

# Exploiting structures in multiobjective simulation optimization problems

Tyler Chang<sup>a</sup> and Stefan Wild<sup>a→b</sup>

<sup>a</sup>Mathematics and Computer Science Division,  
Argonne National Laboratory

<sup>b</sup>Applied Mathematics and Computational Research Division,  
Lawrence Berkeley National Laboratory

SIAM OP 23

# Outlines

Intro, ParMOO, and Problem Types

Initial Results + Optimizer Stalling

Deep-dive into RBFs and Connection to Bayesian optimization

Comparison with Bayesian optimization

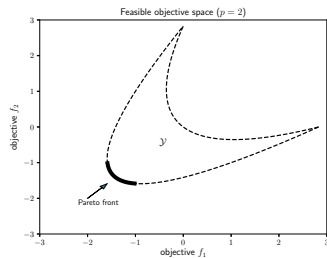
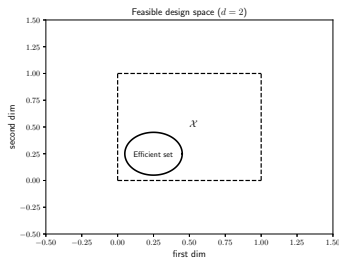
# Multiojective Optimization Problems

$$\min_{x \in \mathcal{X}} F(x)$$



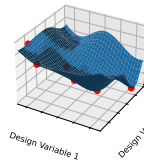
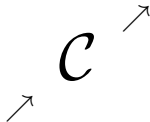
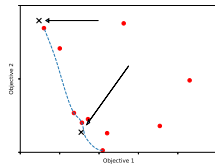
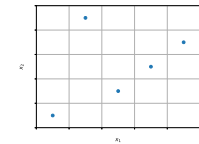
$$F : \mathcal{X} \rightarrow \mathcal{Y}$$

expensive  
blackbox process

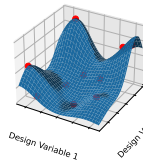


# Multiobjective Response Surface Methodology

or Model-Based Optimization or Active Learning



Simulation 1 output



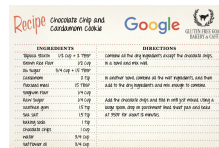
Simulation 2 output

**Challenge 1:**  
**Mixed vars & problem types**  
**+**  
**Unusual computing environments**

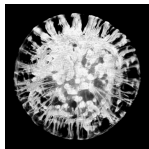
# Commercial solutions



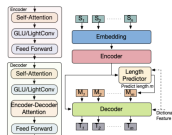
*"Using Bayesian optimization for balancing metrics in recommendation systems" by Yunbao Ouyang et al. on LinkedIn Engineering Blog.*



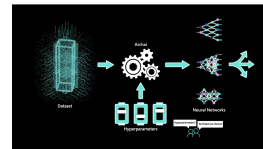
*"The makings of a smart cookie" by Daniel Golovin on Google Research Blog.*



*"Accelerating molecular optimization with AI" by Payel Das et al. on IBM Research Blog.*



*"Optimizing model accuracy and latency using Bayesian multi-objective NAS" by David Eriksson et al. on Meta AI Research Blog.*



*"Archai can design your neural network with state-of-the-art NAS" by Shital Shah et al. on Microsoft Research Blog.*

# Commercial solvers

## General purpose: (solver + backend)

Google – OSS Vizier + Pythia backend

[5] Song et al. OSS Vizier: distributed infrastructure and API for reliable and flexible black-box optimization. In Proc. 2022 AutoML-Conf.

Meta – BoTorch + Ax backend

[6] Balandat et al. BoTorch: a framework for efficient monte-carlo Bayesian optimization. In NeurIPS 2020.

## Special purpose: (solver + special purpose deployment)

IBM – Query-based Molecular Optimization (QMO)

[7] Hoffman et al. Optimizing molecules using efficient queries from property evaluations. Nature Machine Intelligence 4:21–31 (2022).

Microsoft – Archai for NAS

[8] Shah et al. Archai: platform for neural architecture search. Microsoft Research (Jul, 2022).

