Distributed Tracking for Mobile Sensor Networks with Information-Driven Mobility

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Abstract— In this paper, we address distributed target tracking for mobile sensor networks using the extension of a distributed Kalman filtering (DKF) algorithm introduced by the author in [11]. It is shown that improvement of the quality of tracking by mobile sensors (or agents) leads to the emergence of flocking behavior. We discuss the benefits of a flocking-based mobility model for distributed Kalman filtering over mobile networks. This mobility model uses author's flocking algorithm with a natural choice of a moving rendezvous point that is the target itself. As the agents "flock" towards the target, the information value of their sensor measurements improves in time. During this process, smaller flocks merge and form larger flocks and eventually a single flock with a connected topology emerges. This allows the agents to perform cooperative filtering using the DKF algorithm which considerably improves their tracking performance. We show that this flocking algorithm is in fact an information-driven mobility that acts as a cooperative control strategy that enhances the aggregate information value of all sensor measurements. A metric for information value is given that has close connections to Fisher information. Simulation results are provided for a group of UAVs with embedded sensors tracking a mobile target using cooperative filtering.

Index Terms— mobile sensor networks, target tracking, distributed Kalman filters, information-driven mobility, flocking

I. INTRODUCTION

Multi-Sensor Tracking problems have attracted the attention of many researchers in robotics, systems, and control theory over the past three decades [2]. Modern tracking problems are of great importance in surveillance, security, and information systems for monitoring the behavior of adversarial/friendly agents using *sensor networks* [4]. Analogous forms of Kalman filtering algorithms for target tracking, called *process query systems*, have been recently developed for network security applications [17].

In this paper, we focus on *distributed target tracking for mobile sensor networks*. This problem has been mostly considered for static sensor networks and has gone practically unnoticed for the case of mobile networks.

Decentralized estimation using Kalman filters has a long tradition in control theory [1], [2], [21]. Modern implementations of decentralized Kalman filtering and information fusion algorithms for multi-sensor platforms [15] have become more popular due to the emergence of sensor networks as a standard means of distributed sensing and monitoring. In decentralized estimation, each node is permitted to communicate with all other nodes. This overlay network topology

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amounts to a communication cost of $O(n^2)$ for n sensors that means such decentralized algorithms are *not scalable*. An estimation algorithm is called "distributed" if each node only talks to it neighbors. Under the assumption that each node has either $O(\log(n))$ or $O(1)$ neighbors, the communication cost of this class of distributed algorithms is $O(n\log(n))$, or $O(n)$ which are both scalable in n.

Recently, a *distributed Kalman filtering (DKF)* algorithm for sensor networks with a fixed topology was introduced by the author in [11]. The DKF algorithm relies on embedded consensus filters [14], [19] (shown in Fig. 1) that dynamically compute time-varying averages of sensor data and covariance data obtain from all sensors in a distributed way. Consensus filtering algorithms fundamentally build on an earlier work on average-consensus algorithms [13].

An alternative approach to the DKF algorithm is an approximate algorithm for distributed *particle filtering* [18] that is useful for nonlinear estimation problems. The work provides no analysis for the role of the network topology in convergence of particle filtering algorithms.

The problem of distributed estimation for mobile ad hoc networks (MANETs) has received little attention in the past years. The main focus of this paper is to address distributed target tracking for mobile sensor networks with a dynamic topology. A major obstacle in dealing with mobile networks is to guarantee network connectivity. Preserving (or improving) *network connectivity* is a key challenge in this field that has attracted many researchers [6], [10], [20], [3], [22].

We tackle the network connectivity issue using a flocking-based mobility model. This mobility protocol uses a flocking algorithm introduced by the author in [12]. Despite the popularity of Reynolds rules of flocking [16], the author has shown that these heuristic rules lead to fragmented proximity networks that are disconnected for generic initial conditions [12], i.e Reynolds rules are inadequate for creation of connected networks.

From the begining, we do not assume that the mobility model of a group of sensors is flocking-based by any means. Instead, under mild assumptions on sensing model of the agents, we show that a non-cooperative group of self-interested agents with the objective of improving their individual quality of tracking/estimation would end up performing a moving rendezvous in space that eventually leads to emergence of flocking behavior. As a result, flocking is the byproduct of reducing estimation error for the individual mobile agents. Later, we establish that this form of rendezvous is compatible with the flocking algorithms in

[12] which guarantee asymptotic connectivity of the network. This enables performing *cooperative and distributed Kalman filtering* on a mobile network that further improves the performance of tracking.

The main contribution of this paper is to establish rigorous connections between distributed target tracking and flocking-based information-driven mobility for MANETs. In addition, an extension of the DKF algorithm for mobile networks is introduced together with a detailed discussion of the network connectivity issues.

