Lecture 8

2025-10-22

Synaptic transmission; Short-term plasticity; Brian2

- Online slides: lec08-synaptic_brian2.html
- Code: code08.ipynb
- Homework: hw08.pdf

Username: `cns`, Password: `nycu2025`

Laboratory of Computing Biological Networks

Synaptic conductance

$g_\mathrm{s} = \bar{g}_\mathrm{s} P$, Open probability $P = P_\mathrm{s} P_\mathrm{rel}$

Transmitter release probability

Synaptic open probability

Receptor channels

- Ionotropic receptors: Fast
- Metabotropic receptors
  - Typically involves G-protein-mediated receptors
  - Also second messengers
  - Serotonin, dopamine, norepinephrine, acetylcholine

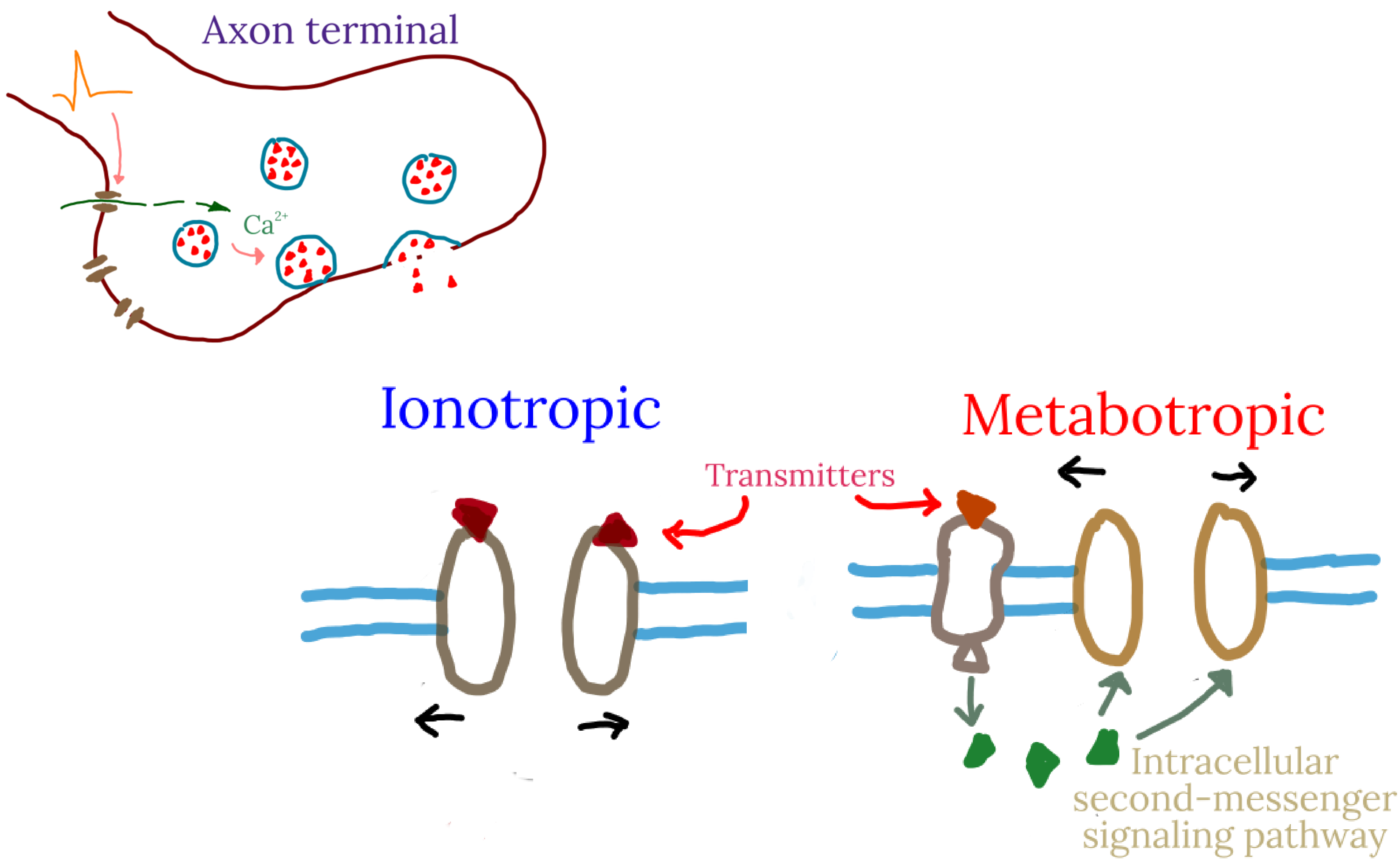
Transmitters

- Excitatory Glutamate

  Receptors: AMPA (rapid), NMDA (slower, Ca^{2+}): Ionotropic
- Inhibitory GABA (γ-aminobutyric acid)

  Receptors: $GABA_A$ (ionotropic Cl^-), $GABA_B$ (metabotropic K^+)

Synaptic actions and opening of ion channels



Postsynaptic conductance

Channel opened by binding k transmitters; closed by unbinding:

$$\frac{dP_\mathrm{s}}{dt} = \alpha_\mathrm{s}(1 - P_\mathrm{s}) - \beta_\mathrm{s} P_\mathrm{s}.$$