**Challenge 2:**  
**SOA blackbox optimization**  
**+**  
**Exploiting problem structure**



# SOA in blackbox optimization



*"Optimization and root finding (scipy.optimize)" in SciPy v1.10.0 [9].*



*Stochastic dimension reduction explained in this context by Stefan [10].*



*SOS structure can be exploited by DFO solver POUNDERS in TAO [11].*

[9] Virtanen et al. *SciPy 1.0: fundamental algorithms for scientific computing in Python*. *Nature Methods* 17:261–272 (2020).

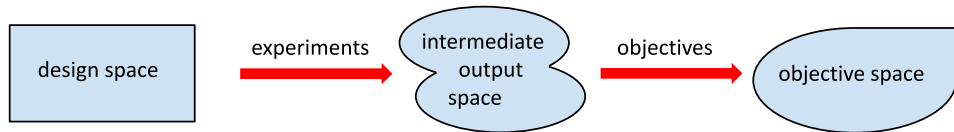
[10] Wild. *Optimization and learning with zeroth-order stochastic oracles*. *SIAM News* 56(1):1,3 (2023).

[11] Wild. *Solving derivative-free nonlinear least squares problems with POUNDERS*. In *Advances and Trends in Optimization with Engineering Applications* (2017).

## Design goals:

1. Highly customizable framework for multiobjective RSM
2. Flexible problem types (mixed-variables, constraints, etc.)
3. Easy to use, deploy, and extend (unforeseen use-cases and environments)
4. Solve large-scale problems + exploit structure and domain knowledge

# Problem structures



**Least-squares structure:**

$$h_i(x, S(x)) = \sum_{j \in N_i} (S_j(x))^2$$

where each  $N_1, \dots, N_o$  is an index set.

Increases order of approximation  $\Rightarrow$   
increases order of convergence

**Heterogeneous MOOPs:**

$$\begin{aligned} h_1(x, S(x)) &= S_1(x) \\ h_2(x, S(x)) &= \|x\|^2 \end{aligned}$$

Use expensive surrogate models for  $h_1$  (i.e.,  $S_1$ ) but not for  $h_2$

## Sample code

```
from parmoo import MOOP
from parmoo.optimizers import LocalGPS as gps
from parmoo.searches import LatinHypercube as lhs
from parmoo.surrogates import GaussRBF as rbf
from parmoo.acquisitions import UniformWeights as wsum
# Create MOOP object with GPS optimizer
moop = MOOP(gps)
# Add a continuous + categorical design variable
moop.addDesign({'name': "x1", 'lb': 0.0, 'ub': 1.0})
moop.addDesign({'name': "x2", 'des_type': "cat", 'levels': 3})
# Define and add a simulation function (with surrogates and search)
def s(x): return [(x["x1"]-.2)**2, (x["x1"]-.8)**2] if x["x2"]==0 else [9,9]
moop.addSimulation({'name': "sim", 'm': 2, 'sim_func': s,
                   'search': lhs, 'surrogate': rbf})
# Add 2 objectives
moop.addObjective({'name': "f1", 'obj_func': lambda x, s: s["sim"][0]})
moop.addObjective({'name': "f2", 'obj_func': lambda x, s: s["sim"][1]})
# Add 3 weighted-sum acquisition functions
for i in range(3):
    moop.addAcquisition({'acquisition': wsum})
# Solve with 5 iterations and fetch numpy struct of solutions
moop.solve(5)
results = moop.getPF()
```

# ParMOO Release



Written in Python

Version 0.2.0 is now available on available on pip,  
conda-forge, and GitHub

<https://github.com/parmoo/parmoo>

<https://parmoo.readthedocs.io>



## Example 1: Material Manufacturing with ParMOO

Choose optimal settings for material manufacturing in a continuous flow reactor (CFR)

We know how to make a desired material, need to produce at scale:

1. **Maximize the product** (battery electrolyte: TFML)
2. Can increase temperature to **reduce reaction time**
3. Too much heat activates a side reaction; need to **minimize unwanted byproduct**

Challenges:

- ▶ Mixed variable types
- ▶ Heterogeneous objectives
- ▶ Must send experiments to run on CFR