The outline of the paper is as follows. In Section II, the connections between information-driven distributed tracking and emergence of flocking behavior is discussed. In Section III, a flocking algorithm is given for an arbitrary dimension. In Section IV, the distributed Kalman filtering algorithm and consensus filters for mobile networks are introduced. In Section V, the role of mobility model in enhancing the information value of the sensor data is explained. Simulation results are presented in Section VI and some concluding remarks are made in Section VII.

II. EMERGENCE OF FLOCKING IN TARGET TRACKING

Ground sensor networks are effective in detection of mobile targets but due to their limited range, any agile or maneuverable target can easily escape their sensing range, particularly in perimeter security applications. This necessitates the use of *mobile sensor network* for tracking agile targets.

By a *mobile sensor*, we mean an unmanned autonomous vehicle (UAV) or mobile robot equipped with various sensors. The type of sensors include both imaging sensors like cameras and non-imaging sensors such as sonar, radar, and thermal signature sensors. A UAV (or mobile robot) embedded with such sensors is called a *mobile agent*.

We argue that motion control for a group of mobile agents with the objective of target tracking and high-performance distributed estimation are "coupled" problems. To get a better sense, let us consider a group of mobile agents with the objective of tracking a moving target called *target γ*. Suppose the following conditions hold:

1) Each agent has a finite *interaction range $r > 0$* that allows that agent to communicate with and sense the presence of other agents;
2) Initially, no agent is within the range r of other agents;
3) None of the agents is aware of the mission of other agents to track target γ;
4) The agents do not communicate their sensed data regarding the target;
5) All agents share a common sensing model in which the quality of sensing improves as the target range $\rho \geq \rho_0$ decreases regardless of target bearing ($\rho_0 \ll r/2$ is a safe distance from the target).

Then, each agent has the incentive to get within a safe close proximity of the target to monitor its behavior for the purpose of increasing its *information value* (the opposite of the uncertainty in sensor measurements that is defined in Section V).

For n mobile sensors ($n \geq 2$), soon all sensors would find themselves near target γ and therefore they become aware of the presence of each other because $\rho_0 \ll r/2$. In other words, the common objective of improving individual information value of the sensors would force then to perform an unplanned *moving rendezvous* near the mobile target.

In real-life, the agents have to *avoid collision* to each other as they get closer to the target and thus each other. The combination of inter-agent "collision avoidance" and "moving rendezvous" in space leads to the emergence of *flocking behavior* as explained in [12]. One concludes that information-driven mobility leads to emergence of flocking and self-assembly of connected networks (to be made precise later).

Interestingly, emergence of flocking does not require any cooperation (or information exchange) among the mobile agents in terms of the task of tracking. This type of flocking closely resembles the unplanned rendezvous in the vehicle routing problem in [7].

Not all sensors in a flock have the same level of information value. We will show that *cooperative filtering* and information fusion can improve the performance of tracking for flock of mobile sensors. From the above argument, it is apparent that "cooperative and distributed tracking" and "flocking" go hand in hand in distributed tracking for mobile sensor networks. This idea is novel and main contribution of this paper.

Another important benefit of using flocking algorithms in [12] as the mobility model of a sensor network is that in steady-state, the self-assembled flock has $O(n)$ links (See Theorem 4 in [12]). This implies the communication cost of the mobile version of the DKF algorithm is $O(n)$.

III. FLOCKING-BASED MOBILITY MODEL FOR MANETs

Consider n mobile agents with equations of motion

$$
\begin{aligned}
\dot{q}_i &= p_i \\
\dot{p}_i &= f_i
\end{aligned}
\tag{1}
$$

where q_i, p_i, and f_i are, respectively, the position, the velocity, and the input control (force) of agent i for $i \in V = \{1, \ldots, n\}$. Given an interaction range $r > 0$, each agent only interacts with a time-varying set of its *neighbors* N_i defined as

$$
N_i = \{j \in V : \|q_j - q_i\| < r\}.
\tag{2}
$$

The set of neighbors that include agent i is denote by $J_i = N_i \cup \{i\}$. The configuration of all agents $q = \text{col}(q_1, \ldots, q_n)$ induces a *proximity net $G(q) = (V, E(q))$* that is a *dynamic graph* [10] with a variable set of spatially-dependent edges

$$
E(q) = \{(i, j) \in V \times V : \|q_j - q_i\| < r\}.
$$

A group of mobile agents with configuration q is called a *flock* over an interval $[t_0, t_f]$ if the proximity net $G(q(t))$ of the agents is *connected* over that interval.

Geometry of flocks can be modeled using lattice-type structures called α-lattices that have an inherent spatial-order.

An α-*lattice* is a configuration q satisfying the following condition:

$$\|q_j - q_i\| = d, \ \forall j \in N_i \tag{3}$$

A structure that approximately satisfies this condition is called a *quasi α-lattice*, i.e. $(\|q_j - q_i\| - d)^2 \delta^2$ for all neighboring agents $(i,j) \in E(q)$ and $\delta \ll d$.

