Abstract

Perspective distortion often appears in the images captured by cameras leading to recognition errors or failures. This project describes a corrective method to rectify said images or documents to minimize distortions.

The procedure illustrated in the following texts portray the problem from a geometric standpoint, applying mathematical concepts and insight in order to arrive at an acceptable conclusion. The fundamental concepts that serve as the cornerstones of perspective rectification will be detailed out to the be Gauss-Jordan method for elimination, transformations, vector basis, and change of basis.

To aid better conceptualization and visualization, abstract concepts have been modularized and then encapsulated into simple, custom definitions. These definitions have been introduced purely for the purpose of helping the reader grasp the respective abstract concepts they represent, better, and can help provide a better picture of the abstract concepts.

Finally, the report goes on to outline the last of salient deliverables that transpired from this investigative analysis; the working model representing how perspective rectification works, and the methodology of rectifying an image with a distorted perspective using the working model in reality.

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1. Introduction

The word “***Perspective***” has multiple definitions, with the most commonly construed definitions being “particular way of considering something” [1]. However, this is not the definition of interest from the perspective of this report. This subject requires *Perspective* to be perceived as the art of representing three-dimensional objects on a two-dimensional surface so as to give the right impression of their height, width, depth, and position in relation to each other. This also helps *Perspective Rectification* be understood as the process of rectifying the existing perspective of an image, so that it may correspond to the required perspective.

A simple understanding of the fundamentals of linear algebra is sufficient to grasp the technique of perspective rectification and its intricacies.

This investigative analysis attempts to address the two façades of perspective rectification; the theoretical postulations, while encompassing the practical implementation as well.

First, a deep dive into the theoretical concepts behind perspective rectification is effectuated. In order to explain perspective rectification, one must familiarize themselves with the principles of perspective rendering as well. Thus, perspective rendering, followed by perspective rectification will been acutely explained.

The ambition of this investigative analysis into perspective rendering and rectification is threefold.

- Discover perspective rendering and rectification, its intricacies, and complications
- Perform a deep dive into the linear algebra techniques that can be adopted to overcome aforementioned complications
- Produce viable working models of a feasible solution to implement perspective rectification to fix perspective distortion.

All the processes and terminology involved, and custom definitions for conceptualization of abstract notions, are defined in a sequential and easy-to-understand manner aided by visual representations wherever necessary in order to build a solid understanding, from a theoretically standpoint. The second façade, i.e., the practical implementation is illustrated and explained as well, in order to provide the reader with a stronger sense of confidence in the implementation and mechanisms of perspective rectification.

This is but one of the many real-world applications of linear algebra which focuses on the theory of linear equations, matrices, determinants, vector spaces, and linear transformations.

2. Review of Literature

Through analyzing principle of imaging, images captured by cameras are perspective in nature, if digital camera's image plane is not parallel to the photographed documents plane. In such images, objects of similar size will appear bigger as it moves closer to view point and smaller as it moves away. So, the rectangular graphics contained scene may not appear rectangular in the image. [2]

In professional architectural photography, the perspective and therefore the control of horizontal architectural features, especially the aesthetic settlement [3], is very important for the purpose of making architectural photographs suitable and correct.

The main tasks of architectural photographers are to present three-dimensional architectural themes in a realistic and impressive way in two-dimensional photography. In this respect, specializing in this area also requires having technical knowledge and experience, as well as general photographic information.

Perspective distortion often appears in the image documents which were token by Digital camera.[4] This phenomenon will lead to recognition errors or failures. Therefore, a correction algorithm is proposed in this project for perspective distortion images of photographed documents. The algorithm makes use of document image's horizontal characteristics of text line and character's features of vertical strokes, to find distortion information and then rectify the perspective image.

In this study, a geometric method used to obtain rectified photographs that will form a base for drawings of building facades are examined. [3] Mathematical model is presented by informing briefly about the method evaluated. [2] This method does not require information of image's edge and paragraph's format. It has a good effect against the incomplete perspective images and irregular paragraph's format. Experiment show that it takes on fast, accurate and high-robust features when using this method to correct perspective distortion in the document images.

The subject is exemplified by a test study. In the test study, firstly, the behavior of the related methods on the created artificial image was examined and the applications on the real photographic sets of the models obtained with different types and different cameras were performed. The obtained results were evaluated statistically and various conclusions were drawn and the suggestions were made for those who want to apply the method and want to do the study in this subject.

The perspective problem is considered as one of the most difficult problems in computer vision. In this paper, the solution of the problem is proposed by recovering the scene viewed from top (top down view) using homography estimation techniques. [5] Three algorithms are tested for the estimation of the homography relation between the perspective view and the top down view. The Direct Linear Transformation (DLT) using point features, the DLT using line features and the robust RANSAC estimation algorithm. A comparative study regarding the noise and the number of correspondences effects on the performances of these estimation methods is presented.