*Design vars and bound constraints*

Parameter	LB	UB
Temp (deg C)	40	150
Reaction time (secs)	60	300
Equivalence ratio (N/A)	0.9	2
Solvent (categorical)	2	lvls
Base (categorical)	2	lvls

# CFR Optimization with ParMOO

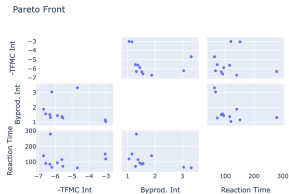
Extend MOOP class to send/receive  
experiment data using MDML library  
(Apache Kafka)

Used embeddings to represent categorical  
variables

Modeled Product/Byprod as sims and  
reaction time using ID mapping from input



Ran to convergence and used data to  
create a surrogate problem for future study

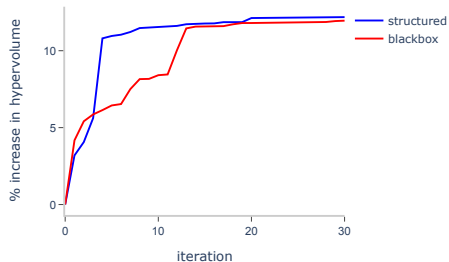


[17] Chang et al. A framework for fully autonomous design of materials via multiobjective optimization and active learning: challenges and next steps. In ICLR 2023, Workshop on ML4Materials.

# CFR Optimization Results

- ▶ 5 mixed-type design variables (including reaction time)
- ▶ 50-pt Latin hypercube design-of-experiments
- ▶ Fit **global** Gaussian RBF surrogate on 2 simulation outputs
- ▶ 3rd obj is identity mapping of reaction time
- ▶ 3  $\varepsilon$ -constraint scalarization functions
- ▶ Multi-start L-BFGS-B global optimization of scalarized objectives
- ▶ Compared against blackbox implementation: all 3 objectives modeled with Gaussian RBFs

Ran 30 iterations (batch size 3) after initial DOE



Notice: structure-exploiting is better early on, then **stalls out** at the end...



## Example 2: Fayans EDF Model Calibration

Find params  $x \in [0, 1]^{13}$  to fit the Fayans model to data  $d_i$ :

$$M(\xi_i; x) \approx d_i \quad i = 1, \dots, 198$$

ParMOO simulation:

$$S_i(x) = M(\xi_i; x) - d_i, \quad i = 1, \dots, 198;$$

Min SOS across 3 observable classes

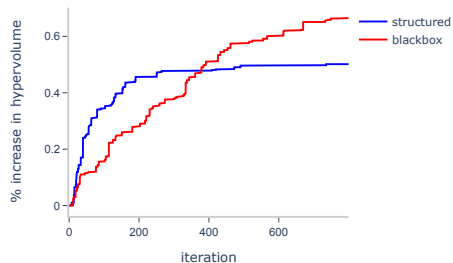
$$F_t = \sum_{i=1}^{m_t} (S_{t,i}(x))^2$$

[16] Bollapragada et al. Optimization and supervised machine learning methods for fitting numerical physics models without derivatives. *Journal of Physics G* 48(2):024001 (2020).

# Fayans Solution with ParMOO

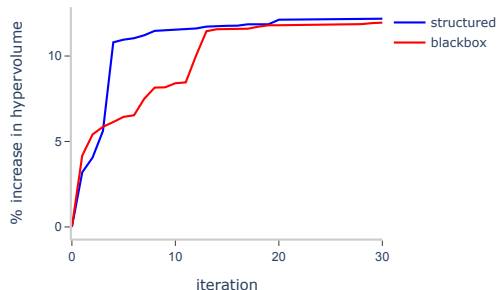
- ▶ Approximated Fayans residuals with MLP trained on existing dataset
- ▶ Implemented batch-parallel **structure-exploiting solver in ParMOO**
  - ▶ 2000-pt LH design-of-experiments
  - ▶ Use **local** Gaussian RBF surrogates to model sim outs
  - ▶ SOS structure applied on top of Gauss RBF surrogate outputs
  - ▶ Solve 10 randomized  $\varepsilon$ -constraint scalarizations per batch
  - ▶ Solve scalarized problem with a trust-region-constrained multi-start L-BFGS-B
- ▶ Compared against **same solver w/o exploiting least-squares structure** (Gaussian RBFs fit directly to three objective values)

