Clique Analysis and Bypassing in Continuous-Time Conflict-Based Search

Supplementary results to the publication at SoCS 2024

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1 Overview

This supplementary material includes full MAPF benchmark results that go with the paper. The first section contains benchmark results and the second section contains significance testing to show the best variant versus previous state-of-the-art.

2 Summary of Benchmark Tests

The following tables show the results of all MAPF benchmarks and tests. The statistics are the total number of problem instances solved in each category in under 30 seconds for each test, with the 95th percentile marked with \pm . The best result in each setting is underlined, and all results that are statistically inside the 95th percentile of the best result are in bold.

Table 1: Total problems solved in under 30 seconds on 4-neighbor grid MAPF benchmarks

Map		CCBS Wi	th Bypass			CCBS With	nout Bypass	
	CCBS	DS	$_{\mathrm{BC}}$	KDB	CCBS	DS^1	$_{\mathrm{BC}}$	KDB
Berlin_1_256	$1,198\pm52$	$1,\!334{\pm}56$	$1,198\pm52$	$\underline{1,334{\pm}56}$	1,182±53	$1,200\pm53$	$1,188\pm53$	$1,200\pm53$
$Boston_0_256$	$1,\!120\!\pm\!34$	$1,\!140{\pm}40$	$1,\!120\!\pm\!34$	$\underline{\textbf{1,152} \!\pm\! 40}$	$1,\!122\!\pm\!34$	$1{,}112{\pm}37$	$1{,}122{\pm}34$	$1{,}112{\pm}37$
Paris_1_256	$1,\!504{\pm}58$	$\underline{1,524{\pm}56}$	$1,\!504{\pm}58$	$\underline{1,524{\pm}56}$	$1,356\pm55$	$1,432\pm 59$	$1,356 \pm 55$	$1,434\pm58$
city	$3,822\pm72$	$3,998{\pm}153$	$3,822 \pm 145$	$4,010{\pm}153$	$3,660\pm142$	$3,744 \pm 150$	$3,666\pm142$	$3,746\pm150$
den520d	858±32	860±33	858±32	868±33	844±32	844±32	844±32	$844{\pm}32$
brc202d	514±12	$604 {\pm} 20$	522 ± 13	$606{\pm}20$	510±15	580 ± 18	510 ± 15	586 ± 17
den 312d	504±16	510 ± 14	504 ± 16	$\underline{552{\pm}15}$	$464{\pm}13$	474 ± 13	464 ± 13	474 ± 13
lak303d	470±15	$538 {\pm} 14$	472 ± 15	$\underline{\textbf{552} {\pm} \textbf{14}}$	410 ± 12	474 ± 14	412 ± 12	476 ± 14
orz900d	524±15	566 ± 15	528 ± 15	564 ± 16	544 ± 20	$\underline{626{\pm}21}$	542 ± 20	$622 {\pm} 21$
ost003d	$572{\pm}22$	$\underline{582 {\pm} 21}$	$572 {\pm} 22$	$\underline{\textbf{582} {\pm} \textbf{21}}$	554 ± 21	$562 {\pm} 21$	554 ± 21	$564 {\pm} 21$
DAO	$3,442\pm57$	$3,\!660\!\pm\!119$	$3,456\pm116$	$\underline{3,724{\pm}121}$	$3,326\pm116$	$3,560\pm122$	$3,326\pm116$	$3,566\pm121$
empty-16-16	516±20	520±20	516±20	546±22	518±20	520±20	518±20	520±20
empty-32-32	930±37	$920{\pm}33$	930 ± 37	$918{\pm}34$	$910{\pm}35$	886 ± 33	$910{\pm}35$	886 ± 33
empty-48-48	1,086±39	$1,110\pm36$	$1,086\pm39$	$\underline{1,144{\pm}33}$	$1,060\pm38$	$1,058\pm35$	$1,056\pm38$	$1,058\pm35$
empty-8-8	292±8	316 ± 8	292 ± 8	$\underline{\textbf{338} {\pm} \textbf{8}}$	296±9	312 ± 8	296 ± 9	314 ± 8
empty	$2,824\pm52$	$2,\!866{\pm}98$	$2,824\pm105$	$\underline{2,946{\pm}98}$	$2,784\pm104$	$2,776\pm97$	$2,780\pm104$	2,778±98
