**(ρ), which required iterating 10-15 times between an Integer Program, a Dynamic Program, and a Stochastic Queuing Model. The main challenge lies in setting appropriate values of the scheduling limits $\widehat{\lambda}_t^{\text{arr}}$ and $\widehat{\lambda}_t^{\text{dep}}$. If set too high, the resulting arrival and departure queue lengths would not

meet the on-time performance targets A_{MAX} and D_{MAX} , respectively. If set too low, they may not minimize the displacement impact on airline schedules of flights. In this paper, we set the scheduling limits $\widehat{\lambda}_t^{\text{arr}}$ and $\widehat{\lambda}_t^{\text{dep}}$ equal to the aggregate schedule (i.e., the vector of the number of scheduled arrivals and departures per time period) obtained by solving Problem **P2**. In other words, we first determine the efficiency-maximizing schedule of flights. We then look for flight schedules that achieve the same aggregate schedule (but not necessarily the same schedule for each individual flight), while yielding a Pareto-optimal solution to the trade-off between weighted efficiency and equity.

By construction, the schedule obtained through this computationally efficient approach meets the delay reduction constraints (10) and (11). On the other hand, Constraints (17) and (18) are more restrictive than Constraints (10) and (11), and may thus yield a sub-optimal solution. Nonetheless, this approach exhibits the following strengths. First, the approach yields improvements in inter-airline equity without sacrificing other objectives, as compared to existing approaches. Second, it starts with the scheduling inputs provided by the airlines to determine the aggregate schedule, which can thus exhibit some peaks and valleys in accordance with airline scheduling preferences and passenger demand. Third, our computational results reported in Section 5 show that it leads to high equity levels. Finally, its reliance on the aggregate schedule of flights (instead of individual flight schedules and expected delay reductions) makes this approach easily communicable and implementable.

Our full solution architecture is shown in Figure 2. It takes as inputs scheduling data, connections data, and flight valuation data, as well as on-time performance targets A_{MAX} and D_{MAX} set by the central decision-maker. First, we successively solve Problems **P1** and **P2**, and we store the optimal efficiency values (δ^* and Δ^*) and the aggregate schedule ($\widehat{\lambda}_t^{\text{arr}}$ and $\widehat{\lambda}_t^{\text{dep}}$). Second, we solve Problems **P3**(ρ) to determine the Pareto frontier of the trade space between weighted efficiency and equity to achieve this aggregate schedule (i.e., $\widehat{\lambda}_t^{\text{arr}}$ and $\widehat{\lambda}_t^{\text{dep}}$). We start by maximizing equity with no weighted efficiency constraint ($\rho = \infty$). We then maximize equity under optimal weighted efficiency ($\rho = 0$), and we relax progressively the weighted efficiency requirements by increasing ρ in increments of 0.001, until optimal equity is reached. We use the following stopping criteria: $\frac{\sigma_a^*(\rho) - \sigma_a^*(\infty)}{\sigma_a^*(\infty)} \leq \varepsilon_\sigma, \forall a \in \mathcal{A}$, i.e., the algorithm terminates when all airlines' disutilities are within ε_σ of their equity-maximizing values. We use here a value of 1% for ε_σ . This algorithm characterizes, for any pair of on-time performance targets A_{MAX} and D_{MAX} , (i) the efficiency-maximizing schedule of flights, and (ii) feasible flight schedules that achieve the same aggregate schedule and yield a set of Pareto-optimal solutions to the trade-off between weighted efficiency and equity.

4. A Theoretical Discussion on Inter-airline Equity

We show that, under some conditions on the type of scheduling interventions or on the scheduling inputs provided by the airlines, efficiency and equity can be jointly optimized. We consider in

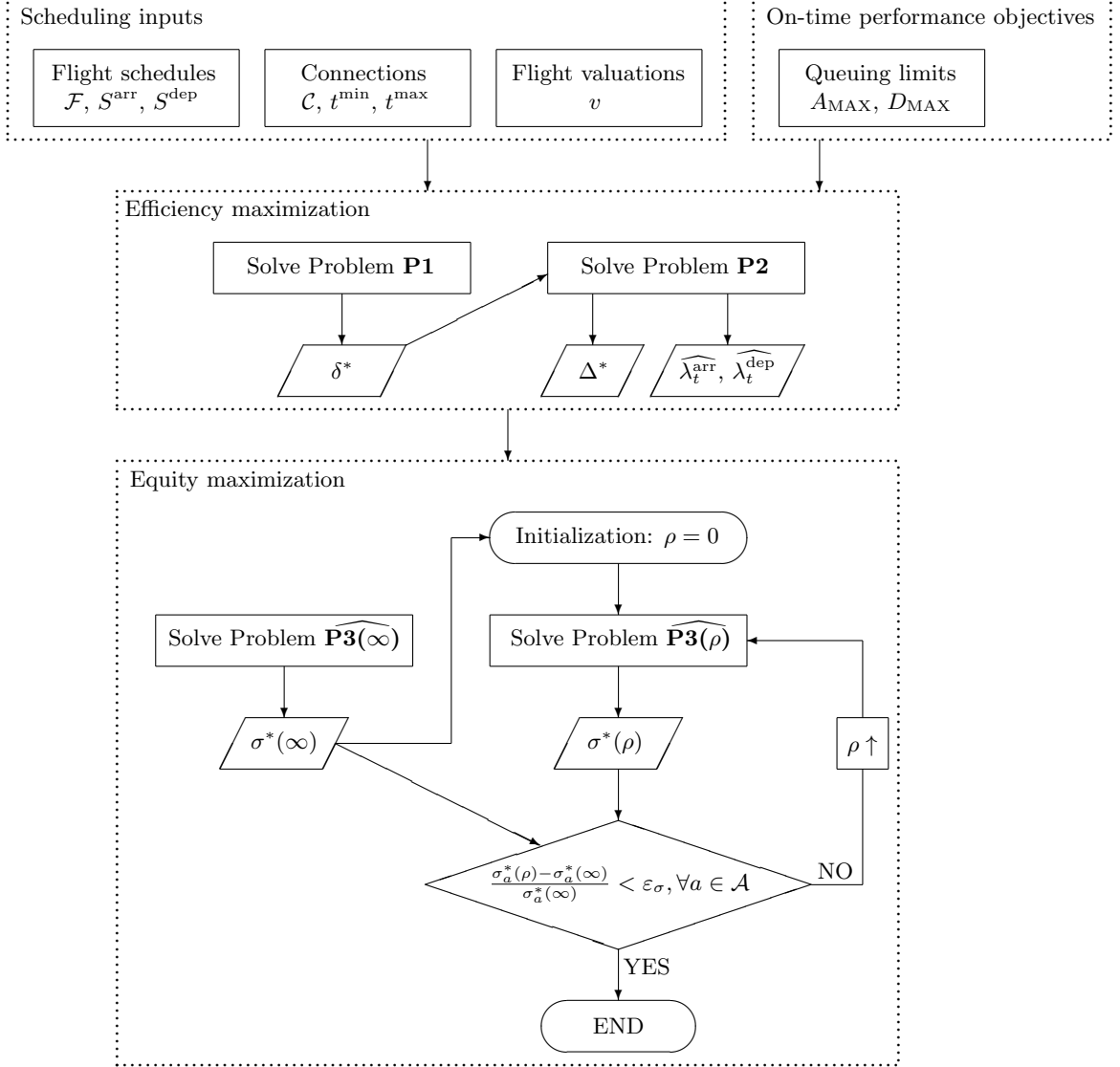


Figure 2: Solution architecture

Section 4.1 the case where no network connections need to be maintained and all flights are equally valued. We first show that efficiency and equity can be jointly optimized if the scheduling interventions involve only reductions in demand (i.e., “accept” or “reject” each flight request). Note that this problem is slightly beyond the scope of the model presented in Section 3, but it guarantees that our major theoretical and computational insights hold for a broader class of scheduling intervention problems than those considered in this paper. We then show that efficiency and equity can also be jointly optimized if the scheduling interventions involve only temporal shifts in demand (i.e., do not eliminate any flights, but may modify their timetabling), if additional scheduling conditions are satisfied. Last, we discuss in Section 4.2 the factors that may violate these conditions, and thus create a trade-off between efficiency and equity in scheduling interventions.

4.1. Cases of Joint Maximization of Efficiency and Equity

We assume in this section that no connections need to be maintained (i.e., $\mathcal{C} = \emptyset$, and therefore the set of relevant flights \mathcal{F} includes only flights scheduled to or from the airport Π under consideration) and that all flights are equally valued (i.e., $v_i = 1, \forall i \in \mathcal{F}$). In the absence of connections, and identical flight valuations, all flights in the set \mathcal{F} can be treated the same irrespective of whether they take-off or land at the airport Π under consideration. So we can simply eliminate the superscripts “arr” and “dep” from all the discussion in this section. We denote by \mathcal{D}_t the set of flights scheduled during period t before the scheduling interventions, i.e., $\mathcal{D}_t = \{i \in \mathcal{F} | S_{it} = 1 \text{ \& } S_{i,t+1} = 0\}$. By convention, we assume that $\mathcal{D}_0 = \mathcal{D}_{T+1} = \emptyset$ and $\hat{\lambda}_0 = \hat{\lambda}_{T+1} = 0$. We also denote the positive part of any number x by $x^+ = \max(x, 0)$.

We first consider the case where the scheduling interventions involve only reductions in demand, i.e., are restricted to accepting or rejecting individual flight scheduling requests, without changing the timetabling of these flights. We define the corresponding problems of efficiency maximization (EFF-AR) and equity maximization (EQ-AR). The decision variable y_i is equal to 1 if flight $i \in \mathcal{F}$ is rejected, or 0 otherwise. Problem (EFF-AR) minimizes the number of flights rejected and Problem (EQ-AR) lexicographically minimizes the proportion of individual airlines’ flights being rejected, subject to the constraint that no more than $\hat{\lambda}_t$ flights may be scheduled during any period t .

$$\begin{array}{ll} \min & \sum_{i \in \mathcal{F}} y_i \quad (\text{EFF-AR}) \\ \text{s.t.} & \sum_{i \in \mathcal{D}_t} (1 - y_i) \leq \hat{\lambda}_t, \forall t \in \mathcal{T} \\ & y_i \in \{0, 1\}, \forall i \in \mathcal{F} \end{array} \quad \text{lex min} \quad \begin{array}{ll} \left(\frac{1}{|\mathcal{F}_a|} \sum_{i \in \mathcal{F}_a} y_i \right)_{a \in \mathcal{A}} & (\text{EQ-AR}) \\ \sum_{i \in \mathcal{D}_t} (1 - y_i) \leq \hat{\lambda}_t, \forall t \in \mathcal{T} & \\ y_i \in \{0, 1\}, \forall i \in \mathcal{F} & \end{array}$$

Proposition 1 shows that, in the case where scheduling interventions are based purely on reductions in demand, efficiency and equity can be jointly maximized. The efficient solution is to reject flights that are in excess of the limit, in each of the time periods. The result below shows that this can be done in a way that also maximizes equity

Proposition 1. *There exists a solution that simultaneously solves (EFF-AR) and (EQ-AR).*

Proof. The constraint in both (EFF-AR) and (EQ-AR) can be re-written as: $\sum_{i \in \mathcal{D}_t} y_i \geq (|\mathcal{D}_t| - \hat{\lambda}_t)^+, \forall t \in \mathcal{T}$. Therefore, any solution that involves rejecting exactly $(|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights in each period t is an optimal solution of (EFF-AR), and the optimal objective function value is $\sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+$.

Let us now consider a solution of (EQ-AR) and assume that $\sum_{i \in \mathcal{F}} y_i > \sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+$, i.e., $\sum_{t \in \mathcal{T}} \left(\sum_{i \in \mathcal{D}_t} y_i - (|\mathcal{D}_t| - \hat{\lambda}_t)^+ \right) > 0$. We denote by $\mathcal{J} = \left\{ t \in \mathcal{T} | \sum_{i \in \mathcal{D}_t} y_i > (|\mathcal{D}_t| - \hat{\lambda}_t)^+ \right\}$. For each $t \in \mathcal{J}$, we select a subset $\mathcal{K}_t \subseteq \{i \in \mathcal{D}_t | y_i = 1\}$, that contains exactly $\left(\sum_{i \in \mathcal{D}_t} y_i - (|\mathcal{D}_t| - \hat{\lambda}_t)^+ \right)$

elements. We now define a solution \bar{y} as follows:

$$\bar{y}_i = \begin{cases} y_i, & \text{if } i \notin \cup_{t \in \mathcal{T}} \mathcal{K}_t \\ 0, & \text{if } i \in \cup_{t \in \mathcal{T}} \mathcal{K}_t \end{cases}$$

We now have, for all periods $t \in \mathcal{T}$, $\sum_{i \in \mathcal{D}_t} \bar{y}_i = (|\mathcal{D}_t| - \hat{\lambda}_t)^+$. Indeed, if $t \notin \mathcal{J}$, then $\sum_{i \in \mathcal{D}_t} \bar{y}_i = \sum_{i \in \mathcal{D}_t} y_i = (|\mathcal{D}_t| - \hat{\lambda}_t)^+$. If $t \in \mathcal{J}$, then $\sum_{i \in \mathcal{D}_t} \bar{y}_i = \sum_{i \in \mathcal{D}_t} y_i - |\mathcal{K}_t| = (|\mathcal{D}_t| - \hat{\lambda}_t)^+$. As a result, the solution \bar{y} is feasible and solves (EFF-AR). Moreover, $\bar{y}_i \leq y_i, \forall i \in \mathcal{F}$, so $\sum_{i \in \mathcal{F}_a} \bar{y}_i \leq \sum_{i \in \mathcal{F}_a} y_i, \forall a \in \mathcal{A}$, so \bar{y} also solves (EQ-AR). \square

We now turn to the case where the scheduling interventions involve temporal shifts in demand—the case considered in the rest of this paper, which is more consistent with current practice at busy airports and with recent research results (see Section 1). We define the following problems of efficiency maximization (EFF) and equity maximization (EQ), subject to the constraint that no more than $\hat{\lambda}_t$ flights may be scheduled during any period t .

$$\begin{array}{ll} \min & \sum_{i \in \mathcal{F}} |u_i| \quad (\text{EFF}) \\ \text{s.t.} & w_{it} \geq w_{i,t+1}, \forall i \in \mathcal{F}, \forall t \in \mathcal{T} \\ & w_{i1} = 1, \forall i \in \mathcal{F} \\ & \sum_{t \in \mathcal{T}} (w_{it} - S_{it}) = u_i, \forall i \in \mathcal{F} \\ & \sum_{i \in \mathcal{F}} (w_{it} - w_{i,t+1}) \leq \hat{\lambda}_t, \forall t \in \mathcal{T} \\ & |u_i| \leq \delta^*, \forall i \in \mathcal{F} \end{array} \quad \begin{array}{ll} \text{lex min} & \left(\frac{1}{|\mathcal{F}_a|} \sum_{i \in \mathcal{F}_a} |u_i| \right)_{a \in \mathcal{A}} \quad (\text{EQ}) \\ \text{s.t.} & w_{it} \geq w_{i,t+1}, \forall i \in \mathcal{F}, \forall t \in \mathcal{T} \\ & w_{i1} = 1, \forall i \in \mathcal{F} \\ & \sum_{t \in \mathcal{T}} (w_{it} - S_{it}) = u_i, \forall i \in \mathcal{F} \\ & \sum_{i \in \mathcal{F}} (w_{it} - w_{i,t+1}) \leq \hat{\lambda}_t, \forall t \in \mathcal{T} \\ & |u_i| \leq \delta^*, \forall i \in \mathcal{F} \end{array}$$

Proposition 2 shows that efficiency and equity can be jointly maximized if the number of flights scheduled over any set of three consecutive time periods is lower than the total number of flights that can be scheduled over the same three periods. In that case, the scheduling interventions in the periods with more than $\hat{\lambda}_t$ flights scheduled can be treated independently and the problem can thus be reduced to a series of one-period problems.