Ran for 800 iterations (10K sim evals)

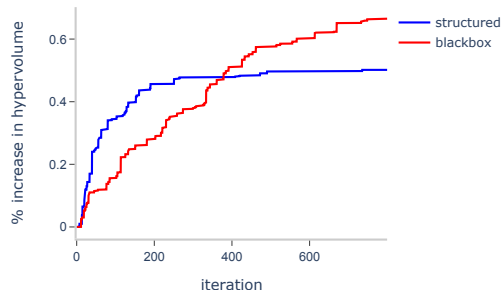


Structure-exploiting is better at small budgets, but **stalls** out...

## Two different problems, same issue...



- ▶ Mixed-variable problem
- ▶ Extremely limited budget
- ▶ Heterogeneous objectives
- ▶ Global Gaussian RBF modeling



- ▶ Continuous optimization problem
- ▶ 13 vars, 3 objs, healthy budget
- ▶ Nonlinear least-squares
- ▶ Local RBF (trust-region method)

# What happened?

# What happened?

Many months of debugging and testing...

## What happened?

Many months of debugging and testing...

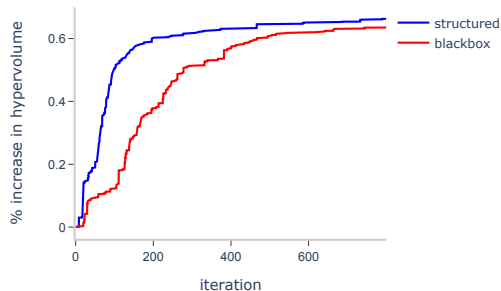
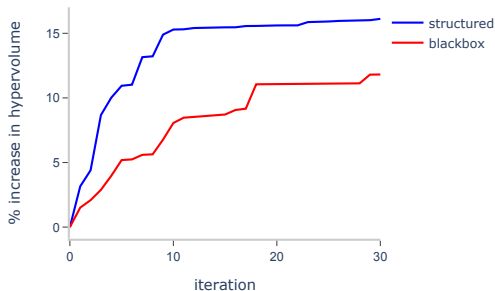
Finally tried centering response values at 0 before fitting surrogate...

# What happened?

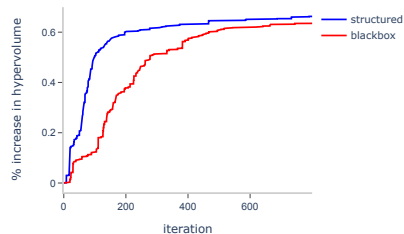
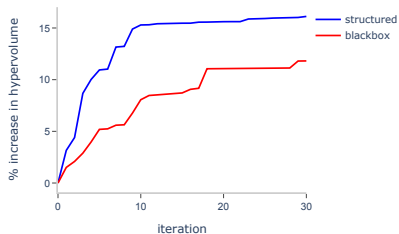
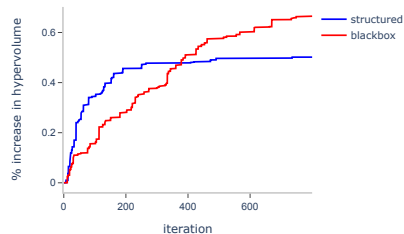
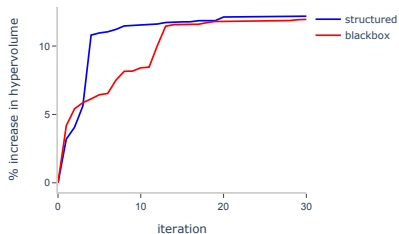
Many months of debugging and testing...

Finally tried centering response values at 0 before fitting surrogate...

**Fixed!**



# Comparison



[12] Chang and Wild. Designing a framework for solving multiobjective simulation optimization problems. ArXiv preprint 2304.06881 (2023).