In most cases, text/object of focus is contained within well-defined rectangular boundaries. Rectangles after undergoing a projective transformation result in quadrilaterals. The vertices of the quadrilaterals could be used to obtain the homography between an arbitrary view to the frontal view. [4] In case the original aspect ratio of the rectangle is known, then the vertices of the quadrilateral in the image can be mapped to the corners of the known rectangle. Thus, exact Rectification could be achieved.

3. First Principles and Preliminaries

3.1. Terminology

The present mental picture presented of perspective rectification is that on a rudimentary level. This mental picture will gradually be remodeled, as the definition is reiterated to encompass the new concepts introduced. This provides a higher level of understanding, and the ability to look at perspective rectifications with multiple outlooks.

Absolute Perspective, and Perspective Distortion

For the purpose this report, the idea of an **absolute perspective**, which is ideally a linear perspective of an image observed with a normal focal length, must be fundamentally conceptualized. The absolute perspective may vary depending upon requirements, but they will always remain parallel to the plane of the photographed image. The **point of absolute perspective** will henceforth be the point in three-dimensional space in reality, from which where the absolute perspective of an image can be observed. An image that has a perspective that does not correspond to the required absolute perspective, is said to have **perspective distortion**. These abstractions will soon prove to be of significant magnitude, as it will help break down complex visualizations and explanations into simpler mental pictures.

Perspective Rectification

Perspective Rectification can be defined as the process of transforming of an image, and its constituent components, into a more authentic depiction of how the image would appear, when observed with a normal focal length, whilst conforming with the commonly accepted distortions in the constructed/initial perspective of the image.
In layman's terms, with the conceptualization of an absolute perspective, perspective rectification can be interpreted as manipulating an image such that it appears to be captured from a position of absolute perspective.

Object, Distorted and Absolute Image

The image that has perspective distortion, in the two-dimensional vector space, will be referred to as the **distorted image**, and the physical object(s) in three-dimensional vector space that is portrayed in the image will be referred to as the **object**.

The **absolute image** can be visualized to be a two-dimensional image of the object when perceived from a point of absolute perspective. The goal of perspective rectification, is to obtain the absolute image, provided the distorted image, and **boundary points.** Boundary points are points, i.e., four sets of coordinates corresponding to the distorted image, which determine the area in the distorted image with respect to which the distorted image must be rectified. In short, boundary points help characterize the absolute perspective.

3.2. Visualization

Perspective rectification now be interpreted as the transformation of a distorted image, to rectify its perspective to be the absolute perspective.

Each distorted image is a combination of pixels. Each pixel can be considered to be a tiny rectangle that has a color assigned to it. Each tiny rectangle in the distorted image corresponds to a parallelogram in the absolute image. The absolute image can thus be formed, by filling each parallelogram with the color of its corresponding rectangle. This reduces the problem of perspective rectification to deriving a function that maps the coordinates of pixels in the distorted image to the corresponding pixels in the absolute image. This function, when applied in the form of a transformation to the distorted image, would invariably produce the expected absolute image.

3.3. Background

Perspective rectification can now be interpreted as the manipulation/transformation of every pixel in the distorted image, to form the absolute image.

The fundamental principle behind performing this transformation lies with the coordinate system. Transforming between the representation of the distorted image in one coordinate system to a representation in another, will help achieve perspective rectification. This transformation can be understood to be like a mapping function, which can be determined by example. Realizing which points in the distorted image plane map to which points in the plane of the absolute image would further lead to the derivation of this mapping function. The boundary for the image, or in some implementations, the part of the image, that is to particularly rectified is specified. This boundary consists of four points, within the distorted image, which will help determine the mapping function, and derive the absolute image from it.

4. Perspective Rendering

4.1. Camera Center and the Image Plane

Consider a simple camera model; the pin hole camera. The pin-hole in the pin-hole camera through which light passes through is known as the **camera center**. [6]
Only the photons that pass linearly through the camera center are detected in the back of the box, by an image sensor array, which is populated by sensor elements, registering an inverted image.
To prevent inversion, consider a simpler camera model, in which the image sensor array is placed between the camera center, and the scene/object being photographed.
The plane containing the image sensor array is called the **image plane**. [6] The sensor element that intersects with the line travelled by a photon, detects the photon that linearly travels to the camera center.

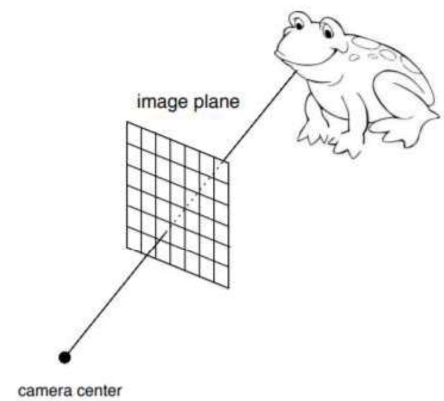

Fig. 4.1.1 Simpler Camera Model [6]

4.2. The Camera Coordinate System and Pixel Coordinates

The Camera Basis [6]

The camera basis defines a basis in the three-dimensional vector space, with respect to the camera. The camera basis is defined to be $[a_1 \quad a_2 \quad a_3]$.