Proposition 2. *If $\sum_{l=t-1}^{t+1} |\mathcal{D}_l| \leq \sum_{l=t-1}^{t+1} \hat{\lambda}_l, \forall t \in \mathcal{T}$, then there exists a solution that simultaneously solves (EFF) and (EQ).*

Proof. Any feasible solution has to displace at least $(|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights in every period t , so $\Delta^* \geq \sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+$. We first construct a feasible solution that reschedules exactly $\sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights. To do so, we reschedule flights recursively from $t = 1, \dots, T$, first to the preceding period (i.e., period $t - 1$), up to capacity, and then to the following period (i.e., period $t + 1$). Specifically,

we select a subset $\mathcal{K}_t^- \subset \mathcal{D}_t$ and then a subset $\mathcal{K}_t^+ \subset \mathcal{D}_t \setminus \mathcal{K}_t^-$ such that:

$$\begin{aligned} |\mathcal{K}_t^-| &= \min \left\{ \hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|), (|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+|)^+ \right\} \\ |\mathcal{K}_t^+| &= (|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-|)^+ \end{aligned}$$

The subsets \mathcal{K}_t^- and \mathcal{K}_t^+ are not uniquely determined, but we can choose any subsets of \mathcal{D}_t that satisfy these properties. We define u^{eff} as follows: $u_i^{\text{eff}} = -1, \forall i \in \mathcal{K}_t^-$ and $u_i^{\text{eff}} = +1, \forall i \in \mathcal{K}_t^+$. We define w^{eff} accordingly (based on the constraints of (EFF)). According to Lemma 2 in Appendix 1, $(w^{\text{eff}}, u^{\text{eff}})$ is a feasible and optimal solution of (EFF). It reschedules exactly $(|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights from any period t by 1 period each, so $\delta^* = 1$ (unless $|\mathcal{D}_t| \leq \hat{\lambda}_t, \forall t \in \mathcal{T}$, in which case $\delta^* = 0$), and $\Delta^* = \sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+$.

We now denote by $(w^{\text{eq}}, u^{\text{eq}})$ an optimal solution of (EQ). If $\sum_{i \in \mathcal{F}} |u_i^{\text{eq}}| = \Delta^*$, then u^{eq} also solves (EFF). We now assume that $\sum_{i \in \mathcal{F}} |u_i^{\text{eq}}| > \Delta^*$. For each $t \in \mathcal{T}$, we define the following set: $\mathcal{I}_t = \{i \in \mathcal{D}_t \mid |u_i^{\text{eq}}| = 1\}$. We have $|\mathcal{I}_t| \geq (|\mathcal{D}_t| - \hat{\lambda}_t)^+, \forall t \in \mathcal{T}$ (otherwise, u^{eq} would not be a feasible solution of (EQ)). We can construct a set $\mathcal{J}_t \subseteq \mathcal{I}_t$ such that $|\mathcal{J}_t| = (|\mathcal{D}_t| - \hat{\lambda}_t)^+$ for all $t \in \mathcal{T}$. As with \mathcal{K}_t^- and \mathcal{K}_t^+ earlier, \mathcal{J}_t is not uniquely determined, but we can choose any subset of \mathcal{I}_t that satisfies this property. Let \mathcal{J} be defined by $\mathcal{J} = \cup_{t \in \mathcal{T}} \mathcal{J}_t$. We construct a solution u^* as follows: $u_i^* = 0, \forall i \notin \mathcal{J}$ and $u_i^* = u_i^{\text{eq}}, \forall i \in \mathcal{J}$. We define w^* accordingly. By construction, for each period t such that $|\mathcal{D}_t| > \hat{\lambda}_t$, this solution displaces exactly $|\mathcal{D}_t| - \hat{\lambda}_t$ flights, so $\sum_{i \in \mathcal{F}} (w_{it}^* - w_{i,t+1}^*) = \hat{\lambda}_t$. Moreover, the number of flights rescheduled to the preceding and following time periods is smaller than under solution $(w^{\text{eq}}, u^{\text{eq}})$ (i.e., $\sum_{i \in \mathcal{F}} (w_{i,t-1}^* - w_{i,t}^*) \leq \sum_{i \in \mathcal{F}} (w_{i,t-1}^{\text{eq}} - w_{i,t}^{\text{eq}})$ and $\sum_{i \in \mathcal{F}} (w_{i,t+1}^* - w_{i,t+2}^*) \leq \sum_{i \in \mathcal{F}} (w_{i,t+1}^{\text{eq}} - w_{i,t+2}^{\text{eq}})$). Therefore, (w^*, u^*) is a feasible solution of (EFF) and (EQ). Moreover, it satisfies: $\sum_{i \in \mathcal{F}} |u_i^*| = \Delta^*$, and $|u_i^*| \leq |u_i^{\text{eq}}|, \forall i \in \mathcal{F}$. Therefore, u^* solves (EFF) and (EQ). \square

Proposition 3 shows that efficiency and equity can be jointly maximized if each airline's share of flights is identical across all periods. Specifically, we assume that the number of flights scheduled by each airline a during each period t is the product of an airline-related factor α_a and a period-related factor β_t . In that case, there is significant flexibility in terms of the airlines whose flights should be rescheduled, which enables equity-maximization at no efficiency loss. For simplicity, we focus on the case of $\delta^* = 1$ period, which is also the most common case encountered with real-world data (see Section 5).

Proposition 3. *If $\delta^* = 1$ period and there exist integers $(\alpha_a)_{a \in \mathcal{A}}$ and $(\beta_t)_{t \in \mathcal{T}}$ such that $|\mathcal{D}_t \cap \mathcal{F}_a| = \alpha_a \beta_t, \forall a \in \mathcal{A}, t \in \mathcal{T}$, then there exists a solution that simultaneously solves (EFF) and (EQ).*

Proof. We consider an optimal solution of (EFF), which we denote by $(w^{\text{eff}}, u^{\text{eff}})$. We denote by

X_t^+ (resp. X_t^-) the number of flights that, under solution $(w^{\text{eff}}, u^{\text{eff}})$, are displaced from period t to period $t+1$ (resp. $t-1$), i.e., $X_t^+ = |\{i \in \mathcal{D}_t | u_i^{\text{eff}} = +1\}|$ (resp. $X_t^- = |\{i \in \mathcal{D}_t | u_i^{\text{eff}} = -1\}|$). We also denote by X_t the total number of flights displaced from period t , i.e., $X_t = X_t^- + X_t^+, \forall t \in \mathcal{T}$. The optimal objective value function of (EFF) is $\Delta^* = \sum_{i=1}^n |u_i^{\text{eff}}| = \sum_{t \in \mathcal{T}} (X_t^- + X_t^+) = \sum_{t \in \mathcal{T}} X_t$. We aim to construct a solution (w^*, u^*) that is feasible, efficient and equitable.

A sufficient condition for (w^*, u^*) to be feasible and efficient is to ensure that, for each period t , the number of flights rescheduled to $t-1$ and to $t+1$, respectively, under solution (w^*, u^*) is equal to that under solution $(w^{\text{eff}}, u^{\text{eff}})$ for every period t , that is $|\{i \in \mathcal{D}_t | u_i^* = -1\}| = X_t^-$ and $|\{i \in \mathcal{D}_t | u_i^* = +1\}| = X_t^+, \forall t \in \mathcal{T}$. Indeed, if this condition is satisfied, the aggregate schedule is identical under solutions $(w^{\text{eff}}, u^{\text{eff}})$ and (w^*, u^*) (i.e., $\sum_{i \in \mathcal{F}} (w_{it}^{\text{eff}} - w_{i,t+1}^{\text{eff}}) = \sum_{i \in \mathcal{F}} (w_{it}^* - w_{i,t+1}^*), \forall t \in \mathcal{T}$), so solution (w^*, u^*) is feasible. Moreover, under this condition: $\sum_{i \in \mathcal{D}_t} |u_i^*| = X_t^- + X_t^+, \forall t \in \mathcal{T}$, and by summing over t we obtain: $\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{D}_t} |u_i^*| = \sum_{t \in \mathcal{T}} X_t$, i.e., $\sum_{i \in \mathcal{F}} |u_i^*| = \Delta^*$, so solution (w^*, u^*) is efficient.

A sufficient condition for a feasible and efficient solution (w^*, u^*) to solve (EQ) is to ensure that the vector U defined by $U_a = \sum_{i \in \mathcal{F}_a} |u_i^*|, \forall a \in \mathcal{A}$ solves the following problem, denoted by $\mathcal{P}(\Delta^*)$:

$$\begin{aligned} \text{lex min} \quad & \left(\frac{U_a}{|\mathcal{F}_a|} \right)_{a \in \mathcal{A}} \\ \text{s.t.} \quad & \sum_{a \in \mathcal{A}} U_a \geq \Delta^* \\ & U_a \geq 0, U_a \text{ integer} \end{aligned}$$

We construct an optimal solution of Problem $\mathcal{P}(\Delta^*)$ in the appendix (Lemma 9), which we will use in this proof to construct a solution of (EQ). First, let us summarize how this solution of Problem $\mathcal{P}(\Delta^*)$ is constructed. We assume without loss of generality that the greatest common divisor (gcd) of $(\alpha_a)_{a \in \mathcal{A}}$ is equal to 1 (if that is not the case then we can adjust the values of $(\alpha_a)_{a \in \mathcal{A}}$ and $(\beta_t)_{t \in \mathcal{T}}$ to ensure that this condition holds). Note that $|\mathcal{F}_a| = \alpha_a (\sum_{t \in \mathcal{T}} \beta_t), \forall a \in \mathcal{A}$ and thus: $\gcd(|\mathcal{F}_a|)_{a \in \mathcal{A}} = \sum_{t \in \mathcal{T}} \beta_t$. We then have: $\frac{|\mathcal{F}_a|}{\gcd(|\mathcal{F}_a|)_{a \in \mathcal{A}}} = \alpha_a, \forall a \in \mathcal{A}$. We denote by $N = \sum_{a \in \mathcal{A}} \alpha_a$. Let $\mathbb{1}$ denote the indicator function. According to Lemma 9 (see appendix), there exists a sequence $(a_1, \dots, a_N) \in \mathcal{A}^N$ such that $\sum_{i=1}^N \mathbb{1}(a_i = a) = \alpha_a, \forall a \in \mathcal{A}$ and the $|\mathcal{A}|$ -dimensional vector U defined by $U_a = q\alpha_a + \sum_{i=1}^r \mathbb{1}(a_i = a), \forall a \in \mathcal{A}$ is an optimal solution of $\mathcal{P}(\Delta^*)$, where q and r denote the quotient and the remainder of the Euclidean division of Δ^* by N (i.e., $\Delta^* = qN + r$). We denote by Ψ the sequence $\Psi = (a_1, \dots, a_N, \dots, a_1, \dots, a_N, a_1, \dots, a_r)$, where the full sequence (a_1, \dots, a_N) is repeated q times. By construction, $\sum_{i=1}^{\Delta^*} \mathbb{1}(\Psi_i = a) = q\alpha_a + \sum_{i=1}^r \mathbb{1}(a_i = a), \forall a \in \mathcal{A}$ and thus the vector U defined by $U_a = \sum_{i=1}^{\Delta^*} \mathbb{1}(\Psi_i = a), \forall a \in \mathcal{A}$ is an optimal solution of $\mathcal{P}(\Delta^*)$.

We now construct a solution (w^*, u^*) that satisfies (i) the sufficient conditions for feasibility and for efficiency maximization: $|\{i \in \mathcal{D}_t | u_i^* = -1\}| = X_t^-, \forall t \in \mathcal{T}$ and $|\{i \in \mathcal{D}_t | u_i^* = +1\}| = X_t^+, \forall t \in \mathcal{T}$, and (ii) the sufficient condition for equity maximization: $\sum_{i \in \mathcal{F}_a} |u_i^*| = \sum_{i=1}^{\Delta^*} \mathbb{1}(\Psi_i = a), \forall a \in \mathcal{A}$.

To do so, we construct a solution that displaces flights from the sequence of airlines Ψ , i.e., a solution that displaces one flight from airline Ψ_1 in period 1, then one flight from airline Ψ_2 in period 1, ..., then one flight from airline Ψ_{X_1} in period 1, then one flight from airline Ψ_{X_1+1} in period 2, then one flight from airline Ψ_{X_1+2} in period 2, ..., then one flight from airline $\Psi_{X_1+X_2}$ in period 2, etc. (of course, each airline may be repeated several times in each sequence). We denote by y_t the total number of flights displaced from period 1 through $t-1$ (both inclusive), i.e., $y_t = \sum_{s=1}^{t-1} X_s$. Note that $y_1 = 0$ and $y_{T+1} = \sum_{s \in \mathcal{T}} X_s = \Delta^*$. We denote by V_{at} the number of times airline a is repeated in the X_t indices between $y_t + 1$ and y_{t+1} (both inclusive), i.e., $V_{at} = \sum_{i=y_t+1}^{y_{t+1}} \mathbb{1}(\Psi_i = a), \forall a \in \mathcal{A}$. Given the periodicity of the sequence Ψ , any consecutive set of $N\beta_t$ values of Ψ_i includes exactly $\alpha_a\beta_t$ elements equal to $a, \forall a \in \mathcal{A}$. Since $y_{t+1} - y_t \leq N\beta_t$, we have $V_{at} \leq \alpha_a\beta_t, \forall a \in \mathcal{A}, t \in \mathcal{T}$. We can thus define a set $\mathcal{J}_{at} \subseteq (\mathcal{D}_t \cap \mathcal{F}_a)$ such that $|\mathcal{J}_{at}| = V_{at}$. As in the proof of Proposition 2, \mathcal{J}_{at} is not uniquely determined, but we can choose any subset of $\mathcal{D}_t \cap \mathcal{F}_a$ that satisfies this property. We construct a solution that displaces the flights in the sets \mathcal{J}_{at} such that the number of flights rescheduled to period $t-1$ (resp. $t+1$) is equal to X_t^- (resp. X_t^+). For each $t \in \mathcal{T}$, we partition $\cup_{a \in \mathcal{A}} \mathcal{J}_{at}$ into two subsets \mathcal{K}_t^+ and \mathcal{K}_t^- such that $|\mathcal{K}_t^+| = X_t^+$ and $|\mathcal{K}_t^-| = X_t^-$. We then define (i) $u_i^* = -1, \forall i \in \mathcal{K}_t^-$, (ii) $u_i^* = +1, \forall i \in \mathcal{K}_t^+$, (iii) $u_i^* = 0, \forall i \notin (\mathcal{K}_t^- \cup \mathcal{K}_t^+)$. We define w^* accordingly (based on the constraints of (EFF) and (EQ)).

By construction, the solution (w^*, u^*) satisfies the sufficient conditions for feasibility and efficiency maximization, so it solves Problem (EFF). Moreover, we have: $\sum_{i \in \mathcal{D}_t \cap \mathcal{F}_a} |u_i^*| = V_{at}, \forall a \in \mathcal{A}, t \in \mathcal{T}$. By summing over $t \in \mathcal{T}$, we obtain: $\sum_{i \in \mathcal{F}_a} |u_i^*| = \sum_{t \in \mathcal{T}} \sum_{i=y_t+1}^{y_{t+1}} \mathbb{1}(\Psi_i = a)$, that is $\sum_{i \in \mathcal{F}_a} |u_i^*| = \sum_{i=y_1+1}^{y_{T+1}} \mathbb{1}(\Psi_i = a) = \sum_{i=1}^{\Delta^*} \mathbb{1}(\Psi_i = a)$. Therefore, the solution (w^*, u^*) solves Problem (EQ). \square

In summary, in the case where scheduling interventions are based on temporal shifts in demand, efficiency and equity can be jointly maximized if (i) no network connections need to be maintained, (ii) all flights are equally valued, and (iii) airline schedules of flights satisfy the conditions of Proposition 2 or Proposition 3 (or both), shown in Figure 3. Under the conditions of Proposition 2 (Figure 3a), the imbalances between demand and capacity are small enough so no time period is such that some flights get displaced *to* that period and some other flights get displaced *from* that period. Under the conditions of Proposition 3 (Figure 3b), the schedules of flights of the airlines exhibit the same intra-day variations. Even though these conditions are usually not exactly satisfied in practice, our computational experiments reported in Section 5 show that the insights derived in these two cases can be relevant and applicable in practical settings.