$\alpha_\mathrm{s} \propto [\mathrm{transmitter}]^k$; β_s: usually assumed constant.

When α_s and β_s are constant, we can solve this exactly:

$$P_\mathrm{s}(t) = \frac{\alpha_\mathrm{s}}{\alpha_\mathrm{s} + \beta_\mathrm{s}} + \left[P_\mathrm{s}(0) - \frac{\alpha_\mathrm{s}}{\alpha_\mathrm{s} + \beta_\mathrm{s}}\right] e^{-(\alpha_\mathrm{s} + \beta_\mathrm{s})t}.$$

We see P_s approaches $P_\mathrm{s}(\infty) = \alpha_\mathrm{s}/(\alpha_\mathrm{s} + \beta_\mathrm{s})$ with the time constant $\tau_\mathrm{s} = 1/(\alpha_\mathrm{s} + \beta_\mathrm{s})$.

Linear ODE and exponential decay

Any linear ODE can be casted into the form:

$$\tau \frac{dx}{dt} = x_\infty - x \Rightarrow \frac{dx}{x - x_\infty} = -\frac{dt}{\tau}$$

$$\Rightarrow \ln(x - x_\infty) = -\frac{t}{\tau} + \text{const.} \Rightarrow x = x_\infty + \text{const.} \times e^{-t/\tau}.$$

Consider the initial condition $x(t=0) = x_0$, the solution is an exponential decay of x to $x = x_\infty$ with the time constant τ:

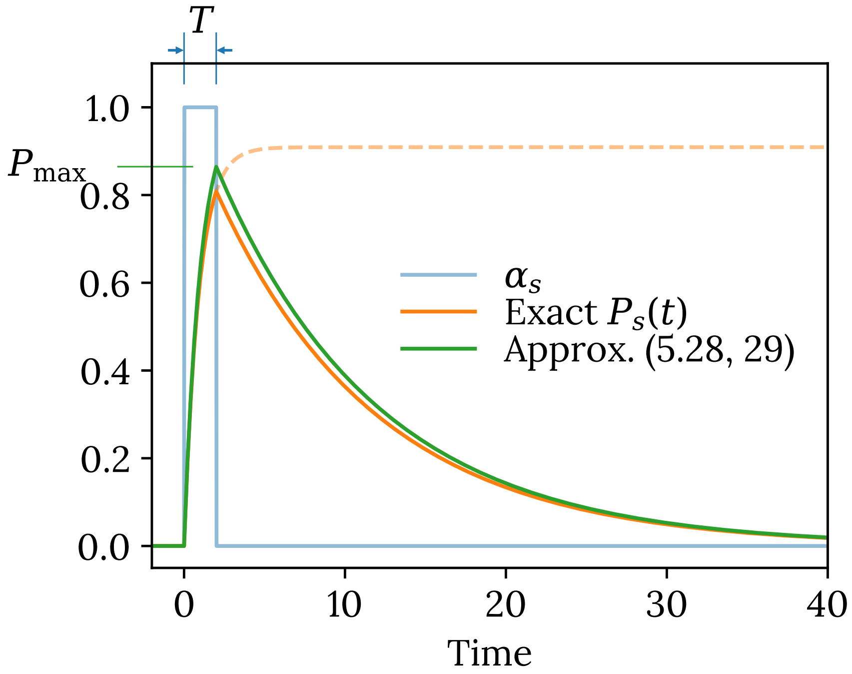
$$x = x_\infty + (x_0 - x_\infty)e^{-t/\tau}$$