Here is the main flocking algorithm[1] for an arbitrary m-dimension space:

Algorithm 1 (Olfati-Saber, 2004 [12]):

$$\dot{q}_i = p_i \tag{4}$$

$$\dot{p}_i = \sum_{j \in N_i} f_{ij} \mathbf{n}_{ij} + \sum_{j \in N_i} a_{ij}(q)(p_j - p_i) + f_i^\gamma \tag{5}$$

where $f_{ij} = \phi_\alpha(\|q_j - q_i\|_\sigma)$ is a gradient-based forced applied to agent i by its neighbor j and

$$f_i^\gamma = -c_1(q_i - \hat{q}_{i,\gamma}) - c_2(p_i - \hat{p}_{i,\gamma}), \ c_1, c_2 > 0 \tag{6}$$

is a tracking feedback applied to agent i by target γ based on *estimated position and velocity* $(\hat{q}_{i,\gamma}, \hat{p}_{i,\gamma})$ of the target. The notations and definitions used in this algorithm are available in [12]. Clearly, the mobility model is coupled with the estimation of the state of the target.

Let $U(q) = \frac{1}{2}\sum_{i,j \neq i} \psi_\alpha(\|q_j - q_i\|_\sigma)$ denote the *collective potential* of a group of mobile agents with configuration q and

$$U_\gamma(q, \hat{q}_\gamma) = \frac{1}{2} \sum_i \rho_i^2 \tag{7}$$

be the *agent-target interaction potential* $T(q)$ between the agents and target γ. Here, $\rho_i = \|q_i - \hat{q}_{i,\gamma}\|$ is the estimated *target range* by sensor i and $\hat{q}_\gamma = \mathrm{col}(\hat{q}_{1,\gamma}, \ldots, \hat{q}_{n,\gamma})$ is the vector of estimated position of the target by all nodes. Then, defining the *augmented potential*

$$U_\lambda(q) = U(q) + \lambda U_\gamma(q, \hat{q}_\gamma) \tag{8}$$

with a weight $\lambda = c_1 > 0$ gives the potential function associated with the gradient-based terms of Algorithm 1. Assuming that all agents can asymptotically reach a consensus regarding the estimated position of the target, we obtain the following result:

Proposition 1. *Consider n mobile agents applying the mobility protocol of Algorithm 1 and assume each agent senses a noisy measurement of position and velocity of a mobile target with the state $(q_{i,\gamma}, p_{i,\gamma})$. Suppose that after some finite time $T > 0$ all agents reach a consensus regarding the estimate $(\hat{q}_{i,\gamma}, \hat{p}_{i,\gamma})$ of the state of mobile target γ and $\hat{q}_{i,\gamma} = \bar{q}, \forall i$. If Conjectures 1 and 2 in [12] hold, then*
  i) *The agents asymptotically self-assemble a flock (i.e. a connected proximity net);*
  ii) *The network topology is asymptotically invariant;*
  iii) *The asymptotic conformation q_λ^* of the agents is a quasi α-lattice (i.e. the inter-agent distances are $d \pm \delta$).*

[1]This algorithm is referred to as Algorithm 2 in [12].

Proof: Given the assumption that after time $T > 0$, $q_{i,\gamma} = \bar{q}$ for all i, we have $U_\gamma(q, \hat{q}_\gamma) = J(q) = \frac{1}{2}\sum_i \|q_i - \bar{q}\|^2$ where $J(q)$ is the moment of inertia of the group of agents. Now, the proof of parts i) and iii) follows from Theorems 2 and 3 in [12]. Part ii) holds because all agents asymptotically align and move with the same velocity. Thus, their inter-agent distances remain invariant and the proximity net becomes fixed. $\square$

IV. DISTRIBUTED KALMAN FILTER FOR MANETS

In this section, we present a slightly modified version of the distributed Kalman filtering (DKF) algorithm in [11] for mobile sensor networks with uncorrelated measurement noise.

A. Target and Sensing Models

The model of the *target* is a dynamic system

$$x(k+1) = A_k x(k) + B_k w(k); \quad x(0) \sim \mathcal{N}(\bar{x}(0), P_0). \tag{9}$$

Every sensor measures the following output

$$z_i(k) = H_i(k)x(k) + v_i(k), \quad z_i \in \mathbb{R}^p \tag{10}$$

Both w_k and v_k are zero-mean white Gaussian noise (WGN) and $x(0) \in \mathbb{R}^m$ is the initial state of the target. The statistics of the measurement noise is given by

$$E[w(k)w(l)'] = Q_k \delta_{kl}, \tag{11}$$

$$E[v_i(k)v_j(l)'] = R_i(k)\delta_{kl}\delta_{ij}. \tag{12}$$

where $\delta_{kl} = 1$ if $k = l$, and $\delta_{kl} = 0$, otherwise.