- The origin is situated at the camera center.
- a_1 goes horizontally from the top left corner of the image plane to the top right corner of one sensor element.
- a_2 goes vertically from the top left corner of the image plane to the bottom left corner of one sensor element.
- a_3 goes from the origin, to the top-left corner of the sensor array (0,0).

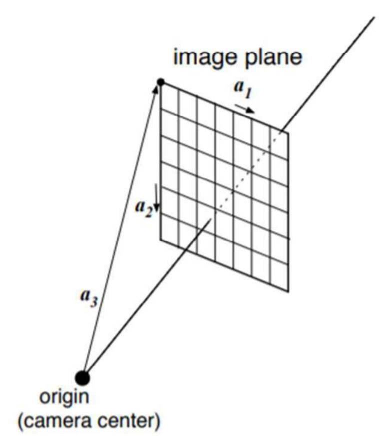

Fig. 4.2.1 The Camera Coordinate System [6]

Scaling Down

Consider the side view of the image plane and any point ***p***, (x_1, x_2, x_3).in the real world.

Expressing ***p*** it in terms of the camera basis, the adjoining figure can be observed.

Since the side view of the image plane is considered, the third coordinate x_1 and its expression in terms of a_1 aren't depicted.

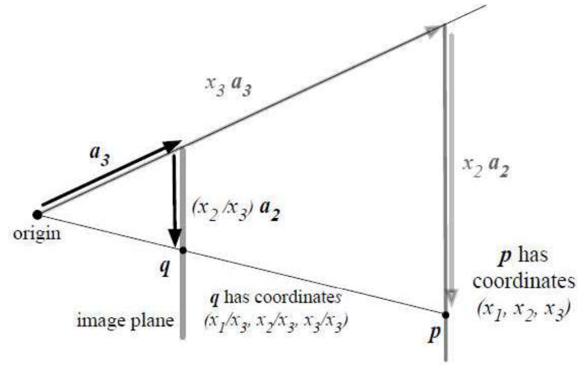

Fig. 4.2.2 Side view of Image Plane while Scaling Down [6]

It has already been established, that for the image to be registered, the ray of light from that point must linearly pass through the origin, and thence be detected by a sensor element in the image plane. Consider the point ***q*** to be the point on the image plane where this ray of light meets.

The coordinates of the point ***q***, that exists on the image plane, can now be easily established by applying the properties of similar triangles, between the two triangle that can be observed.

Since the coordinates of ***p*** is known to be (x_1, x_2, x_3), the coordinates of ***q*** can be calculated to be (x_1/x_3, x_2/x_3, x_3/x_3).

This process can be referred to as **scaling down**.

Obtaining the Pixel Coordinates [6]

After scaling down of an image, it can be observed that the coordinates of ***q*** are determined to be (x_1/x_3, x_2/x_3, 1), where ***p*** is situated at (x_1, x_2, x_3).

Since the third coordinate of ***q*** will always be 1, it can be neglected.

This directly gives the coordinates of the sensor element that detects the ray of light, i.e., the point on the image plane, where the light ray intersects.

Thus, it can be inferred, that ***q*** lies in the pixel coordinate corresponding to (x_1/x_3, x_2/x_3).

5. Perspective Rectification and Homography

5.1. Basis for Perspective Rectification

It has been established that perspective rectification can be achieved by transforming between the representation of the distorted image in one coordinate system to a representation in another. This gives rise to the question regarding which two coordinate systems to be considered, for successfully obtaining the absolute image. The first coordinate system and basis that must be used is the **camera coordinate system**, defined by the **camera basis** $[a_1 \quad a_2 \quad a_3]$ respectively. [7]

The second coordinate system that must be used will be defined by the **object basis**.

The Object Basis [7]

The object basis is a basis defined with respect to the object in the distorted image. The object basis is defined to be $[c_1 \quad c_2 \quad c_3]$.

- The origin is the camera center.
- c_1 goes horizontally from the top left corner to the top right corner of the object.
- c_2 goes vertically from the top left corner to the bottom left corner of the object.
- c_3 goes from the origin, to the top-left corner of the object.

The advantage this camera basis, is that if a point ***q*** corresponding to (y_1, y_2, y_3) on the object is considered, the intersection of the image plane, and a line through the origin (camera center) and ***q***, will have the coordinates (y_1/y_3, y_2/y_3, y_3/y_3).

The coordinate system using the object basis will be referred to as **object coordinate system**.

5.2. Converting between Basis Representations

Consider a point ***p*** in the real world.