4.2. Instances of Efficiency/Equity Trade-off

Based on the discussion above, in the case where the scheduling interventions are based on temporal shifts in demand, a trade-off between efficiency and equity might arise through (i) inter-airline variations in intra-day flight schedule patterns (we refer to it simply by ‘differentiated

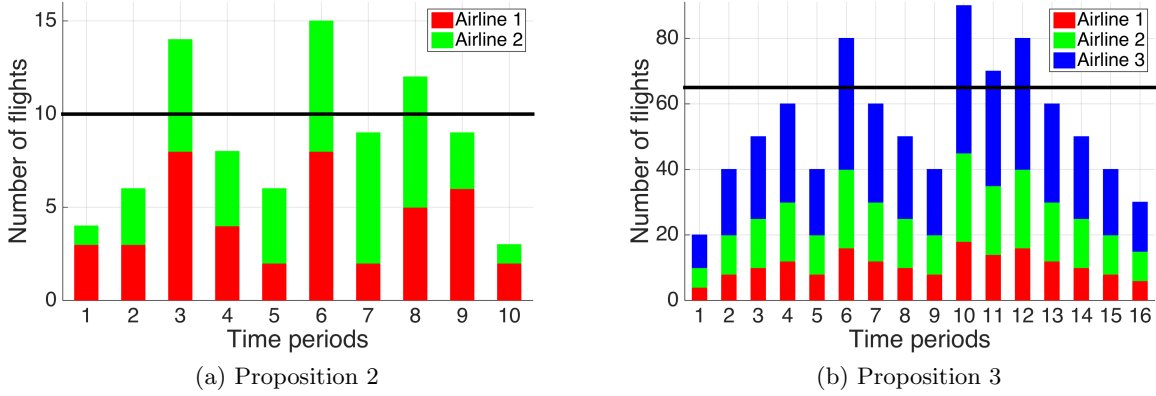


Figure 3: Example of scheduling conditions of Propositions 2 and 3

airline schedules’), (ii) network connections, and (iii) intra-airline variations in flight valuations (we refer to it simply by ‘differentiated flight valuations’). Note that, in the case where the scheduling interventions are based on reductions in demand, a trade-off between efficiency and equity might also arise through network connections, and differentiated flight valuations, which can be similarly demonstrated through examples.

We first provide an example that shows that weighted efficiency and equity may not be jointly maximized in the presence of differentiated airline schedules. Figure 4 shows a hypothetical example with an unconstrained schedule in a 7-period case with 2 airlines and 26 flights per airline, and a simple capacity constraint that ensures that no more than 10 flights may be scheduled per period. We assume that all flights are valued equally and that there are no connections. We also assume that airline 1’s flights (shown in red) are concentrated at earlier periods, and airline 2’s flights (shown in green) are concentrated at later periods. Note that the conditions of either Proposition 2 or 3 are not satisfied here. Figure 4a (resp. Figure 4b) shows which flights are rescheduled to later or earlier times for an efficiency-maximizing solution (resp. an equity-maximizing solution). Since the capacity constraint is only violated during period 5, when all flights scheduled are airline 1’s flights, every efficiency-maximizing solution displaces 4 flights from airline 1 to later times, by 1 period each (one such efficiency-maximizing solution is shown as “+1”s in Figure 4a). The resulting total displacement is equal to 4 periods, and the airline disutilities are equal to $4/26$ for airline 1 and 0 for airline 2. In contrast, every equity-maximizing solution displaces 3 flights of each to earlier times, by 1 period each (one such equity-maximizing solution is shown as “-1”s in Figure 4b). The resulting total displacement is equal to 6 periods, and each airline’s disutility is equal to $3/26$. In turn, the set of efficiency-maximizing solutions and the set of equity-maximizing solutions have no overlap.

We now provide an example that shows that weighted efficiency and equity may not be jointly optimized in the presence of network connections. Intuitively, if one airline’s network is signifi-

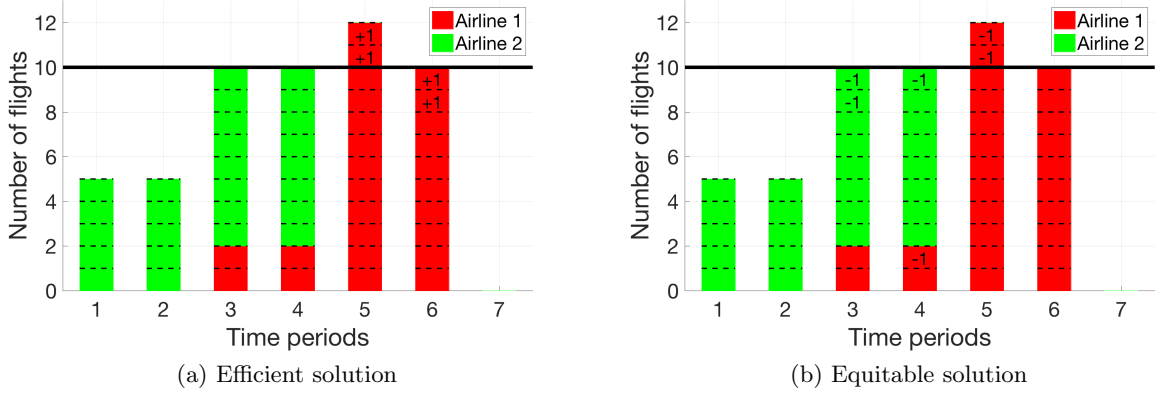


Figure 4: Trade-off between weighted efficiency and equity due to differentiated airline schedules

cantly more connected than another airline's, then the former airline's flights are likely to be more difficult to reschedule. In turn, maximizing efficiency may involve assigning more displacement to the latter airline's flights rather than the former's, at some equity loss. Figure 5 shows such an example with 5 periods, 2 airlines with 13 flights each, and a capacity of 6 flights per period. Note that the conditions of both Propositions 2 and 3 would be satisfied in the absence of network connections. But airline 2's network involves a number of connections, whereas airline 1's network has no connections. We represent connections by dashed, gray "links" between flight pairs, and we assume that each connection requires a 2-period interval between the flights in the connection at a minimum. In this case, every efficiency-maximizing solution displaces 4 of airline 1's flights (the airline with no connections) by 1 period each. The resulting total displacement is equal to 4 periods, and the airline disutilities are equal to $4/13$ for airline 1 and 0 for airline 2. In contrast, every equity-maximizing solution displaces 3 flights of each airline, by 1 period each. The resulting total displacement is equal to 6 periods, and each airline's disutility is equal to $3/13$. Again, the set of efficiency-maximizing solutions and the set of equity-maximizing solutions have no overlap.

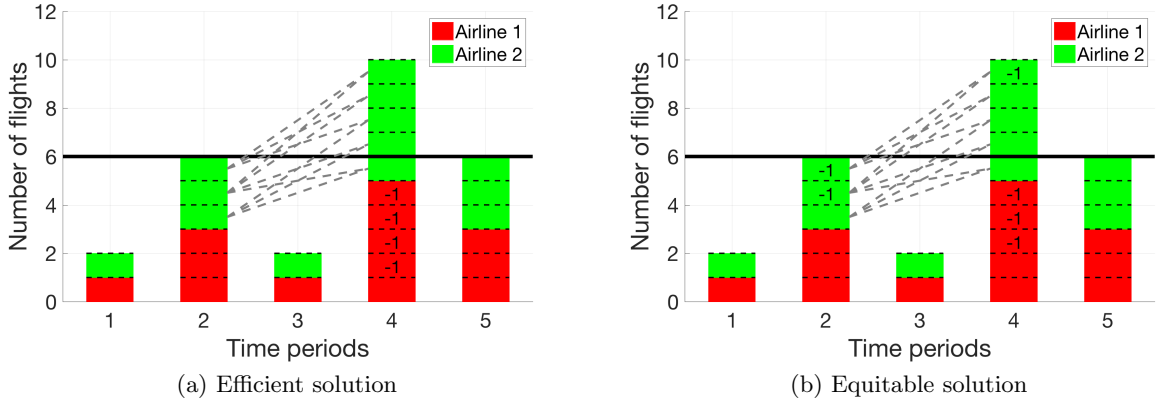


Figure 5: Trade-off between weighted efficiency and equity due to differentiated airline connectivities

Finally, we provide an example that shows that weighted efficiency and equity may not be jointly optimized in the presence of differentiated flight valuations. Figure 6 shows an example with 5 periods, 2 airlines with 10 flights each, and a capacity of 6 flights per period. Note that the conditions of both Propositions 2 and 3 would be satisfied under uniform flight valuations. But we assume that every flight, except the 6 flights scheduled by airline 1 in period 3, has a value equal to $v_i = 1$, and, among the remaining 6 flights, three have a value $v_i = 0.1$ each, and three others have a value $v_i = 1.9$ each (as a result, the average value of airline 1's flights is equal to 1). Every efficiency-maximizing solution displaces the three flights of value $v_i = 0.1$ and three flights of value $v_i = 1$ from period 3. The optimal value of the weighted displacement is equal to 3.3 and the airline disutilities are equal to $0.3/10$ for airline 1 and to $3/10$ for airline 2. In contrast, every equity-maximizing solution displaces four flights of airline 1 and two flights of airline 2. The weighted displacement is equal to 4.2 and the airline disutilities are equal to $2.2/10$ for airline 1 and to $2/10$ for airline 2. Again, the set of efficiency-maximizing solutions and the set of equity-maximizing solutions have no overlap.

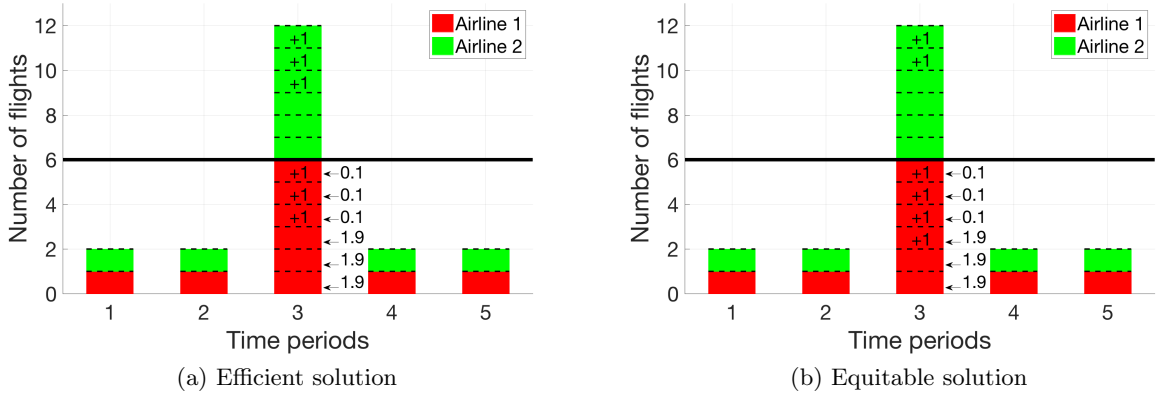


Figure 6: Trade-off between weighted efficiency and equity due to network connections

5. Computational Results

We implement the models developed in Section 3 for a case study at JFK Airport. We show that, in realistic instances, inter-airline equity can be significantly improved at no (or minimal) efficiency losses if flights are equally valued. We then show that significant equity gains can be obtained even under differentiated flight valuations, at small losses in efficiency. In fact, the price of equity is consistently significantly smaller than the price of efficiency even under differentiated flight valuations.

5.1. Experimental Setup

We consider data from September 18, 2007 at the John F. Kennedy Airport (JFK). JFK was chosen as the study airport because it is one of the most congested airports in the US, and its peaked

schedule of flights offers opportunities for delay reductions through scheduling interventions. The day of 09/18/2007 was chosen because no scheduling interventions were in place at JFK in 2007, and because the number of flights scheduled on 09/18 equals the median of the number of daily flights at JFK in 2007. Estimates of JFK’s capacity in various operating conditions were obtained from Simaiakis (2012). Flight schedules were obtained from the Aviation System Performance Metrics (ASPM) database (Federal Aviation Administration, 2013). We group partner airlines together, as major airlines typically coordinate planning and scheduling decisions with their subsidiaries, and passengers can easily connect between flights operated by partner airlines. Specifically, we consider four groups of airlines: (i) Delta Airlines (DAL) and its regional partners (which operated a total of 320 flights on 09/18/2007 at JFK), (ii) American Airlines (AAL) and its regional partners (260 flights), (iii) JetBlue Airways (JBU) (174 flights), and (iv) all other airlines, each of which represents a smaller share of traffic at JFK (408 flights combined). These scheduling data were used to construct sets \mathcal{F} , \mathcal{F}^{arr} , \mathcal{F}^{dep} , \mathcal{F}_a , \mathcal{S}^{arr} , and \mathcal{S}^{dep} .

We reconstructed aircraft and passenger connections to determine \mathcal{C} , t^{\min} , and t^{\max} . Aircraft connections were obtained from the ASPM database (Federal Aviation Administration, 2013). We use the minimum aircraft turnaround time between any pair of flights estimated by Pyrgiotis (2011) as a function of the aircraft type, of the airline and of whether the airport is a hub airport for the airline or not. We use a maximum turnaround time equal to the planned turnaround time plus 15 minutes to maintain comparable aircraft utilization. We obtained passenger connections data from a database developed by Barnhart et al. (2014), based on a discrete choice model for estimating historical passenger flows. We estimate the minimum passenger connection time at JFK as the 5th percentile of the distribution of all planned passenger connection times. Because of data unavailability, we do not reconstruct crew connections here, but their consideration could be easily added as estimates of historical crew schedules become available (Vaze and Wei, 2015).

With the actual schedule of flights on 09/18/2007, the peak expected arrival and departure queue lengths are equal to $\max_{t \in \mathcal{T}} E(A_t) = 14.6$ aircraft and $\max_{t \in \mathcal{T}} E(D_t) = 28.1$, respectively—obtained using the model of airport congestion shown in Equation (9). We vary the expected arrival queue length target A_{MAX} from 15 to 11 aircraft, and the expected departure queue length target D_{MAX} from 30 to 15 aircraft. These are the targets that can be met under scheduling interventions restricted only to temporal shifts in demand and without imposing a prohibitively large set of flight displacements. With these on-time performance targets, the optimal value of the maximum flight displacement δ^* is equal to 1 period, i.e., all on-time performance targets can be achieved without displacing any flight by more than 15 minutes (Jacquillat and Odoni, 2015a). Our computational results will thus focus on weighted efficiency Δ (which, for simplicity, we refer to by “efficiency” in the remainder of this section) and on inter-airline equity Φ (obtained from the vector of airline disutilities, σ), for any set of on-time performance targets.

We implemented the Integer Programming models of scheduling interventions in GAMS 24.0

using CPLEX 12.5 and the Dynamic Programming model of capacity utilization and the Stochastic Queuing Model of airport congestion in MATLAB 8.1. We looked for solutions to the Integer Programming models within an optimality gap of 1%. If none was found after 30 minutes, we accepted the solution obtained at that time.

5.2. Results under Uniform Flight Valuations

We first consider the case where all flights are equally valued, i.e., $v_i = 1, \forall i \in \mathcal{F}$. This corresponds to current practice, where the airlines do not provide any inputs on relative timetabling flexibility of their flight, and scheduling interventions are thus performed under the “a flight is a flight” paradigm. We compare the results obtained under an efficiency-maximization objective (Problems **P1** and **P2**) to those obtained with inter-airline equity objectives (Problems $\widehat{\mathbf{P3}(\rho)}$). This comparison thus shows the extent to which inter-airline equity can be achieved in scheduling interventions under current scheduling conditions and uniform flight valuations.

Note that the solution of Problem **P2** is arbitrarily “chosen” by the optimization solver from the set of (possibly) multiple optimal solutions. In order to characterize the equity range among efficiency-maximizing solutions, we also determine the solution which *minimizes* inter-airline equity, i.e., which lexicographically *maximizes* airline disutilities, while ensuring the optimal value of efficiency. This characterizes the efficiency-maximizing solution that performs the worst in terms of inter-airline equity. We denote this problem by $\overline{\mathbf{P2}}$.