## Equivalence between Gaussian RBFs and GPs

For both Gaussian RBFs and (traditional) Gaussian process mean-function:

$$\hat{F}_{RBF}(x) = \omega^\top \begin{bmatrix} e^{-\|x_1 - x\|^2 / \sigma} \\ e^{-\|x_2 - x\|^2 / \sigma} \\ \vdots \\ e^{-\|x_n - x\|^2 / \sigma} \end{bmatrix}$$

## Equivalence between Gaussian RBFs and GPs

For both Gaussian RBFs and (traditional) Gaussian process mean-function:

$$\hat{F}_{RBF}(x) = \omega^\top \begin{bmatrix} e^{-\|x_1 - x\|^2 / \sigma} \\ e^{-\|x_2 - x\|^2 / \sigma} \\ \vdots \\ e^{-\|x_n - x\|^2 / \sigma} \end{bmatrix}$$

where  $A\omega = y$  and

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2 / \sigma} & \dots & e^{-\|x_1 - x_n\|^2 / \sigma} \\ e^{-\|x_2 - x_1\|^2 / \sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2 / \sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2 / \sigma} & e^{-\|x_n - x_2\|^2 / \sigma} & \dots & 1 \end{bmatrix} \quad y = \begin{bmatrix} F(x_1) \\ F(x_2) \\ \vdots \\ F(x_n) \end{bmatrix}$$

## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

- As we optimize, data points cluster

## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

- ▶ As we optimize, data points cluster
- ▶ As  $\|x_i - x_j\| \rightarrow 0$ ,  $A \rightarrow$  singularity

## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

- ▶ As we optimize, data points cluster
- ▶ As  $\|x_i - x_j\| \rightarrow 0$ ,  $A \rightarrow$  singularity
- ▶ Decrease the shape parameter  $\sigma$  to keep  $A$  nonsingular

## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

- ▶ As we optimize, data points cluster
- ▶ As  $\|x_i - x_j\| \rightarrow 0$ ,  $A \rightarrow$  singularity
- ▶ Decrease the shape parameter  $\sigma$  to keep  $A$  nonsingular
- ▶ Decreasing  $\sigma$  restricts the support of  $\hat{F}_{RBF}$

## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

- ▶ As we optimize, data points cluster
- ▶ As  $\|x_i - x_j\| \rightarrow 0$ ,  $A \rightarrow$  singularity
- ▶ Decrease the shape parameter  $\sigma$  to keep  $A$  nonsingular
- ▶ Decreasing  $\sigma$  restricts the support of  $\hat{F}_{RBF}$ 
  - ▶ With 0-prior, optimizer will be driven to low-support regions
  - ▶ With GP, uncertainty will be maximal in low-support regions (will drive Bayesian optimization to behave similarly?)



## RBF/GP Conditioning and Accuracy

$$A = \begin{bmatrix} 1 & e^{-\|x_1 - x_2\|^2/\sigma} & \dots & e^{-\|x_1 - x_n\|^2/\sigma} \\ e^{-\|x_2 - x_1\|^2/\sigma} & 1 & \dots & e^{-\|x_2 - x_n\|^2/\sigma} \\ \vdots & \vdots & & \vdots \\ e^{-\|x_n - x_1\|^2/\sigma} & e^{-\|x_n - x_2\|^2/\sigma} & \dots & 1 \end{bmatrix}$$

- ▶ As we optimize, data points cluster
- ▶ As  $\|x_i - x_j\| \rightarrow 0$ ,  $A \rightarrow$  singularity
- ▶ Decrease the shape parameter  $\sigma$  to keep  $A$  nonsingular
- ▶ Decreasing  $\sigma$  restricts the support of  $\hat{F}_{RBF}$ 
  - ▶ With 0-prior, optimizer will be driven to low-support regions
  - ▶ With GP, uncertainty will be maximal in low-support regions (will drive Bayesian optimization to behave similarly?)
- ▶ “Uncertainty principle” – for imbalanced datasets, cannot have accuracy and solvability when working with RBF-like models

## Big Question

**Does Bayesian optimization stall out for structure-exploiting solvers?**

# Big Question

**Does Bayesian optimization stall out for structure-exploiting solvers?**

**How to even check something like this?**

# Implemented Bayesian Optimization in ParMOO

To check, we implemented Bayesian optimization in ParMOO

# Implemented Bayesian Optimization in ParMOO

To check, we implemented Bayesian optimization in ParMOO

- ▶ Structure-exploiting Bayesian optimization is hard, **don't try this at home**