The point ***p*** written in terms of the camera coordinates, would be expressed as follows.

$$\mathbf{p} = \left[\begin{array}{c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The same point ***p*** in the object coordinate system, would be expressed as follows.

$$\mathbf{p} = \left[\begin{array}{c|c|c} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{array} \right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Equating the above two expressions, we have,

$$\left[\begin{array}{c|c|c} a_1 & a_2 & a_3 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[\begin{array}{c|c|c} c_1 & c_2 & c_3 \end{array} \right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Let,

$$A = \left[\begin{array}{c|c|c} a_1 & a_2 & a_3 \end{array} \right] \quad \text{and,} \quad C = \left[\begin{array}{c|c|c} c_1 & c_2 & c_3 \end{array} \right]$$

Multiplying with C^{-1} on LHS, and RHS, we obtain,

$$\left[\quad C^{-1} \quad \right] \left[\quad A \quad \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[\quad C^{-1} \quad \right] \left[\quad C \quad \right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$C^{-1} \cdot C = I$, where I is the Identity matrix. [8]

Thus,

$$\left[\quad C^{-1} \quad \right] \left[\quad A \quad \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Let matrix obtained by multiplying $C^{-1} \cdot A = H$.

$$\left[\quad H \quad \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

The matrix H is called the **Homography**.

5.3. Mapping Pixel Coordinates to Absolute Image Coordinates

The Homography can be defined as a transformation (which is a 3x3 matrix) that maps the points in one image, to the corresponding points in another image. For our requirement, we map the points in our distorted image, to the points that are expected to be in the absolute image, with respect to the boundary points in the distorted image specified.

This essentially means that, we now have a function, $f([x_1, x_2, 1]) = [w_1, w_2, 1]$ that can be used for mapping the pixels in the distorted image, to those in the expected final absolute image.

5.4. Homography and its Computation

The homography, H is defined as H = C^{-1}·A, where A represents the matrix with the camera basis as columns, and C represents the matrix with the object basis as columns.

However, since neither C nor A are known with complete certainty, the homography matrix cannot be directly computed, and thus, a slightly different approach is attempted.

The Homography can be explicitly defined as,

$$H = \begin{bmatrix} h_{y_1,x_1} & h_{y_1,x_2} & h_{y_1,x_3} \\ h_{y_2,x_1} & h_{y_2,x_2} & h_{y_2,x_3} \\ h_{y_3,x_1} & h_{y_3,x_2} & h_{y_3,x_3} \end{bmatrix}$$

Let ***p*** be a point on the object, and ***q*** be the corresponding point on the image plane.
Let (x_1, x_2, 1) be the camera coordinates of ***q***.
Let (y_1, y_2, y_3), be the object coordinates of ***q***.

Thus, we obtain,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{y_1,x_1} & h_{y_1,x_2} & h_{y_1,x_3} \\ h_{y_2,x_1} & h_{y_2,x_2} & h_{y_2,x_3} \\ h_{y_3,x_1} & h_{y_3,x_2} & h_{y_3,x_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

Multiply RHS and equate with LHS. [8]

$$y_1 = h_{y_1,x_1}x_1 + h_{y_1,x_2}x_2 + h_{y_1,x_3}$$
$$y_2 = h_{y_2,x_1}x_1 + h_{y_2,x_2}x_2 + h_{y_2,x_3}$$
$$y_3 = h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}$$

Object coordinates of original point, ***p*** is (y_1 /y_3 y_2/y_3,1).

Define w_1, w_2 as follows.

$$w_1 = y_1/y_3$$
$$w_2 = y_2/y_3$$

Object coordinates of original point, ***p*** is (w_1, w_2, 1).

Multiplying by y_3 on both sides,

$$w_1y_3 = y_1$$
$$w_2y_3 = y_2$$

Substituting for y_1, y_2, y_3, we obtain,

$$w_1(h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}) = h_{y_1,x_1}x_1 + h_{y_1,x_2}x_2 + h_{y_1,x_3}$$
$$w_2(h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}) = h_{y_2,x_1}x_1 + h_{y_2,x_2}x_2 + h_{y_2,x_3}$$

Rearranging, we obtain,

$$(w_1x_1)h_{y_3,x_1} + (w_1x_2)h_{y_3,x_2} + w_1h_{y_3,x_3} - x_1h_{y_1,x_1} - x_2h_{y_1,x_2} - 1h_{y_1,x_3} = 0$$
$$(w_2x_1)h_{y_3,x_1} + (w_2x_2)h_{y_3,x_2} + w_2h_{y_3,x_3} - x_1h_{y_2,x_1} - x_2h_{y_2,x_2} - 1h_{y_2,x_3} = 0$$

As explained previously, four distinct points are to be considered, to form a boundary of sorts for rectification of the distorted image.

Substituting the values of each set of coordinates of the in the above equation, we obtain a system 8 distinct linear equations.

However, this poses a problem, since there are 9 unknown variables, and only 8 linear equations. This means that we cannot determine H precisely.

