Electrostatics Problems And Solutions

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Contents

A Problems in Electrostatics

A.1 Problem#1

Problem *1.* (`Point Charges And Force`)

1. Two point charges A and B, having charges +Q and -Q respectively, are placed at certain distance apart and force action between them is F. If 25% charge of A is transferred to B, then force between the charges becomes:

   A. F B. $\frac{9F}{16}$ C. $\frac{16F}{9}$ D. $\frac{4F}{3}$

Solution: Charge on A after the transfer is

$$Q - \frac{1}{4}Q = \frac{3}{4}$$

Charge on B after the transfer is

$$-Q + \frac{1}{4}Q = -\frac{3}{4}Q$$

Now the Force between the charges will become

$$\frac{3}{4} \times \frac{3}{4} \times F = \frac{9}{16}F$$

Therefore option B. is the correct answer.

A.2 Problem#2

Problem *2.* (`Spheres and Surface Charge Densities`)

1. Two metal spheres, one of radius R and the other of radius 2R, both have same surface charge density σ. If they are brought in contact and separated, then the new surface charge densities on each of the sphere are respectively

   A. $\frac{5}{2}\sigma, \frac{5}{4}\sigma$ B. $\frac{5}{3}\sigma, \frac{5}{6}\sigma$ C. $\frac{3}{5}\sigma, \frac{6}{5}\sigma$ D. $\frac{2}{3}\sigma, \frac{1}{2}\sigma$

Solution: Let A be the surface area of the metal sphere of radius R. The surface area of the metal area of radius 2R is $4\pi(2R)^2 = 4 \times 4\pi R^2 = 4A$. The charges will flow until both spheres have the same potential. Also the net charge remains the same before and after contact (as charge is conserved). Charge on sphere of R is

$$\sigma A$$

Charge on sphere of 2R is

$$\sigma \times 4A = 4\sigma A$$

Net charge is conserved. Therefore total charge is 5σ. Potential after contact will be equal. Let σ_1 and σ_2 be the charge densities after contact. Charge on sphere of radius R after contact is $\sigma_1 A$ and on sphere of radius 2R is $\sigma_2 \times 4A$ Therefore,

$$\frac{1}{4\pi\epsilon_0}\frac{\sigma_1 A}{R} = \frac{1}{4\pi\epsilon_0}\frac{4\sigma_2 A}{2R}$$

$$\sigma_1 = 2\sigma_2$$

Net charge after contact is $\sigma_1 A + 4\sigma_2 A$. Charge is conserved (total charge is same before and after). Therefore,

$$\sigma_1 + 4\sigma_2 A = 5\sigma$$

$$2\sigma_2 + 4\sigma_2 A = 5\sigma, \sigma_2 = \frac{5}{6}\sigma$$

$$\sigma_1 = 2\sigma_2 = 2 \times \frac{5}{6}\sigma = \frac{5}{3}\sigma$$

. The new surface charges densities are $\frac{5}{3}\sigma$, $\frac{5}{6}\sigma$. Hence correct option is B.

A.3 Problem#3

Problem *3.* (`Electrostatic and Gravitational Force`)

1. Suppose the charge of a proton and an electron differ slightly. One of them is -e, the other is $(e + \Delta e)$. If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then Δe is of the order of [Given mass of hydrogen $m_h = 1.67 \times 10^{-27}$Kg]

   A. $10^{-47}C$ B. $10^{-20}C$ C. $10^{-23}C$ D. $10^{-37}C$

Solution: Hydrogen atom consists of electron and proton. The net charge on the hydrogen atom is $-e + (e + \Delta e) = \Delta e$. Now the net electrostatic force is

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{\Delta e^2}{d^2}$$

The gravitational force between two hydrogen atoms is given by:

$$F_g = G\frac{m_h^2}{d^2}$$

. As per the problem

$$F_e = F_g$$

$$\frac{1}{4\pi\epsilon_0}\frac{\Delta e^2}{d^2} = G\frac{m_h^2}{d^2}$$

$$\Delta e^2 = 4\pi\epsilon_0 G\frac{m_h^2}{d^2} = 9 \times 10^9 \times 6.67 \times 10^{-11} \times 1.67 \times 10^{-27}$$

$$\Delta e \approx 10^{-37}$$

Hence the correct option is D.