Table 2 shows, for different sets of on-time performance targets A_{MAX} and D_{MAX} , the total schedule displacement faced by each airline (that is, the number of its flights displaced by 15 minutes each, as the maximum displacement δ^* is equal to 1 15-minute period), and each airline’s disutility (i.e., its weighted average per-flight displacement) for Problems **P2**, $\overline{\mathbf{P2}}$ and $\widehat{\mathbf{P3}(\rho^*)}$. It also reports the ratio of the largest to smallest airline disutility. As A_{MAX} and D_{MAX} become smaller, the resulting schedule displacement increases, as noted by Jacquillat and Odoni (2015a), but these results show that, for any set of values of A_{MAX} and D_{MAX} considered, the modeling approach developed in this paper provides strong equity gains at no loss in efficiency. Note, first, that Problem $\overline{\mathbf{P2}}$ results in max-min ratios $\frac{\max_a \sigma_a}{\min_a \sigma_a}$ ranging between 10 and 50. For the cases considered, AAL and JBU tend to be much more significantly penalized than DAL, which is reflected through more of their flights being rescheduled and through much higher disutility values. The set of efficiency-maximizing solutions thus contains highly inequitable outcomes. Problem **P2** does not result in the most inequitable outcome in that set, but provides solutions that still impact some airlines (here, AAL, JBU and the “other” airlines) more negatively than others (here, DAL). Inter-airline equity is achieved only by solving Problem $\widehat{\mathbf{P3}(\rho^*)}$. In that case, airline disutilities are much closer to each other than those obtained by solving Problems $\overline{\mathbf{P2}}$ and **P2**. Note that the differences in airlines’ schedules of flights and network connectivities result in all four airlines *not* having the exact same disutility, but differences are very small (i.e., the max-min ratio $\frac{\max_a \sigma_a}{\min_a \sigma_a}$ is very close to 1) under the equitable solution. Most importantly, the equity-maximizing solution

(Problem $\widehat{\mathbf{P3}(\rho^*)}$) results in the same total displacement as the efficiency-maximizing solution (Problem $\mathbf{P2}$) in all cases considered. Only the distribution of schedule displacement across the airlines is modified. In other words, efficiency and equity can be jointly maximized, and the price of equity ρ^* and the price of efficiency are both zero.

Table 2: Number of flights displaced and airline disutilities per airline under uniform flight valuations

On-time targets			Number of flights displaced					Disutility: $\sigma_a = \frac{1}{ \mathcal{F}_a } \sum_{i \in \mathcal{F}_a} u_i $				
A_{MAX}	D_{MAX}	Model	DAL	AAL	JBU	Others	All	DAL	AAL	JBU	Others	$\frac{\max_a \sigma_a}{\min_a \sigma_a}$
14	23	$\overline{\mathbf{P2}}$	1	13	1	5	20	0.3%	5.0%	0.6%	1.2%	16.00
		$\mathbf{P2}$	1	9	2	8	20	0.3%	3.5%	1.1%	2.0%	11.08
		$\widehat{\mathbf{P3}(\rho^*)}$	4	5	3	8	20	1.3%	1.9%	1.7%	2.0%	1.57
13	20	$\overline{\mathbf{P2}}$	1	29	9	7	46	0.3%	11.2%	5.2%	1.7%	35.69
		$\mathbf{P2}$	7	18	8	13	46	2.2%	6.9%	4.6%	3.2%	3.16
		$\widehat{\mathbf{P3}(\rho^*)}$	13	10	7	16	46	4.1%	3.8%	4.0%	3.9%	1.06
12	18	$\overline{\mathbf{P2}}$	1	28	27	9	65	0.3%	10.8%	15.5%	2.2%	49.66
		$\mathbf{P2}$	10	27	10	18	65	3.1%	10.4%	5.7%	4.4%	3.32
		$\widehat{\mathbf{P3}(\rho^*)}$	18	14	10	23	65	5.6%	5.4%	5.7%	5.6%	1.07
11	15	$\overline{\mathbf{P2}}$	37	113	39	17	206	11.6%	43.5%	22.4%	4.2%	10.43
		$\mathbf{P2}$	50	57	32	67	206	15.6%	21.9%	18.4%	16.4%	1.40
		$\widehat{\mathbf{P3}(\rho^*)}$	57	46	31	72	206	17.8%	17.7%	17.8%	17.6%	1.01

Therefore, joint optimization of efficiency and equity is achievable under current schedules of flights and uniform flight valuations (which is the assumption widely used in current practice). In light of the results from Section 4, this suggests that inter-airline variations in flight schedules and network connectivities are relatively weak and do not create, by themselves, a trade-off between efficiency and equity. This is due to the fact that peak-hour schedules typically include flights from several airlines and the schedules of all airlines exhibit network connections to some extent (so the situations depicted in Figures 4 and 5 are not typical of actual scheduling patterns at busy airports). Under these conditions, incorporating inter-airline equity objectives in scheduling interventions can thus yield significant benefits by balancing scheduling adjustments more fairly among the airlines at no efficiency losses.

5.3. Results under Differentiated Flight Valuations

We now consider the case where all flights are not equally valued, and compare the outcomes of scheduling interventions when only the efficiency objectives are considered to the outcomes when equity objectives are also considered. This captures potential extensions of existing and other previously proposed mechanisms for airport scheduling interventions that would allow the airlines to provide the relative timetabling flexibility of their flights (e.g., auction, credit-based mechanism). Since the flight valuations rely on information that is often private to the airlines and since they are challenging to estimate using available public data, we two different types of approximate

approaches to estimate their impact on our efficiency-equity trade-off results. We first consider the case where the average flight valuation is identical for all airlines, to identify the impact of the *distribution* of flights valuations. We then consider the more general case where average flight valuations may vary across the airlines, by approximating them by revenue estimates.

We first sample $(v_i)_{i \in \mathcal{F}}$ by keeping the average flight valuation of all airlines equal to 1 (without loss of generality), and varying the distribution of flight valuations for one given airline a . We set $v_i = 1, \forall i \notin \mathcal{F}_a$. We partition the set of flights \mathcal{F}_a of airline a into two subsets $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$ such that $\mathcal{F}_a^{(1)} \cap \mathcal{F}_a^{(2)} = \emptyset$ and $\mathcal{F}_a^{(1)} \cup \mathcal{F}_a^{(2)} = \mathcal{F}_a$. We can think of $\mathcal{F}_a^{(1)}$ (resp. $\mathcal{F}_a^{(2)}$) as the set of the more flexible flights (resp. the less flexible flights) of airline a . We choose to represent the valuations of the flights in $\mathcal{F}_a^{(1)}$ (resp. $\mathcal{F}_a^{(2)}$) by a Gamma distribution $\Gamma_1(\mu_1, k)$ (resp. $\Gamma_2(\mu_2, k)$) with mean μ_1 (resp. μ_2) and shape parameter k , with $\mu_1 < \mu_2$. We adjust the shape parameter of these distributions such that the 95th percentile of the former distribution coincides with the 5th percentile of the latter. These choices of distributions and parameters are made in order to provide a transparent and flexible bimodal characterization of flight valuations such that the valuations of flights in $\mathcal{F}_a^{(1)}$ are, in most cases, lower than the valuations of flights in $\mathcal{F}_a^{(2)}$. Finally, we set the values of flights in $\mathcal{F}_a^{(1)}$ (resp. $\mathcal{F}_a^{(2)}$) equal to $\Theta_1^{-1} \left(\frac{1}{(|\mathcal{F}_a^{(1)}|+1)} \right), \Theta_1^{-1} \left(\frac{2}{(|\mathcal{F}_a^{(1)}|+1)} \right), \dots, \Theta_1^{-1} \left(\frac{|\mathcal{F}_a^{(1)}|}{(|\mathcal{F}_a^{(1)}|+1)} \right)$ (resp. $\Theta_2^{-1} \left(\frac{1}{(|\mathcal{F}_a^{(2)}|+1)} \right), \Theta_2^{-1} \left(\frac{2}{(|\mathcal{F}_a^{(2)}|+1)} \right), \dots, \Theta_2^{-1} \left(\frac{|\mathcal{F}_a^{(2)}|}{(|\mathcal{F}_a^{(2)}|+1)} \right)$), where Θ_1 (resp. Θ_2) denotes the cumulative distribution function of $\Gamma_1(\mu_1, k)$ (resp. $\Gamma_2(\mu_2, k)$). This sampling strategy ensures that the resulting set of flight valuations is distributed “smoothly” across the distributions considered without sampling these values multiple times. For each airline, we vary two parameters: (i) the fraction of flights in $\mathcal{F}_a^{(1)}$, denoted by $\eta = \frac{|\mathcal{F}_a^{(1)}|}{|\mathcal{F}_a|}$ (so that $1 - \eta = \frac{|\mathcal{F}_a^{(2)}|}{|\mathcal{F}_a|}$), and (ii) the mean valuations of flights in $\mathcal{F}_a^{(1)}$, i.e., μ_1 (such that $\eta\mu_1 + (1 - \eta)\mu_2 = 1$). Within each set, $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$, we sort flights from the least valuable to the most valuable using 10 random permutations. In other words, the 10 tests have the same sets of flight valuations, but differ in terms of which flights are more flexible and which are less flexible.

Table 3 shows results (with $A_{\text{MAX}} = 11$ and $D_{\text{MAX}} = 15$, which are the most stringent set of on-time targets from those in Table 2) under different sets of flight valuations provided by DAL (left) and AAL (right)—similar results are obtained by varying the flight valuations provided by the other airlines. The first row provides a baseline where all flights are equally valued (i.e., $v_i = 1, \forall i \in \mathcal{F}$). In the top half, we assume that $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$ both comprise 50% of the flights from DAL or AAL, and we progressively increase the valuation differential $\mu_2 - \mu_1$. In the bottom half, we fix $\mu_1 = 0.75$ and we progressively decrease the proportion of flights in $\mathcal{F}_a^{(1)}$ (and we thus decrease μ_2 to ensure that $\eta\mu_1 + (1 - \eta)\mu_2 = 1$). Table 3 reports, in each scenario, the total schedule displacement $\sum_{i \in \mathcal{F}_a} |u_i|$ of *each* airline a obtained in the equity-maximizing scenario (i.e., Problem $\widehat{\mathbf{P3}}(\rho^*)$), as well as the prices of equity and efficiency, averaged across all 10 samples.

The observations from variations in $\mu_2 - \mu_1$ (top) and in η (bottom) are threefold. First, as

an airline’s flight valuations become more differentiated, the displacement of this airline’s schedule increases. In turn, flight valuations create, for each airline, a trade-off between prioritizing *which* flights get rescheduled, on the one hand, and minimizing their total displacement, on the other hand. Second, as the variance in any airline’s flight valuations increases, other airlines’ displacements do not change significantly (in fact, sometimes they decrease a little). In other words, the model can account for any airline’s scheduling preferences without negatively impacting the other airlines. Third, the price of equity is much smaller than the price of efficiency across all the scenarios considered, therefore indicating strong gains in inter-airline equity at small efficiency losses.

Table 3: Average values of the total displacement per airline ($\sum_{i \in \mathcal{F}_a} |u_i|$), the price of equity and the price of efficiency under differentiated flight valuations

Scenario			For variations in DAL’s flight valuations						For variations in AAL’s flight valuations					
μ_1	μ_2	η	DAL	AAL	JBU	Others	P_{eq}	P_{eff}	DAL	AAL	JBU	Others	P_{eq}	P_{eff}
1.0	1.0	–	57.0	46.0	31.0	72.0	0%	0%	57.0	46.0	31.0	72.0	0%	0%
0.9	1.1	50%	57.7	44.0	36.2	70.0	4.4%	15.4%	55.5	46.9	36.0	69.5	2.8%	24.8%
0.8	1.2	50%	57.0	44.7	36.4	70.5	4.5%	15.3%	53.4	50.0	34.6	67.9	2.4%	26.5%
0.7	1.3	50%	58.1	44.1	35.9	70.2	5.6%	14.5%	52.2	50.5	33.1	66.6	2.0%	23.3%
0.6	1.4	50%	59.8	43.5	36.2	69.1	6.8%	14.4%	51.5	59.1	32.7	65.1	2.1%	15.8%
0.5	1.5	50%	63.5	43.4	35.3	68.8	7.2%	13.7%	49.9	64.0	32.1	63.4	2.3%	11.6%
0.4	1.6	50%	67.3	43.5	34.9	68.9	8.1%	13.4%	48.8	70.7	31.4	61.4	2.0%	7.8%
0.3	1.7	50%	73.2	42.3	34.0	67.1	9.3%	13.3%	46.5	78.4	31.7	59.0	2.7%	9.7%
0.2	1.8	50%	77.4	42.4	34.8	67.2	11.2%	13.4%	44.4	87.5	29.9	56.5	2.8%	10.7%
0.1	1.9	50%	84.1	42.9	34.0	67.8	13.5%	11.8%	42.0	95.8	29.6	53.4	3.8%	15.7%
0.75	3.25	90%	57.6	44.4	36.6	70.4	4.1%	16.9%	49.3	42.3	32.2	62.4	2.4%	19.9%
0.75	2.00	80%	57.4	44.7	36.6	70.7	4.4%	17.3%	54.9	48.7	35.2	69.0	2.2%	19.6%
0.75	1.58	70%	57.3	44.4	36.3	70.4	4.3%	17.2%	53.5	49.6	35.1	68.4	2.2%	17.9%
0.75	1.37	60%	57.2	44.7	36.2	70.7	4.5%	16.4%	54.3	50.8	35.1	68.2	2.6%	17.7%
0.75	1.25	50%	57.5	44.1	36.4	69.7	5.1%	15.2%	53.0	52.0	33.8	67.6	1.9%	21.2%
0.75	1.17	40%	57.6	44.3	36.0	70.5	5.5%	13.6%	52.7	54.2	34.4	66.9	2.7%	24.2%
0.75	1.11	30%	58.6	44.1	36.5	70.1	6.2%	12.0%	51.9	55.8	33.4	65.9	2.8%	33.0%
0.75	1.06	20%	59.7	44.0	35.6	69.6	6.6%	10.8%	51.0	58.2	33.0	65.0	2.8%	35.8%
0.75	1.03	10%	66.6	43.1	35.4	67.9	6.9%	9.9%	50.0	62.6	31.8	63.3	2.7%	33.8%

We further investigate the trade-off between equity and efficiency in the case where average flight valuations may vary across the airlines. We estimate $(v_i)_{i \in \mathcal{F}}$ as the product of aircraft sizes and average airfares. This aims to capture variations in operating revenues across flights. Aircraft sizes are obtained from the ASPM database (Federal Aviation Administration, 2013). Airfares are obtained from the Bureau of Transportation Statistics website for each domestic origin-destination market (averaged over the third quarter of 2007) (Bureau of Transportation Statistics, 2013). International airfares are inferred by using a regression model to estimate airfares as a function of flight distance (calibrated using domestic markets data), to which we added a 20% premium, reflecting the fact that international flights are typically more expensive than domestic flights traveling the

same distance. Of course, this procedure can only be treated as highly approximate, and many other drivers of operating profitability and other measures of scheduling flexibility (e.g., load factors, operating margins, connecting passengers) could not be estimated from the publicly available data. Our aim here is to assess the impact of such differentiated flight valuations on the efficiency and equity trade-off in scheduling interventions.