lt_gallowstemplar_n	610±19	654±20	610±19	658±19	562±16	632±19	564±16	634±19
ht_chantry	544±14	$588 {\pm} 17$	540 ± 14	$592 {\pm} 16$	522±11	570 ± 16	518 ± 12	566 ± 16
ht_mansion_n	700±19	726 ± 18	716 ± 19	$\textbf{746} {\pm} \textbf{19}$	662±20	680 ± 19	664 ± 20	680 ± 19
w_{-} woundedcoast	656±19	$698 {\pm} 20$	668 ± 17	$706{\pm}18$	632±20	664 ± 20	636 ± 19	670 ± 20
DAO2	$2,510\pm36$	$2,\!666{\pm}76$	$2,534\pm71$	$\underline{2,702{\pm}73}$	$2,378\pm69$	$2,546\pm75$	$2,382\pm68$	$2,550\pm75$
maze-32-32-2	274±5	278±5	282±5	278±6	272±6	280±5	280±6	286±6
maze-32-32-4	248±7	$256 {\pm} 7$	248 ± 7	$254 {\pm} 7$	246±7	$252 {\pm} 7$	246 ± 7	248 ± 7
maze-128-128-2	210±6	$254 {\pm} 6$	210 ± 6	$256 {\pm} 6$	210±6	248 ± 6	210 ± 6	$252 {\pm} 7$
maze-128-128-10	304±9	$\underline{\textbf{364} \!\pm\! 11}$	304 ± 9	$360{\pm}11$	296±9	340 ± 10	296 ± 9	340 ± 11
maze	$1,226\pm19$	$\underline{1,336{\pm}41}$	$1,234\pm39$	$1,148\pm31$	$1,214\pm39$	$\textbf{1,304} {\pm} \textbf{41}$	$1,222\pm 39$	$1,\!310\!\pm\!41$
random-64-64-10	1,226±43	$1,222\pm40$	$1,226\pm43$	$1,266 \pm 39$	$1,232{\pm}41$	1,210±39	$1,232{\pm}41$	1,210±39
random-64-64-20	856±29	860±30	864±30	$920{\pm}32$	842±28	862±30	842±28	852 ± 30
random-32-32-10	$664{\pm}25$	$674 {\pm} 26$	$664 {\pm} 25$	$674 {\pm} 26$	$_{680\pm 28}$	614 ± 25	$\underline{\textbf{680} {\pm} \textbf{28}}$	614 ± 25
random-32-32-20	450 ± 19	$438 {\pm} 17$	$\underline{\textbf{450} {\pm} \textbf{19}}$	$448 {\pm} 18$	$\underline{\textbf{450} {\pm} \textbf{19}}$	$424{\pm}16$	$450{\pm}19$	$424{\pm}16$
random	$3{,}196{\pm}59$	$3{,}194{\pm}116$	$3,\!204{\pm}119$	$\underline{3,308{\pm}117}$	$3,\!204{\pm}118$	$3,\!110\!\pm\!113$	$3,\!204\!\pm\!118$	$3,100\pm112$
room-64-64-16	428±16	_432±16	428±16	$428{\pm}15$	$418{\pm}15$	$424{\pm}15$	$420{\pm}15$	$424{\pm}16$
room-64-64-8	292±10	288±10	292 ± 10	$\underline{314} \pm \underline{10}$	290±9	290 ± 10	290 ± 9	290 ± 10
room-32-32-4	282±11	$298 {\pm} 11$	286 ± 11	$\underline{\textbf{306} \!\pm\! \textbf{12}}$	282±11	286 ± 11	286 ± 11	286 ± 11
room	1,002±18	$1,018{\pm}38$	$1,006\pm37$	$1,048 \pm 38$	990 ±36	$1,000\pm37$	996 ± 37	1,000±37
w-10-20-10-2-2	1,134±45	$1,\!372{\pm}65$	$1,134\pm45$	$1,390\pm70$	1,138±45	$1,124\pm45$	$1,138\pm45$	1,124±45
w-10-20-10-2-1	1,018±39	$1,078 \pm 43$	$1,018\pm39$	$\overline{1,066\pm 40}$	$1,042{\pm}41$	$1,022\pm40$	$1,042{\pm}41$	$1,022\pm40$
w-20-40-10-2-2	$1,972 \pm 61$	$\overline{1,998 \pm 64}$	$\textbf{1,972} {\pm} 61$	$1,998 \pm 64$	$1,906\pm60$	$1,890\pm61$	$1,906\pm60$	$1,890\pm61$
w-20-40-10-2-1	1,838±47	$1,870\pm45$	$1,838\pm47$	$\overline{2,038\pm53}$	$1,772\pm48$	$1,740\pm38$	$1,772\pm48$	$1,740\pm38$
warehouse	5,962±96	$6,318{\pm}218$	$5,962\pm193$	$6,492{\pm}228$	$5,858\pm196$	5,776±185	$5,858\pm196$	5,776±185
Total	25,572±897	$26,\!816\!\pm\!935$	$25,632\pm899$	$26,860 \pm 934$	$25,014\pm893$	$25,564\pm895$	$25,036\pm894$	25,574±894
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¹ Previous state-of-the-art.