A.4 Problem#4

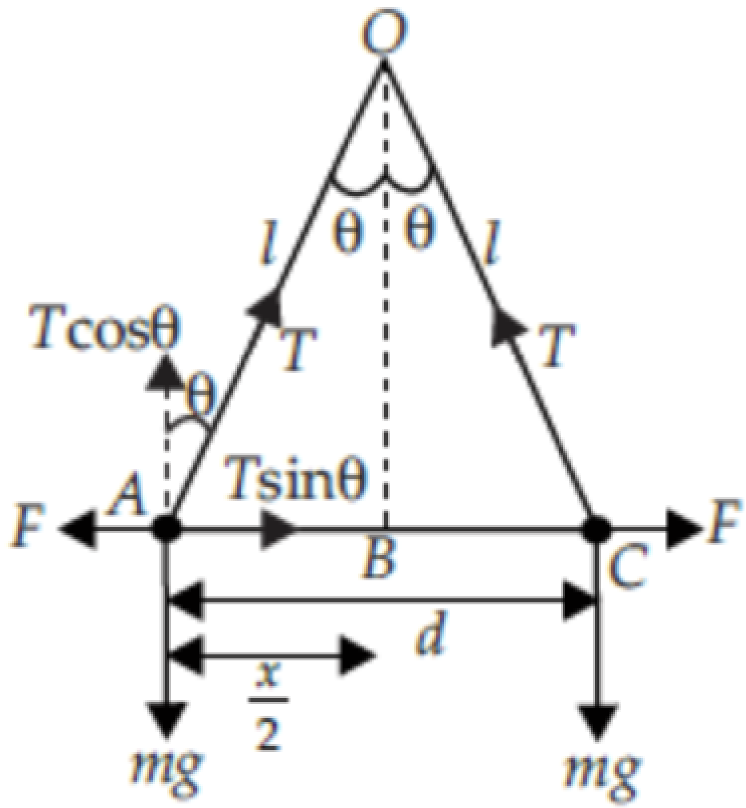
Problem 4. (`Charges Suspended from strings`)

1. Two identical charged spheres suspended from a common point by two massless strings of lengths l, are initially at a distance d ($d \ll l$) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then v varies as a function of the distance x between the spheres, as:

   A. $v \propto x^{\frac{1}{2}}$ B. $v \propto x$ C. $v \propto x^{-\frac{1}{2}}$ D. $v \propto x^{-1}$

Solution:

Draw force diagram showing various forces acting on the spheres (in equilibrium positions).



From the above diagram,

$$T\cos\theta = mg(1) \tag{1}$$

$$T\sin\theta = k\frac{q^2}{mgd^2} \tag{2}$$

Dividing (2) by (1), we get

$$\tan\theta = \frac{kq^2}{mgd^2}$$

If θ is small, then $sin\theta \approx \tan\theta$. Therefore $\tan\theta = \frac{d}{2l}$. The charge is leaking at a constant rate and the both spehres approach and x is the varying as both approach each other. We can write the equation above as:

$$\frac{x}{2l} = \frac{kq^2}{mgx^2}$$

$$\frac{x}{2l} \propto \frac{q^2}{x^2}$$

$$q^2 \propto x^3 \implies q \propto x^{\frac{3}{2}}$$

Differentiation both sides, we get

$$\frac{dq}{dt} \propto \frac{3}{2} x^{\frac{1}{2}} \frac{dx}{dt}$$

Since charge leaks at a constant rate, $\frac{dq}{dt}$ is constant. $\frac{dx}{dt}$ is velocity v. Using this two fact in the above equation,