# Implemented Bayesian Optimization in ParMOO

To check, we implemented Bayesian optimization in ParMOO

- ▶ Structure-exploiting Bayesian optimization is hard, **don't try this at home**
- ▶ When you compose GPs (from simulation outputs) into nonlinear objectives, the posterior depends on the function
  - ▶ May or may not be iid

# Implemented Bayesian Optimization in ParMOO

To check, we implemented Bayesian optimization in ParMOO

- ▶ Structure-exploiting Bayesian optimization is hard, **don't try this at home**
- ▶ When you compose GPs (from simulation outputs) into nonlinear objectives, the posterior depends on the function
  - ▶ May or may not be iid
- ▶ Monte Carlo integrated to evaluate expected-improvement in the  $\varepsilon$ -constraint scalarization
  - ▶ It's very expensive

# Implemented Bayesian Optimization in ParMOO

To check, we implemented Bayesian optimization in ParMOO

- ▶ Structure-exploiting Bayesian optimization is hard, **don't try this at home**
- ▶ When you compose GPs (from simulation outputs) into nonlinear objectives, the posterior depends on the function
  - ▶ May or may not be iid
- ▶ I monte carlo integrated to evaluate expected-improvement in the  $\varepsilon$ -constraint scalarization
  - ▶ It's very expensive
- ▶ Tools like BoTorch are really cool and important



# Bayesian optimization of CFR problem

- ▶ 5 mixed-type design variables (including reaction time)
- ▶ 50-pt Latin hypercube design-of-experiments
- ▶ Fit **global** Gaussian RBF surrogate on 2 simulation outputs
- ▶ 3rd obj is identity mapping of reaction time
- ▶ Compared against blackbox implementation: all 3 objectives modeled with Gaussian RBFs
- ▶ 3 EI of  $\varepsilon$ -constraint acquisition functions
- ▶ Monte carlo integration of 2 Gaussian sim outs
- ▶ Multi-start GPS for global optimization of scalarized objectives

\*Green highlight = changes made from before

# References

- [5] Song et al. *OSS Vizier: distributed infrastructure and API for reliable and flexible black-box optimization*. In *Proc. 2022 AutoML-Conf*.
- [6] Balandat et al. *BoTorch: a framework for efficient monte-carlo Bayesian optimization*. In *NeurIPS 2020*.
- [7] Hoffman et al. *Optimizing molecules using efficient queries from property evaluations*. *Nature Machine Intelligence* 4:21–31 (2022).
- [8] Shah et al. *Archai: platform for neural architecture search*. *Microsoft Research* (Jul, 2022).
- [9] Virtanen et al. *SciPy 1.0: fundamental algorithms for scientific computing in Python*. *Nature Methods* 17:261–272 (2020).
- [10] Wild. *Optimization and learning with zeroth-order stochastic oracles*. *SIAM News* 56(1):1,3 (2023).
- [11] Wild. *Solving derivative-free nonlinear least squares problems with POUNDERS*. In *Advances and Trends in Optimization with Engineering Applications* (2017).
- [12] Chang and Wild. *Designing a framework for solving multiobjective simulation optimization problems*. *ArXiv preprint 2304.06881* (2023).
- [15] Chang and Wild. *ParMOO: A Python library for parallel multiobjective simulation optimization*. *JOSS* 8(82):4468 (2023).
- [16] Bollapragada et al. *Optimization and supervised machine learning methods for fitting numerical physics models without derivatives*. *Journal of Physics G* 48(2):024001 (2020).
- [17] Chang et al. *A framework for fully autonomous design of materials via multiobjective optimization and active learning: challenges and next steps*. In *ICLR 2023 Workshop on ML4Materials*.

## Resources

GitHub: `github.com/parmoo/parmoo`

Docs: `parmoo.readthedocs.io`

PyPI: `pip install parmoo`

Conda: `conda install --channel=conda-forge parmoo`

E-mail: `tchang@anl.gov`

E-mail: `parmoo@mcs.anl.gov`

*Chang and Wild. JOSS 8(82):4468 (2023)*

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research, SciDAC program under contract number DE-AC02-06CH11357.