However, this does not mean that the perspective rectification performed will be incorrect. This only means that the scale of the picture cannot be recovered (for example a tiny building that’s nearby looks similar to a huge building that’s far away), and the image rectified will correspond to the distorted image and boundary coordinates provided as input.

Furthermore, the exact, true H is not necessary.

As long as the H computed is a scalar multiple of the true H (which it will always be), the rectified image will be the best possible version of the absolute image obtainable, with respect to the distorted image and coordinates of the boundary points provided.

To arbitrarily select a scale, one of the unknowns in the homography, H is equated to be 1.

Thus, our problem reduces to solving 8 unknowns while knowing 8 linear equations corresponding to them. This can be easily solved by applying the principles of Gauss-Jordan elimination, [8] thus providing us the complete homography which is a scalar of the true homography.

5.5. Completing Perspective Rectification

The homography matrix provides us the transformation that must be applied to each rectangle of color, i.e., in the distorted image, to find out the corresponding parallelogram/quadrilateral in the absolute image.

For each point ***q*** in the representation of the image, within the specified boundary points, we have the camera coordinates (x_1, x_2, 1) of ***q***. Multiply by the homography, H to obtain the object coordinates (y_1, y_2, y_3) of the same point ***q***.

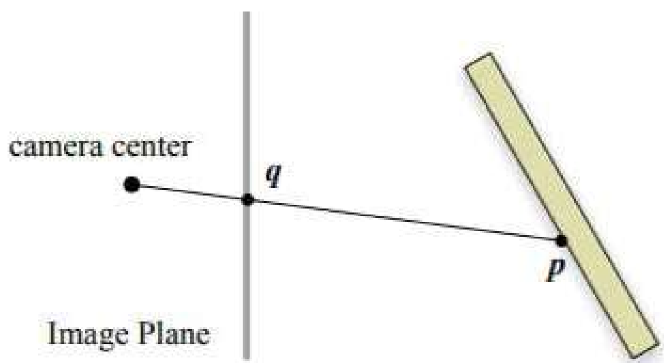

*Fig. 5.5.1 Visualization of the relationship between **p** and **q** [9]*

The object coordinates of the corresponding point ***p*** on the object are (y_1 /y_3 y_2/y_3,1), and can be calculated by scaling down.

Finally, the best version of the absolute image corresponding to the distorted image and boundary points provided can be obtained by updating the points with the same color matrix.

6. Practical Implementation

For the purpose of showing a practical demonstration of perspective rectification, a simple application was made, to perform perspective rectification with respect to the four boundary points on the distorted image specified.

This section will illustrate the functioning of this app, aptly named **Perspective** [10]. Perspective allows users to upload an image and choose the four points that will serve as boundary points, and will render the perspective considering the four chosen points on the distorted image as the extreme corners of the absolute image on the left-hand side of the page. The application has been developed with a minimalistic, efficient and utilitarian approach in mind.

Note: It is inherently implied that greater the size/quality image, proportionally, greater will be the time taken to render the absolute perspective for the image.

1. Opening up the application, we see the following screen. A button to upload an image is present.

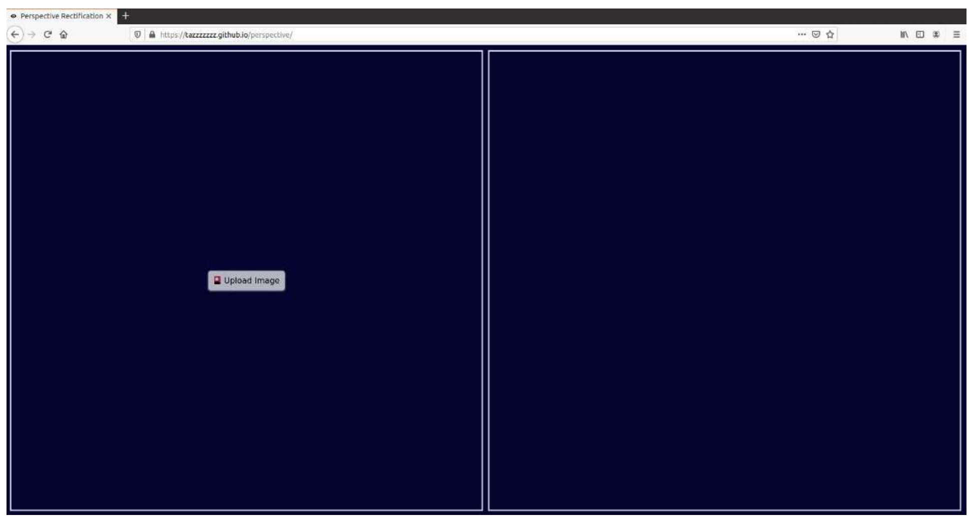

Fig. 6.1 Getting started with Perspective [10]

2. Next, we choose to upload an image using the **Upload Image** button. Notice that there is a light blue box visible, with four circles at all four corners.