Table 4 shows the results for different on-time performance targets A_{MAX} and D_{MAX} . For each set of values for these targets, we report the largest flight displacement (i.e., $\delta^* = \max_{i \in \mathcal{F}} |u_i|$), the total schedule displacement (i.e., $\sum_{i \in \mathcal{F}} |u_i|$), and the disutility of each airline (i.e., the average per-flight weighted displacement $\frac{1}{|\mathcal{F}_a|} \sum_{i \in \mathcal{F}_a} v_i |u_i|$ for each airline a , expressed in \$, since our flight valuations are based on revenue estimates) under the inequitable (Problem $\overline{\mathbf{P2}}$), efficient (Problem $\widehat{\mathbf{P3}}(\mathbf{0})$) and equitable (Problem $\widehat{\mathbf{P3}}(\rho^*)$) solutions. We also report the price of equity and the price of efficiency. First, note that the inequitable solution (obtained by *minimizing* equity, under optimal efficiency) and the efficient solution (obtained by *maximizing* equity, under optimal efficiency) result in the same airline disutilities. This contrasts with the case of uniform valuations (Section 5.2), where the efficient solution vastly improved inter-airline equity. This is because differentiations in flight valuations restrict the set of efficiency-maximizing solutions, thus reducing the flexibility to select the set of flights to be rescheduled in an equitable way. Second, the equitable solution (Problem $\widehat{\mathbf{P3}}(\rho^*)$) balances per-flight weighted displacement much more equitably across the four groups of airlines. Third, inter-airline equity is achieved through moderate increases in the number of flights displaced and, in some instances, by rescheduling *fewer* flights than under the efficient solution. Last, even though the price of equity is higher than in our previous tests, the price of efficiency remains nonetheless significantly higher than the price of equity. This is particularly true for the more aggressive on-time performance targets, which can be explained by the fact that the flexibility to select the set of flights to be displaced in a more equitable manner increases with the schedule displacement. Overall, these results suggest that inter-airline equity can be achieved through comparatively small increases in efficiency, even under strong differentiations in flight valuations across the airlines and across the flights of an airline. Note that such differentiations could arise in many future extensions of the existing mechanisms for airport demand management.

6. Conclusion

Any airport demand management scheme involves a trade-off between mitigating airport congestion, on the one hand, and minimizing interference with airlines' competitive scheduling, on the other hand. In this paper, we have developed, optimized and assessed models for airport scheduling interventions that, for the first time, incorporate inter-airline equity considerations. The resulting Integrated Capacity Utilization and Scheduling Model with Equity Considerations (ICUSM-E) relies on an original lexicographic modeling architecture that optimizes scheduling interventions based on on-time performance, efficiency and inter-airline equity objectives.

Table 4: Displacement, airline disutilities and prices of equity and efficiency under revenue-based flight valuations

Targets		Solution	# flights displaced		Weighted per-flight displacement				Performance
A_{MAX}	D_{MAX}		$\max_{i \in \mathcal{F}} u_i $	$\sum_{i \in \mathcal{F}} u_i $	DAL	AAL	JBU	Others	
14	23	$\overline{\mathbf{P2}}$	1	23	\$119	\$432	\$101	\$218	$\left. \begin{array}{l} P_{\text{eq}} = 20.2\% \\ P_{\text{eff}} = 54.1\% \end{array} \right\}$
		$\widehat{\mathbf{P3(0)}}$	1	23	\$119	\$432	\$101	\$218	
		$\mathbf{P3}(\rho^*)$	1	26	\$267	\$280	\$183	\$218	
13	20	$\overline{\mathbf{P2}}$	1	41	\$368	\$781	\$384	\$257	$\left. \begin{array}{l} P_{\text{eq}} = 17.8\% \\ P_{\text{eff}} = 53.4\% \end{array} \right\}$
		$\widehat{\mathbf{P3(0)}}$	1	41	\$368	\$781	\$384	\$257	
		$\mathbf{P3}(\rho^*)$	1	44	\$455	\$509	\$420	\$489	
12	18	$\overline{\mathbf{P2}}$	1	75	\$809	\$1,566	\$1,596	\$611	$\left. \begin{array}{l} P_{\text{eq}} = 17.6\% \\ P_{\text{eff}} = 33.6\% \end{array} \right\}$
		$\widehat{\mathbf{P3(0)}}$	1	75	\$809	\$1,566	\$1,596	\$611	
		$\mathbf{P3}(\rho^*)$	1	85	\$1,186	\$1,189	\$1,173	\$1,197	
11	15	$\overline{\mathbf{P2}}$	1	226	\$4,389	\$6,530	\$5,909	\$2,393	$\left. \begin{array}{l} P_{\text{eq}} = 6.9\% \\ P_{\text{eff}} = 38.5\% \end{array} \right\}$
		$\widehat{\mathbf{P3(0)}}$	1	226	\$4,389	\$6,530	\$5,909	\$2,393	
		$\mathbf{P3}(\rho^*)$	1	224	\$4,714	\$4,744	\$4,686	\$4,686	

Theoretical results have shown that, in the absence of network connections and under the standard paradigm that “a flight is a flight” (i.e., all flights are equally inconvenient to reschedule), efficiency and inter-airline equity can be jointly maximized if the interventions involve accepting or rejecting flight requests without changing the timetabling of the flights, or if the interventions involve temporal shifts in demand and some additional scheduling conditions are satisfied. Computational results suggested that, under a wide range of realistic and hypothetical scenarios, inter-airline equity can be achieved at small efficiency losses (if any). In other words, achieving maximum equity requires no (or small) sacrifice in terms of efficiency losses. On the other hand, for some of our computational scenarios, our results showed that ignoring inter-airline equity (i.e., considering efficiency-based objectives exclusively, or, in some cases, requiring maximal efficiency) may lead to highly inequitable outcomes. This further highlights that it is critical to explicitly incorporate inter-airline equity objectives in the optimization of scheduling interventions. In turn, this offers the potential to extend existing approaches to airport demand management (either the slot control policies in place at busy airports outside the United States, or the scheduling practices at a few of the busiest US airports where flight caps are in place) in a way that balances scheduling interventions fairly among the airlines, thus considerably enhancing their applicability in practice.

The potential equity benefits of scheduling interventions also motivate future research directions on airport scheduling interventions. Most importantly, this paper has assumed knowledge of the scheduling inputs provided by the airlines. An important opportunity lies in the design and optimization of scheduling intervention mechanisms through which the airlines can provide their preferred schedules of flights (and, potentially, some other inputs as well). The design of such

mechanisms would also create an opportunity to analyze the strategic interactions among the airlines and minimize the potential for gaming. More broadly, this research lays down the modeling framework to optimize and compare non-monetary mechanisms to market-based mechanisms such as congestion pricing and slot auctions, based on common efficiency, inter-airline equity, and on-time performance objectives. The approach developed in this paper provides the methodological foundation to address such problems of airport capacity allocation to mitigate delay externalities, promote airline competition, and maximize social welfare in a way, as the results have shown, that ensures inter-airline equity.

Acknowledgments

The authors would like to thank Prof. Amedeo Odoni and Prof. Cynthia Barnhart for their valuable comments and suggestions which have significantly improved this research and this paper.

Appendix 1: Construction of a solution for Proposition 2

In Proposition 2, we make the following assumption, which we refer to by (\mathcal{H}_t) :

$$(\mathcal{H}_t) : \quad \sum_{l=t-1}^{t+1} |\mathcal{D}_l| \leq \sum_{l=t-1}^{t+1} \hat{\lambda}_l, \forall t \in \mathcal{T}$$

We construct a solution recursively by rescheduling flights from each period $t = 1, \dots, T$ first to the preceding period (i.e., period $t - 1$), up to capacity, and then to the following period (i.e., period $t + 1$). This is done by selecting a subset $\mathcal{K}_t^- \subset \mathcal{D}_t$ and then a subset $\mathcal{K}_t^+ \subset \mathcal{D}_t \setminus \mathcal{K}_t^-$ such that:

$$\begin{aligned} |\mathcal{K}_t^-| &= \min \left\{ \hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|), (|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+|)^+ \right\} \\ |\mathcal{K}_t^+| &= (|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-|)^+ \end{aligned}$$

The subsets \mathcal{K}_t^- and \mathcal{K}_t^+ are not uniquely determined, but we can choose any subsets of \mathcal{D}_t that satisfy these properties. By convention, we set $\mathcal{K}_t^- = \mathcal{K}_t^+ = \emptyset$ if $t < 1$ or $t > T$. We define u as follows: $u_i = -1, \forall i \in \mathcal{K}_t^-$ and $u_i = +1, \forall i \in \mathcal{K}_t^+$. We define w accordingly (based on the constraints $\sum_{t \in \mathcal{T}} (w_{it} - S_{it}) = u_i, \forall i \in \mathcal{F}$).

We want to show that this solution is feasible and optimal for Problem (EFF), defined as follows:

$$\begin{aligned}
\min \quad & \sum_{i \in \mathcal{F}} |u_i| \quad (\text{EFF}) \\
\text{s.t.} \quad & w_{it} \geq w_{i,t+1}, \forall i \in \mathcal{F}, \forall t \in \mathcal{T} \\
& w_{i1} = 1, \forall i \in \mathcal{F} \\
& \sum_{t \in \mathcal{T}} (w_{it} - S_{it}) = u_i, \forall i \in \mathcal{F} \\
& \sum_{i \in \mathcal{F}} (w_{it} - w_{i,t+1}) \leq \hat{\lambda}_t, \forall t \in \mathcal{T} \\
& |u_i| \leq 1, \forall i \in \mathcal{F}
\end{aligned}$$

Lemma 1 first shows that the solution is well defined. To show this, we need to prove that $\hat{\lambda}_t \geq |\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|, \forall t \in \mathcal{T}$, so the cardinality of the set \mathcal{K}_t^- is positive for all $t \in \mathcal{T}$.

Lemma 1. *The solution satisfies: $\hat{\lambda}_t \geq |\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|, \forall t \in \mathcal{T}$*

Proof. We prove the lemma by recursion over $t \in \mathcal{T}$.

First, let us prove the property for $t = 1$. By convention, we have $|\mathcal{K}_0^+| = 0, |\mathcal{D}_0| = 0$ and $\hat{\lambda}_0 = 0$, so $|\mathcal{K}_1^-| = 0$. Then: we have $|\mathcal{K}_1^+| = (|\mathcal{D}_1| - \hat{\lambda}_1)^+ \geq |\mathcal{D}_1| - \hat{\lambda}_1$ and the property holds.

Let us now assume that the property holds for $t - 1$ and prove it for t , i.e., let us now assume that $\hat{\lambda}_{t-1} \geq |\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|$ and prove that $\hat{\lambda}_t \geq |\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|$.

- If $|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| \leq 0$, . The property is clearly satisfied because $|\mathcal{K}_t^-| \geq 0$, and $|\mathcal{K}_t^+| \geq 0$.
- If $|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| > 0$, then:
 - If $|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| \leq \hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|)$, then $|\mathcal{K}_t^-| = |\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+|$ and $|\mathcal{K}_t^+| = 0$. The property is satisfied (as an equality).
 - If $|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| > \hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|)$, then $|\mathcal{K}_t^-| = \hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|)$ and $|\mathcal{K}_t^+| = |\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-|$. Note that \mathcal{K}_t^- is well defined, because its cardinality is positive by the induction hypothesis. Thus the property is still satisfied (as an equality).

This concludes the proof. \square

We now prove that the solution thus constructed is feasible and optimal for Problem (EFF). We partition the set of time periods \mathcal{T} into periods with demand greater than the capacity, and periods with demand less than or equal to the capacity. We denote by $\mathcal{T}^+ = \{t \in \mathcal{T}; |\mathcal{D}_t| > \hat{\lambda}_t\}$, and $\mathcal{T}^- = \mathcal{T} \setminus \mathcal{T}^+$. Lemma 2 shows that the solution displaces exactly $|\mathcal{D}_t| - \hat{\lambda}_t$ flights (i.e., the flights in excess of capacity) from periods in \mathcal{T}^+ (i.e., from periods with demand greater than the capacity), and no flight from periods in \mathcal{T}^- (i.e., periods with demand less than or equal to the capacity). In other words, under hypotheses (\mathcal{H}_t) , the imbalances between demand and capacity are small enough so no period is such that flights get displaced *to* and *from* that period. Moreover,

Lemma 2 shows that the resulting schedule is feasible (i.e., meets the schedule limits constraints of (EFF)) and optimal.

Lemma 2. *For each $t \in \mathcal{T}^+$, the following properties are satisfied: (i) $|\mathcal{K}_{t-1}^+| = 0$, (ii) $|\mathcal{K}_{t+1}^-| = 0$, (iii) $|\mathcal{K}_t^-| + |\mathcal{K}_t^+| = |\mathcal{D}_t| - \hat{\lambda}_t$, and (iv) if $|\mathcal{K}_t^+| > 0$, then $|\mathcal{D}_{t+1}| + |\mathcal{K}_t^+| \leq \hat{\lambda}_{t+1}$ and $|\mathcal{K}_{t+1}^-| = |\mathcal{K}_{t+1}^+| = 0$.*

For each $t \in \mathcal{T}^-$, the following properties are satisfied: (iv) $|\mathcal{K}_t^-| = 0$, (v) $|\mathcal{K}_t^+| = 0$, (vi) $|\mathcal{K}_{t+1}^-| \leq \hat{\lambda}_t - |\mathcal{D}_t| - |\mathcal{K}_{t-1}^+|$, and (vii) if $|\mathcal{K}_{t-1}^+| > 0$, then $|\mathcal{K}_{t+1}^+| = 0$,

Moreover, $|\mathcal{K}_1^-| = |\mathcal{K}_T^+| = 0$.

Therefore, the solution is feasible and optimal for Problem (EFF).

The rationale underlying the lemma is the following. For any initial schedule, flights get rescheduled from period 1 to period T recursively, and, for each period t , the number of flights in the resulting schedule is less than or equal to capacity during each period $s = 1, \dots, t$, since all the excess flights are rescheduled to the “next” period. In the general case, this could lead to unfeasibility if “too many” flights got carried over from period t to period $t + 1$. But under hypotheses (\mathcal{H}_t) , the imbalances are small enough that this situation does not occur and, as Properties (iv) and (vii) show, $|\mathcal{K}_t^+| > 0$ for at most one period t in every set of three consecutive periods. We now prove formally the lemma, and thus the feasibility and optimality of this solution under hypotheses (\mathcal{H}_t) .

Proof. We proceed by induction over $t = 1, \dots, T$.

First, let us prove it for $t = 1$:

- If $|\mathcal{D}_1| - \hat{\lambda}_1 > 0$, then $|\mathcal{K}_1^-| = 0$ and $|\mathcal{K}_1^+| = |\mathcal{D}_1| - \hat{\lambda}_1$ (as in Lemma 1). Also, by definition, $\mathcal{K}_0^+ = \emptyset$. This proves (i) and (iii). Moreover, from (\mathcal{H}_1) , we have: $|\mathcal{D}_2| + |\mathcal{K}_1^+| = |\mathcal{D}_2| + |\mathcal{D}_1| - \hat{\lambda}_1 \leq \hat{\lambda}_2$, so $|\mathcal{K}_2^-| = 0$ and $|\mathcal{K}_2^+| = 0$. This proves (ii) and (iv).
- If $|\mathcal{D}_1| - \hat{\lambda}_1 \leq 0$, then $|\mathcal{K}_1^-| = |\mathcal{K}_1^+| = 0$. Properties (iv), (v) and (vii) are clearly satisfied. This also implies that $|\mathcal{K}_2^-| \leq \hat{\lambda}_1 - |\mathcal{D}_1|$, which proves (vi).

This also proves that $|\mathcal{K}_1^-| = 0$.

We now assume that all properties hold for $s = 1, \dots, t - 1$. We want to prove them for t . To do so, we consider 4 cases: – Case 1.a. $t \in \mathcal{T}^+$ and $t - 1 \in \mathcal{T}^+$ – Case 1.b. $t \in \mathcal{T}^+$ and $t - 1 \in \mathcal{T}^-$ – Case 2.a. $t \in \mathcal{T}^-$ and $t - 1 \in \mathcal{T}^+$ – Case 2.b. $t \in \mathcal{T}^-$ and $t - 1 \in \mathcal{T}^-$

Case 1.a. $t \in \mathcal{T}^+$ and $t - 1 \in \mathcal{T}^+$: In this case, we have $t - 2 \in \mathcal{T}^-$ (otherwise, hypothesis (\mathcal{H}_{t-1}) would clearly not be satisfied). From the induction hypotheses (iv) and (v) applied to $t - 2$, we have $|\mathcal{K}_{t-2}^-| = |\mathcal{K}_{t-2}^+| = 0$. Let us first prove that $|\mathcal{K}_{t-1}^+| = 0$.