☆ Generally, we just need to figure out the asymptotic value x_∞ and the time constant τ.

Pulse action of opening rate

Consider pulse-like α_s that becomes non-zero and stays constant $\alpha_\mathrm{s} \gg \beta_\mathrm{s}$ for a period of time T upon presynaptic action before returning to zero. Approximation:

$$P_\mathrm{s}(t) = \begin{cases} 1 + (P_\mathrm{s}(0) - 1)\exp(-\alpha_\mathrm{s} t), & \text{if } 0 \le t \le T \\ P_\mathrm{s}(T)\exp(-\beta_\mathrm{s}(t-T)), & \text{if } t \ge T \end{cases}$$



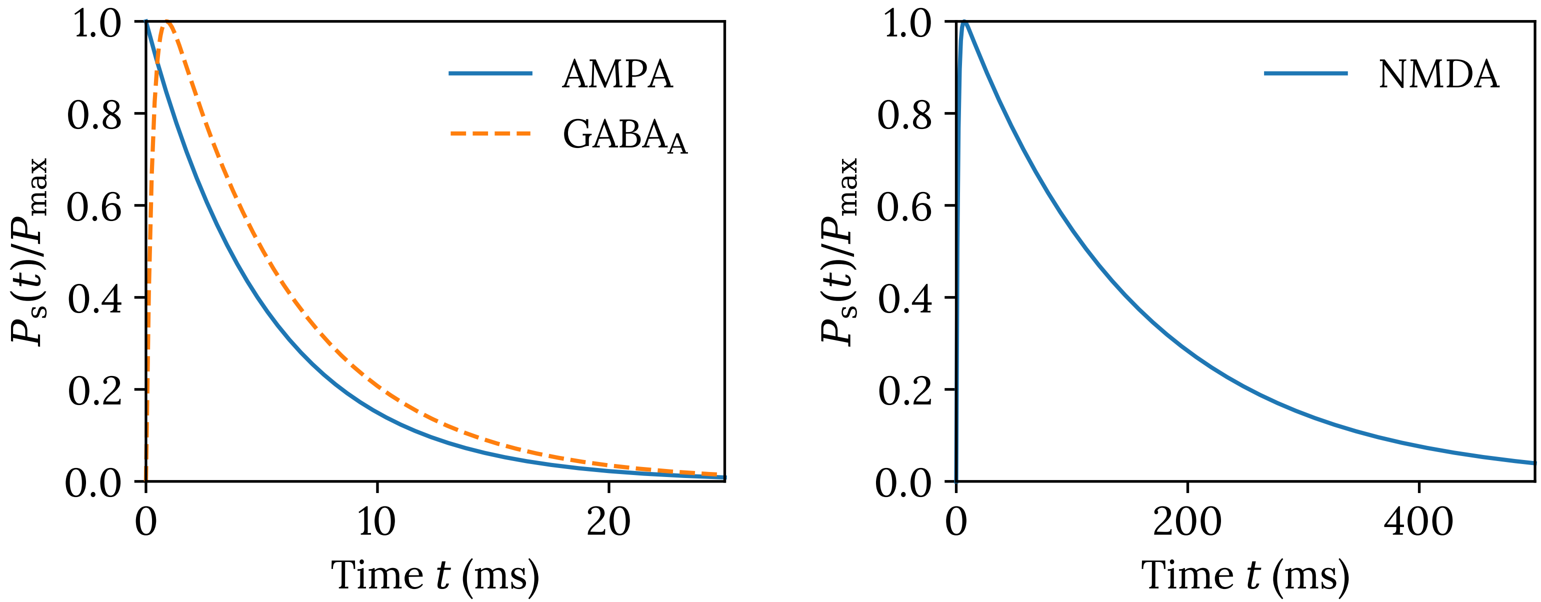
When $P_\mathrm{s}(0) = 0$, peak value is