Table 2: Total problems solved in under 30 seconds on 8-neighbor grid MAPF benchmarks

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Map		CCBS W	ith Bypass				nout Bypass	
	CCBS	DS	BC	KDB	CCBS	DS^1	BC	KDB
Berlin_1_256	$1,654\pm46$	$1,702\pm44$	$1,740 \pm 49$	$1,674\pm44$	$1,576\pm47$	$1,666 \pm 45$	$1,666\pm46$	$1,718 \pm 47$
Boston_0_256	$1,474\pm46$	$1,\!604{\pm}52$	$1,532\pm 50$	$\underline{\textbf{1,626} \!\pm\! 52}$	$1,458\pm46$	$1,554\pm50$	$1,528\pm50$	$\textbf{1,612} {\pm} \textbf{52}$
Paris_1_256	$1,668 \pm 55$	$1,710\pm55$	$1,672 \pm 55$	$1,784 \pm 54$	$1,572\pm53$	$1,600 \pm 51$	$1,608\pm55$	$1,652 \pm 55$
city	$4,796\pm74$	$5{,}016{\pm}153$	$4,944\pm156$	$5{,}084{\pm}150$	$4,606\pm146$	$4,820 \pm 147$	$4,802\pm151$	$4,982\pm155$
den520d	954 ± 31	982 ± 32	$992 {\pm} 32$	$1,022 \pm 34$	934±31	986 ± 30	986 ± 32	$1,004{\pm}30$
brc202d	762 ± 15	854 ± 17	820 ± 16	$876{\pm}15$	752±15	836 ± 17	800 ± 16	$876{\pm}19$
den312d	568 ± 14	$\underline{640{\pm}20}$	572 ± 15	600 ± 17	554±14	600 ± 18	556 ± 14	606 ± 19
lak303d	536 ± 11	572±12	548 ± 11	$\underline{596{\pm}12}$	522±12	562 ± 13	548 ± 11	572 ± 12
orz900d	576 ± 14	634 ± 14	606 ± 15	$656{\pm}15$	578±14	622 ± 14	594 ± 14	628 ± 15
ost003d	646 ± 22	$712 {\pm} 22$	648 ± 22	$\textbf{724} {\pm} \textbf{24}$	640±22	692 ± 21	646 ± 22	696 ± 21
DAO	$4,042 \pm 55$	$4,\!394\!\pm\!119$	$4,\!186\!\pm\!114$	$4,\!\overline{474\pm120}$	$3,980\pm111$	$4,298\pm116$	$4,130{\pm}112$	$4,382{\pm}119$
empty-8-8	400±6	408±6	412±6	$\underline{422 \pm 6}$	402±6	400±6	420±6	408±6
empty-16-16	536 ± 13	594 ± 14	526 ± 13	$\underline{\textbf{658} {\pm} \textbf{16}}$	520±13	616 ± 17	526 ± 13	618 ± 17
empty-32-32	976 ± 29	$1,128\pm30$	$1,008\pm27$	$\underline{\textbf{1,146} \!\pm\! 26}$	922±28	$1,000\pm23$	958 ± 26	$1,032\pm26$
empty-48-48	$1,258\pm39$	$1,\!430\!\pm\!47$	$1,302 \pm 45$	$1,\!402{\pm}43$	$1,288\pm43$	$1,288\pm41$	$1,366\pm51$	$1,338\pm48$
empty	$3,\!170\!\pm\!44$	$3{,}560{\pm}99$	$3,248\pm93$	$\underline{3,628{\pm}92}$	$3,\!132\pm 91$	$3,304\pm89$	$3,270\pm98$	$3,396\pm98$
lt_gallowstemplar_n	738±18	$870 {\pm} 19$	768±20	$864{\pm}21$	748±18	818±18	768±19	820±20
ht_chantry	742 ± 15	774 ± 15	762 ± 16	$\underline{810{\pm}18}$	732±15	768 ± 14	748 ± 15	786 ± 15
ht_mansion_n	800 ± 21	872 ± 22	816 ± 22	$\underline{908{\pm}23}$	762±19	796 ± 21	792 ± 20	818 ± 20
w_woundedcoast	716 ± 14	$802{\pm}14$	740 ± 14	$798{\pm}13$	704±15	792 ± 14	736 ± 14	$818{\pm}14$
DAO2	$2,996\pm35$	$3{,}318{\pm}71$	$3,086\pm74$	$\underline{3,380{\pm}75}$	2,946±69	$3,\!174\pm69$	$3,044\pm70$	$3,242\pm70$
maze-32-32-2	282 ± 4	298 ± 4	290 ± 4	$\underline{308\pm5}$	280±5	$\underline{\textbf{308} {\pm} \textbf{5}}$	286 ± 5	$304 {\pm} 5$
maze-32-32-4	258 ± 8	260 ± 8	260 ± 8	$\underline{274{\pm}9}$	258±8	260 ± 8	260 ± 8	260 ± 8
maze-128-128-2	244 ± 6	$260{\pm}7$	240 ± 6	$\underline{\textbf{264} \!\pm\! 7}$	244±6	$262 {\pm} 7$	242 ± 6	$262 {\pm} 7$
maze-128-128-10	458 ± 10	474 ± 11	458 ± 10	$\underline{490\pm11}$	454±10	474 ± 11	454 ± 10	472 ± 12
maze	1,242±15	1,292±32	1,248±30	$1,336 \pm 33$	1,236±31	$1,\!304\!\pm\!33$	1,242±31	$1,298\pm33$
random-64-64-10	$1,242 \pm 33$	$1,354\pm24$	$1,266 \pm 35$	$\underline{1,448{\pm}27}$	1,162±30	$1,282 \pm 25$	$1,\!196\!\pm\!34$	$1,304 \pm 28$
random-64-64-20	$870{\pm}19$	852 ± 21	$\underline{884{\pm}20}$	$\underline{884{\pm}24}$	862±19	860 ± 21	$870{\pm}19$	858 ± 21
random-32-32-10	878 ± 24	894 ± 25	872 ± 23	$\underline{934{\pm}24}$	840±22	842 ± 22	846 ± 22	$846 {\pm} 22$
random-32-32-20	502 ± 17	$\underline{528 \pm 18}$	$516{\pm}19$	494 ± 17	504 ± 17	502 ± 18	$514{\pm}18$	506±18
random	$3,492\pm47$	$3,628\pm90$	$3,538 \pm 98$	$\underline{3,760{\pm}94}$	$3,368\pm90$	$3,486\pm87$	$3,426 \pm 95$	$3,514\pm90$
room-64-64-16	$466 {\pm} 13$	480 ± 14	$466 {\pm} 14$	500 ± 16	466±13	476 ± 14	$466 {\pm} 14$	$478 {\pm} 14$
room-64-64-8	332 ± 8	$340 {\pm} 8$	326 ± 8	$336{\pm}9$	330±8	$\underline{342 \pm 8}$	324 ± 8	330 ± 8
room-32-32-4	318±9	$328{\pm}9$	316±8	$\underline{332\pm8}$	306±9	$324{\pm}9$	310±9	316±9
room	$1,116\pm15$	$1{,}148{\pm}32$	1,108±31	$1,\!168\pm33$	1,102±31	$1{,}142{\pm}32$	$1,100\pm31$	1,124±32
w-10-20-10-2-2	$1,666 {\pm} 57$	$1,670 \pm 48$	$1,694 \pm 56$	$\underline{1,824{\pm}42}$	1,532±53	$1,558 \pm 51$	$1,538 \pm 52$	$1,568 {\pm} 51$
w-10-20-10-2-1	978 ± 33	$\underline{1,076{\pm}35}$	976 ± 33	$1,\!070\!\pm\!34$	1,012±33	$1,036\pm35$	$1,010\pm33$	$1,030\pm35$
w-20-40-10-2-2	$2,572\pm81$	$2,484\pm75$	$2,602\pm83$	$\underline{\textbf{2,698} \!\pm\! 75}$	$2,524\pm79$	$2,470\pm75$	$2,518\pm77$	$2,466\pm75$
w-20-40-10-2-1	$1,692\pm62$	$1,760{\pm}65$	$1,692\pm62$	$1,688\pm59$	$1,502\pm53$	$1,624\pm58$	$1,516\pm55$	$1,658\pm60$
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warehouse Total	$6,908\pm117$ $27,762\pm811$	$6,990\pm224$ $29,346\pm823$	$6,964\pm236$ $28,322\pm835$	$\frac{7,280\pm212}{30,110\pm813}$	$6,570\pm219$ $26,940\pm792$	$6,688\pm220$ $28,216\pm796$	$6,582\pm219$ $27,596\pm810$	6,722±222 28,660±824

¹ Previous state-of-the-art.