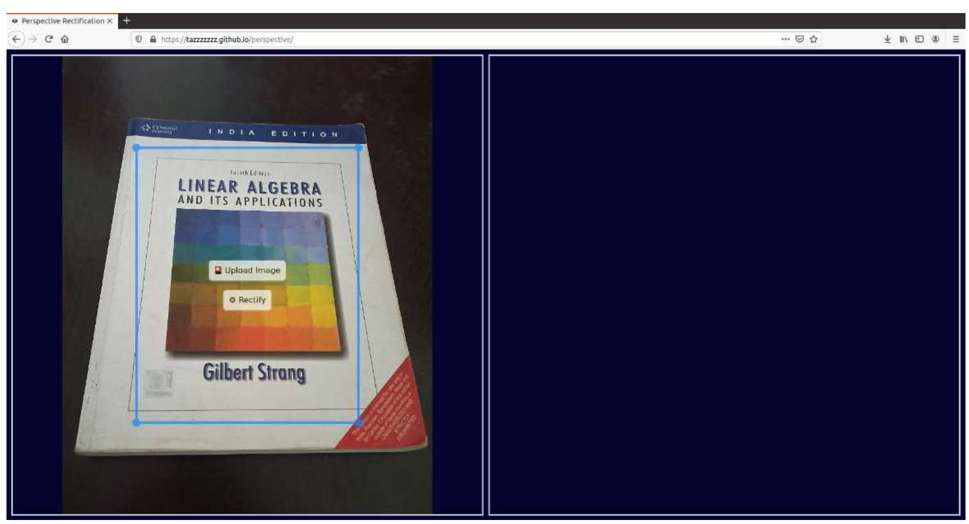

*Fig. 6.2 Uploaded a picture, using **Upload Image** option [10]*

3. Using the four circles, we can determine the four boundary points, or corners.
These boundary points, are used to specify the area/objects in the image whose absolute image must be rendered.

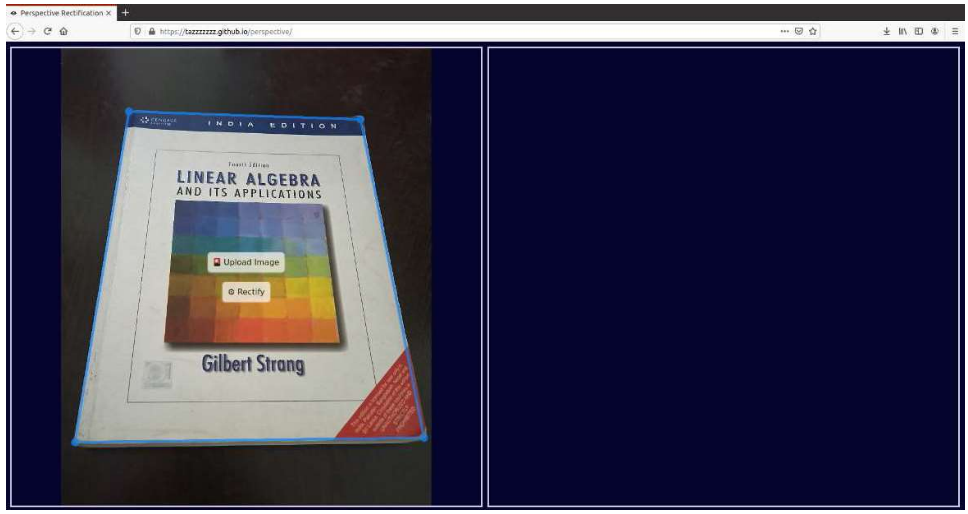

Fig. 6.3 Defining the four boundary points in the distorted image whose absolute image must be rendered [10]

4. After specifying the four corners/boundary points, click on **Rectify** the begin the rendering. With the help of these boundary points, the homography is calculated. The calculated homography is then applied to the area covered by the boundary points in the distorted image. The progress of rendering is specified in real time on the left-hand side of the page.