$$P_\mathrm{s}(T) = P_\mathrm{max} = 1 - \exp(-\alpha_\mathrm{s} T).$$

In general,

$$P_\mathrm{s}(T) = P_\mathrm{s}(0) + P_\mathrm{max}(1 - P_\mathrm{s}(0)).$$

Fitted behavior of various receptors



AMPA: single exponential with $\tau_s = 5.26$ ms.

GABA$_A$: difference of exponentials with $\tau_1 = 5.6$ ms and $\tau_{rise} = 0.3$ ms.

NMDA: $\tau_1 = 152$ ms and $\tau_{rise} = 1.5$ ms.

Dayan & Abbott Figure 5.15, Parameters from Destexhe et al. (1994).

Simplified dynamics: instant rise

When $\alpha_{\mathrm{s}} \to \infty$, rise time $\tau_{\mathrm{rise}} \to 0$. Instantaneous action:

$$P_{\mathrm{s}} \to P_{\mathrm{s}} + P_{\max}(1 - P_{\mathrm{s}})$$

following a presynaptic action potential. While, the decay dynamics remains

$$\tau_{\mathrm{s}} \frac{dP_{\mathrm{s}}}{dt} = -P_{\mathrm{s}}.$$

The dynamics is completely determined by current value of P_{s}. No need to keep track of past firing times – a Markov process.

Simplified dynamics: mixed exponential

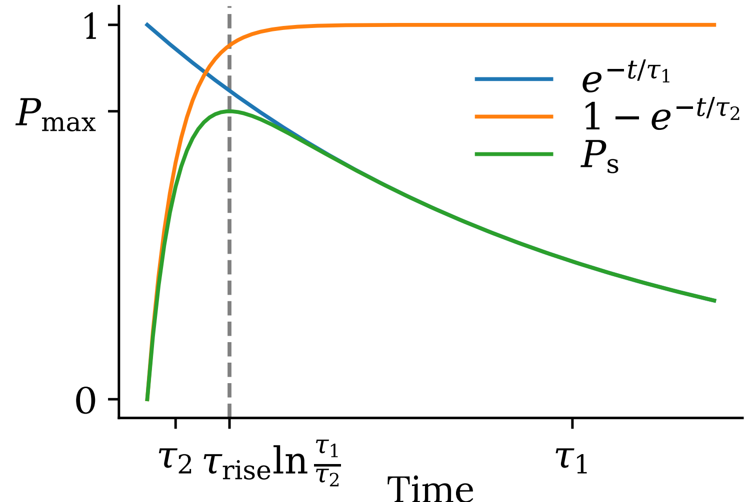
For slow rise, multiple time scales come into play

$$P_\mathrm{s} = P_\mathrm{max} B \left(\exp \frac{-t}{\tau_1} - \exp \frac{-t}{\tau_2} \right),$$