Table 3: Total problems solved in under 30 seconds on 16-neighbor grid MAPF benchmarks

Map		CCBS W	ith Bypass		1		hout Bypass	
Мар	CCBS	DS	BC	KDB	CCBS	DS^1		KDB
Berlin_1_256	1,220±22	1,400±24	1,230±23	1,484±28	1,150±22	1.306±23	1,150±22	1,358±25
Boston_0_256	$1,220\pm22$ $1,176\pm32$	$1,346\pm30$	$1,230\pm23$ $1,190\pm32$	$\frac{1,484\pm28}{1,332\pm37}$	$1,130\pm22$ $1,124\pm31$	$1,300\pm23$ $1,332\pm31$	$1,130\pm22$ $1,130\pm31$	$1,338\pm32$
Paris_1_256	1,170±32 1,196±28	$\frac{1,340\pm30}{1,298\pm35}$	$1,190\pm 32$ $1,204\pm 28$,	$1,124\pm 31$ $1,080\pm 29$	$1,332\pm31$ $1,268\pm31$	$1,130\pm 31$ $1,088\pm 29$	$1,336\pm32$ $1,262\pm31$
city	$3,592\pm41$	1,298±35 4,044±90	$3,624\pm28$	$\frac{1,332\pm32}{4,148\pm99}$	$3,354\pm83$	$3,906\pm86$	$3,368\pm83$	$\frac{1,202\pm31}{3,958\pm89}$
den 520d	784±21	966 ± 23	806 ± 21	$\underline{1,020{\pm}28}$	762±20	946 ± 24	786 ± 21	946 ± 24
brc202d	596±14	646 ± 12	604 ± 14	$\underline{682{\pm}11}$	594 ± 14	662 ± 13	602 ± 14	668 ± 12
den312d	506±12	594 ± 12	516 ± 13	$\underline{\textbf{624} {\pm} \textbf{12}}$	506 ± 12	586 ± 12	514 ± 12	588 ± 12
lak303d	380±9	442 ± 10	384 ± 9	$\underline{478{\pm}12}$	382±9	436 ± 9	386 ± 9	440 ± 9
orz900d	344±10	400 ± 10	354 ± 10	$\underline{\textbf{426} {\pm} 9}$	328±9	400 ± 10	334 ± 9	404 ± 10
ost003d	512±11	$568{\pm}13$	514 ± 11	$570{\pm}11$	506 ± 11	548 ± 13	506 ± 11	556±13
DAO	$3,122\pm40$	$3,616\pm82$	$3,\!178\pm\!81$	$3,\!800\pm\!86$	$3,078\pm78$	$3,578\pm83$	$3{,}128{\pm}79$	$3,602\pm83$
empty-8-8	384±5	390 ± 5	388 ± 5	$404{\pm}5$	384±5	392 ± 5	382 ± 5	392±5
empty-16-16	486±9	582 ± 9	524 ± 10	$\underline{602 {\pm} 9}$	478±10	562 ± 9	504 ± 9	552 ± 10
empty-32-32	826±22	996 ± 21	870 ± 22	$1,008{\pm}21$	792 ± 23	980 ± 20	794 ± 23	$1,\!012\!\pm\!21$
empty-48-48	1,212±30	$1,\!358{\pm}31$	$1,212\pm30$	$1,352 {\pm} 32$	$1,128\pm30$	$1,318\pm25$	$1,128\pm30$	$1,322\pm26$
empty	2,908±34	$3,\!326{\pm}67$	$2,994\pm68$	$3,\!366\!\pm\!68$	2,782±69	$3,252 \pm 61$	$2,808\pm68$	$3,278\pm63$
lt_gallowstemplar_n	656±17	762±18	686±17	794±16	648±19	726±19	662±19	734±19
ht_chantry	582±12	$662 {\pm} 14$	596 ± 12	$\overline{\textbf{676} {\pm} \textbf{14}}$	582±12	$662 {\pm} 14$	592 ± 12	656 ± 14
ht_mansion_n	556±13	710 ± 11	562 ± 13	$\overline{\textbf{738} {\pm} \textbf{12}}$	548±12	698 ± 10	564 ± 12	710 ± 11
w_{-} woundedcoast	450±9	528 ± 10	462 ± 10	568±9	436±9	512 ± 9	454 ± 10	524 ± 9
DAO2	$2,244\pm26$	$2,662 \pm 54$	$2,306\pm54$	$2,776 \pm 53$	$2,214\pm54$	$2,598\pm54$	$2,\!272\pm54$	2,624±53
maze-32-32-2	246±4	270±4	250±4	278±5	246±4	272±4	246±4	$274{\pm}3$
maze-32-32-4	228±6	$\underline{\textbf{250}}\underline{+}6$	234 ± 6	$\overline{248 {\pm} 6}$	224±5	$250{\pm}7$	232 ± 6	$248 {\pm} 6$
maze-128-128-2	194±5	218±5	194 ± 5	$232 {\pm} 4$	194±5	210±5	192 ± 5	210 ± 5
maze-128-128-10	366±12	416 ± 10	376 ± 11	$\overline{430{\pm}11}$	374 ± 11	410 ± 12	376 ± 11	$414{\pm}12$
maze	$1,034\pm13$	$1,154\pm27$	$1,054\pm28$	$1,\!188\pm\!28$	$1,038\pm27$	$1,142\pm28$	$1,046\pm27$	1,146±28
random-64-64-10	968±26	1,060±29	1,022±25	1,108±30	914±24	1,040±28	920±25	1,052±27
random-64-64-20	722±18	790 ± 18	752±18	$\tfrac{2,100\pm30}{810\pm21}$	706±18	778±18	730 ± 18	786±20
random-32-32-10	688±12	784±16	708 ± 12	$824{\pm}16$	690±12	774±16	700 ± 11	778±15
random-32-32-20	478±13	490 ± 14	478±13	518 ± 15	474±13	488±14	472 ± 13	494 ± 14
random	2,856±35	3,124±77	2,960±70	$3,260 \pm 84$	2,784±69	3,080±77	2,822±69	3,110±77
room-64-64-16	370±13	418±14	386±15	416±16	366±13	414±14	380±14	$410{\pm}15$
room-64-64-8	302±7	330±9	306±8	342 ± 10	304±7	322±9	304±8	318±8
room-32-32-4	304±9	310±9	302±8	$\begin{array}{c} 324\pm 9 \end{array}$	302±9	308 ± 9	302±8	312±9
room	976 ± 15	$1,058\pm33$	994 ±31	$1,082\pm36$	972 ±30	1,044±34	986 ±31	1,040±33
w-10-20-10-2-2	1.340±46	1,456±42	1,494±48	$\frac{-}{1,596\pm35}$	1,314±45	1,444±45	1,360±43	1,486±43
w-10-20-10-2-2 w-10-20-10-2-1	984±33	$1,004\pm32$	982±32	$\frac{1,930\pm35}{1,022\pm35}$	958±32	$1,028\pm33$	968 ± 32	$1,012\pm33$
w-20-40-10-2-2	2,350±75	$2,394\pm63$	$2,346\pm74$	$\frac{2,546\pm71}{2}$	$2,166\pm69$	$\frac{1,028\pm 33}{2,266\pm 66}$	$2,196\pm71$	$2,266\pm66$
w-20-40-10-2-2 w-20-40-10-2-1	$1,672\pm43$	$1,796\pm44$	$1,670\pm42$	$\frac{2,340\pm 71}{1,762\pm 41}$	$1,638\pm43$	$1,778\pm43$	$1,636\pm43$	$1,786\pm43$
warehouse	6,346±99	$\frac{1,730\pm44}{6,650\pm183}$	6,492±198	$6,926\pm184$	6.076 ± 191	6,516±188	$6,160\pm191$	$\frac{1,730\pm43}{6,550\pm186}$
Total	23.078 ± 613	$25,634\pm618$	$23,602\pm618$	$\frac{6,526\pm134}{26,546\pm641}$	$22,298\pm604$	$25,116\pm614$	$22,590\pm606$	$\frac{0,330\pm130}{25,308\pm616}$
10001	20,010-010	25,004±010	20,002 ± 010	<u>~0,0±0±0±1</u>	22,200±004	20,1101014	22,000±000	20,000±010