Let $\mathbf{z}(k) = \mathrm{col}(z_1(k), \ldots, z_n(k)) \in \mathbb{R}^{np}$ be the collective measurement data of the entire network at time k. Given the measurements $Z_k = \{\mathbf{z}(0), \mathbf{z}(1), \ldots, \mathbf{z}(k)\}$, the state *estimates* for the target in Kalman filter theory [1] can be expressed as

$$\hat{x}_k = E(x_k|Z_k), \bar{x}_k = E(x_k|Z_{k-1}), \tag{13}$$

$$P_k = \Sigma_{k|k-1}, M_k = \Sigma_{k|k} \tag{14}$$

where $\Sigma_{k|k-1}$ and $\Sigma_{k|k-1}$ are *state covariance matrices* and $\Sigma_{0|-1} = P_0$.

B. Distributed Kalman Filter

The main theorem that forms the basis of consensus-based Kalman filtering for a connected network with is restated in the following:

Theorem 1. *(Distributed Kalman Filter, [11]) Consider a sensor network with n sensors and a connected network topology G observing a target with an m-dimensional state using a p-dimensional vector of observations ($p \leq m$). Suppose the nodes of the network solve two average-consensus problems regarding the fused inverse-covariance matrices*

$$S(k) = \frac{1}{n} \sum_{i=1}^n H_i'(k)R_i^{-1}(k)H_i(k) \tag{15}$$

and fused measurements

$$y(k) = \frac{1}{n} \sum_{i=1}^n H_i'(k)R_i^{-1}(k)z_i(k) \tag{16}$$

at every iteration k. Then, the nodes of the network can calculate identical state estimates $\hat{x}$ of the target using the following micro-Kalman filtering equations

$$
\begin{aligned}
M_\mu(k) &= (P_\mu(k)^{-1} + S(k))^{-1}, \\
\hat{x}(k) &= \bar{x}(k) + M_\mu(k)[y(k) - S(k)\bar{x}(k)], \\
P_\mu(k+1) &= A_k M_\mu(k) A_k' + B_k Q_\mu(k) B_k', \\
\bar{x}(k+1) &= A_k \hat{x}(k).
\end{aligned}
$$

where the μ-indexed matrices satisfy the scaling relationships $M_\mu(k) = nM_k$, $P_\mu(k) = nP_k$, and $Q_\mu(k) = nQ_k$. Furthermore, the obtained estimate is identical to the one computed via a central Kalman filter.

Interestingly, the possibility that the nodes of a sensor network can provide identical state estimates of a mobile target matches the requirement of Proposition 1. The remaining challenge is that all nodes cannot solve such average-consensus problems from the beginning when the network consists of multiple flocks (i.e. is disconnected). The solution to this problem of initial network disconnectivity is that each agent only needs to perform cooperative filtering with the members of its own flock.

The proximity net $G(q(t))$ at any time t consists of $\nu(t) \geq 1$ flocks $F_1(q(t)), \ldots, F_\nu(q(t))$, or connected components. Fragmentation of a flock creates two new flocks and increases $\nu(t)$. In [12], it is discussed in detail that fragmentation does not occur when Algorithm 1 is applied. To take this fact into account, one can equivalently assume that the number of flocks $\nu(t)$ is monotonically decreasing under the mobility protocol specified by Algorithm 1. With this simplifying assumption (that flocks do not fragment), based on Proposition 1, after some finite time $\tau > 0$, only a single flock remains that asymptotically has a fixed connected topology $G^* = G(q_\lambda^*)$. We refer to this simplifying monotonicity condition on $\nu(t)$ as the *sequential merger of flocks*.

According to the above argument, since a flock is a connected network with a variable topology, it makes sense to propose a DKF algorithm that runs in the lth flock $F_l(q)$ of agents with consensus filters that use time-varying sets of neighbors $N_i(t)$.

Fig. 1 shows the architecture of the micro-Kalman filter (MKF) with two embedded consensus filters that run with the same frequency as the MKF at the node level. The low-pass consensus filter is used for fusion of the measurement data to compute an estimate $\hat{y}_i$ and the band-pass consensus filter is used to compute the estimate $\hat{S}_i$ of the average inverse-covariance matrices.