- If $|\mathcal{K}_{t-3}^+| > 0$, then $|\mathcal{K}_{t-1}^+| = 0$ from the induction hypothesis (vii) applied to $t - 2$.
- If $|\mathcal{K}_{t-3}^+| = 0$, then we have, from (\mathcal{H}_{t-1}) : $|\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1} \leq (\hat{\lambda}_{t-2} - |\mathcal{D}_{t-2}|) + (\hat{\lambda}_t - |\mathcal{D}_t|)$, thus, $|\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1} \leq (\hat{\lambda}_{t-2} - |\mathcal{D}_{t-2}|)$ as $t \in \mathcal{T}^+$, and therefore: $|\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1} + |\mathcal{K}_{t-2}^+| \leq \hat{\lambda}_{t-2} - (|\mathcal{D}_{t-2}| + |\mathcal{K}_{t-3}^+| - |\mathcal{K}_{t-2}^-| - |\mathcal{K}_{t-2}^+|)$. In turn, $|\mathcal{K}_{t-1}^-| = |\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1}$, and $|\mathcal{K}_{t-1}^+| = 0$.

This proves (i).

We then have, from the induction hypothesis (iii) applied to $t-1$: $|\mathcal{K}_{t-1}^-| = |\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1}$, so $\hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|) = 0$. As a result, $|\mathcal{K}_t^-| = 0$, and therefore: $|\mathcal{K}_t^+| = |\mathcal{D}_t| - \hat{\lambda}_t$. This proves (iii). Moreover, this implies $|\mathcal{K}_{t+1}^-| \leq \hat{\lambda}_t - (|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|) = 0$, which proves (ii).

Then, $|\mathcal{D}_{t+1}| + |\mathcal{K}_t^+| = |\mathcal{D}_{t+1}| + |\mathcal{D}_t| - \hat{\lambda}_t \leq \hat{\lambda}_{t+1} + (\hat{\lambda}_{t-1} - |\mathcal{D}_{t-1}|)$, from (\mathcal{H}_t) . Since $t-1 \in \mathcal{T}^+$, it implies: $|\mathcal{D}_{t+1}| + |\mathcal{K}_t^+| \leq \hat{\lambda}_{t+1}$, so $|\mathcal{K}_{t+1}^-| = 0$ and then $|\mathcal{K}_{t+1}^+| = 0$. This proves (iv).

Case 1.b. $t \in \mathcal{T}^+$ and $t-1 \in \mathcal{T}^-$: From the induction hypotheses (iv) and (v) applied to $t-1$, we have $|\mathcal{K}_{t-1}^-| = |\mathcal{K}_{t-1}^+| = 0$. This proves (i).

We now prove (iii) and (iv):

- If $|\mathcal{K}_{t-2}^+| > 0$, then from the induction hypothesis (vii) applied to $t-1$, $|\mathcal{K}_t^+| = 0$. This implies that $|\mathcal{D}_t| - \hat{\lambda}_t \leq |\mathcal{K}_t^-|$. But since $|\mathcal{K}_t^-| \leq |\mathcal{D}_t| - \hat{\lambda}_t$, it means that $|\mathcal{K}_t^-| = |\mathcal{D}_t| - \hat{\lambda}_t$. This proves (iii). (Since $|\mathcal{K}_t^+| = 0$, (iv) is trivially satisfied).
- If $|\mathcal{K}_{t-2}^+| = 0$
 - If $|\mathcal{D}_t| - \hat{\lambda}_t \leq \hat{\lambda}_{t-1} - |\mathcal{D}_{t-1}|$, then $|\mathcal{K}_t^-| = |\mathcal{D}_t| - \hat{\lambda}_t$ and $|\mathcal{K}_t^+| = 0$. This proves (iii). (Since $|\mathcal{K}_t^+| = 0$, (iv) is trivially satisfied).
 - If $|\mathcal{D}_t| - \hat{\lambda}_t > \hat{\lambda}_{t-1} - |\mathcal{D}_{t-1}|$, then $|\mathcal{K}_t^-| = \hat{\lambda}_{t-1} - |\mathcal{D}_{t-1}|$ and $|\mathcal{K}_t^+| = |\mathcal{D}_t| - \hat{\lambda}_t - |\mathcal{K}_t^-|$. This proves (iii). Then, we have: $|\mathcal{D}_{t+1}| + |\mathcal{K}_t^+| = |\mathcal{D}_{t+1}| + (|\mathcal{D}_t| - \hat{\lambda}_t) + (|\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1}) \leq \hat{\lambda}_{t+1}$ from (\mathcal{H}_t) . As a result, $|\mathcal{K}_{t+1}^-| = 0$, and $|\mathcal{K}_{t+1}^+| = 0$. This proves (iv).

Moreover, (i) and (iii) imply $|\mathcal{K}_{t+1}^-| \leq \hat{\lambda}_t - (|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|) = 0$, which proves (ii).

Case 2.a. $t \in \mathcal{T}^-$ and $t-1 \in \mathcal{T}^+$: From the induction hypothesis (i) applied to $t-1$, we have $|\mathcal{K}_{t-2}^+| = 0$. From the induction hypothesis (iii) applied to $t-1$, we have: $|\mathcal{K}_{t-1}^-| + |\mathcal{K}_{t-1}^+| = |\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1}$. Therefore, $\hat{\lambda}_{t-1} - (|\mathcal{D}_{t-1}| + |\mathcal{K}_{t-2}^+| - |\mathcal{K}_{t-1}^-| - |\mathcal{K}_{t-1}^+|) = 0$ and thus $|\mathcal{K}_t^-| = 0$. This proves (iv). We now want to prove (v) and (vii).

- If $|\mathcal{K}_{t-1}^+| = 0$, then $|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| = |\mathcal{D}_t| - \hat{\lambda}_t \leq 0$, so $|\mathcal{K}_t^-| = 0$ and $|\mathcal{K}_t^+| = 0$. This proves (v). (Since $|\mathcal{K}_{t-1}^+| = 0$, (vii) is trivially satisfied).
- If $|\mathcal{K}_{t-1}^+| > 0$, then from the induction hypothesis (iv) applied to $t-1$, we have $|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| \leq \hat{\lambda}_t$ and $|\mathcal{K}_t^+| = 0$. This proves (v). Moreover, from induction hypothesis (iii) applied to $t-1$, we have: $|\mathcal{K}_{t-1}^+| \leq |\mathcal{D}_{t-1}| - \hat{\lambda}_{t-1}$. Therefore (since $|\mathcal{K}_t^-| = 0$), we obtain from (\mathcal{H}_t) : $\hat{\lambda}_t - (|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|) = \hat{\lambda}_t - |\mathcal{D}_t| - |\mathcal{K}_{t-1}^+| \geq \hat{\lambda}_t - |\mathcal{D}_t| + \hat{\lambda}_{t-1} - |\mathcal{D}_{t-1}| \geq |\mathcal{D}_{t+1}| - \hat{\lambda}_{t+1} = |\mathcal{D}_{t+1}| - \hat{\lambda}_{t+1} + |\mathcal{K}_t^+|$. This implies $|\mathcal{K}_{t+1}^-| = (|\mathcal{D}_{t+1}| - \hat{\lambda}_{t+1} + |\mathcal{K}_t^+|)^+ = (|\mathcal{D}_{t+1}| - \hat{\lambda}_{t+1})^+$ and $|\mathcal{K}_{t+1}^+| = 0$. This proves (vii).

Finally, (iv) and (v) imply $|\mathcal{K}_{t+1}^-| \leq \hat{\lambda}_t - (|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|) = \hat{\lambda}_t - |\mathcal{D}_t| - |\mathcal{K}_{t-1}^+|$, which proves (vi).

Case 2.b. $t \in \mathcal{T}^-$ and $t-1 \in \mathcal{T}^-$: From the induction hypothesis (v) applied to $t-1$, we have $|\mathcal{K}_{t-1}^+| = 0$, so $|\mathcal{D}_t| - \hat{\lambda}_t + |\mathcal{K}_{t-1}^+| \leq 0$. Then, $|\mathcal{K}_t^-| = 0$ and $|\mathcal{K}_t^+| = 0$. This proves (iv) and (v). As in Case 2.a, (iv) and (v) imply (vi) and, last, since $|\mathcal{K}_{t-1}^+| = 0$, (vii) is trivially satisfied.

Last, in order for the solution to be feasible for Problem (EFF), we investigate the recursion for period T separately to check that the recursion terminates without requiring to displace flights later than period T .

- If $T \in \mathcal{T}^+$, then $T-1 \in \mathcal{T}^-$ (otherwise (\mathcal{H}_T) is not satisfied). Thus, $|\mathcal{K}_{T-1}^-| = |\mathcal{K}_{T-1}^+| = 0$ from the induction hypotheses (iv) and (v). This proves (i).
 - If $|\mathcal{K}_{T-2}^+| > 0$, then $|\mathcal{K}_T^+| = 0$, from the induction hypothesis (vii) applied to $T-1$ (this proves (iv)). This implies that $|\mathcal{D}_T| - \hat{\lambda}_T \leq |\mathcal{K}_T^-|$. But since $|\mathcal{K}_T^-| \leq |\mathcal{D}_T| - \hat{\lambda}_T$, it means that $|\mathcal{K}_T^-| = |\mathcal{D}_T| - \hat{\lambda}_T$. This proves (iii). Since $|\mathcal{D}_{T+1}| = \hat{\lambda}_{T+1} = 0$, we then obtain $|\mathcal{K}_{T+1}^-| = 0$. This proves (ii).
 - If $|\mathcal{K}_{T-2}^+| = 0$, then we have $\hat{\lambda}_{T-1} - (|\mathcal{D}_{T-1}| + |\mathcal{K}_{T-2}^+| - |\mathcal{K}_{T-1}^-| - |\mathcal{K}_{T-1}^+|) = \hat{\lambda}_{T-1} - |\mathcal{D}_{T-1}| \geq |\mathcal{D}_T| - \hat{\lambda}_T$ from (\mathcal{H}_T) . In turn, $|\mathcal{K}_T^-| = |\mathcal{D}_T| - \hat{\lambda}_T$ and $|\mathcal{K}_T^+| = 0$. This proves (iii) and (iv). Since $|\mathcal{D}_{T+1}| = \hat{\lambda}_{T+1} = 0$, we then obtain $|\mathcal{K}_{T+1}^-| = 0$. This proves (ii).
- If $T \in \mathcal{T}^-$, then if $|\mathcal{K}_{T-1}^+| > 0$ (which will require $T-1 \in \mathcal{T}^+$ as per induction hypothesis (v) applied to $T-1$), then $|\mathcal{K}_T^-| = 0$, $|\mathcal{K}_T^+| = 0$, and $|\mathcal{D}_T| + |\mathcal{K}_{T-1}^+| \leq \hat{\lambda}_T$ from the induction hypothesis (iv) applied to $T-1$. Since $|\mathcal{K}_{T+1}^-| = 0$, this proves (i), (ii) and (iii). In the case of $|\mathcal{K}_{T-1}^+| = 0$, $\left(|\mathcal{D}_T| - \hat{\lambda}_T + |\mathcal{K}_{T-1}^+|\right)^+ = 0$, so $|\mathcal{K}_T^-| = |\mathcal{K}_T^+| = 0$ and $|\mathcal{D}_T| + |\mathcal{K}_{T-1}^+| \leq \hat{\lambda}_T$, which proves (i), (ii) and (iii).

This also proves that $|\mathcal{K}_T^+| = 0$ and that the recursion terminates.

This completes the proof of Properties (i) to (vii).

We now use these properties to show that the solution is feasible. Since all flights are rescheduled by at most one period, for each period $t \in \mathcal{T}$, the number of flights scheduled during period t is equal to $|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| + |\mathcal{K}_{t+1}^-| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+|$, and we need to show that it is smaller than $\hat{\lambda}_t$.

- For each period $t \in \mathcal{T}^+$, $|\mathcal{K}_{t-1}^+| = 0$ (Property (i)), $|\mathcal{K}_{t+1}^-| = 0$ (Property (ii)) and $|\mathcal{D}_t| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+| = \hat{\lambda}_t$ (Property (iii)). This proves that $|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| + |\mathcal{K}_{t+1}^-| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+| \leq \hat{\lambda}_t$.
- For each period $t \in \mathcal{T}^-$, $|\mathcal{K}_t^-| = 0$ (Property (iv)), $|\mathcal{K}_t^+| = 0$ (Property (v)), and $|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| + |\mathcal{K}_{t+1}^-| \leq \hat{\lambda}_t$ (Property (vi)). This proves that $|\mathcal{D}_t| + |\mathcal{K}_{t-1}^+| + |\mathcal{K}_{t+1}^-| - |\mathcal{K}_t^-| - |\mathcal{K}_t^+| \leq \hat{\lambda}_t$.

Last, we prove that the solution is optimal. First, $|u_i| \leq 1, \forall i \in \mathcal{F}$. So the maximal flight

displacement $\max_{i \in \mathcal{F}} |u_i|$ is equal to 0 if $|\mathcal{D}_t| \leq \hat{\lambda}_t, \forall t \in \mathcal{T}$, and to 1 otherwise. Moreover,:

$$\begin{aligned}
\sum_{i \in \mathcal{F}} |u_i| &= \sum_{i \in \mathcal{F}} |u_i| \\
&= \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{D}_t} |u_i| \\
&= \sum_{t \in \mathcal{T}} (|\mathcal{K}_t^-| + |\mathcal{K}_t^+|) \\
&= \sum_{t \in \mathcal{T}^+} (|\mathcal{K}_t^-| + |\mathcal{K}_t^+|) \quad \text{from Properties (iv) and (v)} \\
&= \sum_{t \in \mathcal{T}^+} (|\mathcal{D}_t| - \hat{\lambda}_t) \quad \text{from Property (iii)} \\
&= \sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+
\end{aligned}$$

Since any feasible solution of the problem has to displace at least $\sum_{t \in \mathcal{T}} (|\mathcal{D}_t| - \hat{\lambda}_t)^+$ flights by at least 1 period each, we have shown that the proposed solution solves (EFF). \square

Appendix 2: Construction of a solution for Proposition 3

In this Appendix, we characterize the optimal solution of Problem $\mathcal{P}(\Delta)$ defined as follows, where Δ designates any non-negative integer. Problem $\mathcal{P}(\Delta)$ involves allocating a “budget” of Δ items (think of each item as an inconvenience or cost) across “groups” (or airlines, in our case) indexed by $a \in \mathcal{A}$ in a way that lexicographically minimizes the weighted cost borne by any group, where the weight for each group a is given by $\frac{1}{|\mathcal{F}_a|}$.

$$\begin{aligned}
&\text{lex min} \quad \left(\frac{U_a}{|\mathcal{F}_a|} \right)_{a \in \mathcal{A}} \\
&\text{s.t.} \quad \sum_{a \in \mathcal{A}} U_a \geq \Delta \\
&\quad U_a \in \mathbb{Z}^+, \forall a \in \mathcal{A}
\end{aligned}$$

We introduce the following notations. We denote the indicator function by $\mathbb{1}$. For each solution vector $(U_a)_{a \in \mathcal{A}}$, we sort $\left(\frac{U_a}{|\mathcal{F}_a|} \right)_{a \in \mathcal{A}}$ by non-increasing order, and we denote by $s_i(U)$ the i -th element of the resulting vector. In other words, $s_1(U) = \max_{a \in \mathcal{A}} \frac{U_a}{|\mathcal{F}_a|}$. Recursively, if a_1, \dots, a_{i-1} are such that $\frac{U_{a_j}}{|\mathcal{F}_{a_j}|} = s_j(U), \forall j = 1, \dots, i-1$, then $s_i(U) = \max_{a \in \mathcal{A}: a \neq a_1, \dots, a_{i-1}} \frac{U_a}{|\mathcal{F}_a|}$. We denote by $\Theta(U)$ the set of indices that attain the maximum of $\left(\frac{U_a}{|\mathcal{F}_a|} \right)_{a \in \mathcal{A}}$, i.e., $\Theta(U) = \left\{ a \in \mathcal{A}, \frac{U_a}{|\mathcal{F}_a|} = s_1(U) \right\}$. Last, we write $U \succeq_{\text{lex}} V$ (resp. $U \succ_{\text{lex}} V$) to signify that $\left(\frac{U_a}{|\mathcal{F}_a|} \right)_{a \in \mathcal{A}}$ is lexicographically larger (resp. lexicographically strictly larger) than $\left(\frac{V_a}{|\mathcal{F}_a|} \right)_{a \in \mathcal{A}}$. In other words, $U \succ_{\text{lex}} V$ if $\exists i \in \{1, \dots, |\mathcal{A}|\}$, such that $s_1(U) = s_1(V), s_2(U) = s_2(V), \dots, s_{i-1}(U) = s_{i-1}(V)$ and $s_i(U) > s_i(V)$, and $U \succeq_{\text{lex}} V$ if $U \succ_{\text{lex}} V$ or $s_i(U) = s_i(V), \forall i = 1, \dots, |\mathcal{A}|$.