¹ Previous state-of-the-art.

Table 4: Total problems solved in under 30 seconds on 32-neighbor grid MAPF benchmarks

	CCBS With Bypass			CCBS Without Bypass				
Map	CCBS	DS DS	ith Bypass BC	KDB	CCBS	DS ¹	out Bypass BC	KDB
Berlin_1_256	874±20	1,120±26	880±21	1,206±31	846±21	$1,084\pm25$	844±21	1,128±27
	ll .		880±21 880±25				844 ± 21 842 ± 24	
Boston_0_256	854±23	$1,130\pm22$		$\frac{1,242\pm24}{1,222\pm25}$	820±24	$1,122\pm22$		$1,132\pm22$
Paris_1_256	814±26	1,108±25	840±27	$\frac{1,232\pm25}{2,232\pm31}$	810±27	1,104±24	834±27	1,154±25
city	$2,542\pm35$	$3,358\pm74$	$2,600\pm73$	$\underline{3,680{\pm}81}$	$2,476\pm72$	$3,310\pm72$	$2,520\pm74$	$3,414\pm75$
den 520d	484±16	696 ± 14	526 ± 14	$\underline{726 \!\pm\! 17}$	468±15	686 ± 13	528 ± 14	706 ± 13
brc202d	404±13	498 ± 11	420 ± 12	$524 {\pm} 11$	412±13	514 ± 12	428 ± 13	$\underline{530{\pm}11}$
den312d	432±9	544 ± 9	446 ± 10	$\underline{\textbf{564} {\pm} 9}$	426±9	528 ± 9	438 ± 10	550 ± 10
lak303d	306±8	400 ± 9	318 ± 9	$\underline{\textbf{432} {\pm} 9}$	316±9	398 ± 9	320 ± 9	400 ± 9
orz900d	240±8	302 ± 7	252 ± 8	$\underline{\textbf{332} {\pm} 8}$	246±8	300 ± 7	256 ± 8	304 ± 7
ost003d	318±8	444 ± 11	318 ± 8	480 ± 11	326±9	434 ± 11	332 ± 9	450 ± 11
DAO	$2,184\pm33$	$2,\!884{\pm}62$	$2,\!280\!\pm\!64$	$\underline{3,058{\pm}67}$	$2,194\pm66$	$2,\!860\!\pm\!64$	$2,302 \pm 65$	$2,940\pm64$
empty-16-16	410±10	540 ± 13	428±9	$\underline{590\pm10}$	410±10	540 ± 10	430 ± 10	552 ± 12
empty-32-32	0±0	0 ± 0	680 ± 19	$\underline{\textbf{956} {\pm} \textbf{19}}$	640±19	810 ± 18	655 ± 18	840 ± 19
empty-48-48	858±18	$1,130\pm25$	918 ± 19	$\underline{1,}176\pm22$	866±21	$1,066\pm24$	880 ± 20	$1,098\pm24$
empty-8-8	372±5	$388{\pm}5$	384 ± 5	$394{\pm}6$	366±5	382 ± 5	364 ± 5	382 ± 5
empty	$1,640\pm17$	$2,058\pm43$	$2,410\pm54$	$\underline{3,}116{\pm}58$	$2,282\pm57$	$2,798\pm59$	$2,329\pm55$	$2,872\pm60$
lt_gallowstemplar_n	530±13	666±17	548±14	682±15	536±13	654±17	550±14	652±16
ht_chantry	452±12	568 ± 13	468 ± 12	$\overline{604{\pm}13}$	436±13	570 ± 14	458 ± 13	588 ± 15
ht_mansion_n	408±11	586 ± 10	426 ± 11	$612{\pm}10$	390±10	572±9	406 ± 10	578 ± 10
w_woundedcoast	322±8	398±8	332±8	$\overline{444\pm9}$	320±8	394 ± 7	324 ± 8	406±8
DAO2	$1,712\pm22$	2,218±49	$1,774\pm46$	$2,342 \pm 49$	$1,682\pm45$	$2,190\pm48$	1,738±45	2,224±50
maze-32-32-2	214±4	262±4	220±3	_276±4	214±4	260±4	218±3	264±4
maze-32-32-4	176±4	238±6	194 ± 3	$\overline{264\pm6}$	172±3	238±6	192 ± 4	238±6
maze-128-128-2	148±5	194 ± 4	160 ± 5	$210{\pm}5$	148±5	196±4	166 ± 5	198±4
maze-128-128-10	270±9	334 ± 10	294 ± 11	$\overline{374\pm9}$	278±10	336 ± 10	296 ± 11	360 ± 10
maze	808 ±11	1,028±26	868 ±24	$1{,}124{\pm}27$	812 ±23	1,030±26	872 ±23	$1,060\pm26$
random-64-64-10	774±18	966±18	822±20	1,080±16	770±18	968±19	816±19	1,010±20
random-64-64-20	610±11	720 ± 15	638 ± 13	$\tfrac{1,030\pm10}{782\pm17}$	610±11	706 ± 14	634 ± 13	734 ± 16
random-32-32-10	604 ± 15	696 ± 13	628 ± 16	$\frac{782\pm17}{788\pm13}$	590±15	686 ± 13	622 ± 16	702 ± 13
random-32-32-10	438±13	484 ± 12	446 ± 12	$\frac{768\pm15}{536\pm15}$	434±13	480 ± 12	444 ± 12	494 ± 12
random	2,426±29	$2,866\pm60$	$2,534\pm63$	$\frac{330\pm13}{3,186\pm62}$	2,404±18	$2,840\pm59$	2,516±61	$2,940\pm63$
	322±8	408±12	348±8		322±8	402±11	342±8	408±12
room-64-64-16 room-64-64-8	322 ± 8 274 ± 7	408 ± 12 324 ± 8	348 ± 8 278 ± 7	$\tfrac{430\pm14}{352\pm8}$	322 ± 8 274 ± 7	402 ± 11 322 ± 8	342 ± 8 278 ± 7	408 ± 12 320 ± 7
room-32-32-4	304±9	324 ± 6 310 ± 9	302±8	$\frac{332\pm8}{324\pm9}$	302±9	322 ± 6 308 ± 9	302±8	320 ± 7 312 ± 9
room	900 ± 12	1,042±30	928 ±24	$\frac{324\pm 9}{1,106\pm 32}$	898 ±25	1,032±29	922 ±24	$1,040\pm29$
					1	*		*
w-10-20-10-2-2	1,096±28	$1,294\pm29$	$1{,}124{\pm}27$ $980{\pm}32$	$\frac{1,376\pm29}{1,022\pm24}$	1,040±28 958±30	$1,208\pm29$	$1,076\pm28$	$1,244\pm28$
w-10-20-10-2-1	982±32	$\frac{1,040\pm33}{2,084\pm57}$		$1,022 \pm 34$		$1,030\pm33$	962±30	$1,024\pm33$
w-20-40-10-2-2	1,822±55	$2,084\pm57$	1,888±58	$\frac{2,352\pm57}{1,739\pm39}$	1,742±51	$1,974\pm50$	1,812±56	$2,036\pm56$
w-20-40-10-2-1 warehouse	$1,582\pm34$ $5,482\pm75$	$1,710\pm37$ $6,128\pm157$	$1,596\pm35$ $5,588\pm153$	$\frac{1,728\pm38}{6,478\pm160}$	$1,578\pm33$ $5,318\pm144$	$1,620\pm36$ $5,832\pm149$	$1,582\pm33$ $5,432\pm149$	$1,622\pm36$ $5,926\pm154$
	$5,482\pm75$ $17,694\pm473$	0.128 ± 157 21.582 ± 505	$5,588\pm153$ $18,982\pm504$	$\frac{6,478\pm160}{24,000\pm520}$		$5,832\pm149$ 21.892 ± 512	$5,432\pm149$ $18,631\pm501$	
Total	11,094±473	21,582±505	18,982±504	$24,090 \pm 539$	$18,066\pm493$	21,892±512	10,031±501	$22,416\pm524$