Algorithm 2 (Micro-Kalman Filter Iterations): Node i applies the following MKF state estimate updates:

$$
\begin{aligned}
M_i(k) &= (P_i(k)^{-1} + \hat{S}_i(k))^{-1}, \\
\hat{x}_i(k) &= \bar{x}_i(k) + M_i(k)[\hat{y}_i(k) - \hat{S}_i(k)\bar{x}_i(k)], \\
P_i(k+1) &= A_k M_i(k) A_k' + n_i B_k Q_k B_k', \\
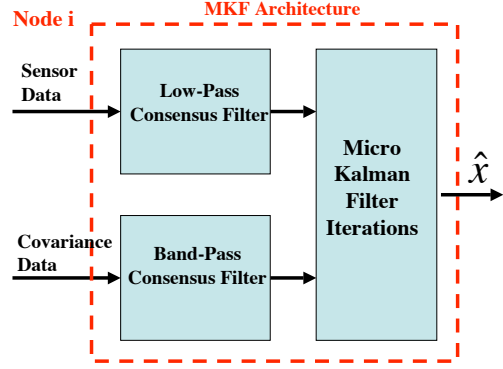\bar{x}_i(k+1) &= A_k \hat{x}_i(k).
\end{aligned}
$$

Fig. 1. Micro-Kalman Filter with embedded consensus filters at every node.

where $n_i = |N_i| + 1$ is an approximation of the number of agents (the transient effects of this choice will go away by time). The effect of the network topology appears nowhere in the MKF iteration (or Algorithm 2) other than in $N_i(t)$.

The MKF iterations for a fixed network and a mobile network are practically the same. The fundamental difference in the DKF algorithm is hidden in the role of embedded consensus filters.

The formal analysis of this DKF algorithm coupled with a flocking-based mobility model is fairly complex and the subject of ongoing research. However, the precise statement of the algorithm is useful and has produced successful numerical estimation results as demonstrated in Section VI.

C. Consensus Filters for Mobile Networks

Based on [14], the dynamics of a discrete-time *low-pass consensus filter (LCF)* for member i of flock F_l can be expressed as follows

$$
\begin{aligned}
\xi_i(k+1) &= \xi_i(k) + \epsilon \sum_{j \in N_i(t)} (\xi_j(k) - \xi_i(k)) \\
&+ \epsilon \sum_{j \in J_i(t)} (y_j(k) - \xi_i(k))
\end{aligned}
$$

where $J_i(t) = N_i(t) \cup \{i\}$. The input data $y_j(k)$ is set to $H_j'(k)R_j(k)^{-1}z_j(k)$ and the step-size ϵ satisfies the bound $\epsilon < 1/\Delta_j(k)$. Here, Δ_j is the maximum node degree of members of the flock that contains node i. The typical value of the maximum degree in 2-D flocking with Algorithm 1 is $\Delta_j = 6$ (due to the triangular crystal structure of the induced proximity graphs of α-lattices).

It turns out that regardless of the connectivity of the network $G(q(t))$, LCF remains a stable filter. In the extreme case that the networks has no links, the dynamics of LCF reduces to

$$
\xi_i(k+1) = \xi_i(k) + \epsilon(y_i(k) - \xi_i(k))
$$

which is a stable low-pass filter for $0 < \epsilon < 1$.

The band-pass consensus filter in the architecture of the MKF is the combination of an LCF and a high-pass

consensus filter (HCF) [19]. The discrete-time *band-pass consensus filter BCF* for fusion of the inverse-covariance data $S_i(k) = H_i'(k)R_i(k)^{-1}H_i(k)$ is in the following form [14]:

$$Y_i(k) = X_i(k) + S_i(k) \qquad (17)$$

$$X_i(k+1) = X_i(k) + \epsilon \sum_{j \in N_i(t)} (Y_j(k) - Y_i(k)) \quad (18)$$

$$Z_i(k+1) = Z_i(k) + \epsilon \sum_{j \in N_i(t)} (Z_j(k) - Z_i(k)) \quad (19)$$

$$+ \epsilon \sum_{j \in J_i(t)} (Y_j(k) - Z_i(k)) \qquad (20)$$

with inputs $S_j(k), j \in J_i(t)$ and output $\hat{S}_i(k) = Z_i(k)$ that is used in Algorithm 2.

The main reason that "network connectivity" is required for the DKF algorithm is because the convergence of the high-pass consensus filter in [19] to the average of the inputs only holds for connected networks.

Since the network connectivity does not initially hold in general, we implement the band-pass consensus filter over each flock. This is justified because as the flocks gradually merge, asymptotically the network becomes a fixed connected network which is compatible with the asymptotic convergence proofs in [19].

V. INFORMATION-DRIVEN MOBILITY

Information-driven mobility for multi-sensor platforms has been recently studied by several researchers [8], [23], [5], [9]. Though, the connections between information-driven mobility and emergence of flocking has not been considered.

We show that adding the agent-target interaction potential $U_\gamma(q, \hat{q}_\gamma)$ to the flocking potential (or cost) of the agents is a way to take the "information value" of sensor measurements into account in motion planning. We introduce a metric that measures the information value of sensor data with similarities to *Fisher Information* [8], [9] that has applications in information theory.