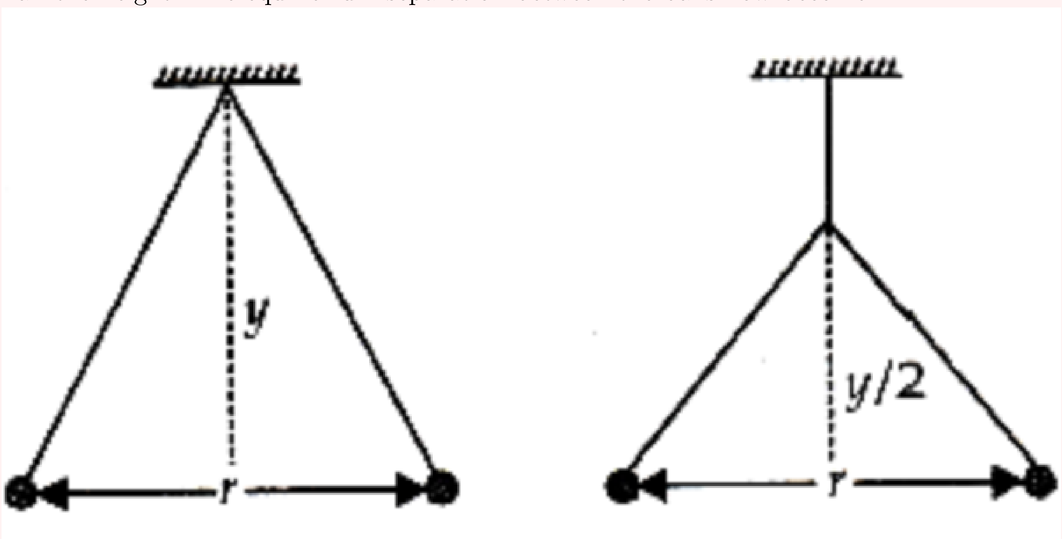
$$v \propto x^{\frac{-1}{2}}$$

. Therefore the correc option is (C).

A.5 Problem#5

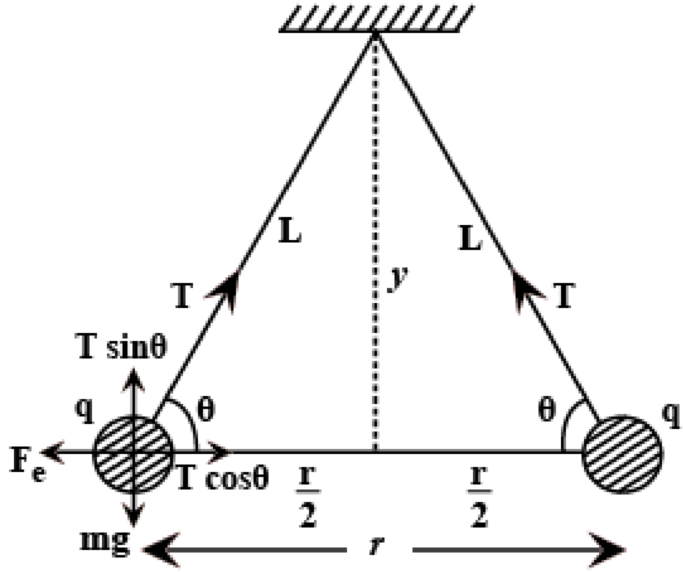
Problem 5. (`Charges Suspended from strings`)

1. Two pith balls carrying equal charges are suspended from a common point by strings of equal length, the equilibrium separation between them is r. Now the strings are rigidly clamped at half the height. The equilibrium separation between the balls now become:



A. $\frac{r}{\sqrt[3]{2}}$ B. $\frac{2r}{\sqrt{3}}$ C. $\frac{2r}{3}$ D. $(\frac{r}{\sqrt{2}})^2$

Solution: Draw force diagram showing various forces acting on the spheres for case #1 (Equilibrium



separation between them is r).

From the above diagram,

$$T\sin\theta = mg(1) \tag{3}$$

$$T\cos\theta = k\frac{q^2}{mgr^2} \tag{4}$$

Dividing (3) by (4), we get

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Also, $\tan\theta = \frac{y}{\frac{r}{2}}$.

$$\frac{2y}{r} = \frac{kq^2}{mgr^2}$$

As $\frac{mg}{kq^2}$ is constant, (rearrange above equation to get):

$$r \propto (y)^{\frac{1}{3}}$$

Let r' be new equilibrium separation when the strings are rigidly clamped at half the height($\frac{y}{2}$). Also

$$r' \propto (\frac{y}{2})^{\frac{1}{3}}$$

Divide both proportionality relations to get r' which is

$$r' = \frac{r}{\sqrt[3]{2}}$$

. Therefore the correct option is A.

A.6 Problem#6

Problem 6. (System of Charges)

1. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to

   A. $\frac{-Q}{4}$ B. $\frac{Q}{4}$ C. $\frac{-Q}{2}$ D. $\frac{Q}{2}$

Solution: For system of charges to be in equilibrium, the net force acting on the charges should be zero. Assume the distance between charge Q and Q be $2a$ and the distance between newly placed charge q and either of the two charges Q shall be a. Let's consider the net force acting on one of the charge Q. This is

$$F_{Qq} + F_{QQ} = \frac{kQq}{a^2} + \frac{kQQ}{(2a)^2}$$

The net force is zero (since the system of charges are in equilibrium.). Hence,

$$\frac{kQq}{a^2} + \frac{kQQ}{4a^2} = 0$$

$$Qq = \frac{-QQ}{4}$$

$$q = \frac{-Q}{4}$$

The correct option is A.