where $\tau_1 > \tau_2$ and the normalization constant

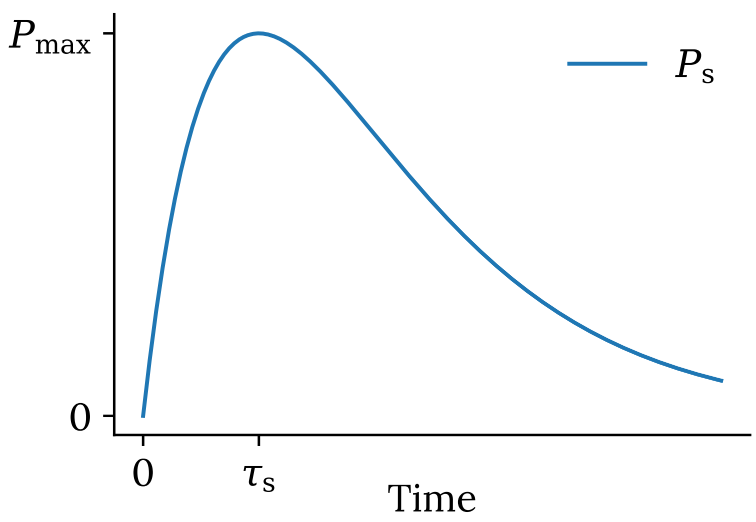
$$B = \left[\left(\frac{\tau_2}{\tau_1} \right)^{\tau_\mathrm{rise}/\tau_1} - \left(\frac{\tau_2}{\tau_1} \right)^{\tau_\mathrm{rise}/\tau_2} \right]^{-1}.$$

$\tau_\mathrm{rise} = \dfrac{\tau_1 \tau_2}{\tau_1 - \tau_2}$, Peak at $\tau_\mathrm{rise} \ln \dfrac{\tau_1}{\tau_2}$.



Considered Markovian, but need to keep two components.

Additional scheme: alpha function



Single-time-scale function with linear rise and exponential decay:

$$P_\mathrm{s} = \frac{P_{\max} t}{\tau_\mathrm{s}} \exp\left(1 - \frac{t}{\tau_\mathrm{s}}\right)$$

which starts at 0, reaches its peak at $t = \tau_\mathrm{s}$, and then decays with a time constant τ_s.

Need to keep the last firing time to know how the opening fraction will go.

NMDA receptor

... with additional dependence on the postsynaptic potential:

channles blocked by Mg^{2+} when the postsynaptic neuron near resting potential

$\bar{g}_{\mathrm{NMDA}} G_{\mathrm{NMDA}}(V) P(V - E_{\mathrm{NMDA}})$ where

$$G_{\mathrm{NMDA}} = \left(1 + \frac{[\mathrm{Mg}^{2+}]}{3.57\mathrm{mM}} \exp(-V/16.13\mathrm{mV})\right)^{-1}$$