We define the *information value* I_i of the measurements of sensor i with covariance R_i, output matrix H_i, and inverse-covariance data $S_i = H_i'R_i^{-1}H_i$ as the following

$$I_i = \text{tr}(S_i) > 0 \qquad (21)$$

($\text{tr}(\cdot)$ denotes the trace of a matrix and the time index k is dropped for clarity). Note that $S = \frac{1}{n}\sum_{i=1}^n S_i$ and the trace operation has an additive property. Hence, denoting the fused information value by $I = \text{tr}(S)$, we obtain the following identity:

$$I = \frac{1}{n}\sum_i I_i \qquad (22)$$

The quantity $1/I$ can be viewed as a metric for the *average information uncertainty* across the sensor network.

To better understand the application of information value as a tool, let us consider a widely used sensor model for target tracking called the *range-bearing model* [1]. Let ρ_i and θ_i, respectively, denote the *estimated* range and bearing of a mobile target with associated Gaussian noise

variances σ_ρ^2 and σ_θ^2. Each sensor makes the measurement $z_i = (\rho_i \cos\theta_i, \rho_i \sin\theta_i)'$ for a target moving in $\mathbb{R}^2$ with the sensing model $z_i = H_i x_i + v_i$ and an output matrix

$$H_i = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]$$

that measures the noisy version of position of the target. The covariance of the noise of the sensor data is given in [1] as

$$R_i = E[v_i v_i'] = \Gamma_i' \left[\begin{array}{cc} \sigma_\rho^2 & 0 \\ 0 & \rho_i^2 \sigma_\theta^2 \end{array} \right] \Gamma_i = \Gamma_i' D(\rho_i)\Gamma_i \quad (23)$$

where Γ_i is the rotation matrix

$$\Gamma_i = \left[\begin{array}{cc} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{array} \right].$$

We obtain

$$S_i = H_i'R_i^{-1}H_i = H_i'\Gamma_i'D(\rho_i)^{-1}\Gamma_i H_i$$

where the rows of $\Gamma_i H_i$ have unit length. Thus, $\text{tr}(S_i)$ can be explicitly calculated and the information value of the data of sensor i can be obtained as

$$I_i = \sum_j D_{jj}^{-1}(\rho_i) = \frac{1}{\sigma_\rho^2} + \frac{1}{\rho_i^2 \sigma_\theta^2}. \qquad (24)$$

After some further simplifying assumptions that hold for sonar sensors, suppose $\sigma_\rho^2 = g(\rho_i)$ and $\rho_i^2 \sigma_\theta^2 = h(\rho_i)$ where $g(\rho)$ and $h(\rho)$ are both increasing and positive functions of ρ, one can explicitly express the information value I_i in terms of functions $g(\rho)$ and $h(\rho)$ as

$$I_i = f(\rho_i) := \frac{1}{g(\rho_i)} + \frac{1}{h(\rho_i)} \qquad (25)$$

where $f(\rho)$ is a decreasing function of the target range ρ. i.e. $\rho_i = f^{-1}(I_i)$. Here is the main result that connects the information value of all nodes to flocking-based mobility.

Proposition 2. *Consider a group of n mobile sensors with individual information value I_i and a nonlinear aggregate information value $I_\gamma = \sum_{j=1}^n (f^{-1}(I_j))^2$. Then, the augmented potential U_λ in (8) is the weighted sum of the structural potential $U(q)$ of a group of particles and the aggregate information value I_γ of the range-bearing sensor data across the mobile network, i.e.*

$$U_\lambda = U(q) + \lambda I_\gamma \qquad (26)$$

(here, $f^{-1}(\cdot)$ is the inverse map of $f(\cdot)$).

Proof: Since $f(\rho)$ is a decreasing function, it is invertible and $\rho_i = f^{-1}(I_i)$. The proof follows from the fact that $U_\gamma(q, \hat{q}_\gamma) = I_\gamma = \sum_j \rho_j^2$. $\qquad \square$

The flocking-based mobility model of the MANETs (i.e. Algorithm 1) seeks to minimize the augmented potential U_λ. As a byproduct, the individual target ranges decrease in time and the information value I_γ increases. Hence, an information-driven flocking algorithm with the coupled cost U_λ will increase the aggregate information value.

The maximum value for I_γ occurs when $\rho_i = 0$ for all agents. This means that the best aggregate information value

is achieved when all agents perform a moving rendezvous at the target location as predicted earlier. However, that is not possible in real-life as all sensors will collide. Thus, the cost of collision-avoidance is taken into account via the addition of $U(q)$. The combination of both costs leads to emergence of flocking and improvement of the aggregate information value.