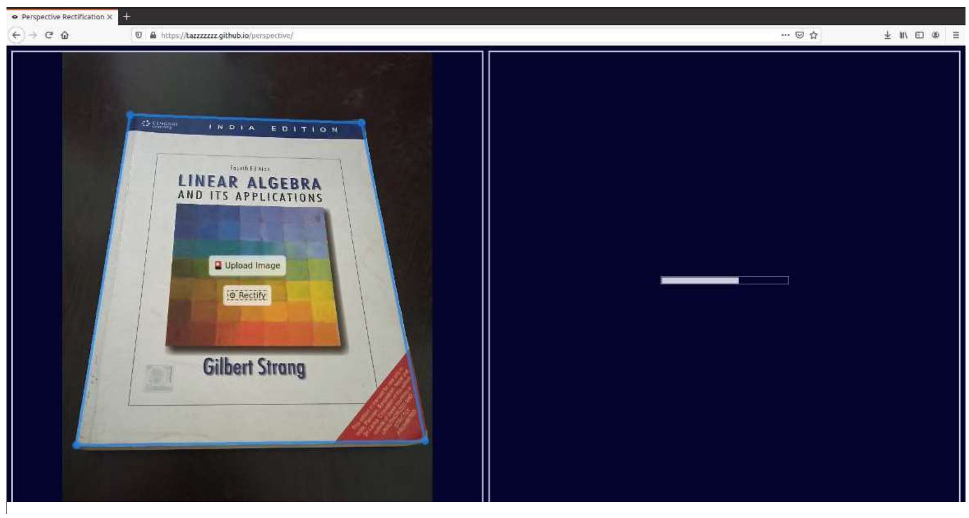

*Fig. 6.4 Rendering the absolute image, after selecting the **Rectify** option [10]*

5. After the absolute image of the specified portion of the distorted image is rendered, the image data is displayed on the left-hand side of the page. The absolute image rendered can be saved onto the local machine by clicking on **Save.**

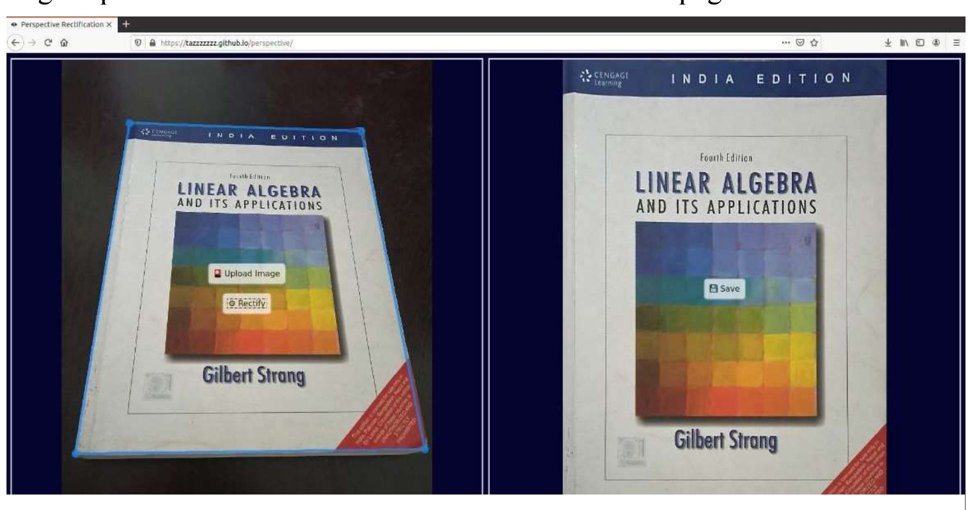

Fig. 6.5 Displaying the rendered absolute image with respect to the boundary points specified in the distorted image [10]

7. Results and Discussions

The principles and philosophy behind perspective rendering and perspective rectification have been expounded upon, down to the finer details. In this section, the results, inferences, and discussions with regards to the implementation will be explained upon in a concise rundown.

7.1. Performing Perspective Rectification

The essence of performing perspective rectification, can be captured in the following steps. [9]

1. Pixel Coordinates of boundary points from distorted image

   $$(x_1, x_2)$$
   $$\downarrow$$

2. Represent pixel in image plane in terms of camera basis

   $$(x_1, x_2, 1)$$
   $$\downarrow$$

3. Change representation into terms of object basis

   $$(y_1, y_2, y_3) \quad = \quad H * (x_1, x_2, 1)$$
   $$\downarrow$$

4. Move corresponding point into plane of absolute image

   $$(y_1/y_3, y_2/y_3, y_3/y_3)$$
   $$\downarrow$$

5. Get Coordinates within Absolute Image

   $$(y_1/y_3, y_2/y_3)$$

6. Calculate Homography, ***H***.

7. Transform the distorted image into the expected absolute image by applying the Homography to every pixel in it.

8. Render the absolute image obtained, and display the same.

7.2. Practical Implementation - The Perspective Application

Perspective Rectification can be implemented in practice using a variety of tools and software. These include, but are not limited to, Python coupled with opencv2, MATLAB, Web Applications, Adobe Photoshop, and so on. The implementation adopted for the purpose of this investigative analysis is that of an application that makes use of client-side processing. These applications are easy to build, easy to scale, give way to no security risks, or threats, and most importantly can be accessed with ease, and are easy to navigate, even for users who are not tech savvy.

Perspective [10] is an application that was developed solely for the purpose of being the cornerstone for the practical façade for this investigative analysis, and providing a demonstration of perspective rectification in reality.

Perspective is an application that can be served using static web pages, and performs all of its computation and operations on client-side. Thus, no data is at risk at any given point due to the app, and Perspective does not give rise to any forms of vulnerabilities.

Perspective is published using the GitHub Pages service.