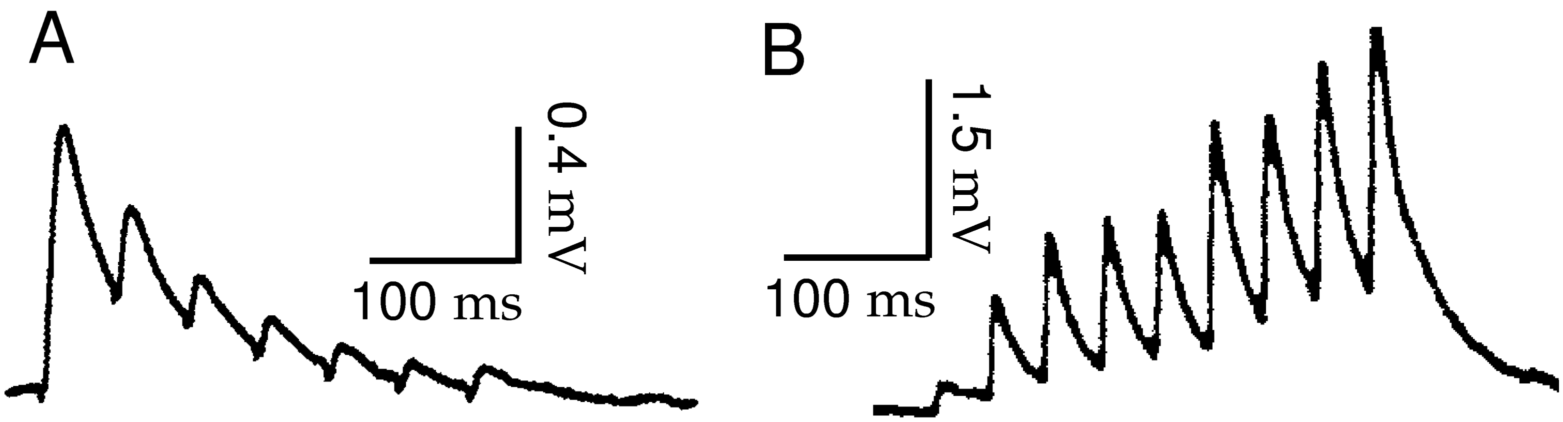
Jahr and Stevens (1990)

- Conducts Ca^{2+} (critical to long-term synaptic modification) and monovalent cations
- Detects coincidence of simultaneous pre- and postsynaptic activity

Release probability and short-term plasticity

Short-term plasticity (ms, s) ⇔ Long-term plasticity (h, life)

P_{rel}, presynaptic factor affecting synaptic transmission



Depression

Facilitation

(A) Layer 5 pyramidal cells in slice of rat somatosensory cortex. Markram and Tsodyks (1996) (B) Pyramidal to interneuron in layer 2 / 3 of rat somatosensory cortex. Markram et al. (1998)

Modeling short-term plasticity

Relaxation to steady state:

$$\tau_{\mathrm{P}} \frac{dP_{\mathrm{rel}}}{dt} = P_0 - P_{\mathrm{rel}}, \quad 0 < P_0 \leq 1.$$

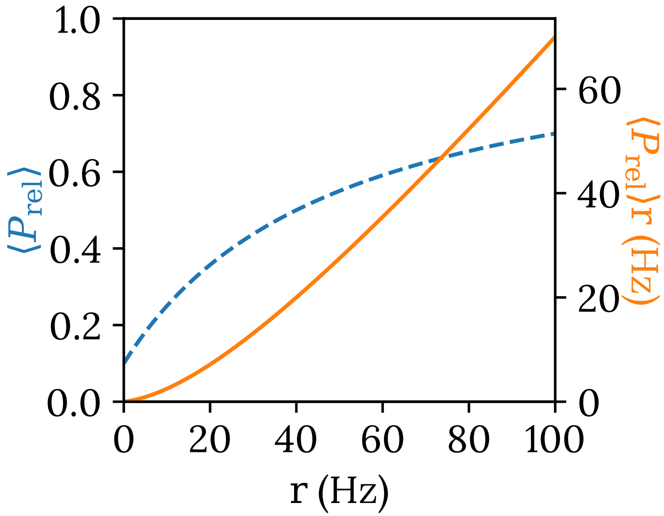
Facilitation:

$$P_{\mathrm{rel}} \rightarrow P_{\mathrm{rel}} + f_{\mathrm{F}}(1 - P_{\mathrm{rel}}), \quad 0 \leq f_{\mathrm{F}} < 1.$$

Depression:

$$P_{\mathrm{rel}} \rightarrow f_{\mathrm{D}} P_{\mathrm{rel}}, \quad 0 < f_{\mathrm{D}} \leq 1.$$