 $[\]overline{\ }^{1}$ Previous state-of-the-art.

Table 5: Total problems solved in under 30 seconds on road maps $\,$

Map		CCBS With Bypass				CCBS Without Bypass			
	CCBS	DS	BC	KDB	CCBS	DS^1	BC	KDB	
sparse	396±7	434±9	400 ± 7	$444{\pm}11$	396±8	434±9	404±8	440±8	
dense	498 ± 13	604 ± 11	536 ± 12	$\underline{712{\pm}12}$	498 ± 13	604 ± 11	536 ± 12	630 ± 14	
super-dense	272 ± 10	402 ± 11	318 ± 10	$474{\pm}11$	272±10	402 ± 11	318 ± 10	$442 {\pm} 11$	
Total	$1,166\pm15$	$1,440\pm32$	$1,254\pm29$	$\underline{1,630{\pm}36}$	$1,\!166\!\pm\!31$	$1,440\pm32$	$1,258\pm30$	$1,512\pm34$	

 $^{^{1}\,\}mathrm{Previous}$ state-of-the-art.

3 Significance Testing Versus Previous State of the Art

The following tables show the p-values from results of significance testing using four tests:

- The paired T-Test, where pairs are the individual results (max number of agents solvable) for the 25 instances from each map.
- The Wilcoxon Ranked Sums test.
- The differential T-Test, where samples are the difference between pairs.
- The Wilcoxon Signed Rank test.

The T-Tests are for data that is normally distributed, or approximately so. The Wilcoxon tests are non-parametric, and likely more appropriate for this data set. The first two tests measure how similar the results are. The second two tests measure the magnitude of the difference between tests. The first two tests will tell us how much the two algorithms differ, e.g., whether one is stronger in certain maps than the other. The second two tests will tell us whether one algorithm significantly outperforms the other. The latter test does not report which algorithm is better, only the p-value that one is better. Thus it is required to state which algorithm has a higher total, this is indicated in the "Winner" column.

P-values of greater than 80% confidence are bold, p-values of greater than 95% confidence have and asterisk (*), p-values of greater than 99% confidence have two asterisks (**), p-values of greater than 99.9% confidence have three asterisks (***) and p-values of greater than 99.99% confidence have four asterisks (****).