The users can upload any image that is believed to require some form of perspective rectification. It is inherently implied that perspectives vary between individuals, and sometimes depend on the requirements as well. The application has been built to cater to these varying needs, and providing this form of flexibility as well.

The users must specify the boundary points, with respect to which the absolute perspective of the image is defined for the users' requirements/from the users' perspective.

This to sum up, means that when the user determines the four boundary points, the user is so expected to do so, when they require the four corners of the absolute image to be defined by these four boundary points in the distorted image. The absolute image rendered as output will not contain image data corresponding to the pixels that fall outside the area bounded by the four boundary points in the distorted image. This provides the user with the absolute image of the only the portion of interest to them, when viewed from what they perceive to be the absolute perspective.

Once the distorted image, and the four boundary points are specified, the perspective rectification is performed as explained, and the absolute image (with respect to the boundary points specified) is rendered onto the page as output.

The ***Perspective*** application can be freely used at its webpage as linked in the references.

7.3. Applications of Perspective Rectification

Through research and study, perspective rectification has emerged as a very dependable, easy and an efficient way to make corrections to images and documents and has wide ranging applications in the modern world. Some of which are mentioned below.

- **Editing text documents:**
  Perspective rectification enhances the readability of documents to the naked eye and information can be comprehended in an easier fashion.
- **Digitization of documents:**
  Digitization of documents can be accurately performed using perspective rectification methods, to provide results similar, and often better quality and standards, to those obtained with a scanner unit.
- **Architecture:**
  Architects are benefitted from this technology as it allows them to focus on the finest of structural details. This helps them to design safer structures and increase the level of accuracy in their work.
- **Photo editing:**
  It adds to the aestheticization of the pictures and increases appeal and presentability.
- **Investigative aid:**
  Perspective rectification has played a major hand in recovering the finest of details in several criminal cases as their use at the crime site has proved pivotal to notice the finest details.
- **Panoramic photography:**
  The principle of perspective rectification has been impressively incorporated into the designing of lenses of certain cameras which allow a spread view(360 degrees). These pictures are referred to as Panoramic pictures.

8. Summary and Conclusions

8.1. The Essence of the Investigative Analysis into Perspective Rectifications

The essence of perspective rectification in itself can be understood, only when its theoretical concepts are completely understood, and this conceptualization is practically experienced.

1. The definition of perspective rectification was first constructed, and the significance of it was gradually and consistently transmuted every time a new train of thought, and new abstractions were conceptualized. These abstractions form a cornerstone of the project, and are vital to deciphering the theoretical aspects of perspective rectification as explained.
2. In order to understand a subject, it is vital to understand the various manners in which the subject has progressed, and having a sense of appreciation for the previous work that has been carried out as well. This has been concisely included, to provide the reader with a sense of appreciation for the same.
3. Perspective rendering, being closely interrelated to perspective rectification, forms a very critical role in understanding perspective rectification correctly. The ideologies and fundamentals of perspective rendering are thus explained in sufficient detail as well.
4. However, although abstractions, appreciation for previous work, fundamentals, exposure to various trains of thought, and strongly conceptualized mental pictures provide a strong foundation to get started, they aren't sufficient to fully understand a topic that is as extensive perspective rectification. It must be possible for one to relate and connect the dots between the aforementioned, to the fundamentals of linear algebra. These concerns are addressed as well. For this purpose, concise explanations connecting the principles and requirements of perspective rectification to the fundamental concepts of linear algebra have been provided. It has been clearly outlined, how to proceed from performing the required transformations, keeping in mind the provided constraints, to obtain the desired output. With this, the exploration of the theoretical concepts and fundamentals of perspective rectification are brought to a close.

5. As previously mentioned, the essence of perspective rectification is captured at its best only when the theoretical conceptualization is put into practical use, and experienced so. For this purpose, Perspective [10] was developed. Perspective, a simple static application that performs perspective rectification, and renders the absolute image, in tune with the provided constraints (four boundary points), helps users firsthand experience, and see how perspective rectification in itself is implemented. It also provides the developers among the readers with the opportunity to observe an approach that can be undertaken to while by attempting to build something similar, with any suite of technologies at their disposal.

6. This brings us to the close of this investigative analysis on perspective rectification. Perspective rectification and its extensive applications have proven to be useful, in numerous ways. Perspective rectification is but one out of the many, many, seemingly arduous operations, that can scaled with ease with the fundamentals of linear algebra, and an open mind. Understanding of the intricacies of perspective rectification is understandably bound to leave one with a little more sense of appreciation for the powerful domain of knowledge that is linear algebra.

8.2. Scope for Future Work – The Perspective Application

- Ability to not only estimate the transformation between frames but also to assess the confidence in these estimates is important in many applications involving the need for perspective rectification. Providing provisions for performing accuracy analysis, using techniques proven to be effective [11], on the homography computed would be a very utilitarian addition.
- Incorporating edge correction, and features for clarity improvement, and noise removal of grainy images are also features that would be a very good addition to the current application.

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