VI. SIMULATIONS

Consider a target moving in a 2-D space with constant velocity driven by zero-mean Gaussian noise [2], i.e.

$$\dot{q}_\gamma = p_\gamma, \; \dot{p}_\gamma = \tilde{w}(t) \tag{27}$$

where $\tilde{w}(t) = (\tilde{w}_1(t), \tilde{w}_2(t))'$ and the $\tilde{w}_i(t)$'s are WGN with variance σ^2. This system with state $x = \text{col}(q_\gamma, p_\gamma) \in \mathbb{R}^4$ can be written in discrete-time as a linear system

$$x(k+1) = \begin{bmatrix} \mathbf{I}_2 & \epsilon\mathbf{I}_2 \\ 0 & \mathbf{I}_2 \end{bmatrix} x(k) + \begin{bmatrix} (\epsilon^2/2)\mathbf{I}_2 \\ \epsilon\mathbf{I}_2 \end{bmatrix} \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$$

where $\mathbf{I}_m$ is an $m \times m$ identity matrix and $w_1(k), w_2(k)$ are zero-mean Gaussian noise with variance σ^2. To consider the role of target range, we assume a sensor model $z_i = Hx + v_i$ with output matrix $H = [\mathbf{I}_2 \; 0]$ and covariance

$$R_i = E[vv'] = \frac{1}{I_0}(a + b + (a-b)\frac{\rho_i - l}{\sqrt{1 + (\rho_i - l)^2}})\mathbf{I}_2$$

where $I_0, > 0$, $l \gg 1$, and $a > b > 0$. The information value of sensor i is $I_i = I_0 f(\rho)$ with

$$f(\rho) = 2[a + b + (a-b)\frac{\rho - l}{\sqrt{1 + (\rho - l)^2}}]^{-1}.$$

For a far away target $\rho \gg 1$, $f(\rho) \approx 1/a$. Thus, parameter a determines the worst level of uncertainty, or the minimum information value of $I_i = I_0/a$.

We consider the task of target tracking using $n = 20$ UAVs (or mobile sensors) with the aforementioned described models with parameters $\sigma^2 = 10$, $\epsilon = 0.05$, $I_0 = 0.1$, $a = 10b$, $b = 1$, $l = 11.5d$, $r = 1.2d$, and $d = 7$. Two cases are considered: 1) cooperative filtering and 2) non-cooperative filtering. In the first case, the agents use the DKF algorithm to estimate the state of the target based on noisy position measurements of the target. In the non-cooperative case case, ever agent assumes that it has no neighbors, i.e. $N_i = \emptyset$, only for the purpose of tracking (not flocking). The agents use Algorithm 1 as the mobility protocol.

Fig. 2 shows the difference in mean-square-error (MSE) of the target state estimates for the cases of cooperative and non-cooperative filtering. The configuration of the sensors with a flocking-based mobility model is depicted in Fig. 3. Fig. 4 demonstrate the snapshots of target position estimates obtained via distributed Kalman filtering (or cooperative filtering) versus non-cooperative filtering. The performance of tracking in the cooperative case is apparently better than the non-cooperative case. According to Fig. 3, a connected mobile network is self-assembled in less than 50 iterations. The network remains connected thereafter (not shown due to space limitations). The estimates in the non-cooperative
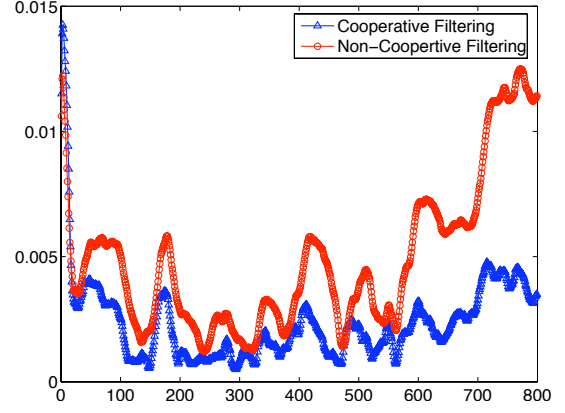

Fig. 2. MSE for tracking using cooperative micro-Kalman filtering vs. non-cooperative filtering.
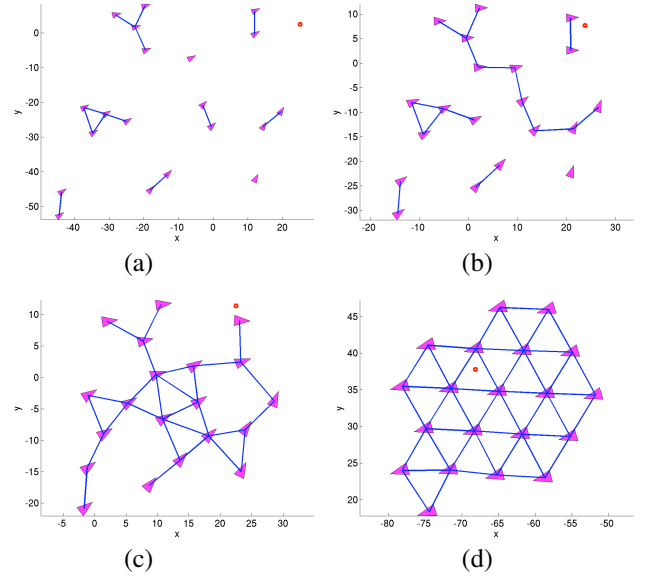