Table 6: Significance Testing on 4-neighbor grid MAPF benchmarks

Table	Paired	Wilcoxon	Differential	Wilcoxon	
Map	T-Test	Ranked Sums	T-Test	Signed Rank Test	Winner
Berlin_1_256	0.5031	0.4669	0.0914	0.0656	BP+KDB
$Boston_0_256$	0.7792	0.7934	0.0278*	0.0412*	BP+KDB
Paris_1_256	0.6597	0.5475	0.1141	0.0925	BP+KDB
den 520d	0.8403	0.869	0.0898	0.1088	BP+KDB
brc202d	0.7092	0.7489	0.2199	0.3499	BP+KDB
den312d	0.1432	0.1595	0.0065**	0.0114*	BP+KDB
lak303d	0.1479	0.1327	0.0023**	0.0036**	BP+KDB
orz900d	0.3776	0.5347	0.1088	0.0986	Base
ost003d	0.7991	0.7269	0.5461	0.5271	BP+KDB
empty-16-16	0.7396	0.7196	0.0849	0.0235*	BP+KDB
empty-32-32	0.794	0.7342	0.3494	0.2476	BP+KDB
empty-48-48	0.4946	0.5157	0.0158*	0.0143*	BP+KDB
empty-8-8	0.3851	0.332	0.1023	0.0679	BP+KDB
$lt_gallowstemplar_n$	0.7078	0.7342	0.0504	0.0412*	BP+KDB
ht_chantry	0.7119	0.6415	0.2623	0.1228	BP+KDB
$ht_{mansion_n}$	0.3459	0.3826	0.0125*	0.0167*	BP+KDB
$w_woundedcoast$	0.5636	0.5869	0.0298*	0.0206*	BP+KDB
maze-32-32-2	0.9267	0.8996	0.8659	0.586	Base
maze-32-32-4	0.9425	0.9459	0.7463	0.7855	BP+KDB
maze-128-128-2	0.7341	0.8234	0.2124	0.2059	BP+KDB
maze-128-128-10	0.6307	0.6415	0.0863	0.0928	BP+KDB
random-64-64-10	0.6958	0.7196	0.2191	0.1088	BP+KDB
random-64-64-20	0.6157	0.6276	0.1078	0.1064	BP+KDB
random-32-32-10	0.5298	0.5737	0.1536	0.1408	BP+KDB
random-32-32-20	0.7079	0.7636	0.13	0.1441	BP+KDB
room-64-64-16	0.9443	0.892	0.7645	0.4386	BP+KDB
room-64-64-8	0.5225	0.4609	0.0559	0.0619	BP+KDB
room-32-32-4	0.6497	0.6695	0.0863	0.1025	BP+KDB
w-10-20-10-2-2	0.2192	0.4551	0.066	0.0464*	BP+KDB
w-10-20-10-2-1	0.7651	0.7415	0.2992	0.1755	BP+KDB
w-20-40-10-2-2	0.6349	0.6624	0.2918	0.285	BP+KDB
w-20-40-10-2-1	0.0842	0.1229	0.0144*	0.0049**	BP+KDB

Table 7: Significance Testing on 8-neighbor grid MAPF benchmarks

	Paired	Wilcoxon	Differential	Wilcoxon	<u> </u>
Map	T-Test	Ranked Sums	T-Test	Signed Rank Test	Winner
Berlin_1_256	0.9608	0.8996	0.7589	0.6219	BP+KDB
Boston_0_256	0.6977	0.522	0.1309	0.0219	BP+KDB
Paris_1_256	0.3417	0.322 0.273	0.0077**	0.0107*	BP+KDB
den520d	0.7574	0.6345	0.2387	0.2456	BP+KDB
brc202d	0.7574	0.5157	0.1367	0.0385*	BP+KDB
den 312d	1.0	0.969	1.0	0.6304	BP+KDB
lak303d	0.4628	0.4492	0.0571	0.0606	BP+KDB
orz900d	0.5351	0.5936	0.01*	0.0066**	BP+KDB
ost003d	0.7046	0.8084	$\boldsymbol{0.1942}$	0.2359	BP+KDB
empty-16-16	0.4905	0.4669	0.154	0.1326	BP+KDB
empty-32-32	0.1088	0.1377	0.0045**	0.0051**	BP+KDB
empty-48-48	0.4597	0.4551	0.0266*	0.0379*	BP+KDB
empty-8-8	0.3866	0.4151	0.1704	0.1813	BP+KDB
lt_gallowstemplar_n	0.5274	0.5803	0.2528	0.2052	BP+KDB
$ht_{-}chantry$	0.4862	0.3721	0.1924	0.2857	BP+KDB
$ht_{mansion_n}$	0.1689	0.1775	0.0031**	0.0022**	BP+KDB
$w_woundedcoast$	0.9068	0.8009	0.6489	0.8104	BP+KDB
maze-32-32-2	1.0	0.9845	1.0	0.927	BP+KDB
maze-32-32-4	0.6726	0.7342	0.1659	0.1797	BP+KDB
maze-128-128-2	0.94	0.8538	0.6639	0.6547	BP+KDB
maze-128-128-10	0.7091	0.7196	0.1185	0.1201	BP+KDB
random-64-64-10	0.0877	0.056	0.0225*	0.0207*	BP+KDB
random - 64 - 64 - 20	0.7749	0.698	0.5532	0.9438	BP+KDB
random - 32 - 32 - 10	0.2813	0.2948	0.0687	0.0388*	BP+KDB
random-32-32-20	0.9022	0.9536	0.6886	0.715	Base
room-64-64-16	0.662	0.7124	0.2328	0.2311	BP+KDB
room-64-64-8	0.8522	0.9227	0.4781	0.5176	Base
room-32-32-4	0.8075	0.7342	0.2563	0.2342	BP+KDB
w-10-20-10-2-2	0.1251	$\boldsymbol{0.0952}$	0.0109*	0.0074**	BP+KDB
w-10-20-10-2-1	0.7882	0.7636	0.1562	0.2059	BP+KDB
w-20-40-10-2-2	0.4068	0.3879	0.0429*	0.0376*	BP+KDB
w-20-40-10-2-1	0.7663	0.7269	0.2065	0.2396	BP+KDB