Effect of STP to neural transmission



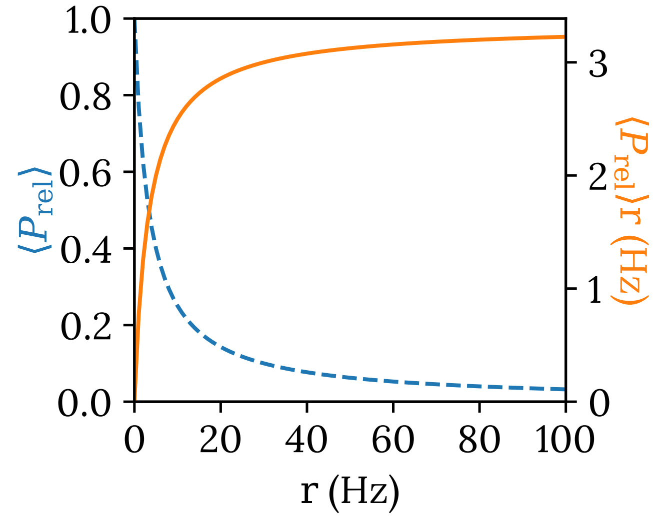


Figure 5.18 Driven by Poisson presynaptic spike train of rate r

$^{\text{Left}}(P_0, f_{\rm F}, \tau_{\rm P}) = (0.1, 0.4, 50\,\mathrm{ms})$; $^{\text{Right}}(P_0, f_{\rm D}, \tau_{\rm P}) = (1, 0.4, 500\,\mathrm{ms})$

Exact solutions

Facilitation:

$$\langle P_{\rm rel}\rangle = \frac{P_0 + \mathrm{r} f_{\rm F}\tau_{\rm P}}{1 + \mathrm{r} f_{\rm F}\tau_{\rm P}}$$

Depression:

$$\langle P_{\rm rel}\rangle = \frac{P_0}{1 + (1 - f_{\rm D})\mathrm{r}\tau_{\rm P}}$$

Depressive synapse driven by stepped firing rate

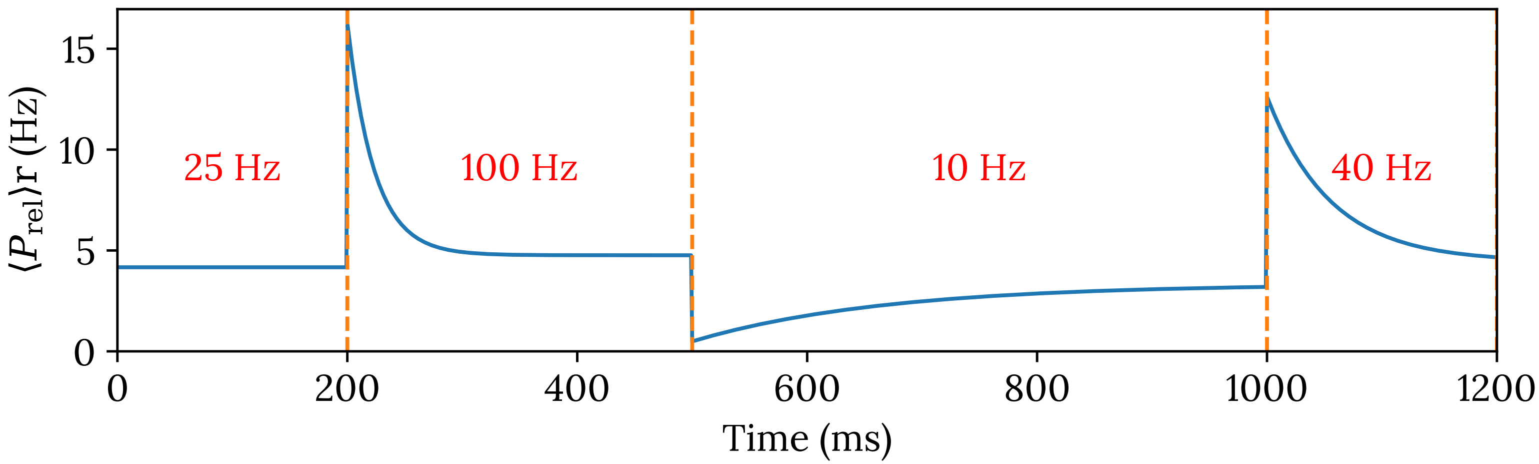


Figure 5.19 Parameters: $(P_0, f_{\rm D}, \tau_{\rm P}) = (1, 0.6, 500\,\rm ms)$

The dynamic of the trial-averaged release probability is described by the ODE:

$$\frac{d\,\langle P_{\rm rel}\rangle}{dt} = \frac{1}{\tau_{\rm P}}\left\{P_0 - \left[1 + (1 - f_{\rm D})\,{\rm r}\right]\langle P_{\rm rel}\rangle\right\}$$

which can be integrated numerically.

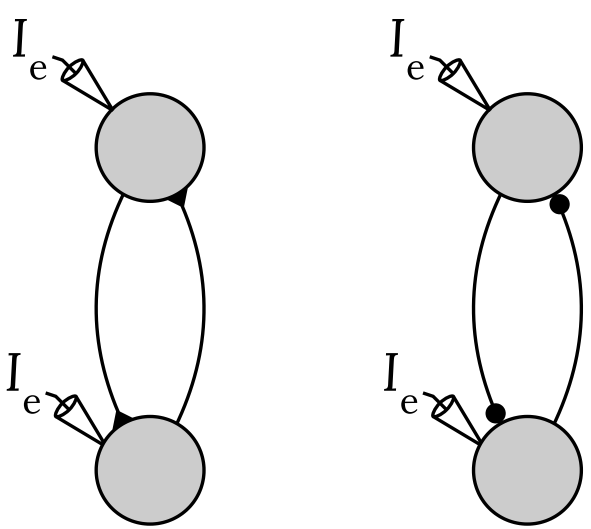
Synapses on integrate-and-fire neurons

Including synaptic conductance in membrane current,

$$\tau_\mathrm{m}\frac{dV}{dt} = E_\mathrm{L} - V - r_\mathrm{m}\bar{g}_\mathrm{s}P_\mathrm{s}(V - E_\mathrm{s}) + R_\mathrm{m}I_\mathrm{e}.$$

and consider coupling of two neurons with α-function synapses,

$$P_\mathrm{s}(t) = \frac{P_\mathrm{max}t}{\tau_\mathrm{s}}\exp\left(1 - \frac{t}{\tau_\mathrm{s}}\right).$$



The coupling between Two neurons are either excitatory ($E_\mathrm{s} > V_\mathrm{th}$) or inhibitory ($E_\mathrm{s} < E_\mathrm{L}$) depending on the reversal potential of the synapses.

Details in the model

Parameters

Parameter	Value	Unit	Variable
E_{L}	-70	mV	`El`
V_{th}	-54	mV	`Vth`
V_{reset}	-80	mV	`Vr`
τ_{m}	20	ms	`tau_m`
$r_{\mathrm{m}}\bar{g}_{\mathrm{s}}$	0.05		`rmgs`
P_{max}	1		`Pmax`
$R_{\mathrm{m}}I_{\mathrm{e}}$	25	mV	`RmIe`
τ_{s}	10	ms	`tau_s`

Variables

Membrane potentials for the two neurons in mV

V_i: `V1`, `V2`

For α function model, we need to keep track the time since last fire for the presynaptic neuron of each synapse

s_i: `s1`, `s2`

In the current two neuron model, s_1 and s_2 keep the time since last firing of neuron 2 and 1 respectively.

Using Brian2 for neural simulations

See code08.ipynb

1. Translate the ordinary differential equations to string text
2. Define parameters with proper units
3. Create NeuronGroup's for different neurons
4. Connect neurons with Synapses
5. Setup StateMonitor for data interested in
6. Run the simulations
7. Extract desirable data from StateMonitor