Lemma 3 shows that, for any optimal solution of $\mathcal{P}(\Delta)$, the constraint $\sum_{a \in \mathcal{A}} U_a \geq \Delta$ is binding.

Lemma 3. *Any optimal solution $(U_a)_{a \in \mathcal{A}}$ of $\mathcal{P}(\Delta)$ satisfies $\sum_{a \in \mathcal{A}} U_a = \Delta$.*

Proof. Let us assume that $(U_a)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta)$ such that $\sum_{a \in \mathcal{A}} U_a > \Delta$. We denote by ε the integer defined by $\varepsilon = \sum_{a \in \mathcal{A}} U_a - \Delta > 0$. We introduce a vector $(\eta_a)_{a \in \mathcal{A}}$ of non-negative integers which satisfies the condition $\sum_{a \in \mathcal{A}} \eta_a = \varepsilon$ (Note that $(\eta_a)_{a \in \mathcal{A}}$ thus defined is not unique, but we can choose any vector that satisfies these properties). We then define $U_a^* = U_a - \eta_a, \forall a \in \mathcal{A}$. By construction, $\sum_{a \in \mathcal{A}} U_a^* = \Delta$, $U_a^* \leq U_a, \forall a \in \mathcal{A}$ and $\exists a \in \mathcal{A}, U_a^* < U_a$. Thus $U \succ_{\text{lex}} U^*$, which contradicts the fact that $(U_a)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta)$. \square

In Lemma 4, we prove an intermediate result that shows that, if we start with an optimal solution of Problem $\mathcal{P}(\Delta)$ and we add one item to any group, then the resulting weighted cost borne by that group is at least equal to the optimal value of the largest weighted cost.

Lemma 4. *If $(U_a)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta)$, then $\frac{U_c+1}{|\mathcal{F}_c|} \geq s_1(U), \forall c \in \mathcal{A}$.*

Proof. Let us consider $c \in \mathcal{A}$. If $\frac{U_c}{|\mathcal{F}_c|} = s_1(U)$, then $\frac{U_c+1}{|\mathcal{F}_c|} > s_1(U)$. If $\frac{U_c}{|\mathcal{F}_c|} < s_1(U)$, then $\exists b \neq c, \frac{U_b}{|\mathcal{F}_b|} = s_1(U)$ as the set \mathcal{A} is non-empty and finite. We define \bar{U} as follows: $\bar{U}_c = U_c + 1$, $\bar{U}_b = U_b - 1$ and $\bar{U}_a = U_a, \forall a \neq b, c$. \bar{U} is a feasible solution of $\mathcal{P}(\Delta)$. If $|\Theta(U)| > 1$, then $U_a = \bar{U}_a, \forall a \in \Theta(U) \setminus \{b\}$ and $s_1(U) = s_2(U) = \dots, s_{|\Theta(U)|-1}(U) = s_1(\bar{U}) = \dots, s_{|\Theta(U)|-1}(\bar{U})$. Since U is an optimal solution of $\mathcal{P}(\Delta)$, $s_{|\Theta(U)|}(\bar{U}) \geq s_{|\Theta(U)|}(U) = s_1(U)$. We have: $s_{|\Theta(U)|}(\bar{U}) = \max \left\{ \max_{a \notin \Theta(U), a \neq c} \frac{U_a}{|\mathcal{F}_a|}, \frac{U_c+1}{|\mathcal{F}_c|}, \frac{U_b-1}{|\mathcal{F}_b|} \right\}$. We know that $\max_{a \notin \Theta(U), a \neq c} \frac{U_a}{|\mathcal{F}_a|} < s_1(U)$ and that $\frac{U_b-1}{|\mathcal{F}_b|} < s_1(U)$. This implies that $\frac{U_c+1}{|\mathcal{F}_c|} \geq s_1(U)$. \square

Lemmas 3 and 4 show that if the constraint $\sum_{a \in \mathcal{A}} U_a \geq \Delta$ is not binding, or if there exists $c \in \mathcal{A}$ such that $\frac{U_c+1}{|\mathcal{F}_c|} < s_1(U)$, then U is not an optimal solution of $\mathcal{P}(\Delta)$. In Lemma 5, we identify necessary conditions over a feasible solution $(V_a)_{a \in \mathcal{A}}$ of Problem $\mathcal{P}(\Delta)$ to ensure that $U \succeq_{\text{lex}} V$ even if $(U_a)_{a \in \mathcal{A}}$ does not satisfy these two conditions.

Lemma 5. *Let $(U_a)_{a \in \mathcal{A}}$ be such that $\sum_{a \in \mathcal{A}} U_a = \Delta$ and $\frac{U_c+1}{|\mathcal{F}_c|} \geq s_1(U), \forall c \in \mathcal{A}$, and V denote any feasible solution of $\mathcal{P}(\Delta)$. If $U \succeq_{\text{lex}} V$, then there exist $b_1, \dots, b_q, c_1, \dots, c_q \in \mathcal{A}$ such that: (i) $\frac{U_{b_i}}{|\mathcal{F}_{b_i}|} = s_1(U), \forall i = 1, \dots, q$, (ii) $\frac{U_{c_i}+1}{|\mathcal{F}_{c_i}|} = s_1(U), \forall i = 1, \dots, q$, (iii) $V_{b_i} = U_{b_i} - 1, \forall i = 1, \dots, q$, (iv) $V_{c_i} = U_{c_i} + 1, \forall i = 1, \dots, q$, and (v) $V_a = U_a, \forall a \neq b_1, \dots, b_q, c_1, \dots, c_q$.*

Proof. Let $(U_a)_{a \in \mathcal{A}}$ be a feasible solution of $\mathcal{P}(\Delta)$ such that $\sum_{a \in \mathcal{A}} U_a = \Delta$ and $\frac{U_c+1}{|\mathcal{F}_c|} \geq s_1(U), \forall c \in \mathcal{A}$. We can partition the set \mathcal{A} into $\Omega_1 = \left\{ a \in \mathcal{A}, \frac{U_a}{|\mathcal{F}_a|} = s_1(U) \right\}$, $\Omega_2 = \left\{ a \in \mathcal{A}, \frac{U_a+1}{|\mathcal{F}_a|} = s_1(U) \right\}$ and $\Omega_3 = \left\{ a \in \mathcal{A}, \frac{U_a}{|\mathcal{F}_a|} < s_1(U) < \frac{U_a+1}{|\mathcal{F}_a|} \right\}$. Let V be any feasible solution of $\mathcal{P}(\Delta)$ such that $U \succeq_{\text{lex}} V$.

First, we note that $V_b \leq U_b, \forall b \in \Omega_1 \cup \Omega_3$ and $V_b \leq U_b + 1, \forall b \in \Omega_2$. Indeed, if $\exists b \in \Omega_1 \cup \Omega_3, V_b > U_b$ (i.e., $V_b \geq U_b + 1$ since V_b and U_b are integer), then $\frac{V_b}{|\mathcal{F}_b|} \geq \frac{U_b+1}{|\mathcal{F}_b|} > s_1(U)$, thus $V \succ_{\text{lex}} U$. Similarly, if $\exists b \in \Omega_2, V_b \geq U_b + 2$, then $\frac{V_b}{|\mathcal{F}_b|} \geq \frac{U_b+1}{|\mathcal{F}_b|} + \frac{1}{|\mathcal{F}_b|} = s_1(U) + \frac{1}{|\mathcal{F}_b|} > s_1(U)$, thus again $V \succ_{\text{lex}} U$. We denote by $X_1^- = \sum_{a \in \Omega_1} \mathbb{1}(V_a < U_a)$ and $X_3^- = \sum_{a \in \Omega_3} \mathbb{1}(V_a < U_a)$, and by $X_2^+ = \sum_{a \in \Omega_2} \mathbb{1}(V_a = U_a + 1)$.

Second, we show that $X_2^+ = X_1^-$ and $X_3^- = 0$. For every $b \in \mathcal{A}$ such that $V_b < U_b$ (i.e., $V_b \leq U_b - 1$), there exists some corresponding $c \in \Omega_2, V_c = U_c + 1$. This is because: (i) $\sum_{a \in \mathcal{A}} U_a = \Delta$ and $\sum_{a \in \mathcal{A}} V_a \geq \Delta$; (ii) $V_a \leq U_a + 1, \forall a \in \mathcal{A}$ (otherwise, $\frac{V_a}{|\mathcal{F}_a|} > \frac{U_a+1}{|\mathcal{F}_a|} \geq s_1(U)$, which contradicts $U \succeq_{\text{lex}} V$); and (iii) $V_a \leq U_a, \forall a \in \Omega_1 \cup \Omega_3$. Therefore, $X_2^+ \geq X_1^- + X_3^-$. Moreover, the number of indices such that $\frac{V_a}{|\mathcal{F}_a|} = s_1(U)$ is equal to: $\sum_{a \in \mathcal{A}} \mathbb{1}\left(\frac{V_a}{|\mathcal{F}_a|} = s_1(U)\right) = \sum_{a \in \Omega_1} \mathbb{1}(V_a = U_a) + \sum_{a \in \Omega_2} \mathbb{1}(V_a = U_a + 1) = (|\Omega_1| - X_1^-) + X_2^+$. Therefore, $s_1(V) = \dots = s_{|\Omega_1| - X_1^- + X_2^+}(V) = s_1(U) = \dots = s_{|\Omega_1|}(U)$, while $s_{|\Omega_1|+1}(U) < s_1(U)$. Thus, if $X_2^+ > X_1^-$, then $V \succ_{\text{lex}} U$, which contradicts our assumption. Therefore, $X_2^+ = X_1^-$, which also implies that $X_3^- = 0$.

Third, we note that $V_b \geq U_b - 1, \forall b \in \Omega_1$. Indeed, if $\exists b_0 \in \Omega_1, V_{b_0} \leq U_{b_0} - 2$, then $\exists c_0, d_0 \in \Omega_2, c_0 \neq d_0, V_{c_0} = U_{c_0} + 1$ and $V_{d_0} = U_{d_0} + 1$, and for each $b \in \Omega_1$ such that $V_b < U_b$, there exists $c \in \Omega_2, c \neq c_0, d_0$ such that $V_c = U_c + 1$ (this is a direct consequence of points (i), (ii) and (iii) in the previous paragraph). This results in $X_2^+ \geq X_1^- + 1$, which contradicts $X_2^+ = X_1^-$.

In summary, we have shown that, if $U \succeq_{\text{lex}} V$, then $V_b = U_b, \forall b \in \Omega_3$ (since $X_3^- = 0$) and for each $b \in \Omega_1$ such that $V_b < U_b$, $V_b = U_b - 1$ and there exists exactly one corresponding $c \in \Omega_2$ such that $V_c = U_c + 1$. Also, for each $c \in \Omega_2$ such that $V_c > U_c$, $V_c = U_c + 1$ and there exists exactly one corresponding $b \in \Omega_1$ such that $V_b = U_b - 1$. Therefore, there exist $b_1, \dots, b_q \in \Omega_1$ and $c_1, \dots, c_q \in \Omega_2$ such that $V_{b_j} = U_{b_j} - 1, \forall j = 1, \dots, q$, $V_{c_j} = U_{c_j} + 1, \forall j = 1, \dots, q$, and $V_a = U_a, \forall a \neq b_1, \dots, b_q, c_1, \dots, c_q$. \square

Lemma 6 then shows that if we start with a solution of Problem $\mathcal{P}(\Delta)$, then we can construct a solution of Problem $\mathcal{P}(\Delta - 1)$ by removing one item from the group (or one of the groups) that bears the largest weighted cost.

Lemma 6. *If $\Delta \geq 1$ and $(U_a)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta)$, then there exists an optimal solution $(U_a^0)_{a \in \mathcal{A}}$ of $\mathcal{P}(\Delta - 1)$ and $a_0 \in \mathcal{A}$ such that: $U_{a_0}^0 = U_{a_0} - 1$ and $U_a^0 = U_a, \forall a \neq a_0$.*

Proof. Let $(U_a)_{a \in \mathcal{A}}$ be an optimal solution of $\mathcal{P}(\Delta)$. We choose $a_0 \in \arg \min_{a \in \Theta(U)} |\mathcal{F}_a|$, and we define U^0 such that: $U_{a_0}^0 = U_{a_0} - 1$ and $U_a^0 = U_a, \forall a \neq a_0$. Note that $U_a^0 \leq U_a, \forall a \in \mathcal{A}$, so $s_i(U) \geq s_i(U^0), \forall i = 1, \dots, |\mathcal{A}|$. Let $(V_a)_{a \in \mathcal{A}}$ be any feasible solution of $\mathcal{P}(\Delta - 1)$ and we need to show that $V \succeq_{\text{lex}} U^0$.

First, we have $\sum_{a \in \mathcal{A}} U_a^0 = \Delta - 1$. According to Lemma 4, we know that $\frac{U_a+1}{|\mathcal{F}_a|} \geq s_1(U), \forall a \in \mathcal{A}$, so $\frac{U_{a_0}^0+1}{|\mathcal{F}_{a_0}|} \geq s_1(U), \forall a \neq a_0$. Since $\frac{U_{a_0}^0+1}{|\mathcal{F}_{a_0}|} = \frac{U_{a_0}}{|\mathcal{F}_{a_0}|} = s_1(U)$, we also have $\frac{U_a^0+1}{|\mathcal{F}_a|} \geq s_1(U) \geq s_1(U^0), \forall a \in \mathcal{A}$. Therefore, U^0 satisfies the conditions of Lemma 5. Thus, $V \succ_{\text{lex}} U^0$ unless there exist $b_1, \dots, b_q, c_1, \dots, c_q \in \mathcal{A}$ such that: (i) $\frac{U_{b_i}^0}{|\mathcal{F}_{b_i}|} = s_1(U^0), \forall i = 1, \dots, q$, (ii) $\frac{U_{c_i}^0+1}{|\mathcal{F}_{c_i}|} = s_1(U^0), \forall i = 1, \dots, q$, (iii) $V_{b_i} = U_{b_i}^0 - 1, \forall i = 1, \dots, q$, (iv) $V_{c_i} = U_{c_i}^0 + 1, \forall i = 1, \dots, q$, and (v) $V_a = U_a^0, \forall a \neq b_1, \dots, b_q, c_1, \dots, c_q$. We now assume these conditions to be satisfied. By construction, $\frac{V_{c_i}}{|\mathcal{F}_{c_i}|} = \frac{U_{b_i}^0}{|\mathcal{F}_{b_i}|}, \forall i = 1, \dots, q$. Therefore, $V \succeq_{\text{lex}} U^0$ if and only if $\left(\frac{V_{b_i}}{|\mathcal{F}_{b_i}|}\right)_{i=1, \dots, q} \succeq_{\text{lex}} \left(\frac{U_{c_i}^0}{|\mathcal{F}_{c_i}|}\right)_{i=1, \dots, q}$.