Fig. 3. Emergence of a connected mobile network (or flock) of 20 sensors (triangles) in less than 50 iterations during cooperative tracking of a mobile target (red square): (a)–(c) $k \leq 50$ iterations and (d) $k = 400$ iterations.

filtering case is slightly delayed and off. Even in case of non-cooperative filtering, a single flock is formed and maintained.

VII. CONCLUSIONS

We addressed distributed tracking for mobile sensor networks with a flocking-based mobility model using a cooperative network of micro-Kalman filters (i.e. the DKF algorithm). A metric for information value of a sensor measurement was introduced. We demonstrated that a flocking-based mobility model that uses the author's flocking algorithm is in fact an information-driven motion control algorithm for cooperative tracking. This done by establishing that the aggregate information value of all sensors appears as part of the cost of flocking. A byproduct of flocking is self-assembly of a connected networks that is ideal for distributed Kalman filtering. The main contribution of the paper is to establish direct connections between distributed tracking for mobile networks and flocking-based information-driven mobility.

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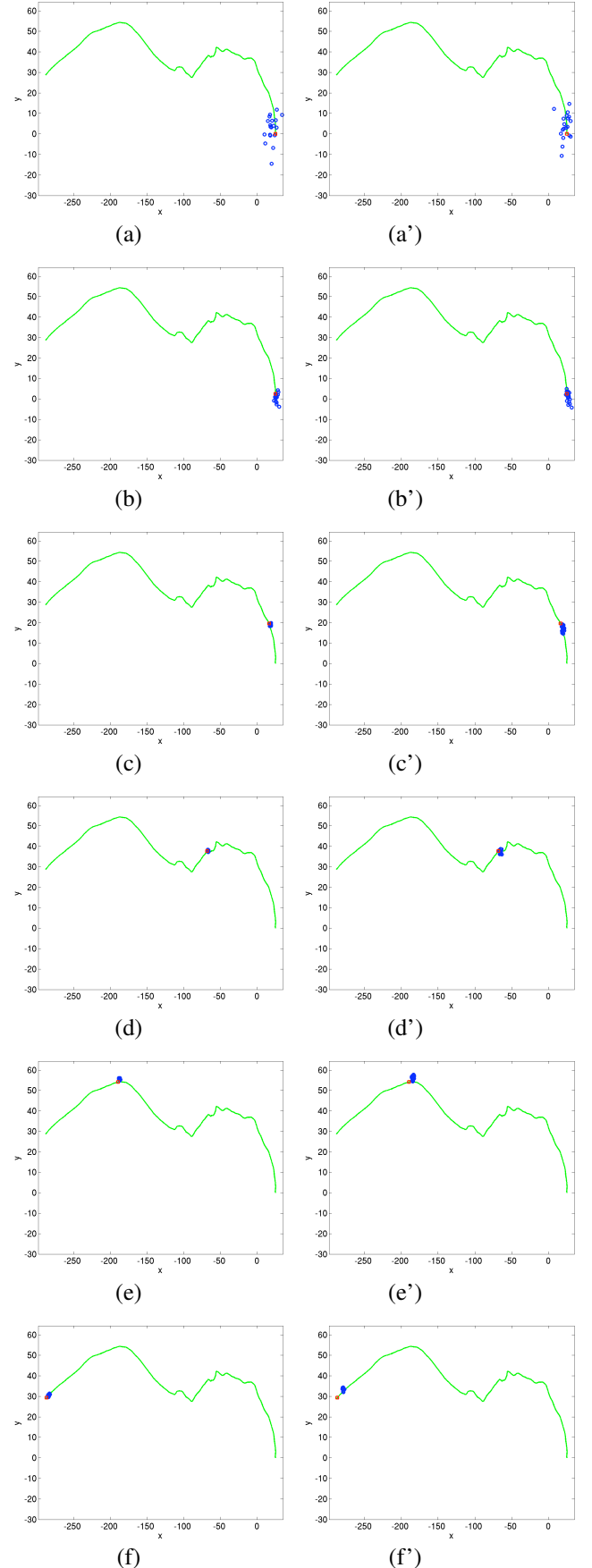
Fig. 4. Position estimates (blue circles) of the target (red square): (a)–(f) cooperative filtering (left) and (a')–(f') non-cooperative filtering (right).