Table 8: Significance Testing on 16-neighbor grid MAPF benchmarks

Table 8: Significance Testing on 16-neighbor grid MAPF benchmarks							
	Paired	Wilcoxon	Differential	Wilcoxon			
Map	T-Test	Ranked Sums	T-Test	Signed Rank Test	Winner		
Berlin_1_256	0.0685	0.0743	0.0049**	0.0083**	BP+KDB		
$Boston_0_256$	1.0	0.9923	1.0	0.5907	BP+KDB		
Paris_1_256	0.5821	0.3826	0.3445	0.3838	BP+KDB		
den 520d	0.4394	0.3084	$\boldsymbol{0.1244}$	0.1194	BP+KDB		
brc202d	0.6521	0.5475	0.1869	0.2031	BP+KDB		
den 312d	0.4051	0.2687	0.0649	0.0719	BP+KDB		
lak303d	0.3106	0.4263	0.012*	0.0033**	BP+KDB		
orz900d	0.4787	0.3933	0.0504	0.056	BP+KDB		
ost003d	0.6316	0.6695	0.1335	0.1646	BP+KDB		
empty-16-16	0.2535	0.2003	0.0106*	0.0088**	BP+KDB		
empty-32-32	0.7115	0.7124	0.4363	0.2935	BP+KDB		
empty-48-48	0.7487	0.8009	0.4326	0.3809	BP+KDB		
empty-8-8	0.5617	0.6004	0.2822	0.2098	BP+KDB		
$lt_gallowstemplar_n$	0.2971	0.3272	0.0124*	0.0072**	BP+KDB		
$ht_chantry$	0.7914	0.7859	0.2153	0.0638	BP+KDB		
$ht_mansion_n$	0.3424	0.3038	0.0202*	0.0176*	BP+KDB		
$w_woundedcoast$	0.12	0.1183	0.0054**	0.0053**	BP+KDB		
maze-32-32-2	0.7275	0.6908	0.6116	0.5827	BP+KDB		
maze-32-32-4	0.935	0.9923	0.7463	0.7855	Base		
maze-128-128-2	0.2447	0.4492	0.0311*	0.0414*	BP+KDB		
maze-128-128-10	0.6425	0.7489	0.218	0.1948	BP+KDB		
random-64-64-10	0.5266	0.4151	0.0249*	0.0178*	BP+KDB		
random-64-64-20	0.661	0.7269	0.1942	0.2115	BP+KDB		
random-32-32-10	0.399	0.4096	0.1178	0.1376	BP+KDB		
random-32-32-20	0.5812	0.522	0.1177	0.1361	BP+KDB		
room-64-64-16	0.9723	0.892	0.9152	0.8878	BP+KDB		
room-64-64-8	0.5777	0.522	0.0475*	0.0473*	BP+KDB		
room-32-32-4	0.6449	0.5869	0.1185	0.1025	BP+KDB		
w-10-20-10-2-2	0.3082	0.3933	0.0409*	0.0374*	BP+KDB		
w-10-20-10-2-1	0.9615	0.9768	0.8845	1.0	Base		
w-20-40-10-2-2	0.2677	0.332	0.0944	0.0992	BP+KDB		
w-20-40-10-2-1	0.9172	0.7562	0.7368	0.7529	Base		

Table 9: Significance Testing on 32-neighbor grid MAPF benchmarks

	Paired	Wilcoxon	Differential	Wilcoxon	
Map	T-Test	Ranked Sums	T-Test	Signed Rank Test	Winner
Berlin_1_256	0.2455	0.2483	0.0046**	0.0005***	BP+KDB
$Boston_0_256$	0.1625	0.1403	0.0034**	0.003**	BP+KDB
Paris_1_256	0.1668	0.0991	0.0009***	0.001**	BP+KDB
den 520d	0.4764	0.4971	0.0912	0.0448*	BP+KDB
brc202d	0.8213	0.7934	0.5261	0.53	BP+KDB
den312d	0.3016	0.332	0.0167*	0.0139*	BP+KDB
lak303d	0.3251	0.2687	0.003**	0.0049**	BP+KDB
orz900d	0.2822	0.3721	0.0011**	0.0025**	BP+KDB
ost003d	0.2733	0.2687	0.0068**	0.0087**	BP+KDB
empty-16-16	0.2029	0.116	0.0009***	0.0018**	BP+KDB
empty-32-32	0.037*	0.0336*	0.0003***	0.0001***	BP+KDB
empty-48-48	0.199	0.2038	0.005**	0.0077**	BP+KDB
empty-8-8	0.5976	0.5283	0.185	0.1927	BP+KDB
$lt_gallowstemplar_n$	0.6365	0.9073	0.309	0.3723	BP+KDB
$ht_chantry$	0.5046	0.4609	0.2023	0.022*	BP+KDB
$ht_{mansion_n}$	0.2837	0.2143	0.002**	0.0028**	BP+KDB
$w_woundedcoast$	0.1108	0.1206	0.0024**	0.0011**	BP+KDB
maze-32-32-2	0.3543	0.4492	0.1476	0.1356	BP+KDB
maze-32-32-4	0.283	0.2903	0.0065**	0.0103*	BP+KDB
maze-128-128-2	0.4627	0.5737	0.05	0.0532	BP+KDB
maze-128-128-10	0.3086	0.1429	0.0019**	0.0035**	BP+KDB
random-64-64-10	0.0857	0.0933	0.0006***	0.0004***	BP+KDB
random-64-64-20	0.1944	0.2563	0.0074**	0.0101*	BP+KDB
random-32-32-10	0.0406*	0.0727	0.0028**	0.0023**	BP+KDB
random-32-32-20	0.281	0.2108	0.0245*	0.0227*	BP+KDB
room-64-64-16	0.5603	0.7342	0.0452*	0.0473*	BP+KDB
room-64-64-8	0.3343	0.3224	0.0105*	0.0109*	BP+KDB
room-32-32-4	0.6449	0.5869	0.1185	0.1025	BP+KDB
w-10-20-10-2-2	0.1218	0.1229	0.0023**	0.0026**	BP+KDB
w-10-20-10-2-1	0.9481	0.9768	0.7512	0.9049	Base
w-20-40-10-2-2	0.0598	0.0991	0.0058**	0.0038**	BP+KDB
w-20-40-10-2-1	0.4278	0.3618	0.1131	0.0452*	BP+KDB

Table 10: Significance Testing on roadmap benchmarks

Man	Paired T-Test	Wilcoxon Ranked Sums	Differential T-Test	Wilcoxon	Winner
Map	1-1est	nanked Sums	1-1est	Signed Rank Test	winner
sparse	0.791	0.6484	0.6252	0.8541	BP+KDB
dense	0.0199*	0.0157*	0.0****	0.0****	BP+KDB
super-dense	0.0914	0.0842	0.0013**	0.0008***	BP+KDB