We now note that if $\frac{U_{a_0}^0}{|\mathcal{F}_{a_0}|} = s_1(U^0)$, then $\frac{U_c^0+1}{|\mathcal{F}_c|} > s_1(U^0), \forall c \neq a_0$. We already know from Lemma 4 that $\frac{U_c^0+1}{|\mathcal{F}_c|} \geq s_1(U) \geq s_1(U^0), \forall c \neq a_0$. Let us assume that $\exists c \in \mathcal{A}, \frac{U_c^0+1}{|\mathcal{F}_c|} = \frac{U_{a_0}^0}{|\mathcal{F}_{a_0}|} = s_1(U^0)$. We can then define \bar{U} such that $\bar{U}_c = U_c + 1, \bar{U}_{a_0} = U_{a_0} - 1 = U_{a_0}^0$ and $\bar{U}_a = U_a, \forall a \neq a_0, c$. \bar{U} is a feasible solution of $\mathcal{P}(\Delta)$. Since $\frac{U_{a_0}^0}{|\mathcal{F}_{a_0}|} = s_1(U^0), \frac{U_a}{|\mathcal{F}_a|} = \frac{U_a^0}{|\mathcal{F}_a|} \leq \frac{U_{a_0}^0}{|\mathcal{F}_{a_0}|}, \forall a \neq a_0$. Therefore, $s_1(\bar{U}) = \frac{U_{a_0}-1}{|\mathcal{F}_{a_0}|} = \frac{U_c+1}{|\mathcal{F}_c|} < \frac{U_{a_0}}{|\mathcal{F}_{a_0}|} = s_1(U)$. This contradicts the fact that U is an optimal solution of $\mathcal{P}(\Delta)$.

Last, we show that for each b such that $\frac{U_b^0}{|\mathcal{F}_b|} = s_1(U^0)$ and for each c such that $\frac{U_c^0+1}{|\mathcal{F}_c|} = s_1(U^0)$, we have $\frac{U_b^0-1}{|\mathcal{F}_b|} \geq \frac{U_c^0}{|\mathcal{F}_c|}$. Based on the previous result, $b \neq a_0$ (otherwise there exists no c such that $\frac{U_c^0+1}{|\mathcal{F}_c|} = s_1(U^0)$), so we need to consider only the following two cases:

- (i) If $b, c \neq a_0$, we define \bar{U} as follows: $\bar{U}_c = U_c + 1, \bar{U}_b = U_b - 1$ and $\bar{U}_a = U_a, \forall a \neq b, c$. Since $\sum_{a \in \mathcal{A}} \bar{U}_a = \Delta$, \bar{U} is a feasible solution of $\mathcal{P}(\Delta)$, so $\bar{U} \succeq_{\text{lex}} U$. Since $\frac{U_b}{|\mathcal{F}_b|} = \frac{\bar{U}_c}{|\mathcal{F}_c|}$ and $\frac{U_a}{|\mathcal{F}_a|} = \frac{\bar{U}_a}{|\mathcal{F}_a|}, \forall a \neq b, c$, this implies that $\frac{\bar{U}_b}{|\mathcal{F}_b|} \geq \frac{U_c}{|\mathcal{F}_c|}$, i.e., $\frac{U_b-1}{|\mathcal{F}_b|} \geq \frac{U_c}{|\mathcal{F}_c|}$, i.e., $\frac{U_b^0-1}{|\mathcal{F}_b|} \geq \frac{U_c^0}{|\mathcal{F}_c|}$.
- (ii) If $c = a_0$, then $\frac{U_b^0}{|\mathcal{F}_b|} = \frac{U_{a_0}}{|\mathcal{F}_{a_0}|}$, i.e., $\frac{U_b}{|\mathcal{F}_b|} = \frac{U_{a_0}}{|\mathcal{F}_{a_0}|}$, and $b \in \Theta(U)$. By construction, $a_0 \in \arg \min_{a \in \Theta(U)} |\mathcal{F}_a|$, so $|\mathcal{F}_{a_0}| \leq |\mathcal{F}_b|$. We thus have $\frac{U_b^0}{|\mathcal{F}_b|} - \frac{1}{|\mathcal{F}_b|} \geq \frac{U_{a_0}}{|\mathcal{F}_{a_0}|} - \frac{1}{|\mathcal{F}_{a_0}|}$, i.e., $\frac{U_b^0-1}{|\mathcal{F}_b|} \geq \frac{U_{a_0}^0}{|\mathcal{F}_{a_0}|}$.

Therefore, $\frac{V_{b_i}}{|\mathcal{F}_{b_i}|} \geq \frac{U_{c_i}^0}{|\mathcal{F}_{c_i}|}, \forall i = 1, \dots, q$. This implies that $V \succeq_{\text{lex}} U^0$. \square

Lemma 7 extends Lemma 6 to construct, from a solution of Problem $\mathcal{P}(\Delta)$, solutions of Problems $\mathcal{P}(\Delta - 1), \dots, \mathcal{P}(0)$ such that each one differs from the following one by only one element.

Lemma 7. *If $\Delta \geq 1$ and $(U_a^\Delta)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta)$, then there exist $(U_a^{\Delta-1})_{a \in \mathcal{A}}, \dots, (U_a^0)_{a \in \mathcal{A}}$ that are optimal solutions of $\mathcal{P}(\Delta - 1), \dots, \mathcal{P}(0)$, respectively, and $a_1, \dots, a_\Delta \in \mathcal{A}$ such that: $U_{a_i}^{i-1} = U_{a_i}^i - 1$ and $U_a^{i-1} = U_a^i, \forall a \neq a_i, \forall i = 1, \dots, \Delta$.*

Proof. This is obtained directly by repeatedly applying Lemma 6 to $\mathcal{P}(\Delta), \mathcal{P}(\Delta - 1), \dots, \mathcal{P}(2)$ and $\mathcal{P}(1)$. \square

We now introduce the following notations. We denote by γ the greatest common divisor of $(|\mathcal{F}_a|)_{a \in \mathcal{A}}$, i.e., $\gamma = \gcd(|\mathcal{F}_a|)_{a \in \mathcal{A}}$. We also introduce $\xi_a = \frac{|\mathcal{F}_a|}{\gamma}$ and $N = \sum_{a \in \mathcal{A}} \xi_a$. We show in Lemma 8 that if we know a solution of Problem $\mathcal{P}(\Delta)$, then we can construct easily the solution of $\mathcal{P}(\Delta + N)$ by adding ξ_a items to each group a .

Lemma 8. *If $(U_a)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta)$, then $(U_a + \xi_a)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(\Delta + N)$.*

Proof. The construction of the proof is very similar to that of Lemma 6. Let $(U_a)_{a \in \mathcal{A}}$ be an optimal solution of $\mathcal{P}(\Delta)$. We have: $\frac{U_a + \xi_a}{|\mathcal{F}_a|} = \frac{1}{\gamma} + \frac{U_a}{|\mathcal{F}_a|}$, $\forall a \in \mathcal{A}$, and thus: $s_i(U + \xi) = \frac{1}{\gamma} + s_i(U)$, $\forall i = 1, \dots, |\mathcal{A}|$. Let $(V_a)_{a \in \mathcal{A}}$ be any feasible solution of $\mathcal{P}(\Delta + N)$ and we need to show that $V \succeq_{\text{lex}} U + \xi$.

First, we have $\sum_{a \in \mathcal{A}} (U_a + \xi_a) = \Delta + N$. According to Lemma 4, we know that $\frac{U_a + 1}{|\mathcal{F}_a|} \geq s_1(U)$, $\forall a \in \mathcal{A}$, so $\frac{U_a + \xi_a + 1}{|\mathcal{F}_a|} \geq s_1(U + \xi)$, $\forall a \in \mathcal{A}$. Therefore, $U + \xi$ satisfies the conditions of Lemma 5.

Thus, $V \succ_{\text{lex}} U + \xi$ unless there exist $b_1, \dots, b_q, c_1, \dots, c_q \in \mathcal{A}$ such that: (i) $\frac{U_{b_i} + \xi_{b_i}}{|\mathcal{F}_{b_i}|} = s_1(U + \xi)$, $\forall i = 1, \dots, q$, (ii) $\frac{U_{c_i} + \xi_{c_i} + 1}{|\mathcal{F}_{c_i}|} = s_1(U + \xi)$, $\forall i = 1, \dots, q$, (iii) $V_{b_i} = U_{b_i} + \xi_{b_i} - 1$, $\forall i = 1, \dots, q$, (iv) $V_{c_i} = U_{c_i} + \xi_{c_i} + 1$, $\forall i = 1, \dots, q$, and (v) $V_a = U_a + \xi_a$, $\forall a \neq b_1, \dots, b_q, c_1, \dots, c_q$. We now assume these conditions to be satisfied. By construction, $\frac{V_{c_i}}{|\mathcal{F}_{c_i}|} = \frac{U_{b_i} + \xi_{b_i}}{|\mathcal{F}_{b_i}|}$, $\forall i = 1, \dots, q$. Therefore, $V \succeq_{\text{lex}} U + \xi$ if and only if

$$\left(\frac{V_{b_i}}{|\mathcal{F}_{b_i}|} \right)_{i=1, \dots, q} \succeq_{\text{lex}} \left(\frac{U_{c_i} + \xi_{c_i}}{|\mathcal{F}_{c_i}|} \right)_{i=1, \dots, q}.$$

We now show that for each b such that $\frac{U_b + \xi_b}{|\mathcal{F}_b|} = s_1(U + \xi)$ and for each c such that $\frac{U_c + \xi_c + 1}{|\mathcal{F}_c|} = s_1(U + \xi)$, we have $\frac{U_b + \xi_b - 1}{|\mathcal{F}_b|} \geq \frac{U_c + \xi_c}{|\mathcal{F}_c|}$. We define \bar{U} as follows: $\bar{U}_c = U_c + 1$, $\bar{U}_b = U_b - 1$ and $\bar{U}_a = U_a$, $\forall a \neq b, c$. Since $\sum_{a \in \mathcal{A}} \bar{U}_a = \Delta$, \bar{U} is a feasible solution of $\mathcal{P}(\Delta)$, so $\bar{U} \succeq_{\text{lex}} U$. Since $\frac{U_b}{|\mathcal{F}_b|} = \frac{\bar{U}_c}{|\mathcal{F}_c|}$ and $\frac{U_a}{|\mathcal{F}_a|} = \frac{\bar{U}_a}{|\mathcal{F}_a|}$, $\forall a \neq b, c$, this implies that $\frac{\bar{U}_b}{|\mathcal{F}_b|} \geq \frac{U_c}{|\mathcal{F}_c|}$ (otherwise $U \succ_{\text{lex}} \bar{U}$), i.e.: $\frac{U_b - 1}{|\mathcal{F}_b|} \geq \frac{U_c}{|\mathcal{F}_c|}$, i.e.: $\frac{U_b + \xi_b - 1}{|\mathcal{F}_b|} \geq \frac{U_c + \xi_c}{|\mathcal{F}_c|}$.

Therefore, $\frac{V_{b_i}}{|\mathcal{F}_{b_i}|} \geq \frac{U_{c_i} + \xi_{c_i}}{|\mathcal{F}_{c_i}|}$, $\forall i = 1, \dots, q$. This implies that $V \succeq_{\text{lex}} U + \xi$. \square

Finally, Lemma 9 uses the result from Lemma 8 to construct a sequence of N elements a_1, \dots, a_N in \mathcal{A} that contains exactly ξ_a repetitions of each $a \in \mathcal{A}$ and from which we can construct the solution of Problem $\mathcal{P}(\Delta)$, for any $\Delta \geq 0$. Their order is chosen such that, for each $\Delta = 1, \dots, N$, we can construct a solution of Problem $\mathcal{P}(\Delta)$ by counting the number of times that a_i is equal to a , for $i = 1, \dots, \Delta$. In other words, $\mathcal{P}(\Delta)$ is solved by the vector U defined by $U_a = \sum_{i=1}^{\Delta} \mathbb{1}(a_i = a)$, $\forall a \in \mathcal{A}$. If $\Delta > N$, then we use a similar process based on the Euclidean division of Δ by N in combination with Lemma 8.

Lemma 9. *There exists a sequence $(a_1, \dots, a_N) \in \mathcal{A}^N$ such that, for any $\Delta \geq 0$, if q and r denote the quotient and remainder of the Euclidean division of Δ by N , then $(U_a)_{a \in \mathcal{A}}$ defined by $U_a = q\xi_a + \sum_{i=1}^r \mathbb{1}(a_i = a)$, $\forall a \in \mathcal{A}$ is an optimal solution of $\mathcal{P}(\Delta)$. (By convention, $\sum_{i=1}^0 \mathbb{1}(a_i = a) = 0$, $\forall a \in \mathcal{A}$).*

Proof. We consider an optimal solution $(U_a^N)_{a \in \mathcal{A}}$ of Problem $\mathcal{P}(N)$. According to Lemma 7, there exist $(U_a^0)_{a \in \mathcal{A}}$, \dots , $(U_a^{N-1})_{a \in \mathcal{A}}$ and $a_1, \dots, a_N \in \mathcal{A}$ such that for all $p = 1, \dots, N$: $(U_a^p)_{a \in \mathcal{A}}$ is an optimal solution of $\mathcal{P}(p)$ and $U_{a_p}^{p-1} = U_{a_p}^p - 1$ and $U_a^{p-1} = U_a^p$, $\forall a \neq a_p$. In other words, there exists a sequence a_1, \dots, a_N such that for all $p = 1, \dots, N - 1$, the vector U defined by $U_a = \sum_{i=1}^p \mathbb{1}(a_i = a)$ solves Problem $\mathcal{P}(p)$. We apply this result to $r \in \{0, \dots, N - 1\}$ to get the optimal solution $(U_a^r)_{a \in \mathcal{A}}$ of Problem $\mathcal{P}(r)$. Then according Lemma 8 (applied q times), the vector U defined by $U_a = q\xi_a + \sum_{i=1}^r \mathbb{1}(a_i = a)$, $\forall a \in \mathcal{A}$ is an optimal solution of $\mathcal{P}(r + qN)$, i.e., of $\mathcal{P}(\Delta)$. \square

We end with a simple example to illustrate how the solution of $\mathcal{P}(\Delta)$ is constructed. We consider a case with three “groups” (i.e., $|\mathcal{A}| = 3$) such that $|\mathcal{F}_1| = 20$, $|\mathcal{F}_2| = 30$ and $|\mathcal{F}_3| = 50$. We have $\gamma = 10$, $\xi_1 = 2$, $\xi_2 = 3$, $\xi_3 = 5$ and $N = 10$. We construct the sequence (a_1, \dots, a_N) by sorting in the increasing order the elements in the following set $\left\{ \frac{1}{\xi_1}, \frac{2}{\xi_1}, \dots, \frac{\xi_1}{\xi_1}, \frac{1}{\xi_2}, \frac{2}{\xi_2}, \dots, \frac{\xi_2}{\xi_2}, \frac{1}{\xi_3}, \frac{2}{\xi_3}, \dots, \frac{\xi_3}{\xi_3} \right\}$, i.e., in the set $\left\{ \frac{1}{2}, 1, \frac{1}{3}, \frac{2}{3}, 1, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1 \right\}$. The sorted set is $\left\{ \frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}, 1, 1, 1 \right\}$. By taking the corresponding indices, we obtain the following sequence: $(a_1, \dots, a_N) = (3, 2, 3, 1, 3, 2, 3, 3, 2, 1)$. In other words, we allocate the first item (if $\Delta = 1$) to $a = 3$ (with a corresponding objective function value equal to $\frac{1}{5}$), the second item (if $\Delta = 2$) to $a = 2$ (with a corresponding objective function value equal to $\frac{1}{3}$), the third item (if $\Delta = 3$) to $a = 3$ (with a corresponding objective function value equal to $\frac{2}{5}$), etc. Note that the last three items (with $\Delta = 8, 9, 10$) are allocated to $a = 3$, $a = 2$ and $a = 1$ in this specific sequence (from the largest to the smallest value of $|\mathcal{F}_a|$) to guarantee that the sequence lexicographically minimizes the elements in the set for $\Delta = 8$ and $\Delta = 9$. Table 5 shows a vector $(U_a)_{a \in \mathcal{A}}$ that solves Problem $\mathcal{P}(\Delta)$ for different values of Δ , based on the sequence (a_1, \dots, a_N) thus constructed.

Table 5: Solution $(U_a)_{a \in \mathcal{A}}$ of $\mathcal{P}(\Delta)$ for $|\mathcal{A}| = 3$, $\xi_1 = 2$, $\xi_2 = 3$, $\xi_3 = 5$, and different values of Δ

	Δ																				
a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	...
1	0	0	0	1	1	1	1	1	1	2	2	2	2	3	3	3	3	3	3	4	...
2	0	1	1	1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	6	6	...
3	1	1	2	2	3	3	4	5	5	5	6	6	7	7	8	8	9	10	10	10	...

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