A.7 Problem#7

Problem 7. (Charge Flow between Spheres)

1. Two metallic spheres of radii 1cm and 3cm are given charges of $-1 \times 10^{-2}C$ and $5 \times 10^{-2}C$, respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is

   A. $2 \times 10^{-2}C$ B. $3 \times 10^{-2}C$ C. $4 \times 10^{-2}C$ D. $1 \times 10^{-2}C$

Solution: Charges flow until the potential is equal. Total charge is conserved. Let x denote the charge on the bigger sphere (which is of radii 3cm). The net charge is $(5-1) \times 10^{-2}C = 4 \times 10^{-}2C$. Therefore the charge on smaller sphere is $4/times 10^{-2} - x$. Now the potential of the metallic sphere are equal (potential is same as that of a point sphere which is $\frac{kq}{r^2}$.

$$\frac{k \times x}{3 \times 10^{-2}} = \frac{k \times (4 \times 10^{-2} - x)}{1 \times 10^{-2}}$$

$$x = 12 \times 10^{-2} - 3x$$

$$4x = 12 \times 10^{-2}$$

$$x = 3 \times 10^{-2}C$$

Therefore the correct option is B.

A.8 Problem#8

Problem 8. (Forces between ions, number of electrons)

1. Two positive ions, each carrying a charge q, are separated by a distance d. If F is the force of repulsion between the ions. The number of electrons missing from each ion will be (e being the charge on an electron)

   A. $\frac{4\pi\epsilon_0 Fd^2}{e^2}$ B. $\sqrt{\frac{4\pi\epsilon_0 Fe^2}{d^2}}$ C. $\sqrt{\frac{4\pi\epsilon_0 Fd^2}{e^2}}$ D. $\frac{4\pi\epsilon_0 Fd^2}{q^2}$

Solution: Let n be the number of electrons missing. Then $q = ne$ where n is the number of electrons missing. By coulomb's law, we have

$$F = \frac{1}{4\pi\epsilon_0}\frac{ne \times ne}{d^2}$$

$$n^2e^2 = 4\pi\epsilon_0 Fd^2$$

$$e = \sqrt{\frac{4\pi\epsilon_0 Fd^2}{e^2}}$$

Therefore the correct option is C.

A.9 Problem#9

Problem 9. (Force between electron and proton)

1. An electron is moving round the nucleus of a hydrogen atom in a circular orbit of radius r. The coulomb force $\vec{F}$ between the two is

   A. $K\frac{e^2}{r^3}\vec{r}$ B. $K\frac{e^2}{r^2}\vec{r}$ C. $-K\frac{e^2}{r^3}\vec{r}$ D. $-K\frac{e^2}{r^2}\vec{r}$

Solution: In an hydrogen atom, we have one electron and one proton. Let e be the charge on the proton and -e will be the charge on the electron. By Coulomb's law force between the two is ,

$$\vec{F} = K\frac{-e \times e}{r^2}\hat{r}$$

Now the unit vector $\hat{r}$ can be written as,

$$\hat{r} = \frac{\vec{r}}{r}$$

$$\vec{F} = -K\frac{e^2}{r^2}\vec{r}$$

Therefore, the correct option is D.

A.10 Problem#10

Problem ***10.*** (`Forces between charges in dielectric medium`)

1. When air is replaced by a dielectric medium of dielectric constant k the maximum force of attraction between the two charges separated by a distance:
   - A. decreases K-times
   - B. increases K-times
   - C. remains unchanged
   - D. becomes $\frac{1}{K^2}$ times

Solution: Recall dielectric constant is the relative permittivity and is the ratio of permittivity of the dielectric medium and permittivity of the air/vacuum $epsilon_0$. (Refer NCERT book 2.13 how introduction of dielectric medium between the plates of capacitor affect the electric field). In short, the the introduction of the dielectric opposes the external electric field.This impact due to orientation of charges oppossite to the charges on the plates of the capacitor. Thus potentail difference decreases and therefore the capacitance raises. It is also the ratio of the capacitance in the medium to the capacitance of vacuum/air. It is the factor by which capacitances increases. Now

$$K = \frac{\epsilon}{\epsilon_0}$$

$$\epsilon = K \times \epsilon_0$$

. Also by Coulomb's law , force between two charges q_1 and q_2 in a dielectric medium which has permittivity ϵ is given by:

$$\vec{F} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \hat{r}$$

Therefore

$$F = \frac{1}{4\pi K \epsilon_0} \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{K} \times F_a$$

where F_a is the force when air is the medium. From the above, the force decreases by K-times. Therefore option A. is the correct answer.